



Article Modeling the Multi-Period and Multi-Classification-Yard Location Problem in a Railway Network

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Abstract: Classification yards are crucial nodes of railway freight transportation network, which plays a vital role in car flow reclassification and new train formation. Generally, a modern yard covers an expanse of several square kilometers and costs billions of Chinese Yuan (CNY), i.e., hundreds of millions of dollars. The determination of location and size of classification yards in multiple periods is not only related to yard establishment or improvement cost, but also involved with train connection service (TCS) plan. This paper proposes a bi-level programming model for the multi-period and multi-classification-yard location (MML) problem. The upper-level is intended to find an optimal combinatorial investment strategy for candidate nodes throughout the planning horizon, and the lower-level aims to obtain a railcar reclassification plan with minimum operation cost on the basis of the strategy given by the upper-level. The model is constrained by budget, classification capacity, the number of available tracks, etc. A numerical study is then performed to evaluate the validity and effectiveness of the model.

Keywords: classification yard; location–allocation problem; multiple periods; bi-level programming model; rail network

1. Introduction

Classification yards are generally referred to as the nerve centers of a railway system, where a great many inbound trains are reclassified, and outbound trains are dispatched. Building or improving a yard constitutes a high portion of capital investment of railroads, and the spatial configuration of yards significantly affects the routing of traffic flows over the whole network. Therefore, classification yard location problem is one of the top-most strategic-level problems for railroads.

Typically, a modern classification yard calls for hundreds of millions of dollars, and takes up a land of several square kilometers. For instance, the length of Maschen Marshalling Yard in Germany reaches 7000 m, the width of which is up to 700 m, covering 2.8 km², and Wuhan North Railway Station in China has a length of over 5000 m and a width of nearly 1000 m, with a land occupation of some 4.5 km². Irrational investment decisions might lead to deficiencies in railroad economies and result in a huge waste of resources. Hence, the research on the multi-classification-yard location (MML) problem is of great significance in practice.

Marshalling stations (also called classification yards) can be divided into different types, based on the number and configuration of yards. Currently, there are four major types of classification yards in China, including single directional lateral-type marshalling station, with three yards in one stage (SDLA); single directional combination-type marshalling station, with four yards in two stages (SDCO); single directional longitudinal-type marshalling station, with three yards in three stages (SDLO); and double directional longitudinal-type marshalling station, with six yards in three stages (DDLO). SDLA yards are small- and medium-sized yards, having advantages of small occupation of area and low capital investment. SDCO yards are more likely to be built in the case of heavy traffic flow and limited building land. SDLO yards are often connected with multiple rail lines and can handle a great many of railcars. It is easier to improve this type of yard to a large-scale marshalling station. DDLO yards has two sets of shunting devices, hence, the classification capacity of this type of yard is the largest.

With the depreciation of devices and advances of technology, existing yards might be improved, and new yards might be built over a period of time. It should be noted that economies of scale exist in the establishment and improvement of yards. For example, the cost of expanding a small yard to a medium-sized yard in the first period, then to a large-scale yard in the second period, is higher than directly improving the small yard to a large-scale yard in the first period. Therefore, the yard investment strategy should be optimized on the basis of the whole planning horizon, and meet the origin-destination (OD) demands in each period at the same time. Furthermore, as the improvement of classification capacity and efficiency provide a solid support for handling more railcars, the change of yards' spatial configuration might invalidate the current train formation plan, and consequently, affect the workload of each yard. If a railcar is reclassified at a certain yard, it will probably not need to be reclassified at other yards. Given the highly-nonlinear interrelation among yards, the freight train formation plan needs to be taken into account in investment analysis, which should be carried out from the perspective of railway network, rather than focusing on a certain yard.

Theoretically, the number of combinatorial strategies of yards' investment grows exponentially with the number of candidate nodes and periods. Furthermore, each strategy corresponds to a railcar reclassification problem, resulting in huge computational cost. Thus, the multi-period and multi-classification-yard location problem is of great significance in theory.

To summarize, quantitatively analyzing the MML problem, from the perspective of capital investment and train formation cost, has already become a theoretically and practically urgent problem.

2. Literature Review

There is a body of literature devoted to the location–allocation problem (LAP). Cooper [1] first presented exact extremal equations and a heuristic method for solving certain classes of LAP. Then, Bongartz et al. [2] proposed a solution method with relaxed 0–1 constraints for solving LAP. Eben-Chaime et al. [3] carried out a study of LAP on a line, formulated appropriate models, and proposed heuristic solution schemes. Brimberg et al. [4] examined an important class of continuous LAP and discussed the advances of new solution methods for this type of problem. Manzini and Gebennini [5] developed and applied innovative mixed integer programming (MIP) models to design and manage dynamic facility LAP. Recently, Gokbayrak and Kocaman [6] formulated a distance-limited continuous LAP and present a three-stage heuristic algorithm. Gupta et al. [7] proposed a solution method combining fuzzy c-means algorithm and particle swarm optimization for common service center location allocation. Cebecauer and Buzna [8] proposed a versatile concept of the adaptive aggregation framework for the facility LAP that kept the problem size in reasonable limits.

Many researchers have studied, in-depth, the multi-period location problem. Wesolowsky and Truscott [9] did some early work in this area. They developed a multi-period location–allocation formulation and presented two methods of solution. Hinojosa et al. [10] established a MIP model for a multi-period facility location problem, and proposed a Lagrangean relaxation together with a heuristic procedure. Canel et al. [11] proposed an algorithm for the capacitated, multi-commodity, multi-period facility location problem. Rajagopalan et al. [12] constructed a multi-period set covering a location model for dynamic redeployment of ambulances. Klibi et al. [13] formulated a stochastic multi-period location transportation problem as a stochastic program with recourse, and proposed a hierarchical heuristic solution approach. Sha and Huang [14] proposed a multi-period location model for the scheduling of engineering emergency blood supply.

Several models which are closely related to classification-yard location problem should be noted. Mansfield and Wein [15] presented the first location model of classification yard in 1958. The model is established to aid a railroad management in candidate location selection when newer facilities were going to be built. However, the background that a large amount of classification yards operated and the freight trains were reclassified almost at every yard they passed through, was completely different from the current situation. Assad [16] proposed the general principles of yard location, but did not give the method of determining the yard quantity and scale. Maji and Jha [17] constructed a location model of classification yard aiming at minimizing the sum of fixed cost and variable cost. Lee et al. [18] developed a marshalling yard location model considering economies of scale due to the consolidation of flows. Lin et al. [19] proposed a multi-project decision model to determine the location and size of new classification yards, as well as the technical improvement plan of existing yards. Other researches about classification yard location problem can be referred to Yan et al. [20], Li et al. [21] and Geng [22]. To our knowledge, there is little literature devoted to the multi-previd location problem of classification yard location problem can be referred to Yan et al. [20], Li et al. [21] and Geng [22]. To our knowledge, there is little literature devoted to the multi-previd location problem of classification yard location problem and this work is presented in Table 1.

Studies	Maji and Jha (1997)	Lin et al. (2002)	Lee et al. (2008)	This Work
Assumptions				
OD Matrix	fixed	fixed	fixed	fixed
Physical Path	fixed	fixed	unfixed	fixed
Decision				
Variables				
Yard Location	\checkmark	\checkmark	\checkmark	\checkmark
Train Service	×	\checkmark	×	\checkmark
Classification of Flow	×	\checkmark	×	\checkmark
Construction of Link	×	×	\checkmark	×
Constraints				
Budget	×		×	
Classification	×	,		,
Capacity	×	\checkmark	X	\checkmark
Track Number	×	\checkmark	×	\checkmark
Link Capacity	×	×	\checkmark	×
Model Structure	Integer Linear Programming	Integer Nonlinear Programming	Integer Nonlinear Programming	Integer Linear Programming
Objective Function	Min of train hauling cost, yard construction and maintenance cost	Min of capital investment and yard operation cost	Min of construction cost of hub and link, transportation cost and delay cost	Min of capital investment and yard operation cost
Multi-period	×	×	×	
Solution Approach	SolutionFloyd algorithm andApproachGenetic algorithm		N/A	Commercial software (Gurobi)

Table 1. Comparisons between a few published studies and this work.

