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Symbiotic Evolution of Rural Settlements and Traditional Agricultural Water Conservancy Facilities Based on the Lotka-Volterra Model

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Abstract

Agricultural water conservancy facilities serve as the foundation and lifeline for the development of sustainable agriculture in a nation. In response to the evolving natural environment and food security demands, ancient agricultural water conservancy facilities coexist with rural areas, establishing a harmonious and sustainable symbiotic coordination mechanism. This study constructs a theoretical framework for the symbiosis between rural areas and ancient agricultural water conservancy facilities based on symbiotic theory. The Lotka-Volterra model is employed to validate the symbiotic evolutionary relationship between rural areas and ancient agricultural water conservancy facilities and to explore the mechanistic patterns of their symbiotic evolution process. Additionally, numerical simulations are conducted using MATLAB software to investigate the optimal solutions for the symbiotic relationship model between rural areas and ancient agricultural water conservancy facilities. The research findings indicate that: (1) The symbiotic model between rural areas and ancient agricultural water conservancy facilities undergoes evolutionary stages, including commensalism, parasitism, competitive symbiosis, and asymmetric mutualism. (2) The evolutionary pattern of their symbiotic relationship is influenced by the symbiotic coefficients of their interactions. (3) The results demonstrate that symmetric mutualism represents the most stable and effective symbiotic model. Therefore, governments and relevant authorities should adopt appropriate measures to guide the evolution of rural areas and agricultural water conservancy facilities toward the symmetric mutualism model. This approach provides a scientific basis for the future development strategies of rural areas and agricultural water conservancy facilities.

Keywords: Lotka-Volterra model; symbiotic model; rural areas; agricultural water conservancy facilities

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1. Introduction

Agriculture is critical to human survival and development and plays a pivotal role in ensuring national food security and promoting sustainable rural development. As global changes driven by human activities and population growth continue, global demand for food is steadily increasing, making food security one of the most significant issues of the 21st century [1]. However, the development of agriculture is often constrained by multiple factors, including extreme climate change, irrigation challenges, soil cultivation conditions, political influences, and various external environmental factors [2,3]. In addition, extreme

climate change, marked by rising temperatures and decreasing water availability, has led to a gradual increase in drought frequency in rural areas, thereby exerting adverse effects on sustainable agriculture and food security [4–6]. Therefore, how to promote the sustainable and healthy development of rural areas has become one of the urgent challenges that must be addressed.

Agricultural water conservancy infrastructure serves as a vital tool for agricultural production in rural areas, providing essential physical support for mitigating natural risks, controlling floods and waterlogging, reducing and preventing disasters, and optimizing the utilization of water resources [7]. Since the Bronze Age, humans have consciously undertaken the construction of water conservancy facilities to secure water resources for local agricultural production [8]. However, with the advancement of modern technology, mechanized water conservancy infrastructure has gradually replaced traditional agricultural water systems, many of which are now facing severe degradation, abandonment, and neglect [9]. In the face of current food security demands, traditional agricultural water conservancy facilities offer distinct advantages over mechanized systems, including lower construction and maintenance costs, longer service life, and broader irrigation coverage. They are also effective in addressing small-scale floods and droughts through water regulation and storage, thereby supporting the sustainable development of rural areas [10]. In China, effectively irrigated farmland accounts for 54% of the country's total arable land, yet it produces over 75% of the nation's grain output and more than 90% of its economic crops [11]. Therefore, exploring the evolutionary relationship between traditional agricultural water conservancy facilities and rural agricultural irrigation is of great significance for building sustainable rural areas in the new era and ensuring national food security.

The concept of "symbiosis" originates from the field of biology and was first proposed by German biologist Anton de Bary [12], It refers to a phenomenon in which organisms of different species coexist and co-evolve through mutual dependence and interaction, surviving together in a state of interrelated influence [13]. Goff proposed that the theory of symbiosis includes three levels: parasitism, symbiosis, and mutualism [13]. The Lotka-Volterra symbiotic model was proposed in the 1920s by scholars Lotka and Volterra. It laid the theoretical foundation for simulating competitive and cooperative dynamics between populations in the field of biology. In recent years, this model has been widely applied in the field of innovative ecological management [14]. In addition, the Lotka-Volterra model has also been applied to research on the evolution of urban innovation development levels, the symbiotic relationship between tourism dynamics and ecology, and the interaction between scientists and social media platforms [14–16]. Moreover, agricultural water conservancy facilities have long been a hot topic of research, with scholars employing advanced methods to assess the operational status and environmental performance of these facilities, thereby ensuring effective water supply options for agricultural regions [12,17]. Researchers have further explored the intrinsic relationship and mechanisms between agricultural water conservancy facilities, environmental changes, and rural population migration [18]. With the increasing frequency of extreme climate events globally, researchers analyze the negative impact of climate change, which leads to reduced water resources and rising drought rates in rural areas, on agricultural irrigation, and propose corresponding countermeasures [19]. In addition, the management and distribution of agricultural water conservancy facilities have also led to numerous social issues. In Peru, the root of these issues lies in the long-standing hegemonic racial and class hierarchies, which have created irreconcilable conflicts between the upper bureaucratic institutions and farmers. Researchers have proposed the establishment of water management agencies led by water conservancy engineers to address these social issues [20]. Therefore, studying ancient agricultural water conservancy facilities holds significant value for current agricultural development and sustainability.

Overall, in recent years, research on agricultural water conservancy facilities has largely focused on individual issues related to the facilities themselves and their associated social impacts. However, the intrinsic connection and co-development between traditional agricultural water conservancy facilities and rural areas have been largely overlooked and have not received sufficient academic attention. Traditional agricultural water conservancy facilities deliver water for rural agricultural irrigation, ensuring grain production and, to a certain extent, fostering the growth of rural populations and economies. In turn, rural areas provide regular maintenance, repair, and management for these facilities. Thus, it is evident that traditional agricultural water conservancy facilities and rural areas coexist in a symbiotic relationship, mutually influencing each other and becoming indispensable components of one another. How to coordinate the future development between traditional agricultural water conservancy facilities and rural areas to achieve maximum benefits has become an urgent issue to address. Therefore, it is essential to clearly understand the long-term evolutionary mechanisms of their relationship and to seek the optimal development model for their interaction, in order to promote the sustainable and healthy development of both. However, current research on the relationship between rural areas and traditional agricultural water conservancy facilities remains insufficient.

In this study, we take the symbiotic model of rural areas and traditional agricultural water conservancy facilities as the entry point to examine their interrelationship and value. The aim of this research is to analyze the symbiosis and evolution of rural areas and traditional agricultural water conservancy infrastructure. First, an evolutionary dynamic model of rural areas and traditional agricultural water conservancy facilities is constructed, and simulation analysis is conducted via MATLAB to verify the scientific validity and feasibility of their symbiotic relationship. Based on the results of the numerical simulations, we propose optimization paths and policy recommendations. This study has practical importance because it contributes to the rational utilization and maintenance of traditional agricultural water conservancy infrastructure, enhances agricultural production efficiency, improves local agricultural output and ecological conditions, and promotes the sustainable development of rural areas.

2. Materials and Methods

2.1. Study Area

We selected Anfengtang as an empirical example of traditional agricultural water conservancy facilities, while the rural areas that benefit from its irrigation coverage serve as the empirical subjects representing rural areas (Figure 1). Anfengtang, which was first constructed during the mid-Spring and Autumn period in China (613 BCE–591 BCE), was the first large-scale water storage and irrigation project in Chinese history. It is also recognized as one of the four major ancient water conservancy projects in China [21]. Anfengtang Town, Yankou Town, Banqiao Town, and Baoyi Town are rural areas within the irrigation coverage area of Anfengtang. These towns are located in one of China's key production regions for rice and wheat and are direct beneficiaries of water conservancy irrigation. Anfengtang was constructed 300 years earlier than the world cultural heritage site of Dujiangyan. It has a perimeter of 24 km, covers an area of 34 square kilometers, and has a water storage capacity of 100 million cubic meters. It has made remarkable contributions to local agricultural development and the ecological environment. In 2015, it was recognized as a "world irrigation project heritage" site and listed among China's important agricultural heritage systems. The International Commission on Irrigation and Drainage identified Shaobei (another name for Anfengtang) as "a global model of sustainable irrigation engineering." Therefore, selecting Anfengtang as an empirical case of traditional agricultural water conservancy facilities holds significant research value [22].

