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Dynamic Intuitionistic Fuzzy Multi-Attribute Group Decision-Making Based on Power Geometric Weighted Average Operator and Prediction Model

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Abstract: With respect to dynamic multi-attribute group decision-making (DMAGDM) problems, where attribute values take the form of intuitionistic fuzzy values (IFVs) and the weights (including expert, attribute and time weights) are unknown, the dynamic intuitionistic fuzzy power geometric weighted average (DIFPGWA) operator and the improved IFVs' GM(1,1) prediction model (IFVs-GM(1,1)-PM) are proposed. First, the concept of IFVs, the operational rules, the distance between IFVs, and the comparing method of IFVs are defined. Second, the DIFPGWA operator and the improved IFVs-GM(1,1)-PM are defined in detail. Third, corresponding decision-making (D-M) steps are proposed. Three kinds of weights are given by the proposed determination method. Finally, an example is given to prove the effectiveness and superiority of the proposed decision-making method.

Keywords: the dynamic intuitionistic fuzzy power geometric weighted average (DIFPGWA) operator; IFVs GM(1,1) prediction model; dynamic intuitionistic fuzzy multi-attribute group decision-making

1. Introduction

Decision-making is an activity frequently occurring in management. This is the basic idea of modern decision theory. One of the important components in modern decision theory is multi-attribute group decision-making (MAGDM). This refers to the decision-making problem of selecting the optimal alternative or scheduling the scenario when multiple decision makers consider multiple attributes. However, many things in real life have fuzziness and uncertainty, and there is no clear statement. In the description of fuzziness and uncertainty, intuitionistic fuzzy sets have unparalleled advantages. If intuitionistic fuzzy sets are applied to MAGDM and generalized to multi-period decision-making, this decision-making process is called dynamic intuitionistic fuzzy multi-attribute group decision-making (DIF-MAGDM).

Since the Bulgarian scholar Atanassov [1] proposed the concept of intuitionistic fuzzy sets (IFSs), the research on the theory of IFSs has come a long way. Intuitionistic fuzzy sets have been widely used in many real-world cases [2–5]. Many scholars have proposed many improvements to MAGDM and multiple criteria decision making (MCDM) on IFS's methods and models. For example, Liu et al. [6] introduced intuitionistic trapezoidal fuzzy number and proposed an intuitionistic trapezoidal fuzzy prioritized ordered weighted aggregation (ITFPOWA) operator; Hashemi et al. [7] combined ELECTRE (originated in Europe in the mid-1960s. The acronym ELECTRE stands for: ELimination Et Choix Traduisant la REalité (ELimination and Choice Expressing REality)), VIKOR (ViseKriterijumska Optimizacija I Kompromisno Resenje, that means: Multicriteria Optimization and Compromise

Solution, with pronunciation: vikor) and GRA (grey relational analysis) theory to propose a new decision model based on IFS; Kahraman et al. [8] proposed an EDAS-based (based on the distance from Average Solution) MCDM method and gave the corresponding sensitivity analysis. In many decision-making methods, it is a very important method to solve problems by using operators. Many operators were proposed based on IFSs, such as an intuitionistic fuzzy ordered weighted averaging (IFOWA) operator, intuitionistic fuzzy weighted averaging (IFWA) operator, intuitionistic fuzzy weighted geometric (IFWG) operator, and intuitionistic fuzzy ordered weighted geometric (IFOWG) operator [9–11]. Although the calculation of the above operator is simple, some facts are ignored. However, these operators ignore a practical problem, that is, decision-makers will have their own preferences, with their own subjectivity. Therefore, Tan and Chen [12] proposed an IF choquet integral operator that fully considers the interaction between decision maker preferences. There are still some problems that have not been taken into consideration. That is, the default alternatives and attributes are both independent. In fact, there is a certain link between them. Xu [13] proposed the IFPWA operators and IFPWG operators, which can capture the delicate nuances of the comprehensive evaluation values that the decision makers need to reflect when evaluating. In addition, IFSs are also combined with grey relational models [14], TOPSIS (Technique for Order Preference by Similarity to an Ideal Solution) methods [15], prospect theories [16], evidential reasoning theories [17], and Frank t-norms family [18], and an improved IF-MADM (multi-attribute decision-making) based on the above theory is constructed, further enriching the IFS theory. However, the above method for IF-MADM only addresses static evaluation information at a certain period. In fact, the evaluation of things at one point in time is one-sided. Things evolve over time, and static evaluation cannot reflect the development trend of things. For DIF-MADM, Xu and Yager [19] proposed a dynamic intuitionistic fuzzy weighted averaging (DIFWA) operator and uncertain dynamic intuitionistic fuzzy weighted averaging (UDIFWA) operator. Wei [20] proposed the DIFWG operator and UDIFWG operator. Su et al. [21] proposed a method that combines TOPSIS method with DIFWA operator and HWA operator to solve DIF-MAGDM. Park et al. [22] extends the VIKOR method and combines (DIFWG) operator and (UDIFWG) operator to solve DIF-MAGDM. Chen et al. [23] proposes a dynamic IF compromise decision method based on time degree. Although the above method solves DIF-MAGDM to some extent from different perspectives, none of them considers the relationship between attribute values. In fact, there are certain links between attributes, alternatives, and periods. Therefore, dynamic intuitionistic fuzzy power geometric weighted average (DIFPGWA) operator that better reflect support relationship between attributes are proposed, which can capture the delicate nuances of the comprehensive evaluation values that the decision makers need to reflect when evaluating. However, it is inaccurate to make judgments by merely aggregating evaluation information from past to present. This may happen because the evaluation of the alternative one by the experts started poorly and then gradually got better. The evaluation of the alternative two started better and then slowly deteriorated. This situation may get the same result through DIFPGWA operator aggregation information. However, this method also has some shortcomings. Therefore, in order to compensate for the lack of DIFPGWA operator, the GM(1,1)-PM based on IFVs that were proposed by Li et al. [24] have been improved. Comprehensive evaluation values of each alternatives could be predicted in the next period, and then make more accurate judgments.

Weight is a relative concept. It is very important to determine the weight in DIF-MAGDM. In this article, attribute weights, expert weights, and time weights are all unknown. For expert weights, Chen and Yang [25] proposed a method for deriving expert weights from evaluation values. Because each expert has his own field and expertise, experts may make irrational comments. If the experts have the same weight, the result will be unreasonable. In addition, if Expert weights are set subjectively, especially in this multi-period situation, not only the workload is large, but also the deviation is large. Therefore, it is generally believed that the more deviated from the common cognition, the smaller the expert weight should be, and the closer to the common cognition, the greater the expert weight should be. For attribute weights, the attribute weights of each period are determined based on the

idea of maximum deviation [26,27]. That is to say, the closer the evaluation value of the attribute is, the smaller the weight should be given. Otherwise, the greater weight should be given. Moreover, due to different evaluations by experts in different periods, attribute weights should also be different. For time weights, Guo et al. [28] proposed a combination of subjective and objective methods to determine the time weights. Subjective can comprehensively consider the expert's knowledge and experience, treat things with their own feelings, and make conclusions. Therefore, the combination of subjectivity and objectiveness can give more reasonable time weights.

In DIF-MAGDM, the proposed DIFPGWA operator not only aggregates the information of each stage, but also captures the fine nuances of the comprehensive evaluation value that the decision makers want to reflect. It is inaccurate to make a judgment based only on the evaluation from past to today. Based on this, this paper forecasts the comprehensive evaluation value of each scheme in the next period through the improved IFVs-GM(1,1)-PM. This method can make up for the lack of DIFPGWA operator. The proposed future adjustment coefficient can combine the two results to make the final result more accurate. Moreover, the multi-period weight determination method proposed in this paper also helps to obtain more accurate results. In the second chapter, this paper introduces the basic knowledge of intuitionistic fuzzy. In the third chapter, this paper constructs the operator and prediction model. In the fourth chapter, the paper gives the corresponding decision steps and predicting steps. In the fifth chapter, the decision method proposed in this paper is applied to the selection of the company's intern employees.

