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Pre-Rationalized Parametric Designing of Roof Shells Formed by Repetitive Modules of Catalan Surfaces

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Abstract: The aim of the study is to develop an original, methodical, and practical approach to the early stages of parametric design of roof shells formed by repetitive modules of Catalan surfaces. It is presented on the example of designing the roof shells compound of four concrete elements. The designing process proposed by us consists in linking geometric shaping of roofs' models with their structural analysis and optimization. Contrary to other methods, which use optimization process in order to find free roof forms, we apply it in order to explore and improve design alternatives. It is realized with the application of designing tools working in Rhinoceros 3D software. The flexible scripts elaborated by us, in order to achieve roofs' models of regular and symmetrical shapes, are converted into simulation models to perform structural analysis. It is mainly focused on how the roof shells perform dependently on their geometric characteristics. The simulation enables one to evaluate various roof shells' shapes, as well as to select an optimal design solution. The proposed approach to the conceptual design process may drive the designing to achieve geometric and structural forms which not only follow the design intentions but also target better results.

Keywords: civil engineering; architecture; conceptual design; parametric design; structural analysis; Grasshopper; optimisation; finite element method (FEM); ruled surface; roof shell; multi criteria decision making

1. Introduction

The underlying compositional rules for architectural design were established by Vitruvius in The Ten Books on Architecture, in the early ages [1]. Over the years, different design methodologies have emerged according to development of design technology. The classical architecture was determined by order and composition of forms being mostly shaped on the basis of platonic solids arranged on the Cartesian grid [2]. However, no linear architecture as well as organic one historically appeared in the Baroque period. This evolution of design activity understood as the process of form generating was caused by wider scientific search for theory of morphogenesis in the natural word. It was related to the profound study of biological organisms, the structure of matter, and application of this knowledge to the design and construction of built environment. Every element of any form was related to each other, according to the symbiotic ordering systems of nature. On the other hand, Modernism's dominant concepts of architectural design were based on industrial technologies of functionalism and universal models [2]. During the last twenty years, the advancement of digital technologies influenced the whole field of the architectural/civil engineering design [3]. Although at first digital media were applied rather as a representation tool, soon they became the means of conceptual design. Due to this fact, the first stage of the architectural design process moved away from a traditional paper-based process consisting in 3D model creation based on the 2D drawing [4,5]. What is more, due to widespread application of information technology tools in designing, the boundary between physical and virtual models began to blur. It was mostly caused by the hybridization of several methods and techniques for acquiring the model's geometry. Such methods were applied among others in reverse engineering [6]. Moreover, thanks to digital tools, the digital architectural/engineering models could be generated. The fast development of computing technologies brought the need of collaboration in various areas of design [7,8]. Building Information Modeling (BIM) as a 3D model-based approach gave architecture and civil engineering possibilities to streamline the design process. BIM, in a way, bridged the gaps in communication between participants of the design process: owners, architects, engineers, and contractors. The important insight into BIM-based design collaboration in the construction industry is given in [9]. However, the development of methodology to analyze the benefits of BIM is presented in [10].

Architectural/civil engineering designing during the last decade was inspired not only by different possibilities of digital technology, but it was also influenced by other disciplines such as mathematics and physics. It helped to introduce the concepts and software enabling creation of dynamic, parametric, and non-linear forms. Although civil engineering industry was among the last to use the new technologies consisting in smooth modeling, that is, digital modeling based on Non-Uniform Rational B-Splines (NURBS), their application resulted in a significant change of the design approach. That was due to the fact that NURBS curves could be controlled during designing, and their flexible shape became a base for creation of various changeable forms. There was also a significant change in the conception of space which started to be treated as a four-dimensional formation—an intersection of space and time. Architectural design established new computational concepts of architecture; topological and parametric architecture, among others [11]. Topological forms described by parametric functions gave a variety of possibilities for structure creation. In parametric design, a geometrical form is shaped not by declaration of its shape and structure, but by parameters and equations describing them. One can distinguish conceptual parametric design and constructive parametric design [12]. Constructive parametric design refers to additional data embedded for 3D determination of the given object. In this context, the architectural/engineering object being designed is influenced by many variables, and designing can be seen as an multi criteria decision making process (MCDM) [13]. Digital environment, however, offers a new approach to designing. On the one hand, it gives much freedom in shaping free forms; on the other hand, it allows designers to use computer software for optimization and simulation of projects. Due to this fact, it helps to make a right decision at the early stage of designing, whereas a digital model becomes a single source of information which can be generated, controlled, and managed by a designer. Digital architecture has profoundly changed the processes of design and construction. After the first generation of digital design processes and application of new digital design tools, new forms and relationships have been developed, and new processes have been emerged. Design tools have been developed that calibrated the digital form with reality on the construction site. Parallel to this, digital fabrication tools have been added to the design process and designing has got a new quality, which has resulted from so called "parametric design thinking" [14–18]. However, "parametric design thinking" entails ability to understand, read, and construct complex and parametrized operations which make a projected object respond and evolve. Alternate terms, such as "digitally intelligent design", "algorithmic design", "object oriented design", have arisen to describe this trend [19,20].

Along this line of thought, the paper discusses a novel parametric approach to conceptual design of roof shells compound of repetitive units of Catalan surfaces. The Catalan surfaces constitute the subset of the ruled surface class, the class of surfaces generated by straight lines. They are worth considering as, due to their striking shape and relative simplicity of construction, they stand out in the architecture of curvilinearity. Modeling of roofs formed by means of Catalan surfaces gives a variety of forms of aesthetic features [21,22]. The method of shaping free form buildings roofed with profiled steel sheets effectively transformed into strips of screw ruled surfaces is presented in [23–25]. Although many works deal with parametric design of beautiful free curvilinear building forms, we have not found examples of parametric design of compound forms of building covers/shells shaped on the base of Catalan surfaces. Therefore, the focus of this research lies on the parametric geometric description of the compound roof shells of symmetrical and regular shape, as well as how these descriptions can be used at early stages of design. The main goal is to elaborate universal scripts for modeling roof shells, which can be next used in various simulations. Furthermore, our approach is to link geometric designing of roofs' models with their structural analysis and optimization. The flexible scripts describing the geometric form are converted to simulation models to perform structural analysis, which is mainly concentrated on how the shells perform dependently on their geometric characteristics. The simulation enables to evaluate various roof shell shapes, as well as the selection of the optimal design solution meeting established criteria. The paper shows that optimization can support conceptual design being an efficient tool for "form-exploration". It also presents rationalized design strategy, which means both shaping roofs of rationalized surface classes, which have positive characteristics for engineering construction, and incorporating knowledge from engineering to architecture. It presents the advantages and shortcomings in parametric geometric modeling of these roofs, discusses their architectural qualities, and their relationship with construction principles. Our proposal targeting the early stages of the design process of roofs based on Catalan surfaces is an approach to make some contribution to the research results, which have been developed so far.