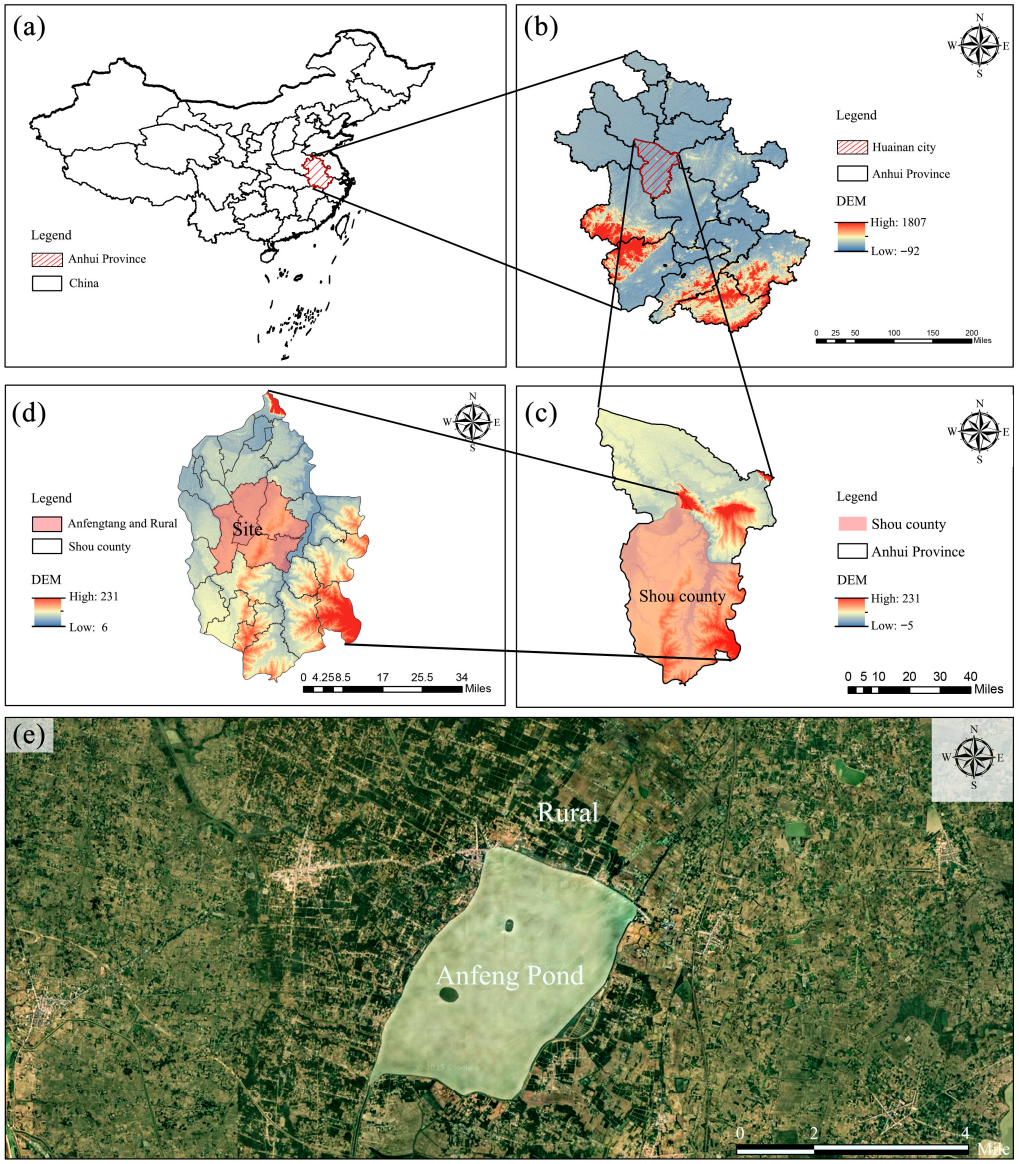


Figure 1. Location of the study area. (**a**) location of Anhui Province in southern China, (**b**) location of Huainan within Anhui Province, (**c**) location of shou county within Huainan city, (**d**) location of rural areas and anfengtang, and (**e**) spatial layout of anfengtang.

Anfengtang has a long history and continues to supply water to the surrounding farmlands, maintaining its irrigation function to this day. Field investigations have confirmed the existence of numerous historical relics at the site, including the Chen Clan Ancestral Hall, the Sun Gong Shrine, the ancient site of Anfeng County (dating back to the Hongwu period of the Ming Dynasty), the ancient sluice gate, and the ruins of an ancient waterway. Among these, the Sun Gong Shrine houses 19 stone inscriptions that provide detailed records of Anfengtang's geographical location, water source distribution, and repair works conducted through successive dynasties. The ancient sluice gate remains operational today, continuing to irrigate the surrounding farmland. The ancient waterway site once featured several stone sculptures of horses, turtles, and sheep, which are now buried in the riverbank silt. In addition, a stele pavilion stands beside Anfengtang, documenting its restoration and maintenance history (Figure 2).



Figure 2. Environmental and historical elements of Anfengtang: (**a**) Chen clan ancestral hall from the Qing dynasty, (**b**) sun gong shrine, (**c**) stone stele, (**d**) anfeng ancient county site, (**e**) ancient water sluice gate, (**f**) tangbei stele pavilion, (**g**) aerial view of Anfengtang and the surrounding village, and (**h**) stone tablet inscribed with the characters "quebei".

2.2. Study Framework

This study uses the Lotka–Volterra model to analyze the symbiotic evolution process between rural areas and Anfengtang and is grounded in socio-ecological symbiosis theory. The concept of symbiosis was first introduced into biology by German mycologist Heinrich Anton de Bary in 1879 to describe various forms of mutualistic, commensal, and parasitic relationships between species [12]. As ecological theories gained increasing application in the social sciences, symbiosis theory was introduced into this field [23]. On this basis, Yuan Chunqing proposed a social symbiosis system composed of "symbiotic units, symbiotic patterns, and symbiotic environments" [24], providing a theoretical basis for applying ecological symbiosis concepts to human-land relationships and historical water conservancy research. In addition, the Lotka–Volterra model is widely applied across ecology, sociology, economics, and other fields to describe population dynamics and distribution patterns of species or ecosystems [14]. Building on these foundations, this study uses the Lotka–Volterra model to analyze the co-evolutionary process between rural villages and Anfengtang. Over time, the evolutionary process of villages and Anfengtang shows population growth constrained by density, ultimately tending toward an equilibrium value consistent with the Logistic growth model. This pattern has been extensively applied and validated in population ecology and sociology research [16,25]. This study constructs a symbiotic model between villages and Anfengtang to validate their symbiotic relationship. Simulation analysis using MATLAB enables quantitative research on the symbiotic evolutionary process.

The experimental steps of this study are as follows. First, the symbiotic system is analyzed and a theoretical framework is constructed. By examining historical documents and on-site stone inscriptions related to Anfengtang, such as the Huainanzi, Shuijingzhu, Song Shi, and Anfengtang Gazetteer, Evolutionary process of symbiosis models between the rural areas and Anfengtang are identified. And we analyze the three elements of symbiosis between rural areas and Anfengtang, encompassing symbiotic units, symbiotic environments, and symbiotic model. Based on these materials, a theoretical framework for the symbiosis between villages and Anfengtang is established, clarifying the symbiotic patterns between the two across different historical periods.

Second, a symbiotic model between rural areas and Anfengtang is constructed. The Lotka-Volterra model is employed to establish a mathematical symbiosis model between rural areas and Anfengtang conservancy facilities. Based on symbiosis theory, the model equations are derived to conduct stability analysis and equilibrium point determination.

The equilibrium points (S1*, S2*) of the symbiosis model are calculated, enabling the computation of eigenvalues and eigenvectors of the Jacobian matrix at these points to confirm the model's equilibrium and stability. If all eigenvalues are <0, the equilibrium point is stable; otherwise, it is unstable. The presence of positive eigenvalues indicates that the rural-hydraulic relationship during that historical period was unstable and susceptible to imbalance due to external disturbances. This analysis facilitates the transition from qualitative historical periodization to quantitative mathematical modeling.

Finally, numerical simulations and scenario analyses are conducted using MATLAB. The constructed symbiotic model is imported into MATLAB to simulate the co-evolutionary patterns between villages and Anfengtang under different historical and management scenarios. Simulation analyses involve inputting various parameter conditions (ranges of symbiosis coefficients) into MATLAB. Ultimately, the simulation results help evaluate the consequences of different decisions, thereby identifying the optimal symbiotic patterns between villages and Anfengtang (Figure 3).