2. Preliminaries

2.1. Concepts and Operational Rules

Definition 1. IFS in space X is defined as $A = \{[x, \mu_A(x), \nu_A(x)] | x \in X\}$. For any $x \in X$, $\mu_A(x)$ and $\nu_A(x)$ was expressed membership degree and non-membership degree of A . Moreover, $\mu_A(x)$ and $\nu_A(x)$ satisfy the following properties: $\mu_A : X \rightarrow [0, 1]$, $\nu_A : X \rightarrow [0, 1]$ and $0 \leq \mu_A(x) + \nu_A(x) \leq 1, \forall x \in X$. For any $x \in X$, $\pi_A(x) = 1 - \mu_A(x) - \nu_A(x)$ was expressed as a hesitancy degree of A [1].

For convenience of computation, IFVs was expressed as $\alpha = (\mu_\alpha, \nu_\alpha)$, where $\mu_\alpha \in [0, 1]$, $\nu_\alpha \in [0, 1]$, $\mu_\alpha + \nu_\alpha \leq 1$ [10].

Definition 2. Let $\alpha = (\mu_\alpha, \nu_\alpha), \beta = (\mu_\beta, \nu_\beta)$ be IFVs, and the operational rules are as follows [19,29]:

$$\bar{\alpha} = (\nu_\alpha, \mu_\alpha), \quad (1)$$

$$\alpha \oplus \beta = (\mu_\alpha + \mu_\beta - \mu_\alpha \mu_\beta, \nu_\alpha \nu_\beta), \quad (2)$$

$$\lambda \alpha = (1 - (1 - \mu_\alpha)^\lambda, \nu_\alpha^\lambda), \lambda > 0, \quad (3)$$

$$\alpha \otimes \beta = (\mu_\alpha \mu_\beta, \nu_\alpha + \nu_\beta - \nu_\alpha \nu_\beta), \quad (4)$$

$$\alpha^\lambda = (\mu_\alpha^\lambda, 1 - (1 - \nu_\alpha)^\lambda), \lambda > 0, \quad (5)$$

$$\begin{aligned} d(\alpha, \beta) &= \frac{1}{2}(|\mu_\alpha - \mu_\beta| + |\nu_\alpha - \nu_\beta| + |(1 - \mu_\alpha - \nu_\alpha) - (1 - \mu_\beta - \nu_\beta)|) \\ &= \frac{1}{2}(|\mu_\alpha - \mu_\beta| + |\nu_\alpha - \nu_\beta| + |\mu_\alpha - \mu_\beta + \nu_\alpha - \nu_\beta|) \end{aligned} \quad (6)$$

Definition 3. Support is a similarity indicator, the closer the two data values, the greater the support. Let $\alpha_1, \alpha_2, \dots, \alpha_n$ be IFVs. Therefore, according to the character of support sets and the operation rules of IFVs, the support is calculated as follows [13]:

$$Sup(\alpha_i, \alpha_j) = 1 - \frac{d(\alpha_i, \alpha_j)}{\sum_{j=1, j \neq i}^n d(\alpha_i, \alpha_j)}. \quad (7)$$

$Sup(\alpha_i, \alpha_j)$ express the support between α_i and α_j , and has the following three properties: $Sup(\alpha_i, \alpha_j) \in [0, 1]$; $Sup(\alpha_i, \alpha_j) = Sup(\alpha_j, \alpha_i)$. If $|\alpha_i - \alpha_j| \leq |\alpha_s - \alpha_t|$, then $Sup(\alpha_i, \alpha_j) \geq Sup(\alpha_s, \alpha_t)$.

2.2. The Comparing Method of IFVs

Definition 4. Let $\alpha = (\mu_\alpha, \nu_\alpha)$, $\beta = (\mu_\beta, \nu_\beta)$ be IFVs, $s(\alpha) = \mu_\alpha - \nu_\alpha$ and $s(\beta) = \mu_\beta - \nu_\beta$ be the scores degrees of α and β . $h(\alpha) = \mu_\alpha + \nu_\alpha$ and $h(\beta) = \mu_\beta + \nu_\beta$ be the accuracy degrees of α and β ; then,

- If $s(\alpha) < s(\beta)$, then $\alpha \prec \beta$,
- If $s(\alpha) > s(\beta)$, then $\alpha \succ \beta$,
- If $s(\alpha) = s(\beta)$, then if $h(\alpha) = h(\beta)$, then $\alpha = \beta$; if $h(\alpha) < h(\beta)$, then $\alpha \prec \beta$; if $h(\alpha) > h(\beta)$, then $\alpha \succ \beta$ [30,31].

3. The Proposed IF-Operators and IFVs-GM(1,1)-PM

3.1. The IFPGWA Operator

Definition 5. Let $a_1, a_2, \dots, a_n \in R$; then, the geometric weighted average operators WGA_w are defined as follows [32]:

$$WGA_w(a_1, a_2, \dots, a_n) = \prod_{i=1}^n a_i^{w_i}, \quad (8)$$

where $w = (w_1, w_2, \dots, w_n)^T$ is the weight vector associated with the operator and $\forall w_i \geq 0, \sum_{i=1}^n w_i = 1$.

Definition 6. Let $a_1, a_2, \dots, a_n \in R$; then, the power geometric weighted average operators $PWGA_w$ is defined as follows [33]:

$$PWGA_w(a_1, a_2, \dots, a_n) = \prod_{i=1}^n a_i^{x_i}, \quad (9)$$

where $w = (w_1, w_2, \dots, w_n)^T$ is the weight vector associated with the operator and $\forall w_i \geq 0, \sum_{i=1}^n w_i = 1$,

$$T(a_i) = \sum_{j=1, j \neq i}^n w_j Sup(a_i, a_j), \quad x_i = \frac{w_i(1+T(a_i))}{\sum_{i=1}^n (1+T(a_i))w_i}.$$

$Sup(a_i, a_j)$ express the support between a_i and a_j , and has the following three properties: $Sup(a_i, a_j) \in [0, 1]$; $Sup(a_i, a_j) = Sup(a_j, a_i)$; If $Sup(a_i, a_j) \geq Sup(a_s, a_t)$, then $|a_i - a_j| \leq |a_s - a_t|$.

Definition 7. Let $\alpha_1, \alpha_2, \dots, \alpha_n$ be IFVs; then, the intuitionistic fuzzy power geometric weighted average (IFPGWA_w) operator is defined as follows:

$$IFPGWA_w(\alpha_1, \alpha_2, \dots, \alpha_n) = \prod_{i=1}^n \alpha_i^{\zeta_i} = \left(\prod_{i=1}^n \mu_{\alpha_i}^{\zeta_i}, 1 - \prod_{i=1}^n (1 - \nu_{\alpha_i})^{\zeta_i} \right) \quad (10)$$

$$\text{where } T(\alpha_i) = \sum_{j=1, j \neq i}^n w_j Sup(\alpha_i, \alpha_j), \quad Sup(\alpha_i, \alpha_j) = 1 - \frac{d(\alpha_i, \alpha_j)}{\sum_{j=1, j \neq i}^n d(\alpha_i, \alpha_j)}, \quad \zeta_i = \frac{w_i(1+T(\alpha_i))}{\sum_{i=1}^n (1+T(\alpha_i))w_i}.$$

It is easy to prove that the IFPGWA_w operator has the following properties:

Theorem 1. (Commutativity). Let $(\alpha_1, \alpha_2, \dots, \alpha_n)$ be a vector of IFVs, and $(\alpha'_1, \alpha'_2, \dots, \alpha'_n)$ be any permutation of $(\alpha_1, \alpha_2, \dots, \alpha_n)$; then, $IFPGWA_w(\alpha_1, \alpha_2, \dots, \alpha_n) = IFPGWA_w(\alpha'_1, \alpha'_2, \dots, \alpha'_n)$.