2. Early Stage of Rationalized Design

A lot of research has tried to outline frameworks for the organization of the decision-making design activities in architecture and civil engineering. The waterfall model proposed in 1970 was a relatively linear sequential design approach [26]. However, this model postulated that the early design process should be organized in a cascading sequence of the designing activities. Another model, closer to design reality, is widely described in [27]. It is a model of different design paths. According to it, the early-stage formation of a building object takes place in the process of successive designing steps with real possibilities of selecting different design ways and paths. These paths form a tree of multivariate opportunity. Comparing a pathfinder model with a waterfall model of design, we can state that a pathfinder model of the conceptual design process has a loose structure. It is characterized by iterations and feedbacks. Iterations and feedbacks assure a practical possibility of multiple modification and verification of the form according to the current need. These are actions in conformity with the design practice. In the architectural design process, it is often assured the possibility of a creative application of a broadly apprehended library of forms, or menu of forms. As a rule, the library of forms is formalized and contains morphems morphemes. They are forms selected and assorted by the designer, which can be ready to use, or can be further modified according to artistic inspiration or need. This permits the designer to think alternatively, by analysis and synthesis, what favors appropriate design decisions. During the design process, the designer examines many possible design decisions and subordinates their selection to his/her proper creative personality, knowledge, and design intuition. The structure of the multivariant decision process permits to perform successive choices according to the vision of the designer. It is in accordance with a statement that "designers work by exploring alternatives", presented in [28,29]. Typically, designers consider several alternatives, and choose the best one comparing the relative benefits across possible alternatives. It permits creative process customization, and does not obstruct thinking and exploration cycles. This is currently an acceptable approach to the problem with conformity with the real aspects of architectural design. Thanks to the development of computer-aided design, the approach based on parametric modeling has found favor in recent years. However, parametric models admit multiple alternatives, each achieved by editing parameters in the digital model, which results in a wider variety of solutions. In this context, parametric design "supports complexity" [15].

The need to develop design frameworks for a complex design process has been noticed by a number of researchers [20]. However, their contribution, although an important one, concerns rather more advanced stages of designing. Nevertheless, the development of the framework for the early stage, responsive and kinetic design of a building skin, is presented in [26]. The authors explore six aspects of designing using diverse means: parametric models, digital simulations, computational analyses, physical models and interactive prototypes. On the other hand, meta-parametric design approach for the concept design stage is proposed in [20]. The purpose of this approach is increasing the number of parametric definitions, which can be implemented at the early stages of design.

The early stage of designing is a conceptual stage when the most important decisions are made for shaping the future of the project. However, at this stage, little is known about objectives that can co-evolve during design development. Therefore, rationalization of this stage of designing is very important. Basically, rationalization in the building industry appears in various forms. It is mostly implemented in order to limit complexity or manufacturing cost. Furthermore, rationalization mostly deals with solving of design problems in relation to construction in the later stages of the design process. This research is focused on rational designing in the sense of incorporating knowledge from engineering to architectural design at the early stage. It is so-called pre-rationalized design insisting on introduction of the active attitude towards geometric design of roof shells.

3. Geometric Properties of Catalan Surfaces

Defining architectural geometric characteristics in an analytical way is becoming, more and more, an area of increasing interest and importance. In general, geometric modeling of surfaces deals with two major aspects: the visual representation focused on aesthetic appearance of surface forms, and analytical representation referring to mathematical descriptions and analysis of their geometric properties. It is also closely linked to the assembly of simple forms into a complex object. In our case, it will be the creation of a multi-shell roof from single shells in order to cover a proper rectangular plan.

The method of geometric modeling of multi-shell roofs depends mostly on the surfaces' properties forming the shell; their curvature, as well as continuity between them. The mathematical development of differential geometry and related general theories provide the needed theoretical basis for understanding surfaces' properties. Catalan surfaces play a specific role, due to their characteristics.

Catalan surfaces are ruled surfaces (scroll surfaces formed by the continuous movement of a straight line) [30]. They are oblique ruled surfaces, which can be divided into two groups:

- oblique ruled surfaces of second order—hyperbolic paraboloid
- oblique ruled surfaces of more than second order—conoids, cylindroids [31,32].

The difference between hyperbolic paraboloid, conoid, and cylindroid results from different path of movement of a surface's ruling during surface's formation. In all cases of Catalan surfaces' creation, the each ruling is parallel to the fixed plane (not containing surface's directrices).

Thin-shell structures are lightweight constructions using shell elements. These elements, typically curved ones, are assembled to make large structures—covers of large buildings free of intermediate supports. The shell material is thin in sections relative to the other dimensions of the roof, and undergoes relatively little deformation under load. All thin shell roofs derive their strength through shape, rather than mass. Considering three various groups of Catalan surfaces, hyperbolic paraboloids are exceptionally stiff, due to their double curvature [33]. What is more, hyperbolic paraboloids exhibit membrane action, wherein internal forces are efficiently transmitted through the surface in an in-plane manner [34]. Due to this fact, they can be analyzed by simple statics.

The design approach to parametric modeling of shell roofs proposed in this research is realized by application of Grasshopper, a parametric plug-in of the 3D modeling software Rhinoceros. The structural analysis is carried out using Karamba 3D, whereas the optimization issues are addressed by means of Galapagos, which work in the same Rhinoceros 3D environment. When it comes to architectural design, Grasshopper is one of the most commonly used generative design editors. It represents geometric shapes, but leaves the mathematical description hidden. However, it allows designers to build form generators from the simple to complicated complex forms.

The goal of our research is to elaborate universal scripts, in order to create digital shell roof models of various forms. Next, the scripts are to be used for modeling four-shell roof composed of repetitive modules of Catalan surfaces. Parametric roofs' models are to be converted to structural models, in order to perform structural analysis. Our comparative criterion to create roofs is the minimum mass and deflection.

4. Parametric Pre-Designing of Roof Shells—Results

4.1. Geometric Modeling by Means of Grasshopper Scripts

Digital modeling always involves the definition of the simple spatial forms, next their transformations and modification. Thanks to Grasshopper plug-in, the form generative algorithm modeling can be applied. Then, the form is generated by means of mathematical operations, functions, and other dependencies, which is shown in a graphical way. Several possibilities exist for defining the motion of a line generating a ruled surface. We can distinguish two methods for ruled surface generation. The first one consists in moving the straight line along generating curves, and the second one consists in connecting the proper corresponding points of two generating curves, which are called surface's directrices [31]. From a geometric point of view, ruled surfaces are infinite surfaces, however, for practical reasons we will consider finite parts of Catalan surfaces composed of rulings' segments.

Each Catalan surface is determined by two directrix lines and the director plane, to which all surface's rulings are parallel. In the case when both directrix lines are curved lines, we obtain a cylindroid; when one directrix is a straight line, we obtain conoid, however, when both directrices are skew straight lines, the Catalan surface is a hyperbolic paraboloid, shown in Figure 1.