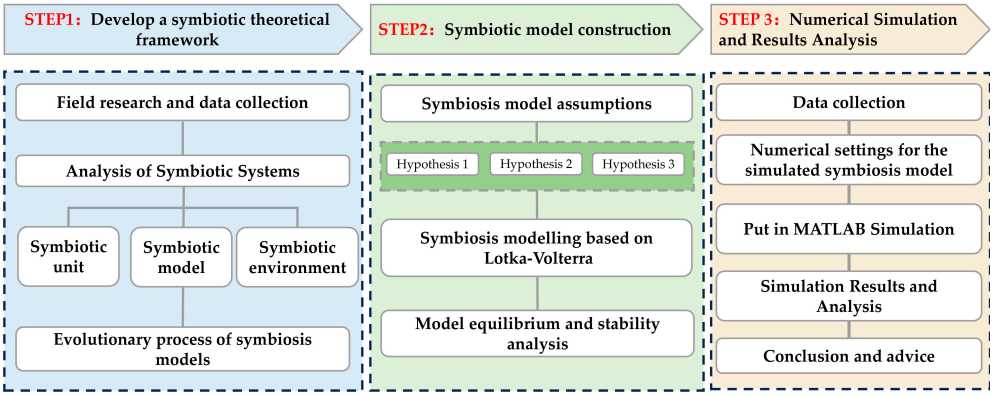


Figure 3. Research framework diagram.

2.3. Symbiotic Theory Framework Construction

2.3.1. Symbiotic Unit

The symbiotic unit is the basic element of energy production and exchange that constitutes the symbiotic relationship and serves as the foundation for its existence. Therefore, there must be an inherent connection between the symbiotic units in order to form a symbiotic relationship [16]. In this study, the symbiotic units are rural areas and anfengtang. The compatibility of their physical parameters represents their mutual existence and interaction. Water is the essential physical parameter for compatibility between rural areas and Anfengtang. The traditional agricultural water conservancy facility, Anfengtang, relies on water resources for its operation and maintenance, whereas rural areas depend on Anfengtang to supply water for irrigation. The sluice, as the medium of symbiosis, transfers water from Anfengtang to the rural farmlands to ensure the production of crops in rural areas.

2.3.2. Symbiotic Model

The symbiotic model refers to the symbiotic relationship between symbiotic units, which represents the interaction or combination of these units [16]. The symbiotic model can be classified into five types of relationships, include Independent coexistence pattern, commensalism pattern, parasitic coexistence pattern, competitive coexistence pattern, asymmetric mutualism pattern [16,21]. The two symbiotic units, rural areas and traditional agricultural water conservancy facilities, are in a mutually influential and interdependent relationship. Anfengtang has played a significant role in the agricultural development of Anfengtang Town and the surrounding villages by providing essential irrigation and flood control functions. The continuous development of rural areas has also contributed

to the protection and maintenance of Anfengtang. Parasitic coexistence pattern refers to a relationship in which one party benefits while the other party is harmed. This model occurred during the Ming Dynasty in China. As the population continued to grow, farmers began to encroach on water bodies for agricultural land. This resulted in compression of the water space of Anfengtang, which damaged both the aquatic and ecological spaces. However, the expansion of land in rural areas led to an increase in agricultural production and yield, which enabled the development of the countryside and benefited rural areas. The asymmetrical mutualistic symbiotic model refers to a relationship where both parties benefit but one parties gains more than the other. In this case, Anfengtang serves the functions of water regulation and drought alleviation. It stores rainwater during periods of abundant precipitation and releases water through the sluices during dry periods. The entire irrigation district, including rural areas, autonomously manages and distributes water resources and provides ecosystem services to rural areas and surrounding regions, thus ensuring the ecological security of the rural areas. Moreover, Anfengtang provides a favorable ecological environment for the growth of aquatic plants and animals, which protects the biodiversity of the region. In terms of agricultural production, Anfengtang ensures a stable water supply for rural agricultural activities that provides irrigation for food crops, thereby supporting crop growth and yield and promoting the sustainable and healthy development of agriculture. This study draws on three types of interactions between populations in biological systems: positive (+), no effect (0), and negative (−) interactions [26]. By referencing the concept of population coexistence in biology, the symbiotic model of rural areas and traditional agricultural water conservancy facilities can be inferred (Figure 4).

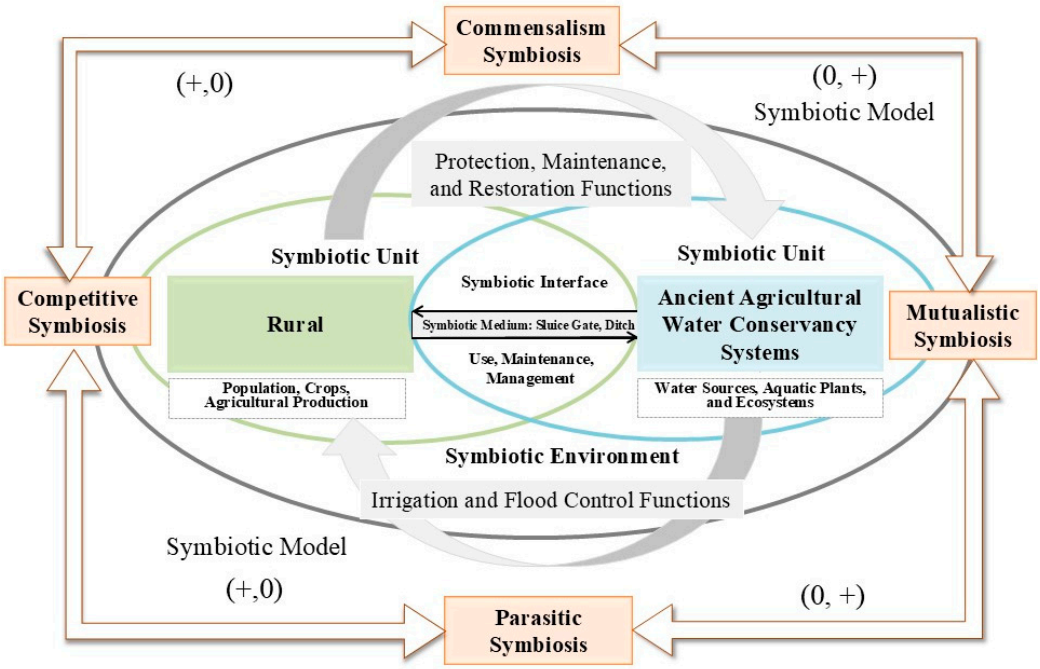


Figure 4. Symbiotic Evolution Process Diagram.

2.3.3. Symbiotic Environment

The symbiotic environment refers to the external conditions that facilitate the existence and development of the symbiotic units. It encompasses all factors, other than the symbiotic units themselves, that collectively constitute the environment in which symbiosis occurs [16]. The symbiotic environment between rural areas and traditional agricultural water conservancy facilities can be analyzed from several aspects. From the perspective of the physical environment, rural areas and traditional agricultural water conservancy

facilities occupy a specific geographical space together. Their symbiotic environment includes physical factors such as air, water, and soil. The symbiotic environment must meet the survival needs of both parties, including physical conditions such as water quality, water quantity, soil fertility, and climate. From an ecological perspective, rural areas and traditional agricultural water conservancy facilities are essential components of the ecosystem. Their symbiotic environment must consider their impact and the roles they play within the ecosystem. For example, the construction and maintenance of traditional agricultural water conservancy facilities may affect the surrounding natural environment, and the ecological practices and management of rural areas may also influence the surrounding ecosystem. From a social environment perspective, rural areas and traditional agricultural water conservancy facilities have coexisted in mutual dependence. Since the Spring and Autumn Period in ancient China, these facilities have been constructed and maintained to the present day. They not only carry profound historical memories but also embody the accumulated repair expertise of rural residents, gradually becoming integral components of local social culture. The symbiotic relationship between rural areas and traditional agricultural water conservancy facilities is built upon shared historical contexts, intangible cultural heritage, and social identity (Figure 5).