Proof.

$$\begin{aligned} IFPGWA_w(\alpha_1, \alpha_2, \dots, \alpha_n) &= \prod_{i=1}^n \alpha_i^{\zeta_i} = \left(\prod_{i=1}^n \mu_{\alpha_i}^{\zeta_i}, 1 - \prod_{i=1}^n (1 - \nu_{\alpha_i})^{\zeta_i} \right) \\ &= \left(\prod_{i=1}^n \mu_{\alpha'_i}^{\zeta'_i}, 1 - \prod_{i=1}^n (1 - \nu_{\alpha'_i})^{\zeta'_i} \right) = \prod_{i=1}^n \alpha'^{\zeta'_i}_i = IFPGWA_{w'}(\alpha'_1, \alpha'_2, \dots, \alpha'_n). \end{aligned}$$

□

Theorem 2. (Idempotency). Let $\alpha_j (j = 1, 2, \dots, n)$ be a collection of IFVs, if $\alpha_j = \alpha$, for all j ; then, $IFPGWA_w(\alpha_1, \alpha_2, \dots, \alpha_n) = \alpha$.

Proof.

$$\begin{aligned} IFPGWA_w(\alpha_1, \alpha_2, \dots, \alpha_n) &= \prod_{i=1}^n \alpha_i^{\zeta_i} = \left(\prod_{i=1}^n \mu_{\alpha_i}^{\zeta_i}, 1 - \prod_{i=1}^n (1 - \nu_{\alpha_i})^{\zeta_i} \right) \\ &= \left(\mu^{\sum_{i=1}^n \left(\frac{w'_i}{\sum_{i=1}^n w'_i} \right)}, 1 - (1 - \nu)^{\sum_{i=1}^n \left(\frac{w'_i}{\sum_{i=1}^n w'_i} \right)} \right) = (\mu, \nu) = \alpha. \end{aligned}$$

□

Theorem 3. (Boundedness). Let $\alpha_j (j = 1, 2, \dots, n)$ be a collection of IFVs, then $\alpha_{\min} \leq IFPGWA_w(\alpha_1, \alpha_2, \dots, \alpha_n) \leq \alpha_{\max}$, where $\alpha_{\min} = \left(\min_j \{ \mu_{\alpha_j} \}, \max_j \{ \nu_{\alpha_j} \} \right)$ and $\alpha_{\max} = \left(\max_j \{ \mu_{\alpha_j} \}, \min_j \{ \nu_{\alpha_j} \} \right)$.

Proof. According to the Theorem 2, the following inequalities can be proved:

$$IFPGWA_w(\alpha_1, \alpha_2, \dots, \alpha_n) \geq IFPGWA_w(\alpha_{\min}, \alpha_{\min}, \dots, \alpha_{\min}) = \alpha_{\min},$$

$$IFPGWA_w(\alpha_1, \alpha_2, \dots, \alpha_n) \leq IFPGWA_w(\alpha_{\max}, \alpha_{\max}, \dots, \alpha_{\max}) = \alpha_{\max}.$$

Then, $\alpha_{\min} \leq IFPGWA_w(\alpha_1, \alpha_2, \dots, \alpha_n) \leq \alpha_{\max}$ can be proved. □

Example 1. Let $\alpha_1 = (0.78, 0.19)$, $\alpha_2 = (0.53, 0.47)$, $\alpha_3 = (0.22, 0.32)$ and $w_1 = 0.259$, $w_2 = 0.296$, $w_3 = 0.445$. The result obtained by $IFPGWA_w$ operator is shown below:

$$T(\alpha_1) = 0.346, T(\alpha_2) = 0.330, T(\alpha_3) = 0.279; \zeta_1 = 0.266, \zeta_2 = 0.300, \zeta_3 = 0.434$$

$$IFPGWA_w(\alpha_1, \alpha_2, \alpha_3) = (0.404, 0.342).$$

3.2. The DIFPGWA Operator

Definition 8. Let $\alpha_i(t_1), \alpha_i(t_2), \dots, \alpha_i(t_g)$ be the i th attribute value at different periods $t_1, t_2, \dots, t_g$, where $\alpha_i(t_l)$ are expressed by IFVs. $\theta(t) = (\theta(t_1), \theta(t_2), \dots, \theta(t_g))^T$ be the weight vector of periods $t_l (l = 1, 2, \dots, g)$ and $\theta(t_l) \geq 0$, $\sum_{l=1}^g \theta(t_l) = 1, l = 1, 2, \dots, g$; then, the dynamic intuitionistic fuzzy power geometric weighted average $DIFPGWA_{\lambda(t)}$ operator is defined as follows:

$$\begin{aligned} DIFPGWA_{\theta(t)}(\alpha_i(t_1), \alpha_i(t_2), \dots, \alpha_i(t_g)) &= \prod_{l=1}^g (\alpha_i(t_l))^{\zeta_l} \\ &= \left(\prod_{l=1}^g \mu_{\alpha_i}^{\zeta_l}(t_l), 1 - \prod_{l=1}^g (1 - \nu_{\alpha_i}(t_l))^{\zeta_l} \right), \end{aligned} \quad (11)$$

$$\text{where } T(\alpha_i(t_l)) = \sum_{l=1, l \neq v}^g \theta(t_l) \text{Sup}(\alpha_i(t_l), \alpha_i(t_v)), \quad \text{Sup}(\alpha_i(t_l), \alpha_i(t_v)) = 1 - \frac{d(\alpha_i(t_l), \alpha_i(t_v))}{\sum_{l=1, l \neq v}^g d(\alpha_i(t_l), \alpha_i(t_v))},$$

$$\xi_l = \frac{\theta(t_l)(1+T(\alpha_i(t_l)))}{\sum_{l=1}^g (1+T(\alpha_i(t_l)))\theta(t_l)}.$$

It is easy to prove that the $DIFPGWA_{\lambda(t)}$ operator has the three properties: Commutativity, Idempotency, and Boundedness. Here no longer proof.

Example 2. Let $\alpha_i(t_1) = (0.48, 0.19)$, $\alpha_i(t_2) = (0.77, 0.10)$, $\alpha_i(t_3) = (0.65, 0.16)$ and $\theta(t_1) = 0.2$, $\theta(t_2) = 0.3$, $\theta(t_3) = 0.5$. The result obtained by the $DIFPGWA_{\theta(t)}$ operator is shown below:

$$T(\alpha_i(t_1)) = 0.426, T(\alpha_i(t_2)) = 0.413, T(\alpha_i(t_3)) = 0.259; \xi_1 = 0.213, \xi_2 = 0.317, \xi_3 = 0.470$$

$$DIFPGWA_{\theta(t)}(\alpha_i(t_1), \alpha_i(t_2), \alpha_i(t_3)) = (0.642, 0.147).$$

3.3. The Improved IFVs-GM(1,1)-PM

Definition 9. Let $\alpha = (\alpha_1, \alpha_2, \dots, \alpha_n)$ be IFVs time series; then, the IFV scores' degrees time series $S(\alpha) = (S(\alpha_1), S(\alpha_2), \dots, S(\alpha_n))$ and IFVs' hesitancy degrees time series $\pi(\alpha) = (\pi(\alpha_1), \pi(\alpha_2), \dots, \pi(\alpha_n))$ can be calculated.