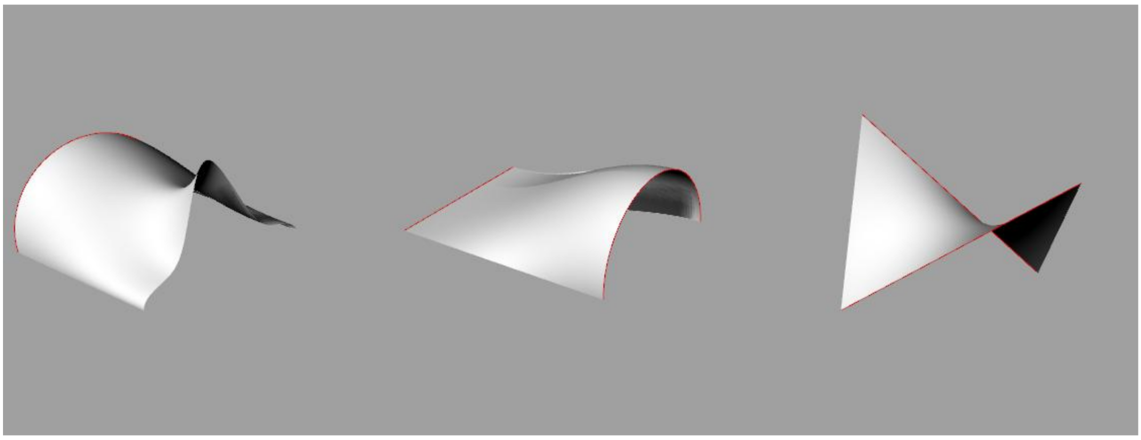
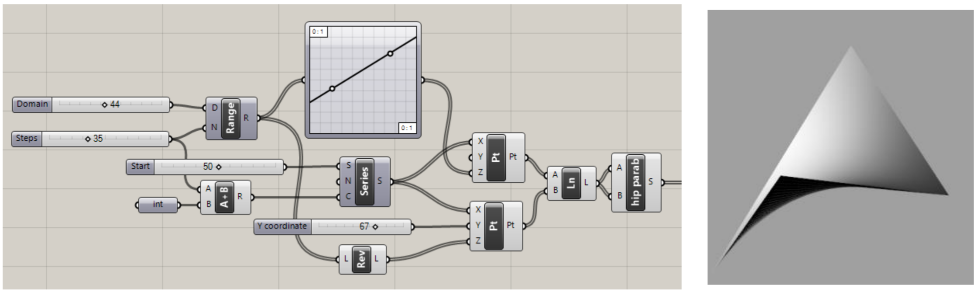


Figure 1. Typical examples of Catalan surfaces; respectively from the left: a cylindroid, a conoid, a hyperbolic paraboloid.

The surfaces as three dimensional objects in three-dimensional space can be described mathematically by a single equation with three space variables (x,y,z). However, to suit the requirements of the Grasshopper's algorithm, the surfaces should be described by two parameters (u,v). The developed Grasshopper's toolbox provides components which perform basic operations. This toolbox allowed us to use a set of functionalities to analyze and generate surfaces within the proposed design approach.

4.1.1. Hyperbolic Paraboloid

We start parametric modeling of a hyperbolic paraboloid from establishing series of points on two arbitrary skew straight lines, however, both contained in vertical parallel planes. The rulings join lines' points, which correspond to the same parameter value along u direction. This guarantees parallelism of all surface's rulings to the same vertical director plane. Application of a graph mapper for one line or for both lines enables steering lines' position in the planes (Figure 2).



(**a**) (**b**)

Figure 2. Single hyperboloid paraboloid form creation: (**a**) Grasshopper script for a single form; (**b**) result.

If all proper pairs of points included in the input curves are joined by straight line segments, a strip of a ruled surface connecting the curves is obtained. It is created as a single unit within Grasshopper domain [0, 1] for u and v variables. Such a unit surface can be a module for complex roof creation. The change of input parameters enables achieving roof forms of different shape, Figure 3.

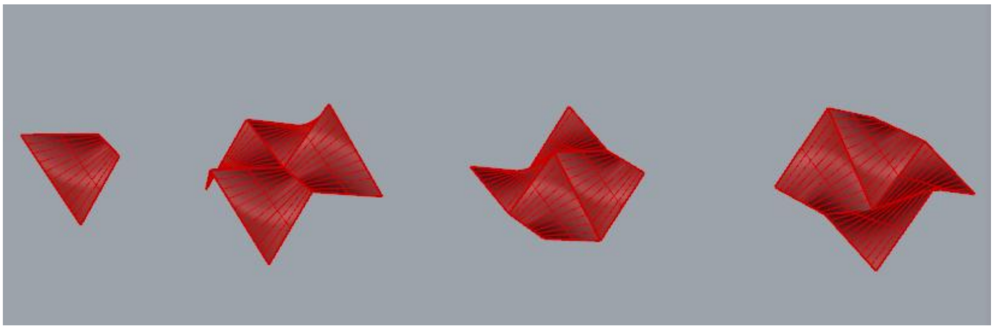
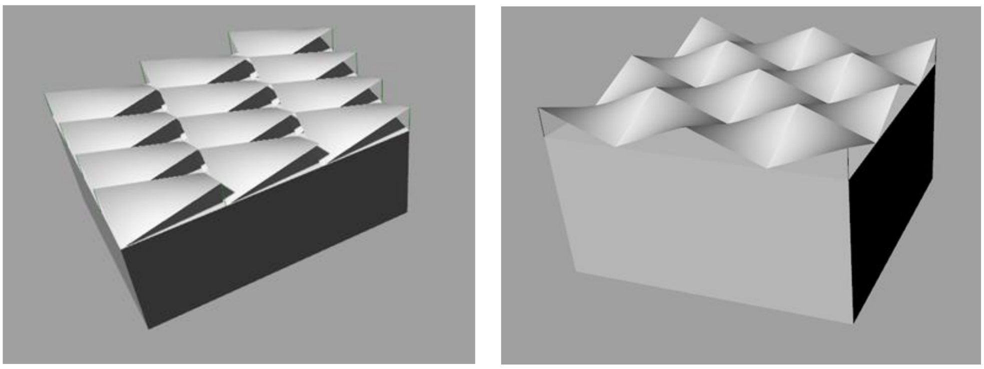


Figure 3. Several compound forms created from four units of the hyperbolic paraboloid surface.

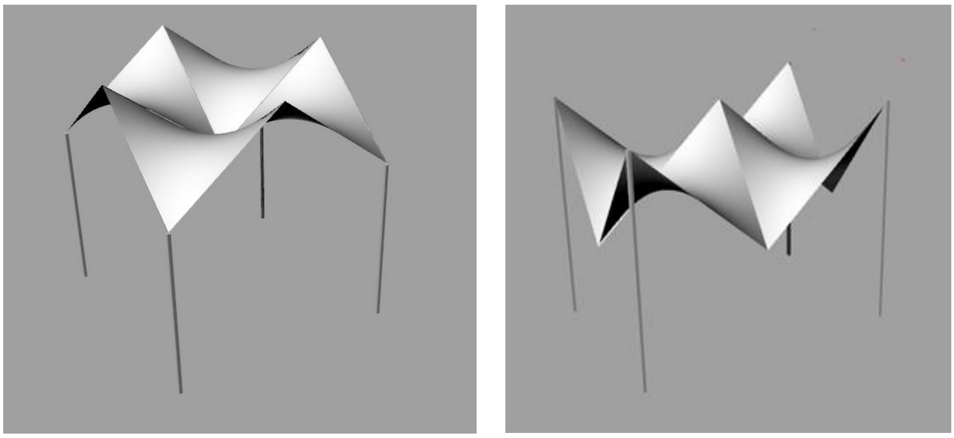
The parametric model of any form can be obtained using different algorithm definitions. That means "that the relationship between a parametric model and its output is many-to-one" [20]. In order to create complex forms from the single one, we have applied various approaches. One of them was morphing box creation on a square surface, which is presented in Figure 3. What is more, thanks to different input parameters which determine both geometric and metric characteristics, it is possible to achieve great amount of complex roofs. They can cover both open and close spaces, Figure 4.