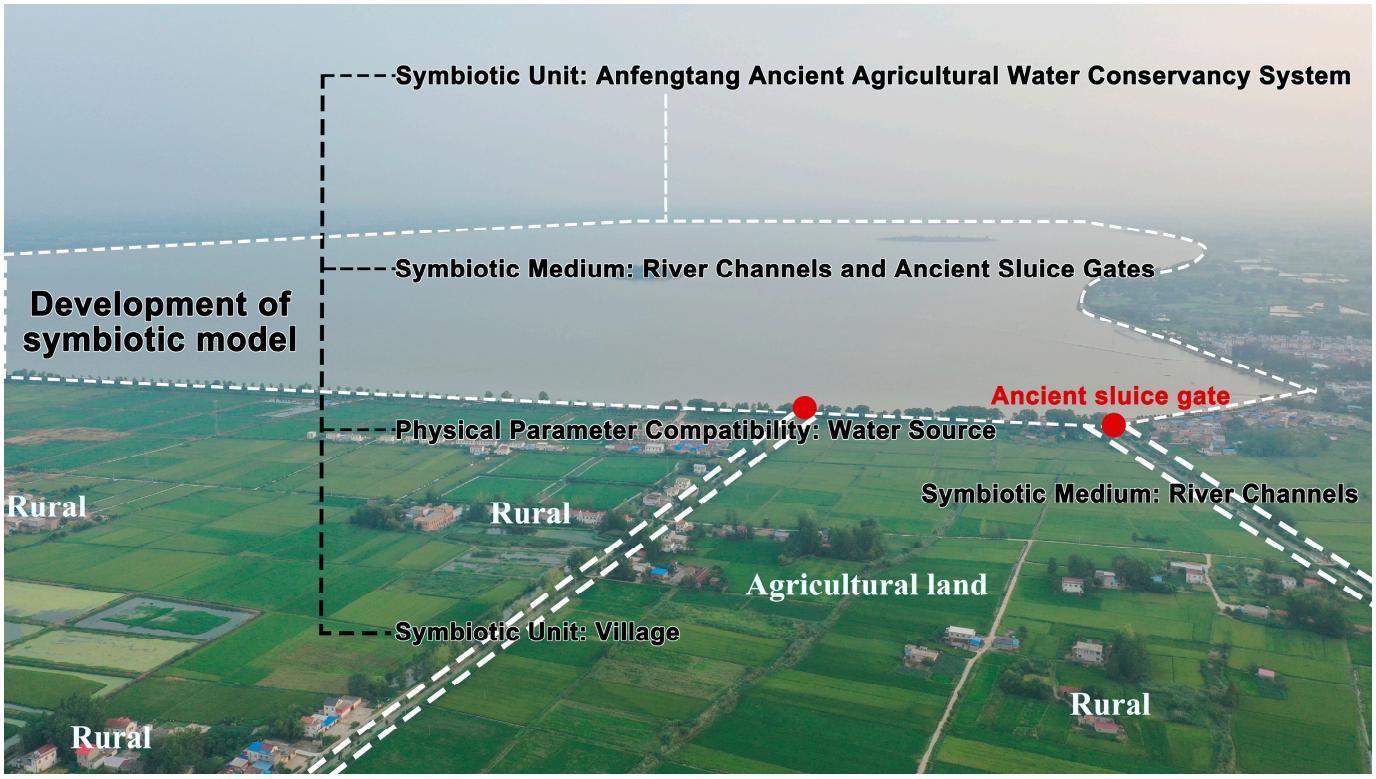


Figure 5. Symbiotic environment development model.

2.3.4. Evolutionary Process of Symbiosis Models

Anfengtang, known as Quebei in ancient China, was constructed between 613 and 591 BC (during the Spring and Autumn Period) and has a history of more than 2600 years [11]. It is regarded as one of the earliest agricultural water conservancy projects in China [21]. According to the *Huainanzi*, "Sun Shuao diverted the Qisi River to irrigate the fields of Yulou. King Zhuang of Chu thus recognized his capability and appointed him as prime minister [27]." This record indicates that the Anfengtang (also known as Quebei) was constructed under the direction of Sun Shuao, the prime minister of the State of Chu. The Shuijing (Feishui zhu) further records that "the reservoir has five gates, which regulate the flow of water, with the northwest being the Xiangmen Reservoir [28]." This shows

that Sun Shuao made use of the south high, north low terrain to divert water into Liu'an, built embankments and five sluice gates, and completed Anfengtang. The Book of Han: Biographies of Model Officials also states that "Sun Shuo'ao, Lord of Chu, constructed the Anfengtang, irrigating tens of thousands of acres of farmland [29]." This historical account confirms that the Anfengtang irrigated nearly ten thousand acres of fertile fields, thereby driving the development of surrounding villages. There are no historical records of the repairs or maintenance of the Anfengtang from the Spring and Autumn period to the Western Han Dynasty. The Liang Shu (Book of Liang), in the Biography of Xiahou Kui, records: "Xiahou Kui led his army to build a dam in Cangling, irrigating over a thousand hectares of farmland and producing more than one million shi of grain annually [30]." This document indicates that after the construction of the Anfengtang, the surrounding farmland benefited from extensive irrigation, resulting in increased agricultural yields and promoting the economic development of Shou County [31,32]. Therefore, it can be analyzed that during this period, the benefits derived from the Anfengtang were greater than those derived from the villages. The coefficient of influence of the Anfengtang on the villages was higher than the coefficient of mutual influence between the villages and the Anfengtang. During this period, the symbiotic relationship between the two followed a commensalism symbiosis model.

During the Song Dynasty, local landlords began to occupy the Anfengtang. According to the Song Shi, in Volume 291, Biography of Li Ruogu: "He was promoted to an official position in Shouzhou. Many local powerful families divided and occupied the Quebei [33]." This is the first recorded instance of villagers occupying the Anfengtang. During the Ming and Qing dynasties, the phenomenon of peasants and landlords occupying ponds for farming became increasingly severe. The stele inscription "Record of the Reconstruction of Sha Pond by Lord Wei" by Jin Xie in 1483 AD states: "As generations changed and responsibilities shifted, no one was specifically tasked with maintenance. Water lost its original course, and the pond gradually fell into ruin. Residents took advantage of this, encroaching day by day and month by month, converting it into private property." This historical inscription, currently housed within the Anfengtang Sun Clan Ancestral Hall, documents how during the Ming Dynasty, the absence of local governmental oversight led to widespread occupation of Anfengtang by peasants for agricultural use. The Records of the Quebei states: "At that time, Anfengtang had been abandoned, and the Quebei lacked a dedicated administrator. Malicious people from Liu'an built dams on the upper reaches of Zhu Huige and Li Ziwan in the middle reaches, diverting water for their own benefit. In the southern part of the reservoir, the stubborn villagers, including Dong Yuan, occupied the land, building houses. The reservoir was gradually destroyed [34]." This document further supports the phenomenon of villagers occupying and damaging the Anfengtang (Quebei) for personal gain. In the late Qing Dynasty, the Annals of Anfengtang recorded that the area around the Anfengtang had become "crowded with people, and the silted land within the reservoir was all cultivated into farmland, while the areas turned into pastures for livestock [35]." This document highlights the increase in population and the changes in land use around the Anfengtang, where silted land was reclaimed for agriculture, and the depressions were transformed into grazing areas; therefore, this document indicates that the encroachment on Anfengtang during the Qing Dynasty became even more severe. In conclusion, it can be analyzed that during the Northern Song to Ming and Qing Dynasties, a symbiotic conflict emerged between the villages and the ancient agricultural irrigation systems. The villages and the Anfengtang competed for land resources, which led to an increase in the village population, creating a significant demand for land. Farmers continuously encroached on the Anfengtang's water areas for cultivation. As a result, the surface area of the Anfengtang decreased year by year, and its

irrigation efficiency gradually declined. During this period, the Anfengtang was the victim, as its land was continually occupied, causing a reduction in its ecological functions. On the other hand, the villages benefited, gaining large amounts of land resources to develop agricultural production. During this period, the relationship evolved into a parasitic pattern, in which rural communities expanded at the expense of the anfengtang's ecological and hydraulic functions.

During the Republic of China period (1912–1949), the gates and embankments of the Anfengtang were damaged, with a water storage capacity of only about 17 million cubic meters and an irrigation area of less than 200,000 acres. The Chronicle of Chinese Water Resources records: "The effectiveness of water storage was almost entirely lost [36]." The Anfengtang lost its original irrigation and ecological functions, and both parties were harmed. As a result, the original positive interaction collapsed, and the relationship degenerated into a competitive coexistence pattern, in which neither the rural areas nor the facility could maintain their former functions, the symbiotic relationship between the villages and the ancient agricultural irrigation system shifted from a parasitic symbiosis model to a competitive symbiosis model (Table 1).

Table 1. Symbiotic Model between Anfengtang and Rural areas.