The paper is organized as follows: Section 1 introduces the significance and complexity of the MML problem. Section 2 provides a brief survey of the literature devoted to the LAP and multi-period location problem, as well as classification-yard location problem. In Section 3, we describe the MML problem in detail. In Section 4, a bi-level programming model is established. A numerical example and conclusions are presented in Sections 5 and 6, respectively.

3. Problem Description

In this section, we first construct a simple line network to facilitate the description of the MML problem. Then, we analyze two extreme combinatorial investment strategies and the complexity of this problem.

3.1. An Example of Multi-Period and Multi-Classification-Yard Location Problem

To describe the MML problem clearly, we construct a simple line network consisting of five nodes and four rail lines, which is shown in Figure 1.



Figure 1. A simple line railway network.

It is assumed that stations S1 through S5 are all SDLA yards, denoted as green circle, and S2 and S4 are candidate nodes for investment. For simplification, we analyze the MML problem in just two periods (Period 1 and Period 2). In addition, we assume that S2 and S4 both have two investment plans for selection in Period 1, including remaining as a SDLA yard and improved to a SDCO yard (expressed by yellow circle in Figure 2). It should be noted that, if the candidate yard remains a SDLA yard in Period 1, it still has two investment plans to choose in Period 2; otherwise, it can only remain a SDCO yard in Period 2, i.e., yard downsizing and closure are not considered (SDLA \rightarrow SDCO is feasible, not vice versa). Moreover, we assume that the budget in each period is only enough for expanding one candidate yard from SDLA to SDCO.



Figure 2. Cont.



Figure 2. Seven combinatorial strategies for investment.

As shown in Figure 2, there are seven combinatorial strategies. The train services between every two adjacent stations are created and expressed by a black solid line with two arrows, while the train services between non-adjacent stations are expressed by blue solid lines with red arrows, which constitute the optimal TCS plan together. In fact, improved yards are capable of handling more traffic flows with lower reclassification cost per car, and creating more blocks simultaneously. In this context, part of shipments originally reclassified at high-cost yards might be transferred to low-cost yards, and more train services might be provided. Therefore, the optimum TCS plan of a network might change with the spatial configuration of yards.

Strategy 1 (see Figure 2a): None of the candidate stations are improved in two periods. In this case, the capital investment is the lowest, while the total operating cost is the highest. There are only three train services between non-adjacent stations, namely, $S_1 \rightarrow S_3$, $S_3 \rightarrow S_5$ and $S_4 \rightarrow S_2$.

Strategy 2 (see Figure 2b): Although station S_4 remains a SDLA yard in Period 1, it is expanded to a SDCO yard in Period 2. However, no capital is invested to station S_2 in both two periods. A new train connection service $S_4 \rightarrow S_1$ is provided in Period 2 on the basis of Strategy 1.

Strategy 3 (see Figure 2c): Station S₄ is expanded to a SDCO yard in Period 1 and remains a SDCO yard in Period 2. Station S₂ remains a SDLA yard in two periods. The capital investment of this strategy is equal to that of Strategy 2. As a more efficient yard S₄ comes into use since Period 1, the total operation cost of Strategy 3 is lower than Strategy 2. Additionally, a Strategy 1-based new train service $S_4 \rightarrow S_1$ is created in both periods.

Strategy 4 (see Figure 2d): Only station S_2 is improved to a SDCO yard in Period 2. And a new direct train service $S_2 \rightarrow S_5$ is provided in Period 2 in comparison with Strategy 1.

Strategy 5 (see Figure 2e): S₂ is improved to a SDCO yard in Period 2, and S₄ is expanded to a SDCO yard in Period 1. In this case, a new direct train service $S_4 \rightarrow S_1$ is provided in both periods, and another new train service $S_2 \rightarrow S_5$ is added in Period 2 based on Strategy 1.

Strategy 6 (see Figure 2f): Station S₂ is expanded to a SDCO yard in Period 1, while station S₄ remains a SDLA yard throughout the planning horizon. The capital investment of this strategy is equal to Strategy 4, but the total operation cost is lower than Strategy 4. Two periods of Strategy 6 share the common train services and a new service S₂ \rightarrow S₅ is added on the basis of Strategy 1.

Strategy 7 (see Figure 2g): In contrast to Strategy 5, S₂ is improved to a SDCO yard in Period 1, and S₄ is expanded to a SDCO yard in Period 2. In this context, a new direct train service S₂ \rightarrow S₅ is

provided both in Period 1 and Period 2, and another new train service $S_4 \rightarrow S_1$ is created in Period 2 based on Strategy 1. This strategy calls for the equivalent capital investment of Strategy 5.

3.2. Analysis of Two Extreme Combinatorial Strategies of the Multi-Period and Multi-Classification-Yard Location Problem

- (1) None of the candidate stations are invested in the whole planning horizon. This strategy calls for the minimum capital investment. However, the total operation cost is at the highest level due to the highest unit reclassification cost in average. Moreover, congestion may occur at some yards as the classification capacity of yards does not necessarily meet the increasing OD demands.
- (2) All candidate stations are expanded to their largest scale at the first period. Although this strategy significantly reduces the operation cost and satisfies the demands of all shipments, it calls for a significant amount of capital investment, which quite possibly exceeds the budget, and results in a huge waste of resources.

It should be noted that the solution to the MML problem is a trade-off between capital investment and operation cost. The evaluation of this trade-off is the core of our study.

3.3. The Complexity of Multi-Period and Multi-Classification-Yard Location Problem

The number of combinatorial investment strategies grows exponentially with the number of candidate nodes and periods. For a network containing five candidate nodes, if each of them has three optional investment plans in each period, then the number of combinatorial strategies (without consideration of budget constraint) will reach 243 for one period, 59,049 for two periods, 14,348,907 for three periods, and over 3.4 billion for four periods. Furthermore, each investment strategy is involved with a TCS problem, which is also a very challenging problem. For instance, there are 10 combinations for routing shipments over a line network with four yards, 150 for five yards, 7800 for six yards, 1,575,600 for seven yards, and over 1.3 billion for eight yards (Lin et al. [23]). Therefore, the MML problem is highly complex.

4. Mathematical Model

In this section, we formulate a bi-level programming model for the MML problem. The upper-level aims to find an optimal investment strategy for candidate nodes, i.e., which type of yards should the candidate nodes be built into or improved to. The lower-level is intended to obtain a least costly train connecting service plan, considering reclassification cost and accumulation delay. The model is constrained by capital budget, classification capacity, the number of available tracks, etc. For simplification, following assumptions are made in this paper.

Assumption 1. We assume that train services must be provided between adjacent stations. It simplifies the problem by avoiding considering which pairs of adjacent stations should be provided with train services, hence reducing the complexity in making train formation plans to some extent. In fact, as positive traffic flows do not necessarily exist between every two adjacent stations, there might be no train services between some of them, in practice. To relax this assumption, the train services between adjacent stations should also be treated as variables.

Assumption 2. It is assumed that the physical path of each OD demand is predefined. Although this is a standard practice used in China railway system in making a TCS plan, it might result in a higher operation cost compared with joint optimization of railcar itinerary and train formation plan. One solution to this problem is to optimize train paths and railcar reclassification plans simultaneously.