(**a**) (**b**)

Figure 4. Building covers composed of hyperbolic paraboloid units: (**a**) with the same direction; (**b**) with different directions.

The geometric characteristic describes the shape of roof's repetitive units, their number, and arrangement (Figure 5). However, the metric characteristic determines the roof's span and height.

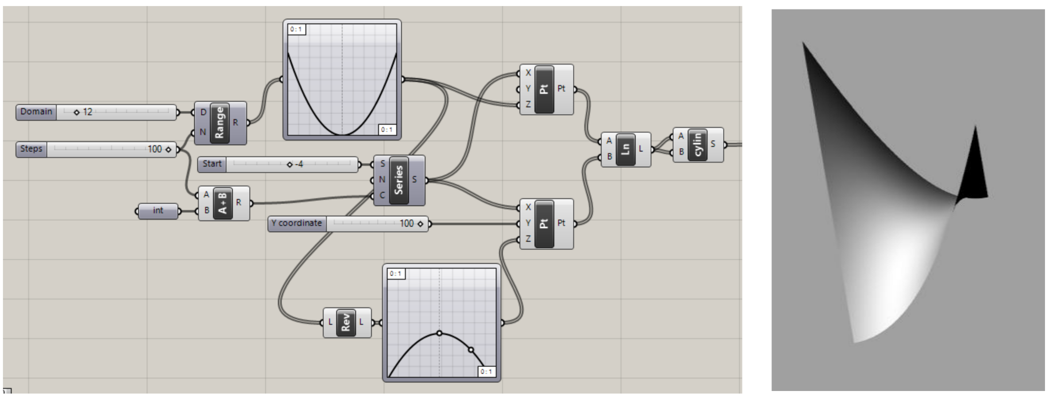


(**a**) (**b**)

Figure 5. The compound roof shell: (**a**) concave upwards; (**b**) concave downwards.

4.1.2. Cylindroid and Conoid

According to the geometrical definition presented above, the cylindroid's rulings join points located on two curved lines. Therefore, in the script for hyperbolic paraboloid creation, we used curved lines as directrix lines, instead of straight lines. These curves are plane lines included in the vertical parallel planes. The shape of the directrix line can be differentiated dependently on the modification by graph mapper (Figure 6).



(**a**) (**b**)

Figure 6. Single cylindroid form creation: (**a**) Grasshopper script for a single form; (**b**) result.

Thanks to it, it is possible to achieve various form creations dependently on the parameters' input, as well as directrices' curvature. However, due to their shape, not all generated forms can be suitable to form a shell roof. Some example forms are presented in Figure 7.

In order to create a parametric conoid surface, only slight modification of our script is necessary to make one directrix line be straight, which gives variety of interesting solutions (Figure 8).

The compound roofs shaped by means of repetitive units of conoid elements are very popular in building industry, and can be also generated by our script (Figure 9).

The principal advantage of parametric designing is the flexibility to perform transformations, which results in various configurations of the same geometrical components, different alternatives.

In order to evaluate roofs' alternatives, an optimization procedure will be applied by means of Galapagos, as well as structural analysis by means of Karamba 3D.

In our further considerations, we take into account the roofs composed of four identical modules of Catalan surfaces covering a rectangular plan.

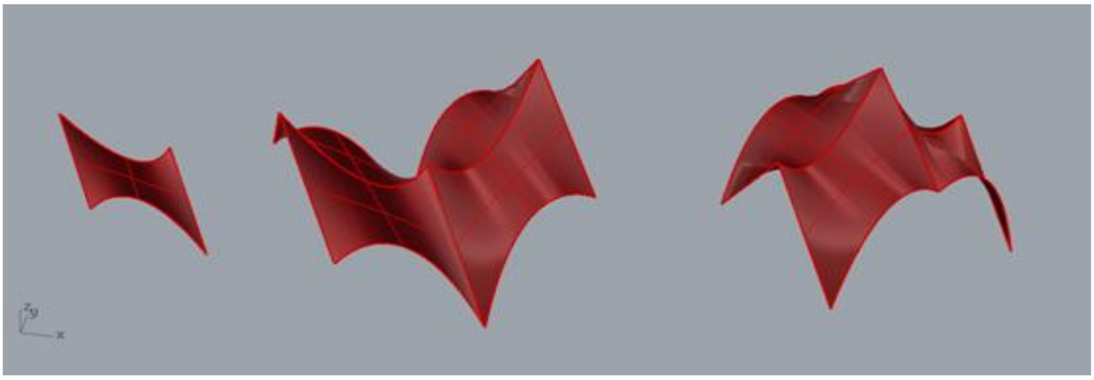


Figure 7. Several compound forms created from the units of a cylindroid surface.

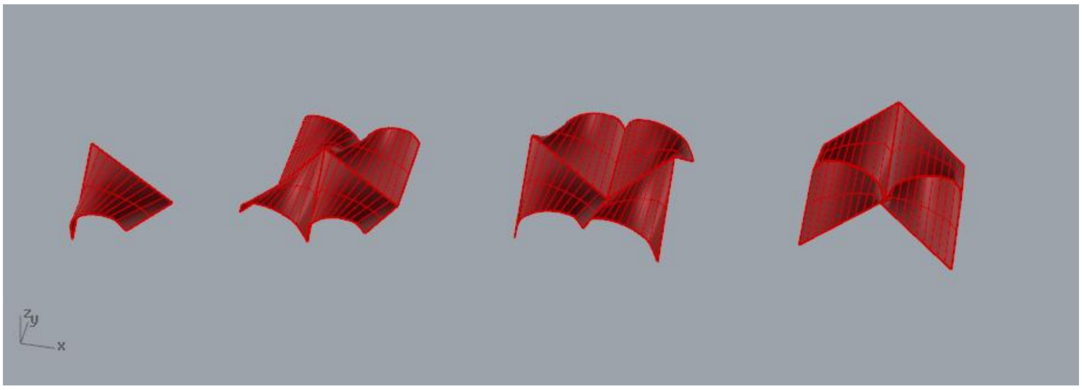
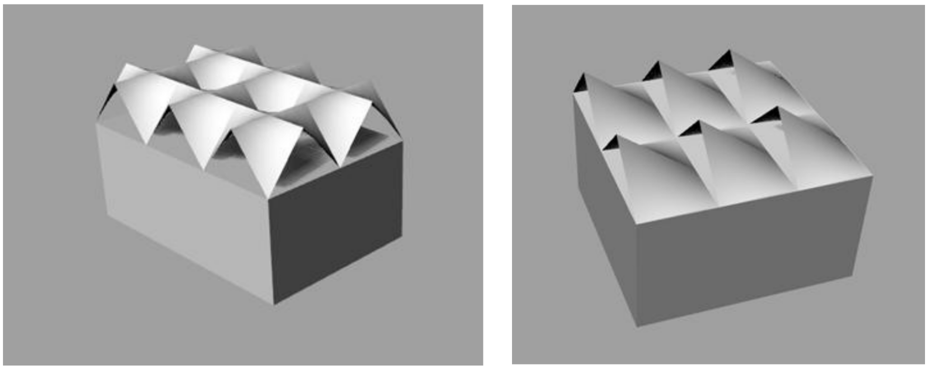


Figure 8. Several compound forms created from the units of a conoid surface.