Period	Phenomenon	Symbiotic Model	References
Spring and Autumn—Western Han	The creation of Anfengtang facilitated the irrigation of tens of thousands of hectares of fertile land, significantly promoting the development of rural agriculture.	Commensalism pattern	The Book of Han: biographies of model officials records [29]
Northern Wei—Yuan dynasty	Water source blockage, drying of the reservoir surface, and farmers occupying the embankments for farmland, intensifying land resource conflicts.	Parasitic coexistence pattern	Jin Xie, Inscription of the reconstruction of quebei by wei gong in the ming dynasty
Mid-Ming Dynasty	The government set boundaries and constructed ditches to prevent farmers from occupying the embankments, with weak irrigation benefits from Anfengtang.		
Republic of China period	Anfengtang lost its irrigation function, leading to disasters in rural areas and damage to both parties.	Competitive coexistence pattern	The chronicle of Chinese water resources [36]
1953	The government undertook channel restoration and repairs, gradually restoring the irrigation and ecological functions.	Asymmetric mutualism pattern	Shou County Water Resources Bureau
1976	Anfengtang underwent slope protection works; although the surface area was 34 square kilometers, the irrigated area increased to 630,000 mu.		
2022	The irrigation area of Anfengtang was approximately 980,000 mu		

In 1951, the Shou County Government began managing and renovating the Anfengtang. By 1954, its irrigation coverage reached 200,000 mu. In November 1976, slope

protection works were carried out on Anfengtang, increasing its irrigation coverage to 630,000 mu. Currently, the reservoir holds approximately 73 million cubic meters of water, irrigating an area of 42,000 hectares. In 2022, the irrigation area of Anfengtang was approximately 980,000 mu. Although smaller than its original size, Anfengtang now functions as a medium sized counter regulation reservoir capable of adjusting water storage levels as needed. Its effective irrigation coverage exceeds previous farmland areas, driving agricultural tourism and aquaculture development in surrounding villages. This has amplified its irrigation benefits, ecological value, and economic returns. Rural areas and Anfengtang interact in a mutually beneficial symbiosis. Villages maintain and protect the ecological and water resources of ancient agricultural water conservancy facilities while developing aquaculture, thereby enhancing the ecological system of Anfengtang. In turn, the reservoir supplies water for agricultural production in the villages, safeguarding the local ecological environment and biodiversity. This model has gradually evolved into an asymmetric mutualistic symbiosis. Figure 6 presents the evolution of the Anfengtang area. The diagram is redrawn based on the scale change map of Quebi from the Anhui Museum, the "General Report on the Protection and Development of Quebi in China" provided by the Huainan Water Conservancy Bureau, and the historical evolution maps of Quebi created by scholar Zhudiqin [37].

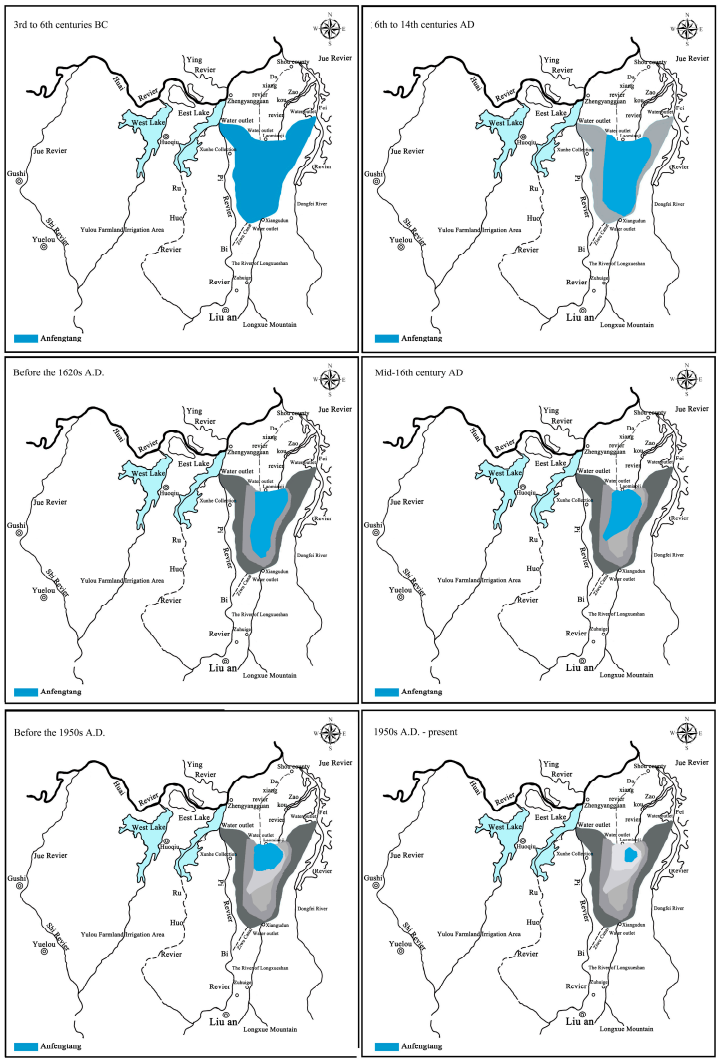


Figure 6. Schematic diagram of the Anfengtang Ponding Process [37].

2.4. Symbiotic Model Construction

2.4.1. Symbiosis Model Assumptions

Hypothesis 1. *Ancient agricultural water conservancy facilities are an integral part of the rural agricultural ecosystem and form a symbiotic unit with rural areas. Subject to constraints from the natural environment, socioeconomic factors, and ecological conditions, their development scale follows a logistic growth pattern, indicating that the organizational scale will not grow indefinitely.*

Hypothesis 2. *There is a relationship of mutual dependence and mutual promotion between ancient agricultural water conservancy facilities and the rural symbiotic system. The water resources provided by ancient agricultural water conservancy facilities ensure rural agricultural production and maintain the ecological environment, whereas rural development relies on the support and maintenance of these facilities. The symbiotic coefficient reflects the strength of this relationship: a coefficient greater than zero indicates a positive interaction, whereas a coefficient less than zero implies a negative impact.*

Hypothesis 3. *In the early stages of symbiotic evolution, there was a competitive relationship between rural areas and ancient agricultural water conservancy facilities. Over time, this relationship has generally progressed toward symbiosis and development. However, when the water volume of ancient agricultural water conservancy facilities declines or their ecological condition deteriorates, rural agricultural production is impacted negatively, leading to a decline in rural socioeconomic conditions. A reduction in the ecological quality and water resources of these facilities can harm both the agricultural productivity and the ecological environment of rural areas. Moreover, the healthy development of rural agricultural ecosystems requires the effective management and preservation of ancient agricultural water conservancy facilities to maintain the stability and sustainability of the symbiotic system.*

2.4.2. Symbiosis Modelling Based on Lotka-Volterra

Let S_1 (t) be the growth rate of the size of the village (countryside) and S_2 (t) be the growth rate of the size of the Amphion Pond (an ancient agricultural water facility) at time t. R_1 represents the natural growth rate of the size of the countryside, R_2 represents the natural growth rate of the irrigation area of the ancient agricultural water conservancy facilities, V_1 represents the maximum size of the development of the countryside, and V_2 represents the maximum value of the area of Anfengtang. Symbiosis models (1) and (2) for the countryside and ancient agricultural water conservancy facilities are independent of each other and do not affect each other's symbiotic relationship. The symbiosis model in which these are independent of each other and do not affect each other is constructed as follows [38].

$$\frac{dS_1(t)}{dt} = r_1 S_1 \left(1 - \frac{S_1}{V_1}\right) \tag{1}$$

$$\frac{dS_2(t)}{dt} = r_2 S_2 \left(1 - \frac{S_2}{V_2}\right) \tag{2}$$

During the development of the countryside and ancient agricultural water facilities, in most cases, they were not independent of each other; instead, they developed each other. The production facilities in the countryside were dependent on the irrigation function of the ancient agricultural water facilities. As the population grew, the countryside had to expand, which increased the demand for land space by the rural indigenous people. Therefore, the rural indigenous people adopted a method of weir creation to transform part of the water area of Anfengtang into farmland. This led the land space of Anfengtang to become a competing resource. In addition, Anfengtang provided an irrigation function for the villages by watering the farmland in the villages and ensuring the food yield. Residents of the countryside also regularly repaired and protected Anfengtang so that it

could be preserved. The symbiotic pattern during this period involved mutual cooperation and competition. Therefore, on the basis of this symbiotic relationship, the coefficients of competition and cooperation are added on the basis of models (1) and (2) to form symbiotic evolution extension models (3) and (4) of the countryside and ancient agricultural water conservancy facilities. The symbiosis model is as follows [39].