According to $S(\alpha_k) + \widehat{a}z^{(1)}(k) = b$, the time response function of $S(\alpha) = (S(\alpha_1), S(\alpha_2), \dots, S(\alpha_n))$ can be calculated as follows:

$$S^{(1)}(\widehat{\alpha}_{k+1}) = \left(S(\alpha_1) - \frac{b}{a} \right) e^{-ak} + \frac{b}{a}, \quad (12)$$

where $z^{(1)}(k) = \frac{S(\alpha_k) + S(\alpha_{k-1})}{2}$. Thus, the predictive value of IFVs scores degree can be calculated as follows:

$$S(\widehat{\alpha}_{k+1}) = (1 - e^a) \left(S(\alpha_1) - \frac{b}{a} \right) e^{-ak}. \quad (13)$$

In addition, the predictive value of hesitancy degree can be calculated as follows:

$$\pi(\widehat{\alpha}_{k+1}) = \max\{\pi(\alpha_1), \pi(\alpha_2), \dots, \pi(\alpha_n)\}. \quad (14)$$

According to the definition of IFVs, the IFVs-GM(1,1)-PM can be drawn as [24]

$$\widehat{\alpha}_{k+1} = (\widehat{\mu}_{\alpha_{k+1}}, \widehat{\nu}_{\alpha_{k+1}}) = \left(\frac{1 + S(\widehat{\alpha}_{k+1}) - \pi(\widehat{\alpha}_{k+1})}{2}, \frac{1 - S(\widehat{\alpha}_{k+1}) - \pi(\widehat{\alpha}_{k+1})}{2} \right). \quad (15)$$

However, this IFVs-GM(1,1)-PM cannot ensure that the degree of membership and non-membership are both in the range of $[0, 1]$. Therefore, the above IFVs-GM(1,1)-PM needs to be improved as follows:

$$\begin{aligned} \widehat{\alpha}_{k+1} = (\widehat{\mu}_{\alpha_{k+1}}, \widehat{\nu}_{\alpha_{k+1}}) = & \left(\min \left(\left(\max \left(\frac{1 + S(\widehat{\alpha}_{k+1}) - \pi(\widehat{\alpha}_{k+1})}{2}, 0 \right) \right), 1 \right), \rightarrow \right. \\ & \left. \rightarrow \min \left(\left(\max \left(\frac{1 - S(\widehat{\alpha}_{k+1}) - \pi(\widehat{\alpha}_{k+1})}{2}, 0 \right) \right), 1 \right) \right). \end{aligned} \quad (16)$$

The best using condition of IFVs-GM(1,1)-PM is: $a \geq -0.5$ [24].

Example 3. Let $\alpha_1 = (0.58, 0.17)$, $\alpha_2 = (0.63, 0.19)$, $\alpha_3 = (0.72, 0.20)$ be IFVs' time series. IFV scores' degrees time series $S(\alpha) = (0.41, 0.44, 0.52)$ and IFVs hesitancy degrees time series $\pi(\alpha) = (0.25, 0.18, 0.08)$. The result obtained by IFVs-GM(1,1)-PM is: $\widehat{\alpha}_4 = (0.681, 0.069)$.

3.4. Future Adjustment Coefficient

Definition 10. Let ψ be future adjustment coefficient in the range of $[0, 1]$. Assuming that $\alpha = (\mu_\alpha, \nu_\alpha)$ is the aggregated decision value in period of $[0, t]$, $\beta = (\mu_\beta, \nu_\beta)$ is the predicted decision value in the period $t + 1$. The formula of the comprehensive evaluation value is as follows:

$$\phi = (1 - \psi)\alpha \oplus \psi\beta = \left(1 - (1 - \mu_\alpha)^{(1-\psi)}, \nu_\alpha^{(1-\psi)}\right) \oplus \left(1 - (1 - \mu_\beta)^\psi, \nu_\beta^\psi\right) \\ \left(\left(1 - (1 - \mu_\alpha)^{(1-\psi)}\right) + \left(1 - (1 - \mu_\beta)^\psi\right) - \left(1 - (1 - \mu_\alpha)^{(1-\psi)}\right)\left(1 - (1 - \mu_\beta)^\psi\right), \nu_\alpha^{(1-\psi)} \nu_\beta^\psi\right) \cdot \quad (17)$$

Example 4. Suppose there are two sets of time series, as shown in Table 1.

Table 1. Time series A and B.

Time Series A	Scores Degree of A	Time Series B	Scores Degree of B
(0.4, 0.1)	0.3	(0.60, 0.10)	0.50
(0.5, 0.1)	0.4	(0.55, 0.10)	0.45
(0.6, 0.1)	0.5	(0.50, 0.10)	0.40

The time weight vector is $(0.2, 0.3, 0.5)$. By observing the score degree of the Time series A and B, it is obvious that A is in the rising period and B is in the recession period. The results shown in Table 2, which is calculated by DIFPGWA $_{\lambda(t)}$ operator IFVs-GM(1,1)-PM and Future Adjustment Coefficient. Let $\psi = 0.5$.

Table 2. Results of time series A and B.

Time Series	Type	Result	Scores Degree
A	Result of aggregation	(0.52, 0.10)	0.42
	Result of prediction	(0.50, 0.00)	0.50
	comprehensive evaluation	(0.51, 0.00)	0.51
B	Result of aggregation	(0.53, 0.10)	0.43
	Result of prediction	(0.48, 0.12)	0.36
	comprehensive evaluation	(0.506, 0.11)	0.396

The result of using the DIFPGWA $_{\lambda(t)}$ operator is $A \prec B$. The result obtained using the IFVs-GM(1,1)-PM is $A \succ B$. The comprehensive evaluation is $A \succ B$. Obviously, the results obtained using the IFVs-GM(1,1)-PM are more accurate. However, the advantage of the DIFPGWA $_{\lambda(t)}$ operator could not be negated. There are also errors in the IFVs-GM(1,1)-PM. Obviously, through the adjustment coefficient in the future, the results of the two can be integrated to get more accurate comprehensive evaluation value.

4. Decision-Making Steps

Description 1. In a MAGDM process, $E = \{e_k | k = 1, 2, \dots, p\}$ is a set of experts, $A = \{A_i | i = 1, 2, \dots, m\}$ is an alternative set, and $C = \{C_j | j = 1, 2, \dots, n\}$ is a set of attributes. Expert's evaluation values are expressed by IFVs. The weight vector of time under different period t_l is $\theta(t)$, and $\theta(t) = (\theta(t_1), \theta(t_2), \dots, \theta(t_g))^T$, $\theta(t_l) \geq 0$, $\sum_{l=1}^g \theta(t_l) = 1$, $l = 1, 2, \dots, g$. The attribute weight vector in period t_l is $w(t_l) = (w_1(t_l), w_2(t_l), \dots, w_n(t_l))^T$, $l = 1, 2, \dots, g$, where $w_j(t_l) \geq 0$, $\sum_{j=1}^n w_k = 1$. The expert weight vector in period t_l is $\lambda(t_l) = (\lambda_1(t_l), \lambda_2(t_l), \dots, \lambda_p(t_l))^T$, $l = 1, 2, \dots, g$, where $\lambda_k(t_l) \geq 0$, $\sum_{k=1}^p \lambda_k = 1$. The evaluation matrix made by expert e_k at time t_l is $A_{ijk}(t_l) = (\alpha_{ijk}(t_l))_{m \times n}$.

• **Step 1.** Determine expert weights and group decision matrix

Because each expert has his own field and expertise, experts may make irrational comments. If the experts have the same weight, the result will be unreasonable. On the other hand, if expert weights are set subjectively, especially in this multi-period situation, not only the workload is large, but also the deviation is large. Therefore, the following group mean evaluation matrix $A'_{ij}(t_l)$ is constructed. Assessments close to the average will have a greater weight, while assessments away from the average will have smaller weight [25]:

$$A'_{ij}(t_l) = (\alpha'_{ij}(t_l))_{m \times n} = (\mu'_{ij}(t_l), \nu'_{ij}(t_l)) = \left(\frac{1}{p} \sum_{k=1}^p \mu_{ijk}(t_l), \frac{1}{p} \sum_{k=1}^p \nu_{ijk}(t_l) \right). \quad (18)$$

The degree of similarity between $\alpha_{ijk}(t_l)$, $k = 1, 2, \dots, p$ and $\alpha'_{ij}(t_l)$ is defined as $S(\alpha_{ijk}(t_l), \alpha'_{ij}(t_l))$

$$S(\alpha_{ijk}(t_l), \alpha'_{ij}(t_l)) = 1 - \frac{d(\alpha_{ijk}(t_l), \alpha'_{ij}(t_l))}{\sum_{k=1}^p d(\alpha_{ijk}(t_l), \alpha'_{ij}(t_l))}, k = 1, 2, \dots, p, i = 1, 2, \dots, m, j = 1, 2, \dots, n, \quad (19)$$

where $\sum_{k=1}^p d(\alpha_{ijk}(t_l), \alpha'_{ij}(t_l)) \neq 0$.