(**a**) (**b**)

Figure 9. Building covers composed of conoid units: (**a**) with the same direction; (**b**) with different directions.

4.2. Form Optimisation by Means of Galapagos

In general, the optimal realization of the designer's idea from both an aesthetic and functional point of view is a difficult and a complex problem. Aesthetics and functionality are mutually dependent issues, as functional optimization cannot be separated from shape optimization, and vice versa. "Any optimum is only an optimum within the conceptualization of the problem space and the boundary conditions applied" [31]. The algorithmic methods and tools proposed by Grasshopper allow for a surfaces' analysis and modeling the rationalized forms, which match the initial ideas of design. Grasshopper enables generation of forms of a predefined logic. However, Galapagos, a module included in Grasshopper, is a numeric approach to drive controlled, suitable, and optimal results within the iterative design process. It enables the user to achieve results that best meet design criteria.

In our case study, we have taken into account the roof shells composed of four identical modules of Catalan surfaces that are four units of a hyperbolic paraboloid surface, a conoid surface, and a cylindroid surface, covering the same rectangular plan. Our optimization criterion was a minimum area of the roof shell, which covers a rectangle of a unit area. The height of the roof was assumed as the integer value within a fixed range of 1 m to 10 m.

Each optimization problem requires defining a fitness function, a sort of problem which is suited for Galapagos Evolutionary Solver. In our function, "each instance"—an alternative roof area (A)—was dependent on the rectangular area (B) covered by this roof, as well as the roof's height. The roof's area (A) was compared further with the area (B) of the covered rectangular plan. Therefore, in our fitness function, the difference, A−B, between the above areas was optimized (Figure 10).

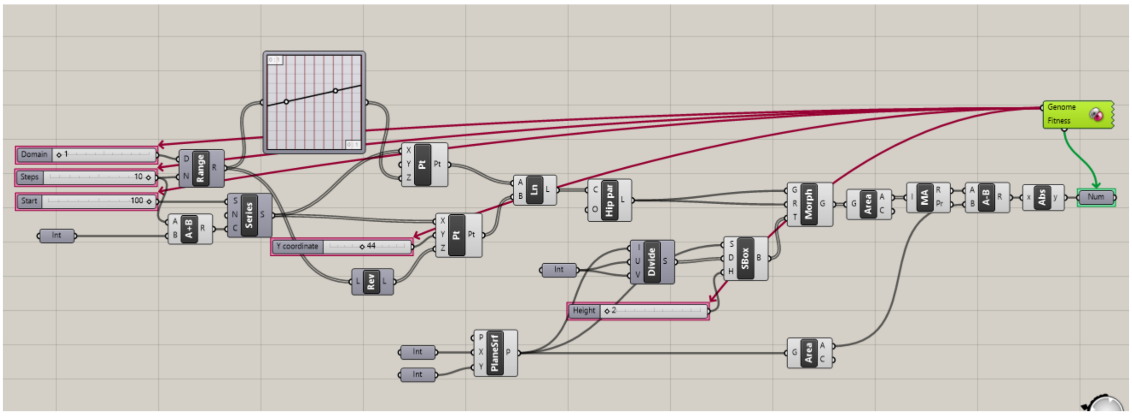


Figure 10. Optimization of a roof shell by Galapagos.

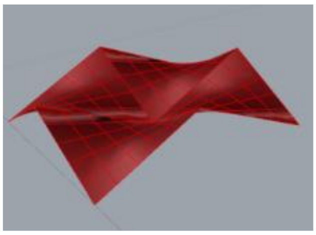
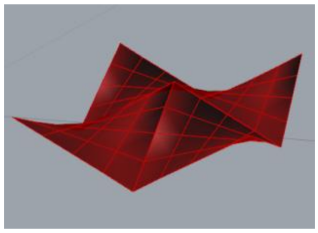
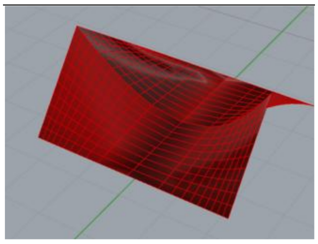
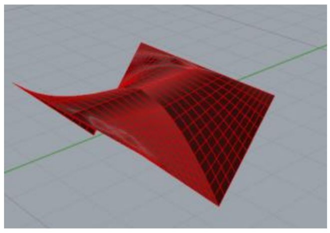
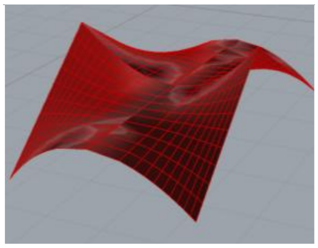
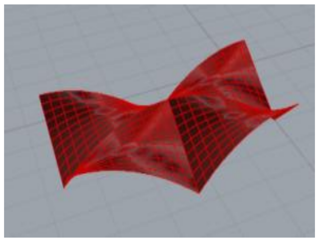
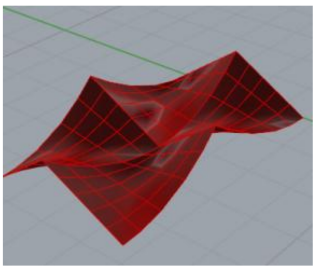
The minimal value of A−B meant the best solution. Thanks to Galapagos Evolutionary Solver, we have found the optimal height for each roof shell, as well as the minimal area of the Catalan surface which covers the unit area. The worst optimization result that is the biggest area of the roof shell was achieved for the roof shell composed from conoid elements. However, the best result that is the smallest roof shell area was obtained in the case of the roof shell composed from cylindroid elements determined by two parabolas as surfaces' directrices. These results were obtained for the height of each roof equal to 1 m.

4.3. Optimization Based on Structural Analysis

The above optimization results mean that the weight of the considered cylindroid shell roof should be the smallest compared to the weight of the other roof types, assuming the same specific height of all roofs equal to 1 m. This issue can be further checked by means of structural analysis. Structural analysis consists in determination of the effects of loads on a designed roof shell. In order to perform an accurate structural analysis, it is necessary to determine geometry of a roof's surface, structural loads, support conditions, as well as material properties. The results of such an analysis typically include support stresses, displacements, deformations, as well as support reactions. This information can be compared to the criteria that indicate the conditions of failure. Structural analysis is thus a key part of the engineering design. As far as the roof shell is concerned, we can state that it is a form-based structure, which means that its shape influences its load carrying capacity. Therefore, shell's geometry along the boundary conditions, as well as the type of loading applied, dictate the way the shell transfers load, or the way it fails. Shell can fail due to increasing deformations, failure of material, or a combination of both. Due to this fact we have taken into consideration shell's deflection as the main criterion for assessing structural stability.