Assumption 3. We assume that candidate nodes can only be built into new yards or improved to larger yards, *i.e.*, yard downsizing and closure are not permitted. This assumption simplifies the problem by excluding many combinatorial investment strategies. In the real world, with the configuration change of yards, the workload of some yards might decline dramatically, which might be closed or downsized by railroad for optimization. To relax this assumption, we should consider yard downsizing and closure as potential plans for candidate yards.

4.1. Notations

The notations used in this paper are listed in Table 2.

Table 2	. N	Jotations	used	in	this	paper.
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$v_{Ploential}$ The set of candidate nodes, including all potential locations for new yards, and existing yards that might be improved; v_{Pli} v_{Pli} The set of existing yards in a rall network; $P(k)$ $P(k)$ The set of investment plans for candidate node k ; The set of yards through which a flow from <i>i</i> to <i>j</i> passes on its itinerary, excluding yard <i>i</i> and yard <i>j</i> ; The set of periods in a planning horizon; N ParametersDescriptionsParametersDescriptionsParametersDescriptions $p_{i}^{p_{i} \rightarrow p_{i}}$ $p_{i}^{p_{i}}$ The extra investment plans, i.e., $p_{a}, p_{b} \in P(k)$; $p_{i}^{p_{i}}$ $p_{i}^{p_{i}}$ The discount rate of yard operation cost; The discount rate of yard operation cost; γ The discount rate of yard operation cost; γ The discount rate of yard operation cost into economic cost; α 	Sets	Descriptions								
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4.2. Model Descriptions

In each period, only one investment plan can be selected for each candidate node, which can be described by

$$\sum_{p_a, p_b \in P(k)} y_{ks}^{p_a \to p_b} = 1 \qquad \forall k \in V^{\text{Potential}}; s \in S$$
(1)

It should be noted that, in the first period, if $p_a \neq p_k^{\text{Original}}$, then $y_{ks}^{p_a \to p_b} = 0$. In addition, according to Assumption 3, if the yard scale corresponding to plan p_b is smaller than p_a , then $y_{ks}^{p_a \to p_b} = 0$. Additionally, a logical constraint indicating the relation of selected plans in consecutive periods should be considered:

$$\sum_{p_a \in P(k)} y_{ks}^{p_a \to p_b} = \sum_{p_t \in P(k)} y_{k(s+1)}^{p_b \to p_t} \qquad \forall k \in V^{\text{Potential}}; p_b \in P(k); s, s+1 \in S.$$
(2)

The daily operation cost of all nodes, including accumulation delay and reclassification cost, is denoted by Z(Y, s), which is converted to the PDV by λ_s :

$$\lambda_s = \frac{(1+\gamma)^{T_s} - 1}{\gamma(1+\gamma)^{\sum\limits_{u=1}^s T_u}} \quad \forall s \in S.$$
(3)

Therefore, an upper-level formulation can be constructed to describe the MML problem:

Upper-level program:

p

$$\min \sum_{k \in V^{\text{Potential}}} \sum_{p_a, p_b \in P(k)} \sum_{s \in S} I_k^{p_a \to p_b} y_{ks}^{p_a \to p_b} + 365 \alpha \sum_{s \in S} \frac{(1+\gamma)^{T_s} - 1}{\gamma (1+\gamma)^{\sum\limits_{u=1}^s T_u}} Z(Y, s)$$
(4)

s.t.

$$\sum_{p_a, p_b \in P(k)} y_{ks}^{p_a \to p_b} = 1 \qquad \forall k \in V^{\text{Potential}}; s \in S$$
(5)

$$\sum_{p_a \in P(k)} y_{ks}^{p_a \to p_b} = \sum_{p_t \in P(k)} y_{k(s+1)}^{p_b \to p_t} \quad \forall k \in V^{\text{Potential}}; p_b \in P(k); s, s+1 \in S$$
(6)

$$\sum_{k \in V^{\text{Potential}}} \sum_{p_a, p_b \in P(k)} I_k^{p_a \to p_b} y_{ks}^{p_a \to p_b} \le B_s \quad \forall s \in S$$
(7)

$$y_{ks}^{p_a \to p_b} \in \{1, 0\} \qquad \forall k \in V^{\text{Potential}}; p_a, p_b \in P(k); s \in S$$
(8)

The first term of the objective function is the capital investment for all candidate nodes throughout the planning horizon. The second term is the PDV of total operation cost of nodes in all periods. The constraint (7) ensures that the capital investment would not exceed the budget in each period. Apparently, without consideration of the operation cost reduction due to yard establishment or improvement, the upper-level reaches its optimality when the investment for all candidate nodes is zero. In fact, building or improving a yard will not only increase the classification capacity and the number of tracks, but also raise operation efficiency and reduce classification cost, significantly affecting the routing of car flows over the network. Therefore, the selection of investment strategies for candidate yard locations can be viewed as a location problem, while the determination of train connecting services and distribution of classification workload can be referred as an allocation problem. If a direct train service is provided between two yards, then an accumulation delay $c_i m_{ij}$ will be incurred at the origin yard. Note that, the charge of providing a train service is related to the size of train, rather than traffic volume (details can be referred to Lin et al. [23]). If some shipments are reclassified at yard k, reclassification cost will be incurred. Symmetry 2018, 10, 135

The workload of node *k* in period *s*, i.e., the number of cars reclassified at node *k* per day in period *s*, can be expressed by

$$F_k^s = \sum_i \sum_j f_{ij}^{ks} \quad \forall i \neq j, k \in V^{\text{Original}} \cup V^{\text{Potential}}; s \in S.$$
(9)

Then, the service flow from *i* to *j* in period *s*, which is equivalent to the number of cars shipped by train service $i \rightarrow j$, can be expressed by

$$D_{ij}^{s} = f_{ij}^{s} + \sum_{t \in V^{\text{Original}} \cup V^{\text{Potential}}} f_{it}^{js} \quad \forall i \neq j \in V^{\text{Original}} \cup V^{\text{Potential}}; s \in S.$$
(10)

In this way, the lower-level program can be expressed by

Lower-level program:

$$\min \sum_{s \in S} \frac{(1+\gamma)^{T_s} - 1}{\sum\limits_{i \neq j \in V^{\text{Original}} \cup V^{\text{Potential}}} c_i m_{ij} x_{ij}^s} + \sum_{k \in V^{\text{Original}} - V^{\text{Potential}} \cap V^{\text{Original}}} F_k^s \tau_k + \sum_{k \in V^{\text{Potential}}} \sum_{p_a, p_b \in P(k)} F_k^s \tau_k^{p_b} y_{ks}^{p_a \to p_b} \right]$$
(11)

$$x_{ij}^{s} + \sum_{k \in \rho_{ij}} x_{ij}^{ks} = 1 \qquad \forall \ i \neq j \in V^{\text{Original}} \cup V^{\text{Potential}}; s \in S$$
(12)

$$x_{ij}^{ks} \le x_{ik}^{s} \quad \forall i \neq j \in V^{\text{Original}} \cup V^{\text{Potential}}; k \in \rho_{ij}; s \in S$$
(13)

$$f_{ij}^{s} + \sum_{k \in \rho_{ij}} f_{ij}^{ks} = N_{ij}^{s} + \sum_{w \in V^{\text{Original}} \cup V^{\text{Potential}}} f_{wj}^{is} \qquad \forall i \neq j \in V^{\text{Original}} \cup V^{\text{Potential}}; s \in S$$
(14)

$$f_{ij}^{s} \le M x_{ij}^{s} \quad \forall i \ne j \in V^{\text{Original}} \cup V^{\text{Potential}}; s \in S$$
 (15)