If $\sum_{k=1}^p d(\alpha_{ijk}(t_l), \alpha'_{ij}(t_l)) = 0$, then $S(\alpha_{ijk}(t_l), \alpha'_{ij}(t_l)) = 1 - \frac{1}{p}$.

Then, the expert weight for $\lambda_{ijk}(t_l)$ is defined as:

$$\lambda_{ijk}(t_l) = \frac{S(\alpha_{ijk}(t_l), \alpha'_{ij}(t_l))}{\sum_{k=1}^p S(\alpha_{ijk}(t_l), \alpha'_{ij}(t_l))}, k = 1, 2, \dots, p, i = 1, 2, \dots, m, j = 1, 2, \dots, n. \quad (20)$$

In period t_l , the group decision matrix $B_{ij}(t_l) = (\beta_{ij}(t_l))_{m \times n}$ is obtained by aggregating the information of each expert with $IFPGWA_{\lambda_{ijk}(t_l)}$ operator, where

$$\begin{aligned} \beta_{ij}(t_l) &= IFPGWA_{\lambda_{ijk}(t_l)}(\alpha_{ij1}(t_l), \alpha_{ij2}(t_l), \dots, \alpha_{ijp}(t_l)) = \prod_{k=1}^p \alpha_{ijk}^{\zeta_k}(t_l) \\ &= \left(\prod_{k=1}^p \mu_{\alpha_{ijk}}^{\zeta_k}(t_l), 1 - \prod_{k=1}^p (1 - \nu_{\alpha_{ijk}}(t_l))^{\zeta_k} \right), \end{aligned} \quad (21)$$

where $T(\alpha_{ijk}(t_l)) = \sum_{k=1, k \neq x}^p \lambda_{ijk}(t_l) \sup(\alpha_{ijk}(t_l), \alpha_{ijx}(t_l))$, $\sup(\alpha_{ijk}(t_l), \alpha_{ijx}(t_l)) = 1 - \frac{d(\alpha_{ijk}(t_l), \alpha_{ijx}(t_l))}{\sum_{k=1, k \neq x}^p d(\alpha_{ijk}(t_l), \alpha_{ijx}(t_l))}$, $\zeta_k = \frac{\lambda_{ijk}(t_l)(1+T(\alpha_{ijk}(t_l)))}{\sum_{k=1}^p (1+T(\alpha_{ijk}(t_l)))\lambda_{ijk}(t_l)}$.

• **Step 2.** Determine attribute weights

In period t_l , for attribute C_j , specifies that $D_{ij}(w_j(t_l)) = \sum_{l=1}^m d(\beta_{ij}(t_l), \beta_{sj}(t_l))w_j(t_l)$ represents the total deviation.

Then, for all attributes $D(w_j(t_l)) = \sum_{j=1}^n \sum_{i=1}^m D_{ij}(w_j(t_l)) = \sum_{j=1}^n \sum_{i=1}^m \sum_{l=1}^m d(\beta_{ij}(t_l), \beta_{sj}(t_l))w_j(t_l)$ represents the total deviation. Then, the following model is constructed:

$$\begin{aligned} \max D(w_j(t_l)) &= \sum_{j=1}^n \sum_{i=1}^m D_{ij}(w_j(t_l)) = \sum_{j=1}^n \sum_{i=1}^m \sum_{l=1}^m d(\beta_{ij}(t_l), \beta_{sj}(t_l)) w_j(t_l) \\ \text{s.t. } \begin{cases} \sum_{j=1}^n (w_j(t_l))^2 = 1 \\ w_j(t_l) \geq 0, j = 1, 2, \dots, n \end{cases} \end{aligned} \quad (22)$$

According to Label (22), the results are shown below:

$$w_j(t_l) = \frac{\sum_{i=1}^m \sum_{l=1}^m d(\beta_{ij}(t_l), \beta_{sj}(t_l))}{\sqrt{\sum_{j=1}^n \left(\sum_{i=1}^m \sum_{l=1}^m d(\beta_{ij}(t_l), \beta_{sj}(t_l)) \right)^2}}. \quad (23)$$

Then, the results of standardization are as follows:

$$w_j(t_l) = \frac{\sum_{i=1}^m \sum_{l=1}^m d(\beta_{ij}(t_l), \beta_{sj}(t_l))}{\sum_{j=1}^n \sum_{i=1}^m \sum_{l=1}^m d(\beta_{ij}(t_l), \beta_{sj}(t_l))}. \quad (24)$$

Therefore, the attribute weight vector in period t_l is $w(t_l) = (w_1(t_l), w_2(t_l), \dots, w_n(t_l))^T$, $l = 1, 2, \dots, g$, where $w_j(t_l) \geq 0$, $\sum_{j=1}^n w_k = 1$.

- **Step 3.** Determine all alternative's comprehensive evaluation value by operator in each period

All the alternative's comprehensive evaluation value by operator $X(t_l) = (\chi_1(t_l), \chi_2(t_l), \dots, \chi_m(t_l))^T$ is obtained by aggregating the information of each attribute with $IFPGWA_{w_j(t_l)}$ operator in period t_l , $l = 1, 2, \dots, g$, where

$$\begin{aligned} \chi_i(t_l) &= IFPGWA_{w_j(t_l)}(\beta_{i1}(t_l), \beta_{i2}(t_l), \dots, \beta_{in}(t_l)) = \prod_{j=1}^n \beta_{ij}^{\zeta_j}(t_l) \\ &= \left(\prod_{j=1}^n \mu_{\beta_{ij}}^{\zeta_j}(t_l), 1 - \prod_{j=1}^n (1 - \nu_{\beta_{ij}}(t_l))^{\zeta_j} \right), \end{aligned} \quad (25)$$

where $T(\beta_{ij}(t_l)) = \sum_{j=1, j \neq s}^n w_j(t_l) Sup(\beta_{ij}(t_l), \beta_{sj}(t_l))$, $Sup(\beta_{ij}(t_l), \beta_{sj}(t_l)) = 1 - \frac{d(\beta_{ij}(t_l), \beta_{sj}(t_l))}{\sum_{j=1, j \neq s}^n d(\beta_{ij}(t_l), \beta_{sj}(t_l))}$,

$$\zeta_j = \frac{w_j(t_l)(1+T(\beta_{ij}(t_l)))}{\sum_{j=1}^n (1+T(\beta_{ij}(t_l)))w_j(t_l)}.$$

- **Step 4.** Determine time weights

In order to prevent the error caused by the weight setting of the supervisor and objective, the two methods should be used together. The time weight is obtained as follows [28]:

$$\begin{aligned} &\max \left[-\sum_{l=1}^g \theta(t_l) \ln(\theta(t_l)) \right] \\ \text{s.t. } \begin{cases} \delta = -\sum_{l=1}^g \left(\frac{g-l}{g-1} \theta(t_l) \right) \\ \sum_{l=1}^g \theta(t_l) = 1, \theta(t_l) \in [0, 1], g = 1, 2, \dots, p \end{cases} \end{aligned} \quad (26)$$

$\theta(t) = (\theta(t_1), \theta(t_2), \dots, \theta(t_g))^T$ are the weight of each period and $\theta(t_l) \geq 0$, $\sum_{l=1}^g \theta(t_l) = 1$.

δ represents subjective emphasis on recent data. $\delta \in [0, 1]$, if δ is closer to 1, there is more emphasis on recent data. If δ is closer to 0, there is more emphasis placed on forward data.