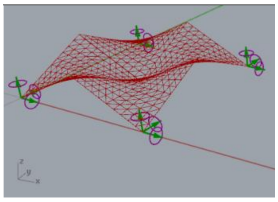
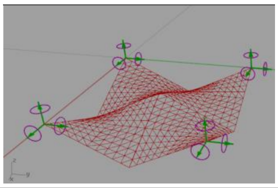
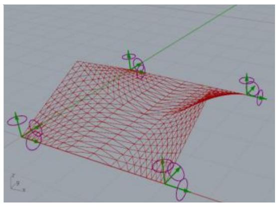
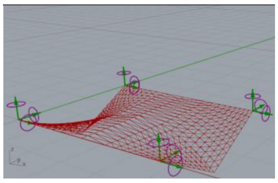
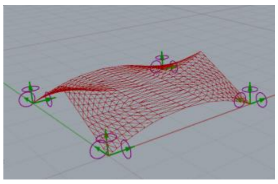
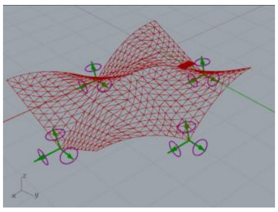
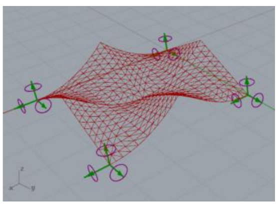
In our research, several representative roof shells have been subjected to structural analysis. Each of them was compound of four identical concrete unit surfaces of the same class Table 1. It was assumed that each roof shell covered a horizontal rectangle with the area of 16 m.

Table 1. The types of the analyzed compound roof shells.

Kind of Thse Urface	Type of the Compoud Roof Shell	
Hyperbolic paraboloid	 1	 2
Conoid	 3	 4
Cylindroid	 5	 6
	 7	

Each compound roof shell made up of four concrete surface units has been treated as one coherent shell element for calculation. Therefore, for each roof type, the calculations have been carried out as for a single concrete shell structure supported on four corners with circulation space in the middle. It is a common practice to apply approximate solutions of differential equations as the basis for structural analysis, which can be prepared using numerical approximation techniques. In our structural analysis, we used the finite element method (FEM) as the most commonly used numerical approximation in structural analysis. However, we carried out the analysis by application of Karamba 3D, an interactive parametric finite element program, which works in environment of Rhinoceros 3D. FEM simulation represents physical objects as a collection of discrete components or "elements". Due to this fact, the geometry of shells was presented by meshes, and each mesh face corresponded to the constant strain finite element, Table 2. In order to achieve comparable results each shell's mesh was divided into the same number of 400 quads, which were automatically decomposed to triangles.

Table 2. The scheme of the mesh, supports' location, as well as the obtained simulation results.

Kind of the Surface	The Scheme of the Mesh and Supports' Location	Dimmentions (m)	Mass (kg)	Displacement (m)
Hyperbolic paraboloid 1		$a = b = 4.00$ $h = 1$	3021.93	0.0004
Hyperbolic paraboloid 2		$a = b = 4.00$ $h = 1$	3021.93	0.0000
Conoid 3		$a = 4.75$ $b = 3.35$ $h = 1$	2993.24	0.0042
Conoid 4		$a = 4.75$ $b = 3.35$ $h = 1$	3993.52	0.0035
Cylindroid 5		$a = 3.47$ $b = 4.61$ $h = 1$	2901.03	0.0033
Cylindroid 6		$a = 4.69$ $b = 3.41$ $h = 1$	3113.00	0.0033
Cylindroid 7		$a = 4.67$ $b = 3.47$ $h = 1$	3107.00	0.0008

In order to perform a structural analysis by Karamba 3D, we converted our Grasshopper models given by the scripts to a simulation models (Figure 11).

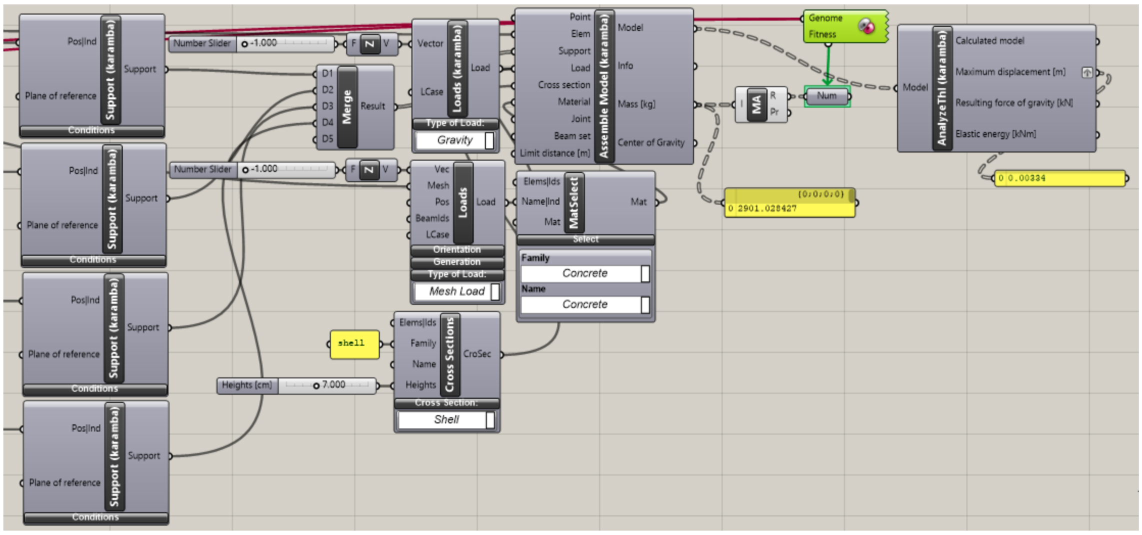


Figure 11. The assembly of load and supports in a simulation model in order to check roof shell behavior under self-load.

The first simulation was performed for the shell submitted to the dead load, which was the self-weight of concrete used in the shell construction. The dead load acted on the surface of the shell in negative z direction. The thickness of each shell was assumed to be of 7 cm. We have examined the behavior of each roof under self-weight, and established both the minimal mass for each roof and deflection assuming that the roof height $h \geq 1$ m. The simulation was performed by means of Galapagos Evolutionary Solver, and a minimum mass was an optimization criterion. The results are presented in Table 2. For each case of roof, there were also established dimensions of the rectangular plan covered by the roof: its length a according to direction x, and its width b according direction y, Table 2. Analyzing the achieved results, we can state that the construction of each roof is stable. However, the minimal mass of roof's construction was obtained in the case of the roof shell number 5, which was composed of the cylindroid units. This result confirmed the results of the previous analysis performed in point 3.2. In order to perform further structural analysis, we have chosen two of the roof shells' alternatives presented in Table 2. The first one was the cylindroid roof number 5 as a roof of a minimal mass. The second one was the roof shell number 2 compound of hyperbolic paraboloid units. It is a regular shaped roof, which covers a square plan. It also has a more favorable shape than other roof of hyperbolic paraboloid shape (the roof number 1), due to less possibility of the accumulation of precipitation (Table 2).