$$\frac{dS_1(t)}{dt} = r_1 S_1 \left(1 - \frac{S_1}{V_1} + \theta_{21} \frac{S_2}{V_2}\right) \tag{3}$$

$$\frac{dS_2(t)}{dt} = r_2 S_2 \left(1 - \frac{S_2}{V_2} + \theta_{12} \frac{S_1}{V_1}\right) \tag{4}$$

where $1 - \frac{S_x}{V_x}$ is the logistic model coefficient. θ mn (m $\neq$ n, m = 1,2, n = 1,2) is the coefficient of symbiotic effect of subject m on subject n. In the formula, θ_{12} is the coefficient of symbiotic effect of Anfengtang 2 on Countryside 1, which indicates the coefficient of symbiotic effect of agricultural water facilities on Countryside. Equation θ_{21} is the coefficient of symbiotic effect of village1on Amphitheatre2. As the values of θ mn (m $\neq$ n, m = 1,2, n = 1,2) keep changing, different symbiotic patterns of rural. When $\theta_{21} = \theta_{12} = 0$, the symbiotic model between the rural area and Anfengtang is characterized by mutual independence and no interaction. When the symbiotic coefficient $\theta_{21} = 0$ and $\theta_{12} > 0$, the influence of the rural area on Anfengtang exceeds the influence of Anfengtang on the rural area. Conversely, when $\theta_{12} = 0$, $\theta_{21} > 0$, Anfengtang's influence on the rural area exceeds that of the rural area on Anfengtang. Both cases represent a commensalism model. When $\theta_{21} > 0$ or $\theta_{12} < 0$, it is a parasitic symbiosis model. When $\theta_{21} < 0$, $\theta_{12} < 0$, it is a competitive symbiosis mode. When $\theta_{21} > 0$, $\theta_{12} > 0$, it is a reciprocal symbiosis mode. When the coefficient of symbiosis $\theta_{21} = \theta_{12} = 0$, the symbiosis pattern between the countryside and Amphion Pond is independent of each other and does not affect each other [40].

2.4.3. Model Equilibrium and Stability Analysis

The equilibrium point is a state of dynamic equilibrium when the growth rates of two species are equal and their population sizes no longer change [39]. Therefore, linear stability analysis methods can be used to determine the stability of equilibrium points. By calculating the Jacobian matrix, the eigenvectors at the equilibrium points can be identified. When the eigenvalues of the eigenvectors are negative, the equilibrium point is stable; otherwise, it is unstable. Suppose that at the equilibrium point, the scale of the rural community is S_1, and the scale of the traditional agricultural water conservancy facility is S_2. The equilibrium point satisfies the following system of equations. To further explore the stability of the rural community and traditional agricultural water conservancy facilities, we conduct a stability analysis of model Equations (3) and (4), which requires calculation of the equilibrium points of these equations. The system can be represented by the following differential equations.

$$\mathrm{E}(S_1, S_2) = \frac{dS_1(t)}{dt} = r_1 S_1 (1 - \frac{S_1}{V_1} + \theta_{21} \frac{S_2}{V_2}) = 0 \tag{5}$$

$$\mathrm{F}(S_1, S_2) = \frac{dS_2(t)}{dt} = r_2 S_2 \left(1 - \frac{S_2}{V_2} + \theta_{12} \frac{S_1}{V_1}\right) = 0 \tag{6}$$

The term $\left(1 - \frac{S_x}{V_x} + \theta_{mn} \frac{S_x}{V_x}\right)$ represents the logistic growth model. Therefore, when S_1 is at [0,V_1] and S_2 is at [0,V_2], the four equilibrium points for the symbiotic evolution between rural areas and traditional agricultural water conservancy facilities are as follows: F_1(0,0), F_2(0,V_2), F_3(V_1,0), F_4 $\left(\frac{(1+\theta_{21})V_1}{1-\theta_{21}\theta_{12}}, \frac{(1+\theta_{21})V_1}{1-\theta_{21}\theta_{12}}\right)$. In the next step, the Jacobi matrix is

calculated, the growth rate of the size of the rural and ancient agricultural water facilities is derived, and a 2 × 2 rectangle is constructed, which results in the Jacobi matrix for the symbiotic evolutionary model of the rural and ancient agricultural water facilities.

$$J = \begin{bmatrix} r_1\left(1 - \frac{2S_1}{V_1} + \theta_{21}\frac{S_2}{V_2}\right) & \frac{r_1\theta_{21}S_1}{V_2} \\ \frac{r_2\theta_{12}S_2}{V_1} & r_2\left(1 - \frac{2S_2}{V_2} + \theta_{12}\frac{S_1}{V_1}\right) \end{bmatrix} \quad (7)$$

On the basis of the Jacobi matrix, the stability of each local point can be calculated and determined. The specific determination method is as follows: when the equilibrium point P satisfies det (J) > 0 and tr(J) < 0, it is considered stable; otherwise, it is unstable [40]. By substituting the equilibrium points P_1, P_2, P_3, and P_4 into the Jacobian matrix, the symbiotic evolution and stability analysis of rural areas and ancient agricultural water conservancy facilities can be summarized as shown in Table 2.

Table 2. Equilibrium points and stability conditions for the symbiotic evolution of rural areas.

Equilibrium	Tr (*J*)	Det (*J*)	Stability Condition
F_1 (0, 0)	$r_1 + r_2$	$r_1 r_2$	unstable
F_2 (0, V_2)	$-r_1 r_2(1+\theta_{12})$	$-r_1 + r_2(1+\theta_{12})$	$\theta_{12} < -1$, stable
F_3 (0, V_1)	$-r_1 r_2(1+\theta_{21})$	$-r_1 + r_2(1+\theta_{21})$	$\theta_{21} < -1$, stable
$F_4\left(\frac{(1+\theta_{21})V_1}{1-\theta_{21}\theta_{12}}, \frac{(1+\theta_{21})V_1}{1-\theta_{21}\theta_{12}}\right)$	$\frac{r_1(-1-\theta_{12})+(-1-\theta_{12})r_2}{1-\theta_{21}\theta_{12}}$	$\frac{r_1(-1+\theta_{12})+(-1+\theta_{12})r_2}{1-\theta_{21}\theta_{12}}$	$\theta_{12} > -1, \theta_{21} > -1$

3. Results

3.1. Numerical Settings for the Simulated Symbiosis Modell

As shown in Table 2, the symbiosis coefficient influences the symbiotic relationship between the evolution paths of rural areas and agricultural water conservancy facilities. Therefore, this paper employs Matlab R2019b to simulate five symbiosis patterns. This study references the parameter settings outlined in relevant simulation literature [16,41], establishing the symbiotic interaction coefficient between rural areas and water conservancy facilities as θ_{12} = 0.1, and the symbiotic interaction coefficient from water conservancy facilities to rural areas as θ_{21} = 0.2. Based on the natural population growth rate data for 2019–2022 released by the Shou County People's Government of Anhui Province, the irrigated area data benefiting from water conservancy facilities provided by the Shou County Water Resources Bureau, and historical records of the irrigated area benefiting from the Anfengtang documented in historical texts, the following parameters were established: initial population size, natural growth rates for villages and agricultural water conservancy facilities, maximum village development scale, and maximum irrigated area. This approach ensures that parameter settings are scientifically grounded and possess practical value to reference. Therefore, the natural growth rate of the rural population is set as r_1 = 0.23%, and the natural growth rate of the irrigated area benefiting from water conservancy facilities is set as r_2 = 2.4%. Based on the 2022 rural population data provided by the Shou County People's Government and the beneficiary irrigation data from the Shou County Water Resources Bureau, the maximum rural development population is set as V_1 = 132,745, and the maximum beneficiary irrigation capacity of agricultural water conservancy facilities is set as 980,000.

3.2. Analysis of the Independent Coexistence Pattern

When $\theta_{nm} = 0$ and $\theta_{mn} = 0$, this corresponds to $\theta_{21} = 0$ and $\theta_{12} = 0$, this indicates that rural areas and agricultural water conservancy facilities operate in an independent

coexistence model. Set the symbiotic coefficients to $\theta_{12} = 0$ and $\theta_{21} = 0$. Simulation analysis using MATLAB R2019b reveals (Figure 7) that the rural population grows slowly from an initial value of approximately 13,274 people, reaching only about 15,900 by the 1000th iteration. The overall increase is less than 20%. However, the irrigated area benefiting from traditional agricultural water conservancy facilities approached a capacity of 980,000 after approximately the 200th iteration and remained stable thereafter. This result indicates that when the symbiosis coefficient is zero, rural areas and agricultural water conservancy facilities are governed by their respective endogenous parameters. The lack of coupling elasticity between systems means that agricultural water conservancy expansion is unaffected by population growth rates.