- **Step 5.** Determine all alternative's comprehensive evaluation value by operator

All alternative's comprehensive evaluation value by operator $Y = (Y_1, Y_2, \dots, Y_m)^T$ is obtained by aggregating the information of each attribute with $DIFPGWA_{\theta(t)}$ operator, where

$$Y_i = DIFPGWA_{\theta(t)}(\chi_i(t_1), \chi_i(t_2), \dots, \chi_i(t_g)) = \prod_{l=1}^g (\chi_i(t_l))^{\xi_l} \\ = \left(\prod_{l=1}^g \mu_{\chi_i}^{\xi_l}(t_l), 1 - \prod_{l=1}^g (1 - \nu_{\chi_i}(t_l))^{\xi_l} \right), \quad (27)$$

where $T(\chi_i(t_l)) = \sum_{l=1, l \neq v}^g \theta(t_l) Sup(\chi_i(t_l), \chi_i(t_v))$, $Sup(\chi_i(t_l), \chi_i(t_v)) = 1 - \frac{d(\chi_i(t_l), \chi_i(t_v))}{\sum_{l=1, l \neq v}^g d(\chi_i(t_l), \chi_i(t_v))}$,
 $\xi_l = \frac{\theta(t_l)(1+T(\chi_i(t_l)))}{\sum_{l=1}^g (1+T(\chi_i(t_l)))\theta(t_l)}.$

- **Step 6.** Predicting the $g + 1$ period comprehensive evaluation value

In Step 3, all alternative's comprehensive evaluation value is calculated by operator $X(t_l) = (\chi_1(t_l), \chi_2(t_l), \dots, \chi_m(t_l))^T$, $l = 1, 2, \dots, g$, which is $X_i = (\chi_i(t_1), \chi_i(t_2), \dots, \chi_i(t_g))$, $i = 1, 2, \dots, m$.

Then, the IFVs scores degrees time series $S(X_i) = (S(\chi_i(t_1)), S(\chi_i(t_2)), \dots, S(\chi_i(t_g)))$, $i = 1, 2, \dots, m$ and the IFV hesitancy degrees time series $\pi(X_i) = (\pi(\chi_i(t_1)), \pi(\chi_i(t_2)), \dots, \pi(\chi_i(t_g)))$, $i = 1, 2, \dots, m$ could be calculated.

According to $S(\chi_i(t_k)) + \widehat{a}z^{(1)}(k) = b$, the time response function of $S(X_i) = (S(\chi_i(t_1)), S(\chi_i(t_2)), \dots, S(\chi_i(t_g)))$ is $S^{(1)}(\widehat{\chi}_i(t_{g+1})) = (S(\chi_i(t_1)) - \frac{b}{a})e^{-ag} + \frac{b}{a}$, where $z^{(1)}(k) = \frac{S(\chi_i(t_k)) + S(\chi_i(t_{k-1}))}{2}$. Thus, the predictive value of IFVs scores degree can be calculated as follows:

$$S(\widehat{\chi}_i(t_{g+1})) = (1 - e^a) \left(S(\chi_i(t_1)) - \frac{b}{a} \right) e^{-ag}. \quad (28)$$

In addition, the predictive value of hesitancy degree is $\pi(\widehat{\chi}_i(t_{g+1})) = \max\{\pi(\chi_i(t_1)), \pi(\chi_i(t_2)), \dots, \pi(\chi_i(t_g))\}$.

According to the definition of IFVs, the IFVs-GM(1,1)-PM can be calculated as follows:

$$\widehat{\chi}_i(t_{g+1}) = (\widehat{\mu}_{\chi_i(t_{g+1})}, \widehat{\nu}_{\chi_i(t_{g+1})}) \\ = \left(\min \left(\max \left(\frac{1+S(\widehat{\chi}_i(t_{g+1}))-\pi(\widehat{\chi}_i(t_{g+1}))}{2}, 0 \right), 1 \right), \rightarrow \right. \\ \left. \rightarrow \min \left(\max \left(\frac{1-S(\widehat{\chi}_i(t_{g+1}))-\pi(\widehat{\chi}_i(t_{g+1}))}{2}, 0 \right), 1 \right) \right) \quad (29)$$

Then, all alternative's predictive comprehensive evaluation value are obtained as follows:

$$\widehat{Y} = (\widehat{\chi}_1(t_{g+1}), \widehat{\chi}_2(t_{g+1}), \dots, \widehat{\chi}_m(t_{g+1}))^T = (\widehat{Y}_1, \widehat{Y}_2, \dots, \widehat{Y}_m)^T. \quad (30)$$

- **Step 7.** Comprehensive evaluation

Let ψ be future adjustment coefficient in the range of $[0, 1]$. The formula for calculating the comprehensive evaluation values of all alternatives is as follows:

$$\bar{Y} = \left((1 - \psi)Y_1 \oplus \psi\hat{Y}_1, (1 - \psi)Y_2 \oplus \psi\hat{Y}_2, \dots, (1 - \psi)Y_m \oplus \psi\hat{Y}_m \right)^T = (\bar{Y}_1, \bar{Y}_2, \dots, \bar{Y}_m)^T. \quad (31)$$

All alternatives are sorted according to the ranking method of IFVs. If $\max(\bar{Y}_1, \bar{Y}_2, \dots, \bar{Y}_m) = \bar{Y}_i$, the i th alternative is best in period $g + 1$. If $\max(\bar{Y}_1, \dots, \bar{Y}_{i-1}, \bar{Y}_{i+1}, \dots, \bar{Y}_m) = \bar{Y}_j$, then the j th alternative is suboptimal in now.

5. Example

A company plans to recruit two employees. A total of five interviewees (A_1, A_2, A_3, A_4, A_5) received internship opportunities. The internship lasted for three months (t_1, t_2, t_3). At the end of each month, three experts (e_1, e_2, e_3) evaluated their performance during the month. These candidates (A_1, A_2, A_3, A_4, A_5) are evaluated in terms of practical ability C_1 , learning ability C_2 and innovation ability C_3 . Each expert uses the IFVs to evaluate five candidates in each period, as shown in Tables 3–5.

Table 3. The three experts' evaluation value in the first month t_1 .

Period t_1		A_1	A_2	A_3	A_4	A_5
Experts e_1	C_1	(0.56, 0.33)	(0.56, 0.16)	(0.78, 0.13)	(0.73, 0.27)	(0.88, 0.12)
	C_2	(0.57, 0.32)	(0.58, 0.16)	(0.74, 0.17)	(0.48, 0.52)	(0.82, 0.18)
	C_3	(0.57, 0.31)	(0.61, 0.17)	(0.83, 0.07)	(0.02, 0.40)	(0.36, 0.24)
Experts e_2	C_1	(0.63, 0.34)	(0.65, 0.16)	(0.82, 0.07)	(0.72, 0.14)	(0.77, 0.16)
	C_2	(0.63, 0.35)	(0.64, 0.16)	(0.81, 0.08)	(0.70, 0.15)	(0.58, 0.42)
	C_3	(0.57, 0.31)	(0.63, 0.17)	(0.78, 0.13)	(0.78, 0.22)	(0.16, 0.47)
Experts e_3	C_1	(0.66, 0.33)	(0.52, 0.17)	(0.80, 0.09)	(0.58, 0.42)	(0.70, 0.30)
	C_2	(0.63, 0.32)	(0.53, 0.16)	(0.79, 0.09)	(0.76, 0.11)	(0.28, 0.72)
	C_3	(0.63, 0.34)	(0.48, 0.19)	(0.77, 0.10)	(0.65, 0.16)	(0.19, 0.25)

Table 4. The three experts' evaluation value in the second month t_2 .

Period t_2		A_1	A_2	A_3	A_4	A_5
Experts e_1	C_1	(0.62, 0.33)	(0.51, 0.17)	(0.75, 0.12)	(0.50, 0.50)	(0.57, 0.11)
	C_2	(0.60, 0.35)	(0.51, 0.17)	(0.72, 0.14)	(0.85, 0.15)	(0.85, 0.15)
	C_3	(0.66, 0.32)	(0.62, 0.17)	(0.70, 0.14)	(0.74, 0.13)	(0.41, 0.06)
Experts e_2	C_1	(0.61, 0.32)	(0.61, 0.32)	(0.68, 0.15)	(0.40, 0.39)	(0.40, 0.60)
	C_2	(0.55, 0.18)	(0.65, 0.16)	(0.67, 0.16)	(0.74, 0.26)	(0.24, 0.31)
	C_3	(0.53, 0.16)	(0.61, 0.32)	(0.64, 0.16)	(0.87, 0.13)	(0.84, 0.16)
Experts e_3	C_1	(0.77, 0.10)	(0.71, 0.14)	(0.68, 0.15)	(0.44, 0.20)	(0.72, 0.28)
	C_2	(0.71, 0.14)	(0.74, 0.13)	(0.72, 0.13)	(0.51, 0.18)	(0.75, 0.25)
	C_3	(0.70, 0.16)	(0.77, 0.10)	(0.63, 0.17)	(0.70, 0.30)	(0.43, 0.39)

Table 5. The three experts' evaluation value in the third month t_3 .