In the second structural analysis performed by us, we considered not only dead loads, but also live loads acting on the construction. Both snow and wind loads can have a considerable effect on shell structures. These loads are calculated in the form of pressure coefficients acting over the surface of the shell. We assumed snow pressure of 1.3 kN/m^2 and wind pressure of 1 kN/m^2, and applied different load combinations, one of them is shown in Figure 12.

We could do this using the load component of the Karamba 3D toolbar. This is a multi-use component which allowed us to specify a number of various types and different load combinations for the model. We have oriented self-load and snow load globally to a system of axes x, y, z, whereas a wind load locally to the mesh. The assemble component gathered all necessary information and created a static model. Based on simulation performed by Karamba 3D, it was possible to predict the behavior of the construction under loads, and the stress that each construction element experienced. Both roof shells were stable under the dead and live loads, however, the best optimization results, that is, the smallest deflection (equal to 0 mm) has been achieved in the case of the roof shell number 2 (Figure 13).

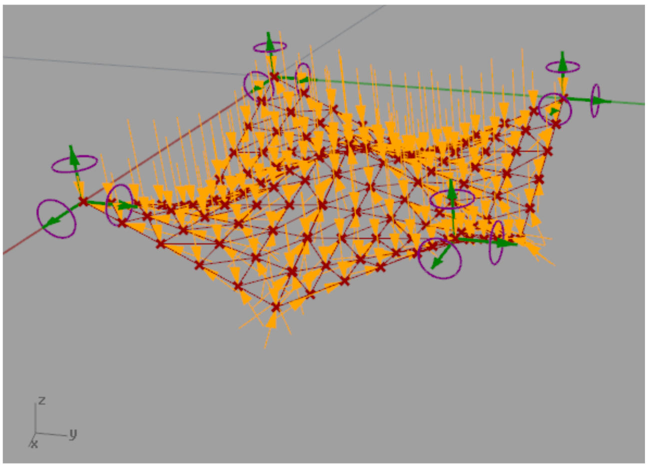


Figure 12. The schema of the application of live loads acting on the structure.

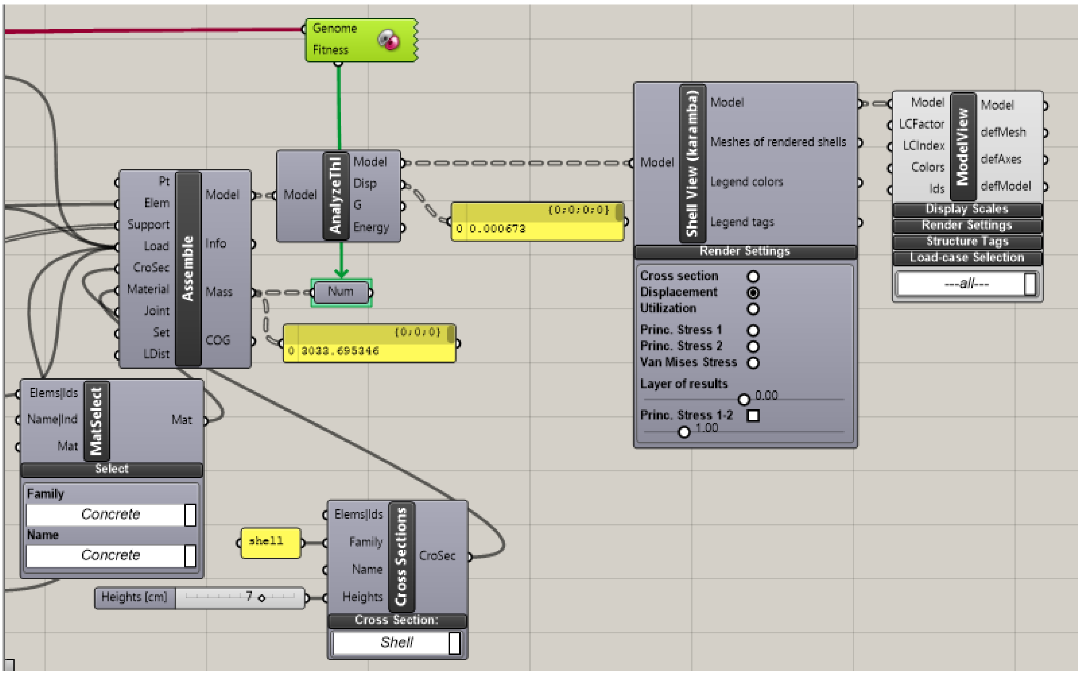


Figure 13. The best simulation result—behavior under dead and live loads for the shell roof number 2.

Due to this fact, the roof shell number 2, which is composed of four parts of a hyperbolic paraboloid surface, has been chosen as the best one from seven alternatives, and can be taken for further consideration in the more advanced process of design.

4.4. Discussion

The elaborated method of conceptual designing of roof shells composed of several parts of Catalan surfaces works well. The scripts for parametric designing developed by us seem to be universal, as they allow for the generation of various roof forms. These flexible scripts are the key for further simulation and optimization processes. Thanks to their application, we could find the roof shell of the best characteristic, that is, the roof compound of four parts of a hyperbolic paraboloid surface, which can be taken into consideration in a further design process. It is evident that in parametric design, shape and metric properties are the most important aspects of the design framework. They should be controlled at all times during the design process. This is due to the fact that they allow automatic evaluation of the performance of various design options in order to target better results. Karamba 3D includes a number of analysis components for performing different types of structural calculations: deflections and mass, among others. We could minimize the mass, deflection, and find out proper parameters to optimize the construction's right shape. However, due to the fact that we propose a structural analysis at the early stages of designing, this analysis is treated rather as an estimated analysis, not an accurate one.

The research shows that uncomplicated scripts give more flexibility in further optimization process. Due to this fact, they are of considerable importance, in particular, for the design of rationalized surfaces. It is worth mentioning, that in all presented plug-in tools for parametric design such as Grasshopper and Karamba 3D, the interactive display of the designed form is generated parallel with an interactive window for visual scripting modification. It helps in generation of complicated geometry and facilitates the re-editing process. What is important is that Karamba 3D and Galapagos work in the same Rhinoceros 3D environment. This minimalizes problems that could occur during application of various toolsets working in different software environments.

5. Conclusions

The research shows that it is possible to deploy structural analysis of the designed form at the early stages of design. What is more, it is very useful as it enables to estimate object's performance in different conditions, as well as generation and testing of the design variants based on various criteria and a specialist input.

However, our aim was not only designing of the optimal roof's form, but also methodology formulation, as well as indication of the approach and tools for the initial designing of roof shells. Due to this fact, we tried to elaborate scripts for further implementation.

The study shows that optimization can be not only a tool to search for optimal form, but can also be applied to check the performance of existing forms. Therefore, it can be applied as an efficient tool for form exploration and improvement in order to support conceptual design.

The rationalized design strategy presented in the paper consists in both application of rational surface classes, such as Catalan surfaces, simple and fundamental to roof design, and the active and complex attitude to the design concept. Such an attitude strengthens and facilitates sharing information, and co-operation between an architect and a civil engineering designer.