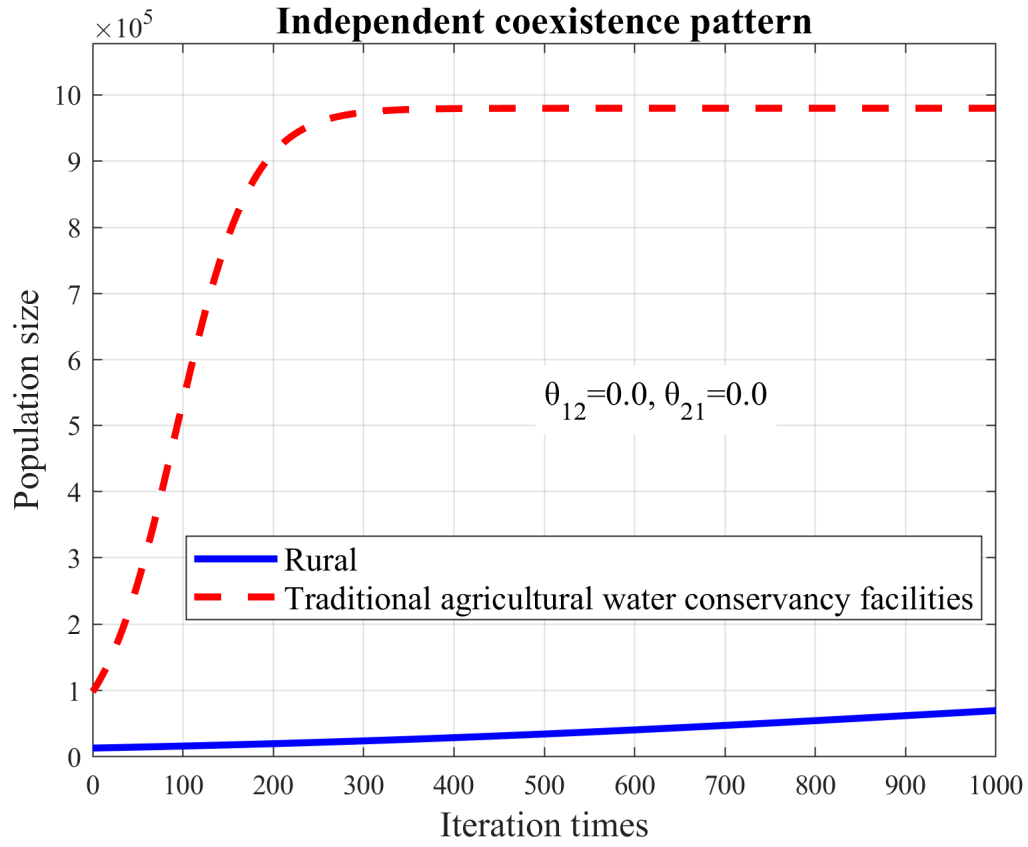


Figure 7. Independent coexistence pattern. Changes in the size of the two populations after 1000 iterations (θ_{12} = 0.0 and θ_{21} = 0.0).

3.3. Analysis of the Commensalism Pattern

When $\theta_{mn} = 0$, $\theta_{nm} > 0$, this corresponds to $\theta_{12} = 0$, $\theta_{21} > 0$. The results indicate a mutually beneficial symbiotic relationship between rural areas and agricultural water conservancy facilities. The symbiosis coefficients are set as $\theta_{12} = 0$ and $\theta_{21} = 0.1$. Simulation results from MATLAB R2019b analysis indicate (Figure 8) that the hydraulic facility curve nearly overlaps with the independent model, rapidly stabilizing after approximately 200 iterations. In contrast, the rural population exhibits significant acceleration, reaching approximately 27,800 individuals by the 1000th iteration—an increase of about 75% compared to the independent model. The data characteristics from simulation results validate this mechanism: with external support, the effective growth rate of the rural system increases, while the water conservancy system remains stable regardless of population expansion. The underlying mechanisms involve two key points: first, comprehensive water conservancy infrastructure significantly enhances the output capacity of cultivated land; second, the flood storage and regulation functions of agricultural water conservancy facilities reduce the

risks of drought and flood disasters, thereby safeguarding rural population accumulation and industrial stability.

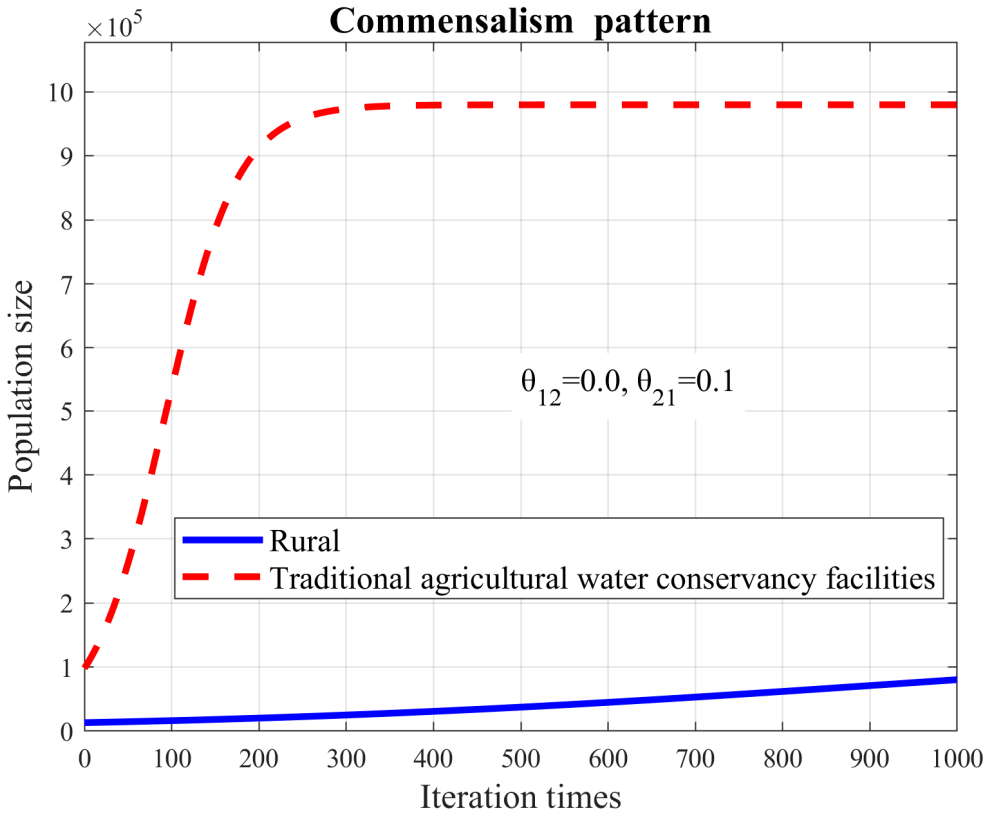


Figure 8. Commensalism pattern. Changes in the size of the two populations after 1000 iterations (θ_{12} = 0.0 and θ_{21} = 0.1).

3.4. Analysis of the Parasitic Coexistence Pattern

When $\theta_{nm} > 0$, $\theta_{mn} < 0$, this corresponds to $\theta_{21} > 0$, $\theta_{12} < 0$. Figure 9 shows that the symbiosis pattern between the rural population and agricultural water conservancy facilities systems is a parasitic coexistence pattern. Set the symbiosis coefficients to $\theta_{12} = -0.1$ and $\theta_{21} = 0.2$. Simulation analysis using MATLAB R2019b indicates (Figure 9) that the steady-state value of the rural population reaches 91,034 individuals, representing a 585.8% increase from the initial value. A turning point occurs near step 880, after which the growth enters a plateau phase. Meanwhile, the steady-state value of the irrigated area reached 916,278 mu, representing an 835.0% increase from the initial value. A growth inflection point appeared as early as step 93, followed by a declining trend thereafter.

The parasitic coexistence pattern reflects a dynamic where "one party benefits while the other suffers," aligning with the historical context of rural areas and ancient agricultural water conservancy facilities in China from the Northern Wei Dynasty to the early ming dynasty. Historical records indicate that due to the blockage of Anfengtang's water source, its water volume gradually decreased, causing the pond's surface area to shrink continuously. During dry seasons, parts of the pond bed were even exposed. Simultaneously, the demand for rural land surged, prompting farmers to occupy the Anfeng tang area and convert the pond surface into farmland. This process drastically reduced the area benefiting from Anfengtang's irrigation, while the villages gained additional arable land, expanded their populations, and became the sole beneficiaries.