$$f_{ij}^{ks} \le M x_{ij}^{ks} \qquad \forall i \ne j \in V^{\text{Original}} \cup V^{\text{Potential}}; k \in \rho_{ij}; s \in S$$
(16)

$$F_k^s = \sum_i \sum_j f_{ij}^{ks} \quad \forall i \neq j, k \in V^{\text{Original}} \cup V^{\text{Potential}}; s \in S$$
(17)

$$D_{ij}^{s} = f_{ij}^{s} + \sum_{t \in V^{\text{Original}} \cup V^{\text{Potential}}} f_{it}^{js} \quad \forall i \neq j \in V^{\text{Original}} \cup V^{\text{Potential}}; s \in S$$
(18)

$$200\left(z_{ij}^{s}-1\right) < D_{ij}^{s} \qquad \forall i \neq j \in V^{\text{Original}} \cup V^{\text{Potential}}; s \in S$$
(19)

$$D_{ij}^{s} \leq 200 z_{ij}^{s} \quad \forall i \neq j \in V^{\text{Original}} \cup V^{\text{Potential}}; s \in S$$
 (20)

$$F_k^s \le \theta_k \Big(C_k^{\text{Total}} - C_{ks}^{\text{Local}} \Big) \qquad \forall \, k \in V^{\text{Original}} - V^{\text{Original}} \cap V^{\text{Potential}}; s \in S$$
(21)

$$F_k^s \le \theta_k \left(C_k^{\text{Total}} - C_{ks}^{\text{Local}} + \sum_{p_a, p_b \in P(k)} y_{ks}^{p_a \to p_b} \Delta C_k^{p_b} \right) \quad \forall k \in V^{\text{Potential}}; s \in S$$
(22)

$$\sum_{V^{\text{Original}} \cup V^{\text{Potential}}} z_{ij}^{s} \le \theta_{i} \left(L_{i}^{\text{Total}} - L_{is}^{\text{Arrival}} \right) \qquad \forall i \in V^{\text{Original}} - V^{\text{Original}} \cap V^{\text{Potential}}; s \in S$$
(23)

j∈

$$\sum_{j \in V^{\text{Original}} \cup V^{\text{Potential}}} z_{ij}^{s} \le \theta_{i} \left(L_{i}^{\text{Total}} - L_{is}^{\text{Arrival}} + \sum_{p_{a}, p_{b} \in P(k)} y_{is}^{p_{a} \to p_{b}} \Delta L_{i}^{p_{b}} \right) \qquad \forall i \in V^{\text{Potential}}; s \in S$$
(24)

$$f_{ij}^s \ge 0 \qquad \forall i \neq j \in V^{\text{Original}} \cup V^{\text{Potential}}; s \in S$$
 (25)

$$f_{ij}^{ks} \ge 0 \quad \forall i \ne j, k \in V^{\text{Original}} \cup V^{\text{Potential}}; s \in S$$
 (26)

$$x_{ii}^{s}, x_{ii}^{ks} \in \{1, 0\} \qquad \forall i \neq j, k \in V^{\text{Original}} \cup V^{\text{Potential}}; s \in S$$
(27)

$$z_{ij}^s \in N \quad \forall i \neq j \in V^{\text{Original}} \cup V^{\text{Potential}}; s \in S$$
 (28)

The objective function of the lower-level consists of three terms. The first term is the total accumulation delay of all train services. The second term is the classification cost of yards not included in $V^{\text{Potential}}$, while the third term is the classification cost of nodes included in $V^{\text{Potential}}$.

The constraint (12) guarantees that a car flow can either be directly shipped to the destination or classified at not less than one intermediate yard on its itinerary in each period. The constraint (13) ensures that if a car flow from i to j is initially reclassified at yard k, there must be a direct train service from *i* to *k*. The constraint (14) guarantees that the volume of car flows from *i* to *j*, carried by direct trains or reclassified at one or more yards on their itinerary, is equal to the total volume of N_{ii}^{s} (railcars which originate at *i* and are destined to *j* in period *s*), and the railcars which are reclassified at *i* and destined to *j*. The constraints (15) and (16) are logical constraints, indicating the relations between x_{ij}^s and f_{ij}^s , x_{ij}^{ks} , and f_{ij}^{ks} , respectively. For example, if $x_{ij}^s = 0$, there will be no direct train service from *i* to *j* in period *s*, and no shipments will be directly shipped from *i* to *j* in this period, i.e., $f_{ij}^s = 0$. By contrast, if $x_{ij}^s = 1$, the shipments from *i* to *j* will be delivered by direct trains. As f_{ij}^s is less than the total traffic volume in the network, hence, $f_{ii}^s \leq M x_{ii}^s$ (where *M* is a large enough positive number). It should be noted that, one classification track can generally store, at most, 200 railcars which are destined to other yards, simultaneously, in practice. In this case, the constraints (19) and (20) imply the relation between service flow D_{ij}^s and the number of tracks needed. For the nodes not included in V^{Potential}, the constraint (21) ensures that the workload would not exceed their available classification capacity, and the constraint (23) guarantees that the number of used tracks is less than the number of available tracks. Similarly, for the nodes included in V^{Potential}, the constraint (22) states that the capacity of these nodes is large enough to handle the traffic flows after possible establishment or improvement, and the constraint (24) represents that there are enough classification tracks for storing railcars in each period after possible investment.

In fact, the vast majority of car flows originating at yard i is actually obtained by aggregating the shipments from the small stations around yard i (see Lin et al. [23] for details). In the process of aggregation, these shipments should be reclassified and assigned to classification tracks (the storage capacity of this type of track is about 200 railcars in practice), together with car flows from other yards. Similarly, when the shipments arrive at the destination yard j, they should also be reclassified and assigned to the tracks reserved specially for them (the capacity of this type of track is usually set to 150 railcars), and then delivered to the final destinations (small stations surrounding yard j) by local trains (with smaller train size and higher frequency).

5. Numerical Studies

In this section, a numerical example is carried out test our method, on the basis of a small railway network containing nine yards, and the results are subsequently analyzed.

5.1. The Input Data

To test the effectiveness and validity of our model, a small railway network containing nine yards (denoted Y_1 through Y_9) and ten small stations (denoted S_1 through S_{10}) is constructed (see Figure 3). It is assumed that these nine yards are all SDLA yards at present, while nine stations have no classification capacity. The parameters of nine yards, such as accumulation parameter, classification cost per railcar, etc., are listed in Table 3.



Figure 3. A small railway network.

Yards	c _i	ø _k	C_k^{Total}	C_{k1}^{Local}	C_{k2}^{Local}	L_i^{Total}	L_{i1}^{Arrival}	L_{i2}^{Arrival}
Y ₁	10.2	3.9	1850	1175.4	1410.48	15	4	4
Y ₂	10.1	4.0	1600	1313.17	1575.80	13	6	7
Y3	10.3	3.9	1800	1383.56	1660.27	14	4	5
Y_4	10.3	3.9	1820	1473.57	1768.28	15	5	6
Y_5	10.4	3.8	1900	1339.21	1707.05	15	5	6
Y ₆	10.5	3.8	1950	1213.86	2056.63	16	5	10
Y_7	10.7	3.6	2300	1528.48	2034.18	17	6	8
Y_8	10.6	3.7	2100	1315.27	1778.32	16	5	6
Y9	10.1	3.9	1750	1485.4	1682.48	13	6	6

Table 3. The parameters of nine yards.

For simplification, Y3 and Y6 are defined as two candidate yards for improvement, and we intend to obtain the optimal investment strategy in just two periods. The OD demands (the number of railcars from origin to destination per day in average) in Period 1 and Period 2 are shown in Tables 4 and 5, respectively, and the physical path (listed in Appendix A) of each OD pair is specified in advance, which is a standard practice in China railway system, thus, the operation mileage in total is a constant.