Period t_3		A_1	A_2	A_3	A_4	A_5
Experts e_1	C_1	(0.62, 0.32)	(0.72, 0.14)	(0.56, 0.16)	(0.39, 0.61)	(0.91, 0.09)
	C_2	(0.72, 0.14)	(0.83, 0.07)	(0.53, 0.17)	(0.13, 0.33)	(0.15, 0.53)
	C_3	(0.68, 0.25)	(0.62, 0.32)	(0.54, 0.16)	(0.63, 0.21)	(0.07, 0.16)
Experts e_2	C_1	(0.62, 0.32)	(0.67, 0.25)	(0.55, 0.16)	(0.47, 0.11)	(0.87, 0.02)
	C_2	(0.67, 0.25)	(0.61, 0.32)	(0.49, 0.18)	(0.41, 0.49)	(0.35, 0.30)
	C_3	(0.61, 0.32)	(0.74, 0.17)	(0.40, 0.22)	(0.21, 0.44)	(0.92, 0.08)
Experts e_3	C_1	(0.82, 0.13)	(0.61, 0.32)	(0.38, 0.23)	(0.42, 0.50)	(0.51, 0.21)
	C_2	(0.83, 0.13)	(0.81, 0.13)	(0.36, 0.24)	(0.14, 0.59)	(0.81, 0.19)
	C_3	(0.62, 0.32)	(0.82, 0.13)	(0.37, 0.23)	(0.23, 0.65)	(0.26, 0.74)

- **Step 1.** Based on the above evaluation information given by the experts in each period, the group mean evaluation matrix could be constructed. Then, using Equations (18)–(20), to get the expert weights. The group decision matrices for each period are calculated by Equation (21), which is shown in Table 6.

Table 6. Group decision matrix in each month.

		A_1	A_2	A_3	A_4	A_5
Period t_1	C_1	(0.62, 0.33)	(0.57, 0.16)	(0.80, 0.09)	(0.68, 0.28)	(0.78, 0.19)
	C_2	(0.61, 0.33)	(0.58, 0.16)	(0.78, 0.11)	(0.65, 0.25)	(0.53, 0.47)
	C_3	(0.58, 0.31)	(0.58, 0.17)	(0.79, 0.10)	(0.29, 0.25)	(0.22, 0.32)
Period t_2	C_1	(0.65, 0.27)	(0.61, 0.21)	(0.70, 0.14)	(0.44, 0.37)	(0.56, 0.35)
	C_2	(0.62, 0.22)	(0.63, 0.15)	(0.70, 0.14)	(0.70, 0.20)	(0.58, 0.23)
	C_3	(0.63, 0.21)	(0.66, 0.20)	(0.65, 0.16)	(0.77, 0.19)	(0.52, 0.23)
Period t_3	C_1	(0.66, 0.27)	(0.66, 0.24)	(0.50, 0.18)	(0.42, 0.47)	(0.77, 0.10)
	C_2	(0.73, 0.17)	(0.75, 0.16)	(0.46, 0.19)	(0.19, 0.49)	(0.33, 0.36)
	C_3	(0.63, 0.30)	(0.73, 0.20)	(0.42, 0.21)	(0.30, 0.47)	(0.26, 0.43)

- **Step 2.** The closer the evaluation value of the attribute is, the smaller the weight should be given. Otherwise, the greater weight should be given. The attribute weights for each period are calculated by Equations (22)–(24), which are shown in Table 7.

Table 7. Attribute weights in each month.

	w_1	w_2	w_3
Period t_1	0.25888	0.29594	0.44518
Period t_2	0.43661	0.23824	0.32415
Period t_3	0.27301	0.37785	0.34914

- **Step 3.** The comprehensive evaluation values are calculated by Equation (25) in each period, which are shown in Table 8.

Table 8. Comprehensive evaluation value by operator in each month.

	A_1	A_2	A_3	A_4	A_5
Period t_1	(0.599, 0.322)	(0.576, 0.166)	(0.789, 0.099)	(0.458, 0.258)	(0.404, 0.342)
Period t_2	(0.635, 0.242)	(0.634, 0.193)	(0.683, 0.146)	(0.588, 0.276)	(0.554, 0.284)
Period t_3	(0.675, 0.244)	(0.716, 0.196)	(0.458, 0.193)	(0.279, 0.478)	(0.383, 0.324)

- **Step 4.** In determining the time weight, experts think the recent data is more important. The time weight vectors that are calculated by Equation (26) are $\theta(t) = (0.154, 0.292, 0.554)^T$, where $\delta = 0.3$.
- **Step 5.** The comprehensive evaluation values are calculated by Equation (27), which are shown in Table 9.

Table 9. Comprehensive evaluation value by operator in $\delta = 0.3$.

	Comprehensive Evaluation Value by Operator	Scores Degree	Accuracy Degree
A_1	(0.649, 0.257)	0.392	0.906
A_2	(0.665, 0.191)	0.474	0.856
A_3	(0.563, 0.164)	0.399	0.727
A_4	(0.379, 0.389)	−0.01	0.768
A_5	(0.433, 0.315)	0.118	0.748

- **Step 6.** The predict comprehensive evaluation values are calculated by Equations (28)–(30), which are shown in Table 10.

Table 10. Predict comprehensive evaluation value.

	<i>a</i>	<i>b</i>	Predict Comprehensive Evaluation Value	Scores Degree	Accuracy Degree
A_1	−0.09	0.35	(0.674, 0.204)	0.470	0.878
A_2	−0.17	0.33	(0.679, 0.064)	0.615	0.743
A_3	0.68	1.19	(0.392, 0.258)	0.134	0.650
A_4	9.04	3.53	(0.358, 0.358)	0.000	0.716
A_5	1.28	0.52	(0.363, 0.344)	0.019	0.707

- **Step 7.** The comprehensive evaluation values are calculated by Equation (31), which are shown in Table 11. Let $\psi = 0.3$.

Table 11. Comprehensive evaluation value in $\delta = 0.3$.

	Comprehensive Evaluation Value	Scores Degree	Accuracy Degree
A_1	(0.657, 0.240)	0.417	0.896
A_2	(0.669, 0.138)	0.532	0.807
A_3	(0.517, 0.188)	0.330	0.705
A_4	(0.373, 0.379)	−0.007	0.752
A_5	(0.413, 0.323)	0.089	0.736

All alternatives are sorted according to the ranking method of IFVs. When $\delta = 0.3$, the final sort result is $A_2 \succ A_1 \succ A_3 \succ A_5 \succ A_4$.

- **Step 8.** Result Analysis

Result Analysis

$\delta = 0.3$ shows that experts pay more attention to recent data and $\psi = 0.3$ shows that experts are hesitant about future forecast data. The result of aggregation by operators is $A_2 \succ A_3 \succ A_1 \succ A_5 \succ A_4$, and the prediction result is $A_2 \succ A_1 \succ A_3 \succ A_5 \succ A_4$. This shows that, as time goes on, experts are better at evaluating A_1 than A_3 . This means that A_1 is a growth candidate and A_3 is a negative candidate. Thus, companies should choose A_1 instead of A_3 . This also shows that A_2 , A_4 and A_5 are stable candidates. This means that performance of candidate A_2 is not only the best but also the most stable. Performances of A_4 and A_5 have been poor and do not recommend admission. The above analysis results are consistent with the comprehensive evaluation results obtained by future adjustment coefficients. The final result is $A_2 \succ A_1 \succ A_3 \succ A_5 \succ A_4$.

6. Comparative Analysis

- (1) The proposed decision method in this paper was applied to the Case of [34], where $\delta = 0.3$ and $\psi = 0.3$. The result is as shown in Table 12:

Table 12. The final sort results of the case in [34].