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References

1. Vitruvius, P. *Vitruvius: The Ten Books on Architecture*, 1st ed.; Harvard University Press: Cambridge, UK, 1914; pp. 13–31.
2. Biermann, V.; Borngasser, B. *Architectural Theory from the Renaissance to the Present*, 1st ed.; Taschen: Koln, Germany, 2003; pp. 6–20.
3. Oxman, R. Theory and design in the first digital age. *Des. Stud.* **2006**, *27*, 229–265. [CrossRef]
4. Unwin, S. Nalyzing architecture through drawing. *Build. Res. Inf.* **2007**, *35*, 101–110. [CrossRef]
5. Hewitt, M. Representational Forms and Modes of Conception; an Approach to the History of Architectural Drawing. *J. Archit. Educ.* **2014**, *39*, 2–9.

6. Biagini, C.; Donato, V. Behind the complexity of a folded paper. In Proceedings of the Mo. Di. Phy. Modeling from Digital to Physical. Innovation in design languages and project procedures, Milano, Italy, 11–12 November 2013; pp. 160–169.
7. Kim, D.-Y.; Lee, S.; Kim, S.-A. Interactive decision Making Environment for the Design optimization of climate Adaptive building Shells. In Proceedings of the International Conference on Cooperative Design, Visualization and Engineering (CDVE 2013), Mallorca, Spain, 22–25 September 2013; Springer: Berlin/Heindelberg, Germany, 2013.
8. Luo, Y.; Dias, J.M. Development of a Cooperative Integration System for AEC Design. In Proceedings of the International Conference on Cooperative Design, Visualization and Engineering (CDVE 2004), Palma de Mallorca, Spain, 19–22 September 2004; Springer: Berlin/Heidelberg, Germany, 2004; Volume 3190.
9. Wang, J.; Chong, H.-Y.; Shou, W.; Wang, X.; Guo, J. BIM- Enabled design Collaboration for Complex Building. In Proceedings of the International Conference on Cooperative Design, Visualization and Engineering (CDVE 2013), Mallorca, Spain, 22–25 September 2013; Springer: Berlin/Heindelberg, Germany, 2013.
10. Barlish, K.; Sullivan, K. How to measure the benefits of BIM- A case study approach. *Autom. Constr.* **2012**, *24*, 149–159. [CrossRef]
11. Kolarevic, B. *Architecture in the Digital Age: Design and Manufacturing*, 1st ed.; Spon Press: London, UK, 2003; pp. 20–98.
12. Stravic, M.; Marina, O. Parametric Modeling for Advanced Architecture. *Int. J. Appl. Math. Inform.* **2011**, *5*, 9–16.
13. Elango, M.; Devadas, M.D. Multi-Criteria Analysis of the Design Decisions In architectural Design Process during the Pre-Design Stage. *Int. J. Eng. Technol.* **2014**, *6*, 1033–1046.
14. Bhooshan, S. Parametric design thinking: A case study of practice-embedded architectural research. *Des. Stud.* **2017**, *52*, 115–143. [CrossRef]
15. Wortmann, T.; Tuncer, B. Differentiating parametric design: Digital Workflows in Contemporary Architecture and Construction. *Des. Stud.* **2017**, *52*, 173–197. [CrossRef]
16. Oxman, R. Thinking difference: Theories and models of parametric design thinking. *Des. Stud.* **2017**, *52*, 4–39. [CrossRef]
17. Woodburg, R.; Arish, R.; Kilian, A. Some Patterns for Parametric Modeling. In Proceedings of the 27th Annual Conference of the Association for Computer Aided Design in Architecture, Halifax, NS, Canada, 1–7 October 2007; pp. 222–229.
18. Turrin, M.; von Buelow, P.; Stouffs, R. Design explorations of performance driven geometry in architectural design using parametric modeling and genetic algorithms. *Adv. Eng. Inform.* **2011**, *25*, 656–675. [CrossRef]
19. Moussavi, F. Parametric software is not substitute for parametric thinking. *Archit. Rev.* **2011**, *230*, 39–40.
20. Hardling, J.E. Meta-parametric Design. *Des. Stud.* **2017**, *52*, 73–95. [CrossRef]
21. Tofli, J. Application of Catalan surface in designing roof structures—An important issue in the education of the future architect engineer. In Proceedings of the International Conference on Engineering Education (ICEE 2007), Coimbra, Portugal, 3–7 September 2007.
22. Przewlocki, S. *Descriptive Geometry in Construction*, 1st ed.; Arkady: Warsaw, Poland, 1982; pp. 57–209.
23. Abramczyk, J. *Shaping Free Forms of Buildings Roofted with Transformed Corrugated Sheeting*; Publishing House of Rzeszow University of Technology: Rzeszow, Poland, 2017.
24. Abramczyk, J. Shaping Innovative Forms of Buildings Roofted with Corrugated Hyperbolic Paraboloid Sheeting. *Procedia Eng.* **2016**, *161*, 60–66. [CrossRef]
25. Abramczyk, J. Building Structures Roofted with Multi-Segment Corrugated Hyperbolic Paraboloid Steel Shells. *Procedia Eng.* **2016**, *161*, 1545–1550. [CrossRef]
26. Zboinska, M.A.; Cudzik, J.; Juchnevic, R.; Radziszewski, K. A Design Framework and Digital Toolset Supporting the Early-Stage Explorations of Responsive Kinetic Building Skin Concepts. In Proceedings of the 33rd Annual eCAADe Conference, Real Time: Extending the Reach of Computation, Vienna, Austria, 16–18 September 2015; Volume 2, pp. 715–724.
27. Prokopska, A. *Methodology of Architectural Design, Preliminary Phases of the Architectural Process*; Publishing House of Rzeszow University of Technology: Rzeszow, Poland, 2015; pp. 39–121. (In Polish)
28. Prokopska, A. Morphology of the Architectural Achievement. A Methodological Analysis of Selected Morphological Systems of the Natural and Architectural Environments. *Syst. J. Transdiscipl. Syst. Sci.* **2002**, *7*, 1–116.

29. Woodburg, R.; Mohiuddin, A. Interactive design galleries: A general approach to interacting with design alternatives. *Des. Stud.* **2017**, *52*, 40–72. [CrossRef]
30. Dzwierzynska, J.; Abramczyk, J. *Engineering Descriptive Geometry*; Publishing House of Rzeszow University of Technology: Rzeszow, Poland, 2015.
31. Pottman, H.; Asperl, A.; Hofer, M.; Kilian, A. *Architectural Geometry*, 1st ed.; Bentley Institute Press: Exton, PA, USA, 2007; pp. 35–194.
32. Krivoshapko, S.; Ivanov, V.N. *Encyclopedia of Analytical Surfaces*; Springer International Publishing: Cham, Switzerland, 2015; pp. 60–77.
33. Chudley, R.; Greeno, R. *Advanced Construction Technology*, 4th ed.; Dorset Press: Dorchester, UK, 2006; pp. 528–529.
34. Min, C.S.; Gupta, A.K. Inelastic behavior of reinforced concrete hyperbolic paraboloid saddle shell. *Eng. Struct.* **1994**, *16*, 227–237. [CrossRef]