Table 4. The OD matrix in Period 1 (car per day).

	Y ₁	Y ₂	Y ₃	Y ₄	Y ₅	Y ₆	Y ₇	Y ₈	Y9
Y ₁	_	24.29	96.56	84.57	124.53	111.72	143.20	40.38	67.44
Y ₂	66.07	_	55.60	140.04	79.41	61.04	44.14	21.76	67.47
Y ₃	138.30	79.20	_	124.14	42.80	71.24	156.01	154.65	100.50
Y_4	102.33	127.32	29.03	_	150.37	56.32	92.28	122.06	85.90
Y_5	48.71	67.40	74.58	146.57	_	23.07	126.98	55.51	157.59
Y ₆	12.74	69.84	25.65	30.51	104.09	—	150.20	66.54	93.35
Y_7	77.88	112.41	53.20	79.65	86.17	71.50	_	113.21	56.08
Y_8	13.93	156.21	48.50	15.18	30.80	106.56	121.61	_	152.30
Y9	22.75	140.97	133.60	87.30	20.63	159.49	43.96	96.07	_

	Y_1	Y ₂	Y ₃	Y_4	Y_5	Y ₆	Y_7	Y_8	Y9
Y ₁	_	29.15	115.87	101.48	149.44	134.06	171.84	48.46	80.93
Y ₂	79.28	—	66.72	168.05	95.29	73.25	52.97	26.11	80.96
Y ₃	165.96	95.04	_	148.97	51.36	85.49	187.21	185.58	120.60
Y_4	122.80	152.78	34.84	_	180.44	67.58	110.74	146.47	103.08
Y_5	58.45	80.88	89.50	175.88	_	227.68	152.38	66.61	89.11
Y ₆	15.29	83.81	30.78	36.61	124.91	—	180.24	79.85	112.02
Y_7	93.46	134.89	63.84	95.58	103.40	285.80	_	135.85	67.30
Y_8	16.72	187.45	58.20	18.22	36.96	327.87	145.93	_	182.76
Y9	27.30	169.16	160.32	104.76	24.76	191.39	52.75	115.28	_

Table 5. The OD matrix in Period 2 (car per day).

We assume that both Y_3 and Y_6 have three investment plans as options, which are remaining a SDLA yard, improved to a SDCO yard, and expanded to a SDLO yard, respectively, i.e., $P(3) = \{1,2,3\}$, $P(6) = \{1,2,3\}$. As we do not take yard downsizing and closure into consideration, there are six combinations that might occur for each candidate yard in two periods. The capital investment, increase of classification capacity and track number, and decrease of unit classification cost of these combinations, are listed in Table 6.

Table 6. Information of six investment combinations for a SDLA yard in two periods.

Investment Combinations	Investment (Billion CNY)	Increase of Capacity (Car per Day)	Increase of Track Number	Decrease of $ø_k$
SDLA→SDLA	0	0	0	0
SDLA→SDCO	0.7	1500	10	-0.4
SDLA→SDLO	1.0	2500	18	-0.6
SDCO→SDCO	0	0	0	0
SDCO→SDLO	0.5	1000	8	-0.2
SDLO→SDLO	0	0	0	0

Furthermore, the budgets of investment in Period 1 and Period 2 are set to 1.5 billion CNY and 1.0 CNY respectively, i.e., $B_1 = 1.5$, $B_2 = 1.0$; the time span of each period is set as five years, i.e., $T_1 = 5$, $T_2 = 5$; the discount rate of capital investment is equal to 0.02 ($\gamma = 0.02$); the coefficient α is set to 20, i.e., the economic cost of one car-hour is equivalent to 20 CNY; and the proportional factor of classification capacity and tracks for nine yards are all set to 0.9 ($\theta_k = 0.9$). In addition, OD demands satisfying the sufficient condition for providing a train service, that they must be delivered to their destinations directly without optimization. The sufficient condition can be described as follows:

$$N_{ii}^{s}\tau_{k}^{\min} \ge c_{i}m_{ij} \qquad \forall i \neq j \in V^{\text{Original}} \cup V^{\text{Potential}}, k \in \rho_{ij}, s \in S,$$
(29)

where τ_k^{\min} denotes a minimum classification cost of yards on the itinerary of shipment N_{ij}^s , i.e.,

$$\tau_k^{\min} = \min\{\tau_k | k \in \rho_{ij}\}.$$
(30)

In other words, if shipment N_{ij}^s is reclassified at a certain intermediate yard with minimum relative delay on its itinerary, and the classification cost is greater than the cost of dispatching direct trains, then a direct train service should be provided without optimization.

5.2. Results and Discussion

The MML problem mentioned above is solved by Gurobi 7.5.2 on a 2.20 GHz Intel (R) Core (TM) i5-5200U CPU computer with 4.0 GB of RAM. As each yard has six investment combinations in total throughout the planning horizon, there are 36 (6×6) combinatorial investment strategies for two

yards, in theory. However, in light of the budget constraints in two periods, there are only 17 strategies that are available, whose capital investment, operation cost, etc., are listed below.

As shown in Table 7, Strategy 3 is the optimal one with a minimum total cost of 2.643 billion CNY. In this strategy, yard Y_3 remains a SDLA yard in two periods, while yard Y_6 is expanded to a SDCO yard in Period 1 and remains unchanged in Period 2. The capital investment of this strategy is 0.7 billion CNY, and the overall operation cost of nine yards in two periods is equal to 1.943 billion CNY. Additionally, the daily workload and track utilization of nine yards in two periods are shown in Table 8.

Combinatorial Strategy	Y	3	У	6	Total Investment	Operation Cost	Total Cost
Combinatorial Strategy	Period 1	Period 2	Period 1	Period 2	(Billion)	(Billion)	(Billion)
1	SDLA	SDLA	SDLA	SDCO	0.7	1.962	2.662
2	SDLA	SDLA	SDLA	SDLO	1	1.947	2.947
3	SDLA	SDLA	SDCO	SDCO	0.7	1.943	2.643
4	SDLA	SDLA	SDCO	SDLO	1.2	1.928	3.128
5	SDLA	SDLA	SDLO	SDLO	1	1.919	2.919
6	SDLA	SDCO	SDCO	SDCO	1.4	1.888	3.288
7	SDLA	SDCO	SDLO	SDLO	1.7	1.873	3.573
8	SDLA	SDLO	SDCO	SDCO	1.7	1.879	3.579
9	SDLA	SDLO	SDLO	SDLO	2	1.864	3.864
10	SDCO	SDCO	SDLA	SDCO	1.4	1.874	3.274
11	SDCO	SDCO	SDLA	SDLO	1.7	1.868	3.568
12	SDCO	SDCO	SDCO	SDCO	1.4	1.862	3.262
13	SDCO	SDCO	SDCO	SDLO	1.9	1.856	3.756
14	SDCO	SDLO	SDCO	SDCO	1.9	1.852	3.752
15	SDCO	SDLO	SDCO	SDLO	2.4	1.846	4.246
16	SDLO	SDLO	SDLA	SDCO	1.7	1.853	3.553
17	SDLO	SDLO	SDLA	SDLO	2	1.847	3.847

Table 7. The cost of available combinatorial strategies.

Table 8. Workload and track utilization of nine yards.