Decision Method	Weight	Result
Decision method in [34]	Weights used in [34]	$A_2 \succ A_3 \succ A_1 \succ A_4$
the proposed DIFPGWA operator	Weights used in [34]	$A_2 \succ A_3 \succ A_1 \succ A_4$
The TFVs-GM(1,1)-PM	Weights used in [34]	$A_2 \succ A_1 \succ A_3 \succ A_4$
Comprehensive evaluation results	Weights used in [34]	$A_2 \succ A_3 \succ A_1 \succ A_4$
the proposed DIFPGWA operator	Weights are unknown	$A_2 \succ A_3 \succ A_4 \succ A_1$
The TFVs-GM(1,1)-PM	Weights are unknown	$A_2 \succ A_1 \succ A_4 \succ A_3$
Comprehensive evaluation results	Weights are unknown	$A_2 \succ A_3 \succ A_4 \succ A_1$

The original result is $A_2 \succ A_3 \succ A_1 \succ A_4$. If the weight information is unchanged, the result of using the proposed DIFPGWA operator is also $A_2 \succ A_3 \succ A_1 \succ A_4$. The prediction result is $A_2 \succ A_1 \succ A_3 \succ A_4$. The above results show that A_2 is not only the most stable but also the best. The performance of A_1 has been getting better, but the performance of A_3 has deteriorated. However, the comprehensive evaluation results are consistent with the original text, which shows that although the evaluation value of A_1 shows an upward trend, but it can not completely exceed A_3 . If the weights are unknown, the result of using the proposed DIFPGWA operator is $A_2 \succ A_3 \succ A_4 \succ A_1$. The prediction result is $A_2 \succ A_1 \succ A_4 \succ A_3$. The final result is $A_2 \succ A_3 \succ A_4 \succ A_1$. This result is slightly different from before because the weights are different. However, the same place is that A_2 is the best and most stable.

- (2) The proposed decision method in this paper was applied to the case of [35], where $\delta = 0.3$ and $\psi = 0.3$. The result is as shown in Table 13:

Table 13. The final sort results of the case in [35].

Decision Method	Weight	Result
Decision method in [35]	Weights used in [35]	$A_4 \succ A_3 \succ A_1 \succ A_2$
the proposed DIFPGWA operator	Weights used in [35]	$A_4 \succ A_3 \succ A_1 \succ A_2$
The TFVs-GM(1,1)-PM	Weights used in [35]	$A_1 \succ A_3 \succ A_4 \succ A_2$
Comprehensive evaluation results	Weights used in [35]	$A_1 \succ A_3 \succ A_4 \succ A_2$
the proposed DIFPGWA operator	Weights are unknown	$A_3 \succ A_4 \succ A_1 \succ A_2$
The TFVs-GM(1,1)-PM	Weights are unknown	$A_3 \succ A_1 \succ A_4 \succ A_2$
Comprehensive evaluation results	Weights are unknown	$A_3 \succ A_1 \succ A_4 \succ A_2$

The original result is $A_4 \succ A_3 \succ A_1 \succ A_2$. If the weight information is unchanged, the result of using the proposed DIFPGWA operator is also $A_4 \succ A_3 \succ A_1 \succ A_2$. The prediction result is $A_1 \succ A_3 \succ A_4 \succ A_2$. The comprehensive evaluation result is $A_1 \succ A_3 \succ A_4 \succ A_2$. A_1 and A_3 rose the fastest, and A_1 exceeded A_3 . A_4 changed from first to fourth. If the weights are unknown, the result of using the proposed DIFPGWA operator is $A_3 \succ A_4 \succ A_1 \succ A_2$. The prediction result is $A_3 \succ A_1 \succ A_4 \succ A_2$. The comprehensive evaluation result is $A_3 \succ A_1 \succ A_4 \succ A_2$. This result is slightly different from before because the weights are different. When weights are unknown, both A_3 and A_1 perform better than A_4 . To sum up, the performance of A_3 and A_1 is not only the best but has been in a rising stage.

- (3) The proposed decision method in this paper was applied to the case of [36], where $\delta = 0.3$ and $\psi = 0.3$. The result is as shown in Table 14.

Table 14. The final sort results of the case in [36].

Decision Method	Weight	Result
Decision method in [36]	Weights used in [36]	$A_1 \succ A_2 \succ A_3$
the proposed DIFPGWA operator	Weights used in [36]	$A_1 \succ A_2 \succ A_3$
The TFVs-GM(1,1)-PM	Weights used in [36]	$A_2 \succ A_1 \succ A_3$
Comprehensive evaluation results	Weights used in [36]	$A_2 \succ A_1 \succ A_3$
the proposed DIFPGWA operator	Weights are unknown	$A_1 \succ A_2 \succ A_3$
The TFVs-GM(1,1)-PM	Weights are unknown	$A_1 \succ A_2 \succ A_3$
Comprehensive evaluation results	Weights are unknown	$A_1 \succ A_2 \succ A_3$

The original result is $A_1 \succ A_2 \succ A_3$. If the weight information is unchanged, the result of using the proposed DIFPGWA operator is $A_1 \succ A_2 \succ A_3$. The prediction result is $A_2 \succ A_1 \succ A_3$. The comprehensive evaluation result is $A_2 \succ A_1 \succ A_3$. This shows that the rise of A_2 exceeds A_1 , and the problem is not considered in the original text. If the weights are unknown, the result of using the

proposed DIFPGWA operator is $A_1 \succ A_2 \succ A_3$. The prediction result is $A_1 \succ A_2 \succ A_3$. The above results are basically the same—that A_1 is the best, A_2 is the second, and A_3 is the worst.

After the above comparative analysis, it is obvious that the decision method proposed in this paper is not only effective but also more advantageous.

7. Conclusions

DIF-MAGDM is an important branch of modern decision theory. Because of this, the DIFPGWA operator is proposed. The operator makes up for the shortcomings of the previous decision-making method and better reflects the support relationship between attributes, between substitutions, and between periods. The DIFPGWA operator can capture the subtle details of the comprehensive evaluation value that the decision maker needs to reflect. However, judging only by aggregating evaluation information from past to present is inaccurate. Therefore, the TFVs-GM(1,1)-PM is improved in this paper. The previous IFVs-GM(1,1)-PM predicts that the membership and membership of IFVs may be out of range. Our improved IFVs-GM(1,1)-PM limits membership and non-affiliation within [0,1]. The DIFPGWA operator and the IFVs-GM(1,1)-PM complement each other. The future adjustment coefficient is proposed, which combines the two results. This method can help decision-makers to make more accurate judgments. It is very important to determine the weights in DIF-MAGDM. In this article, attribute weights, expert weights, and time weights are all unknown. This paper adopts the idea of deriving expert weight from the evaluation value and improves the previous method. At each period, therefore, it is generally agreed that the more deviated from the common cognition, the smaller the expert weight should be, and, the closer to the common cognition, the greater the expert weight should be. However, if the experts evaluate the same value, the sum of the deviations will be zero. Therefore, it will lead to no solution to the weight of experts. Our improved approach avoids this error. For attribute weights, the decision maker can determine the weight of each period's attributes based on the idea of maximum deviation. That is, the closer the evaluation value of the attribute is, the smaller the weight that should be given. Otherwise, it should be given greater weight. Moreover, due to different evaluations of experts in different periods, attribute weights should also be different. For time weights, the decision maker can use subjective and objective methods to determine the time weights. Subjective can fully consider the knowledge and experience of experts, treat things with their own feelings, and draw conclusions. Objective can make full use of all the time information. Therefore, the combination of subjectivity and objectivity can give more reasonable time weights. The decision method proposed in this paper is applied to the selection of the company's intern employees. After three months of internship, two excellent and up-scaling employees were selected. Not only that, this article also applies the proposed decision method to other cases. The final comparative analysis shows that the decision-making method is more effective and superior. In the future, we will first apply the decision method proposed in this paper to practical cases, and second apply this method to more fields.

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