Period	Classification Yards	Y ₁	Y ₂	Y ₃	Y_4	Y ₅	Y ₆	Y ₇	Y ₈	Y9
	F_k^1	285.95	84.57	366.83	287.63	76.07	1156.09	0	0	0
	Available Capacity	674.60	286.83	416.44	346.43	560.79	2236.14	771.52	784.73	264.60
1	Workload (%)	42.39	29.48	88.09	83.03	13.56	51.70	0	0	0
1	Used Tracks	6	4	9	8	6	12	4	5	5
	Available Tracks	11	7	10	10	10	21	11	11	7
	Track Percentage (%)	54.55	57.14	90.00	80.00	60.00	57.14	36.36	45.45	71.43
	F_k^2	343.14	0	95.56	0	91.29	1204.93	0	0	0
	Available Capacity	439.52	24.20	139.73	51.72	192.95	1393.37	265.82	321.68	67.52
r	Workload (%)	78.07	0	68.39	0	47.31	86.48	0	0	0
2	Used Tracks	9	5	8	7	7	13	8	7	5
	Available Tracks	11	6	9	9	9	16	9	10	7
	Track Percentage (%)	81.82	83.33	88.89	77.78	77.78	81.25	88.89	70.00	71.43

In Table 8, the available capacity of a certain yard is obtained by classification capacity in total minus the capacity reserved for local car flows, which can be expressed by

$$C_{ks}^{\text{Available}} = C_k^{\text{Total}} - C_{ks}^{\text{Local}} \quad \forall k \in V^{\text{Original}} - V^{\text{Original}} \cap V^{\text{Potential}}; s \in S,$$
(31)

$$C_{ks}^{\text{Available}} = C_k^{\text{Total}} - C_{ks}^{\text{Local}} + \sum_{p_a, p_b \in P(k)} y_{ks}^{p_a \to p_b} \Delta C_k^{p_b} \quad \forall k \in V^{\text{Potential}}; s \in S.$$
(32)

Similarly, the available track number of a certain yard is equal to the total tracks minus the tracks reserved for arrival car flows, which can be expressed by

$$L_{is}^{\text{Available}} = L_i^{\text{Total}} - L_{is}^{\text{Arrival}} \qquad \forall i \in V^{\text{Original}} - V^{\text{Original}} \cap V^{\text{Potential}}; s \in S,$$
(33)

$$L_{is}^{\text{Available}} = L_i^{\text{Total}} - L_{is}^{\text{Arrival}} + \sum_{p_a, p_b \in P(k)} y_{is}^{p_a \to p_b} \Delta L_i^{p_b} \quad \forall \ i \in V^{\text{Potential}}; s \in S.$$
(34)

As shown in Table 8, the workload of yards Y_7 , Y_8 , and Y_9 are equal to zero both in Period 1 and Period 2. This is because no car flow passes through these yards according to the predefined physical paths (i.e., none of them is intermediate yard). In addition, the workload of yards Y_2 and Y_4 decrease to zero in Period 2. This is mainly due to the increase of car flow volume, i.e., it is more favorable to provide more direct trains than reclassify railcars at Y_2 and Y_4 . The train connection services in the railway network in Period 1 are depicted in Figure 4.



Figure 4. Direct train services among nine yards in Period 1.

Theoretically, the potential train services for nine yards are 72 (9 × 8). However, in Figure 2, there are only 39 direct train services provided in this railway network, 22 of which are created between adjacent yards (Type 1 service); nine of which satisfy the sufficient condition (Type 2 service); and eight of which are obtained by the optimization from the remaining 41 potential train services (Type 3 service). Table 9 lists all the direct train services, and the volume of D_{ij}^1 and f_{ij}^1 in Period 1, while Table 10 lists all the car flow variables and corresponding traffic volume in Period 1.

Origin	Destinatio	D_{ij}^1	f_{ij}^1	Origin	Destinatio	D_{ij}^1	f_{ij}^1
Y_1	Y ₂	176.26	91.69	Y_4	Y ₇	136.42	136.42
Y_1	Y ₃	171.14	171.14	Y_5	Y_1	204.62	62.64
Y_1	Y_5	308.88	246.74	Y_5	Y_4	146.57	146.57
Y_1	Y ₆	179.16	111.72	Y_5	Y ₆	150.05	23.07
Y_1	Y_7	143.2	143.2	Y_5	Y ₈	117.65	117.65
Y ₂	Y_1	167.24	66.07	Y_5	Y9	157.59	157.59
Y ₂	Y ₃	184.11	55.6	Y ₆	Y ₃	202.42	74.15
Y ₂	Y_4	268.75	224.61	Y ₆	Y_5	210.89	210.89
Y ₃	Y_1	216.59	173.79	Y ₆	Y ₇	442.75	442.75
Y ₃	Y ₂	149.04	149.04	Y ₆	Y ₈	397.88	397.88
Y ₃	Y_4	257.13	257.13	Y ₆	Y9	455.07	455.07
Y ₃	Y ₆	132.28	132.28	Y_7	Y_4	323.14	79.65
Y ₃	Y ₇	156.01	156.01	Y_7	Y ₆	326.96	71.5
Y ₃	Y ₈	154.65	154.65	Y_8	Y ₂	156.21	156.21
Y ₃	Y9	167.97	167.97	Y_8	Y_5	44.73	30.8
Y_4	Y_1	180.21	180.21	Y_8	Y ₆	444.15	106.56
Y_4	Y ₂	239.73	239.73	Y_9	Y ₂	140.97	140.97
Y_4	Y ₃	82.23	82.23	Y9	Y ₃	243.65	133.6
Y_4	Y_5	150.37	150.37	Y9	Y ₆	320.15	159.49
Y_4	Y ₆	264.28	56.32				

Table 9. Information of direct train services in Period 1.

Origin	First Classification	Destinat	f_{ij}^{k1}	Origin	First Classification	Destinat	f_{ij}^{k1}
Y ₁	Y ₂	Y ₄	84.57	Y ₇	Y ₄	Y ₁	77.88
Y_1	Y_5	Y_8	62.14	Y_7	Y_4	Y2	112.41
Y_1	Y ₆	Y9	67.44	Y_7	Y_4	Y ₃	53.2
Y ₂	Y ₁	Y_5	79.41	Y_7	Y ₆	Y_5	86.17
Y ₂	Y_1	Y ₈	21.76	Y_7	Y ₆	Y ₈	113.21
Y ₂	Y ₃	Y ₆	61.04	Y_7	Y ₆	Y9	56.08
Y ₂	Y ₃	Y9	67.47	Y ₈	Y_5	Y_1	13.93
Y ₂	Y_4	Y_7	44.14	Y ₈	Y ₆	Y3	48.5
Y ₃	Y ₁	Y_5	42.8	Y ₈	Y ₆	Y_4	15.18
Y_4	Y ₆	Y_8	122.06	Y ₈	Y ₆	Y ₇	121.61
Y_4	Y ₆	Y9	85.9	Y ₈	Y ₆	Y9	152.3
Y_5	Y ₁	Y ₂	67.4	Y9	Y ₃	Y_1	22.75
Y_5	Y ₁	Y ₃	74.58	Y9	Y ₃	Y_4	87.3
Y_5	Y ₆	Y ₇	126.98	Y9	Y ₆	Y_5	20.63
Y ₆	Y ₃	Y_1	12.74	Y9	Y ₆	Y ₇	43.96
Y ₆	Y ₃	Y ₂	69.84	Y9	Y ₆	Y ₈	96.07
Y ₆	Y_3	Y_4	45.69				

Table 10. Car flow variable x_{ij}^{k1} and corresponding traffic flow volume f_{ij}^{k1} in Period 1.

As shown in Table 9, the direct train service with maximum traffic volume is $Y_6 \rightarrow Y_9$, whose service flow D_{ij}^1 is up to 455.07 cars per day, i.e., on average dispatching 9.1 (455.07/50) trains each day. Conversely, the direct train service with minimum volume is $Y_8 \rightarrow Y_5$, whose service flow is just 44.73 cars per day, less than one train per day.

Additionally, the reclassification strategy of each OD demand can be obtained on the basis of Tables 9 and 10. For example, to get the reclassification strategy of the OD demand N_{19}^1 , we should first determine whether direct train service $Y_1 \rightarrow Y_9$ exists in Table 9. If the answer is NO, then we turn to Table 10. It can be found that car flow variable x_{19}^{61} (Y_1 , Y_6 , Y_9) exists in Table 10, which indicates that shipment N_{19}^1 will be first reclassified at classification yard Y_6 , and merged into the car flow from Y_6 to Y_9 . Next, we look up Table 10 to determine whether train service $Y_6 \rightarrow Y_9$ exists or not. Obviously, the answer is YES, i.e., N_{19}^1 will be consolidated into the direct train service $Y_6 \rightarrow Y_9$ at yard Y_6 and delivered to its destination Y_9 . Finally, the reclassification strategy of OD demand N_{19}^1 is $Y_1 \rightarrow Y_6 \rightarrow Y_9$.

Figure 5 depicts all the direct train services (48 services) provided in Period 2, 12 of which satisfy the sufficient condition; 22 of which are created between adjacent yards; and 14 of which are obtained by the optimization from the remaining 38 train services.

Table 11 lists all the direct train services, and the volume of D_{ij}^2 and f_{ij}^2 in Period 2, while Table 12 lists all the car flow variables and corresponding traffic volume in Period 2.

Origin	Destinat	D_{ij}^2	f_{ij}^2	Origin	Destinat	D_{ij}^2	f_{ij}^2
Y ₁	Y ₂	110.03	110.03	Y_5	Y ₆	316.79	227.68
Y_1	Y ₃	205.37	205.37	Y_5	Y_7	152.38	152.38
Y_1	Y_4	101.48	101.48	Y_5	Y_8	141.18	141.38
Y_1	Y_5	370.66	296.09	Y ₆	Y ₂	83.81	83.81
Y_1	Y ₆	214.99	134.06	Y_6	Y ₃	104.27	88.98
Y_1	Y_7	171.84	171.84	Y ₆	Y_4	159.59	159.59
Y ₂	Y_1	200.68	79.28	Y ₆	Y_5	253.07	253.07
Y ₂	Y3	119.69	66.72	Y_6	Y_7	232.99	232.99
Y ₂	Y_4	168.05	168.05	Y ₆	Y_8	380.71	380.71
Y ₂	Y ₆	154.21	73.25	Y ₆	Y9	654.00	654.00
Y3	Y_1	259.91	208.55	Y_7	Y_1	93.46	93.46
Y ₃	Y ₂	95.04	95.04	Y_7	Y ₂	134.89	134.89
Y ₃	Y_4	148.97	148.97	Y_7	Y ₃	63.84	63.84
Y ₃	Y ₆	391.67	85.49	Y_7	Y_4	95.58	95.58
Y ₃	Y_7	240.18	240.18	Y_7	Y ₆	456.5	285.8
Y_4	Y_1	122.8	122.8	Y_7	Y_8	135.85	135.85
Y_4	Y ₂	152.78	152.78	Y ₈	Y ₂	187.45	187.45

Table 11. Information of direct train services in Period 2.

Y_4	Y3	34.84	34.84	Y ₈	Y ₅	53.68	36.96
Y_4	Y_5	180.44	180.44	Y_8	Y ₆	404.29	327.87
Y_4	Y ₆	170.66	67.58	Y_8	Y_7	145.93	145.93
Y_4	Y_7	110.74	110.74	Y_8	Y ₉	182.76	182.76
Y_4	Y_8	146.47	146.47	Y9	Y ₂	169.16	169.16
Y_5	Y_1	245.55	75.17	Y9	Y3	187.62	160.32
Y_5	Y_4	175.88	175.88	Y9	Y ₆	488.94	191.39

Table 11. Cont.



Figure 5. Direct train services among nine yards in Period 2.

Origin	First Classification	Destinat	f_{ij}^{k2}	О	First Classification	Destinat	f_{ij}^{k2}
Y ₁	Y_5	Y ₈	74.57	Y_5	Y ₆	Y9	89.11
Y_1	Y_6	Y9	80.93	Y_6	Y_3	Y_1	15.29
Y ₂	Y_1	Y_5	95.29	Y_7	Y ₆	Y_5	103.4
Y ₂	Y_1	Y_8	26.11	Y_7	Y ₆	Y9	67.3
Y ₂	Y ₃	Y_7	52.97	Y_8	Y_5	Y_1	16.72
Y ₂	Y_6	Y9	80.96	Y_8	Y ₆	Y ₃	58.2
Y_3	Y_1	Y_5	51.36	Y_8	Y ₆	Y_4	18.22
Y ₃	Y_6	Y_8	185.58	Y9	Y_3	Y_1	27.3
Y_3	Y_6	Y9	120.6	Y9	Y ₆	Y_4	104.76
Y_4	Y_6	Y9	103.08	Y9	Y ₆	Y_5	24.76
Y_5	Y_1	Y ₂	80.88	Y9	Y ₆	Y_7	52.75
Y_5	Y_1	Y3	89.5	Y9	Y_6	Y_8	115.28

Table 12. Car flow variable x_{ij}^{k2} and corresponding traffic flow f_{ij}^{k2} in Period 2.

In Table 11, the direct train service with maximum traffic volume is $Y_9 \rightarrow Y_6$, reaching 488.94 cars per day, i.e., on average dispatching 9.8 (488.94/50) trains each day. In contrast, the direct train service with minimum traffic volume is $Y_4 \rightarrow Y_3$, whose service flow is just 34.84 cars per day, less than one train per day. The reclassification strategies of OD demands in Period 2 can also be obtained by the method mentioned above.

6. Conclusions

In this paper, we formulate the MML problem as a multi-period location-allocation problem with railway characteristics, and established a bi-level programming model constrained by budget, classification capacity, and number of available tracks. The upper-level is intended to find an optimal combinatorial investment strategy for all candidate nodes throughout the planning horizon, and the lower-level aims to obtain a least costly railcar reclassification plan on the basis of the strategy given by the upper-level. In light of the economies of scale in yard establishment and improvement, and the highly-nonlinear interrelation among yards, the investment strategy should be analyzed from the perspective of the entire planning horizon and the whole railway network, rather than focusing on a certain period or a certain yard. To test the effectiveness and validity of our model, a numerical study with two candidate yards in two periods is carried out and solved by using Gurobi 7.5.2. In the optimal investment strategy we obtained, i.e., yard Y₃ remains a SDLA yard in two periods, while yard Y_6 is expanded to a SDCO yard in Period 1 and remains unchanged in Period 2, all the yards can handle their workload in the near future and keep an appropriate capacity utilization ratio, which indicates that the proposed model can serve as a solid aid in the decision-making of classification yard location. With the building of new railway lines, such as Lanzhou-Chongqing Railway and Afuzhun Railway, some yards might be improved to match with the expanded railway network. In this case, railway departments might achieve substantial cost savings by applying our method in their five-year plans for yard improvement.

In the long term, researchers can focus on the simultaneously addressing of car flow routing and classification yards location. We identify this a promising area for future research.

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No.	Origin	Destinat	Physical Path	No.	Origin	Destination	Physical Path
1	Y ₁	Y ₂	$Y_1 \rightarrow Y_2$	37	Y ₅	Y ₆	$Y_5 \rightarrow Y_6$
2	Y_1	Y ₃	$Y_1 {\rightarrow} Y_2 {\rightarrow} Y_3$	38	Y_5	Y ₇	$Y_5 {\rightarrow} Y_6 {\rightarrow} Y_7$
3	Y_1	Y_4	$Y_1 {\rightarrow} Y_2 {\rightarrow} Y_3 {\rightarrow} Y_4$	39	Y_5	Y ₈	$Y_5 \rightarrow Y_8$
4	Y_1	Y_5	$Y_1 \rightarrow Y_5$	40	Y_5	Y9	$Y_5 {\rightarrow} Y_6 {\rightarrow} Y_9$
5	Y_1	Y ₆	$Y_1 {\rightarrow} Y_2 {\rightarrow} Y_3 {\rightarrow} Y_6$	41	Y ₆	Y_1	$Y_6 {\rightarrow} Y_3 {\rightarrow} Y_2 {\rightarrow} Y_1$
6	Y_1	Y_7	$Y_1 {\rightarrow} Y_2 {\rightarrow} Y_3 {\rightarrow} Y_4 {\rightarrow} Y_7$	42	Y ₆	Y ₂	$Y_6 \rightarrow Y_3 \rightarrow Y_2$
7	Y_1	Y_8	$Y_1 {\rightarrow} Y_5 {\rightarrow} Y_8$	43	Y ₆	Y ₃	$Y_6 \rightarrow Y_3$
8	Y_1	Y9	$Y_1 {\rightarrow} Y_2 {\rightarrow} Y_3 {\rightarrow} Y_6 {\rightarrow} Y_9$	44	Y ₆	Y_4	$Y_6 {\rightarrow} Y_3 {\rightarrow} Y_4$
9	Y ₂	Y_1	$Y_2 \rightarrow Y_1$	45	Y ₆	Y_5	$Y_6 \rightarrow Y_5$
10	Y ₂	Y ₃	$Y_2 \rightarrow Y_3$	46	Y ₆	Y ₇	$Y_6 \rightarrow Y_7$
11	Y ₂	Y_4	$Y_2 {\rightarrow} Y_3 {\rightarrow} Y_4$	47	Y ₆	Y_8	$Y_6 \rightarrow Y_8$
12	Y ₂	Y_5	$Y_2 {\rightarrow} Y_1 {\rightarrow} Y_5$	48	Y ₆	Y9	$Y_6 \rightarrow Y_9$
13	Y ₂	Y ₆	$Y_2 {\rightarrow} Y_3 {\rightarrow} Y_6$	49	Y_7	Y_1	$Y_7 {\rightarrow} Y_4 {\rightarrow} Y_3 {\rightarrow} Y_2 {\rightarrow} Y_1$
14	Y ₂	Y_7	$Y_2 {\rightarrow} Y_3 {\rightarrow} Y_4 {\rightarrow} Y_7$	50	Y_7	Y ₂	$Y_7 {\rightarrow} Y_4 {\rightarrow} Y_3 {\rightarrow} Y_2$
15	Y ₂	Y_8	$Y_2 {\rightarrow} Y_1 {\rightarrow} Y_5 {\rightarrow} Y_8$	51	Y_7	Y ₃	$Y_7 {\rightarrow} Y_4 {\rightarrow} Y_3$
16	Y ₂	Y ₉	$Y_2 {\rightarrow} Y_3 {\rightarrow} Y_6 {\rightarrow} Y_9$	52	Y_7	Y_4	$Y_7 \rightarrow Y_4$
17	Y ₃	Y_1	$Y_3 {\rightarrow} Y_2 {\rightarrow} Y_1$	53	Y_7	Y_5	$Y_7 {\rightarrow} Y_6 {\rightarrow} Y_5$
18	Y ₃	Y ₂	$Y_3 \rightarrow Y_2$	54	Y_7	Y ₆	$Y_7 \rightarrow Y_6$
19	Y ₃	Y_4	$Y_3 \rightarrow Y_4$	55	Y_7	Y_8	$Y_7 {\rightarrow} Y_6 {\rightarrow} Y_8$
20	Y ₃	Y_5	$Y_3 {\rightarrow} Y_2 {\rightarrow} Y_1 {\rightarrow} Y_5$	56	Y_7	Y9	$Y_7 {\rightarrow} Y_6 {\rightarrow} Y_9$
21	Y ₃	Y ₆	$Y_3 \rightarrow Y_6$	57	Y_8	Y_1	$Y_8{\rightarrow}Y_5{\rightarrow}Y_1$
22	Y ₃	Y_7	$Y_3 {\rightarrow} Y_4 {\rightarrow} Y_7$	58	Y_8	Y ₂	$Y_8 {\rightarrow} Y_5 {\rightarrow} Y_1 {\rightarrow} Y_2$

Appendix A. The Physical Paths of OD Demands

23	Y ₃	Y ₈	$Y_3 \rightarrow Y_6 \rightarrow Y_8$	59	Y ₈	Y3	$Y_8 \rightarrow Y_6 \rightarrow Y_3$
24	Y ₃	Y9	$Y_3 {\rightarrow} Y_6 {\rightarrow} Y_9$	60	Y_8	Y_4	$Y_8 {\rightarrow} Y_6 {\rightarrow} Y_3 {\rightarrow} Y_4$
25	Y_4	Y_1	$Y_4 {\rightarrow} Y_3 {\rightarrow} Y_2 {\rightarrow} Y_1$	61	Y ₈	Y_5	$Y_8 \rightarrow Y_5$
26	Y_4	Y ₂	$Y_4 {\rightarrow} Y_3 {\rightarrow} Y_2$	62	Y_8	Y ₆	$Y_8 \rightarrow Y_6$
27	Y_4	Y3	$Y_4 \rightarrow Y_3$	63	Y_8	Y_7	$Y_8 \rightarrow Y_6 \rightarrow Y_7$
28	Y_4	Y_5	$Y_4 {\rightarrow} Y_3 {\rightarrow} Y_2 {\rightarrow} Y_1 {\rightarrow} Y_5$	64	Y_8	Y9	$Y_8{\rightarrow}Y_6{\rightarrow}Y_9$
29	Y_4	Y ₆	$Y_4 {\rightarrow} Y_3 {\rightarrow} Y_6$	65	Y9	Y_1	$Y_9 {\rightarrow} Y_6 {\rightarrow} Y_3 {\rightarrow} Y_2 {\rightarrow} Y_1$
30	Y_4	Y_7	$Y_4 \rightarrow Y_7$	66	Y9	Y ₂	$Y_9 {\rightarrow} Y_6 {\rightarrow} Y_3 {\rightarrow} Y_2$
31	Y_4	Y_8	$Y_4 {\rightarrow} Y_3 {\rightarrow} Y_6 {\rightarrow} Y_8$	67	Y9	Y3	$Y_9 \rightarrow Y_6 \rightarrow Y_3$
32	Y_4	Y9	$Y_4 {\rightarrow} Y_3 {\rightarrow} Y_6 {\rightarrow} Y_9$	68	Y9	Y_4	$Y_9 {\rightarrow} Y_6 {\rightarrow} Y_3 {\rightarrow} Y_4$
33	Y_5	Y_1	$Y_5 \rightarrow Y_1$	69	Y9	Y_5	$Y_9 \rightarrow Y_6 \rightarrow Y_5$
34	Y_5	Y ₂	$Y_5 {\rightarrow} Y_1 {\rightarrow} Y_2$	70	Y9	Y ₆	$Y_9 \rightarrow Y_6$
35	Y_5	Y ₃	$Y_5 {\rightarrow} Y_1 {\rightarrow} Y_2 {\rightarrow} Y_3$	71	Y9	Y_7	$Y_9 {\rightarrow} Y_6 {\rightarrow} Y_7$
36	Y_5	Y_4	$Y_5 {\rightarrow} Y_1 {\rightarrow} Y_2 {\rightarrow} Y_3 {\rightarrow} Y_4$	72	Y9	Y_8	$Y_9{\rightarrow}Y_6{\rightarrow}Y_8$

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