This pattern reveals the mechanism by which excessive occupation of embankment land in rural areas leads to the degradation of water conservancy facilities. While it may benefit short-term population expansion in villages, unregulated practices will cause irre-

versible damage to agricultural ecosystems and undermine the system's long-term carrying capacity. Several factors contribute to this pattern: First, the headwaters of Anfeng tang have become blocked, causing water sources to gradually diminish and exposing the riverbed during dry periods. Second, local governments' non-interventionist policies and lack of oversight have enabled farmers to encroach excessively on Anfengtang's boundaries. Therefore, government departments must intervene and implement institutionalized management to strengthen supervision, restoration, and maintenance of agricultural water conservancy facilities.

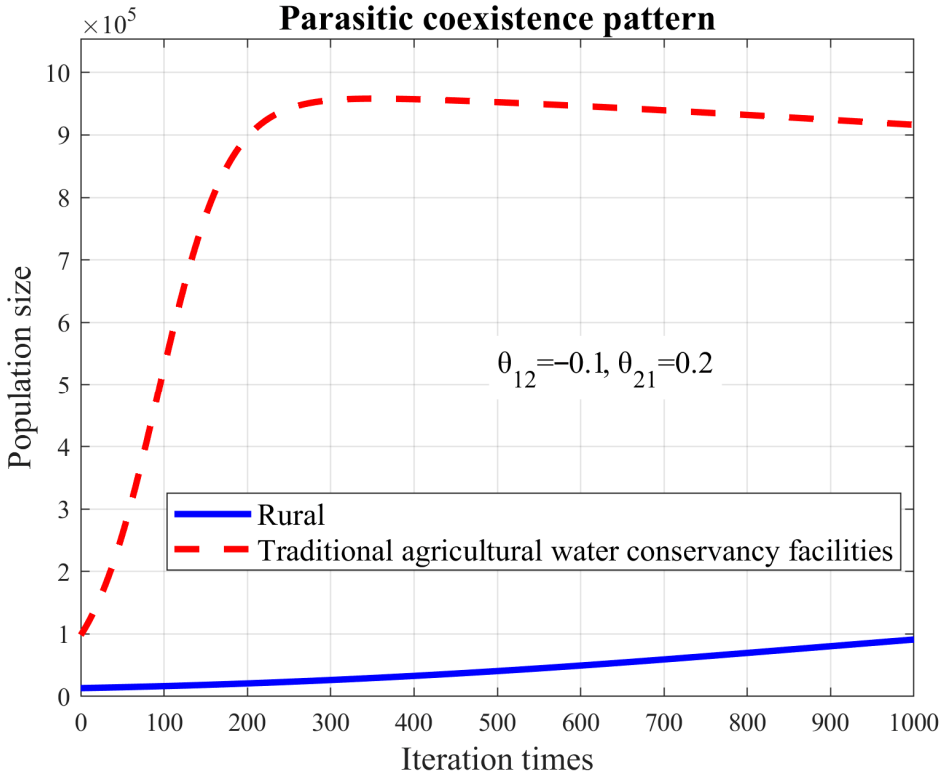


Figure 9. Parasitic coexistence pattern. Changes in the size of the two populations after 1000 iterations ($\theta_{12} = -0.1$ and $\theta_{21} = 0.2$).

3.5. Analysis of the Competitive Coexistence Pattern

When $\theta_{nm} < 0$, $\theta_{mn} < 0$, this corresponds to $\theta_{21} < 0$, $\theta_{12} < 0$. Figure 10 shows that the symbiotic relationship between rural areas and traditional agricultural water conservancy facilities is a competitive coexistence pattern. The symbiosis coefficients are set as $\theta_{12} = -0.1$ and $\theta_{21} = -0.2$. Simulation analysis using MATLAB R2019b indicates (Figure 10) that the steady-state rural population reaches 52,041 individuals, representing a 292.0% increase from the initial value, the lowest among the five models. The growth curve begins to flatten after 1000 iterations, lagging behind other symbiotic models. The steady-state value of the irrigated area reaches 943,157 mu, representing an 862.4% increase from the initial value. The competitive coexistence pattern aligns with the historical context of rural areas and agricultural water conservancy facilities in China from the mid-Ming Dynasty to the Republican era. Historical records indicate that the failure to curb the previous symbiotic model led to a drastic reduction in the area of Anfengtang and a gradual decrease in irrigable land. Consequently, the village's farmland could not be effectively irrigated, and the interests of the rural community were compromised. Additionally, several factors contributed to this pattern. First, amid national social unrest, local nobility disregarded the interests of commoners and privately seized exposed marshlands from Anfengtang to convert them into farmland. Second, with the government overwhelmed and

lacking oversight, a wave of land grabs for agricultural use surged. Finally, as Anfeng tang's area steadily diminished year after year, the irrigated land shrunk, causing agricultural water conservancy facilities to become blocked at one point.

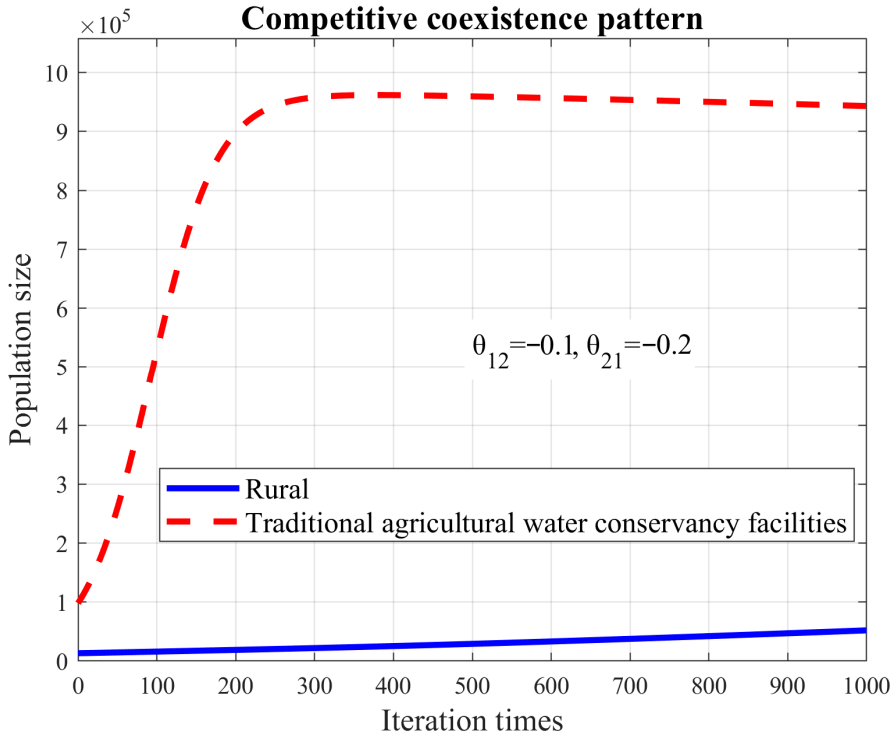


Figure 10. Competitive coexistence pattern. Changes in the size of the two populations after 1000 iterations ($\theta_{12} = -0.1$ and $\theta_{21} = -0.2$).

3.6. Analysis of the Asymmetric Mutualism Pattern

When $\theta_{nm} > 0$, $\theta_{mn} > 0$, this corresponds to $\theta_{21} > 0$, $\theta_{12} > 0$. The symbiosis coefficients are set as $\theta_{12} = 0.1$ and $\theta_{21} = 0.2$. Figure 11 shows that the symbiotic relationship between rural areas and traditional agricultural water conservancy facilities exhibits asymmetric mutualism pattern.

Simulation results from MATLAB R2019b analysis indicate (Figure 11) that the steady-state rural population reached 92,874 individuals, representing a 599.6% increase from the initial value. Its growth curve entered the inflection point interval at iteration 896, significantly earlier than other models and rapidly stabilizing thereafter. The steady-state value of the irrigated area reached 1,045,359 mu, representing a 966.7% increase from the initial value. This value is the highest among all symbiotic models. This model corresponds to the contemporary phase of conservation and utilization at Anfengtang. During this period, local governments and residents established a co-governance and co-management mechanism for water conservancy maintenance. They regularly dredged and repaired Anfengtang, protected the ecological integrity of its watershed, prevented encroachment on the pond surface, and ensured the sustained operation of the Anfengtang irrigation system.

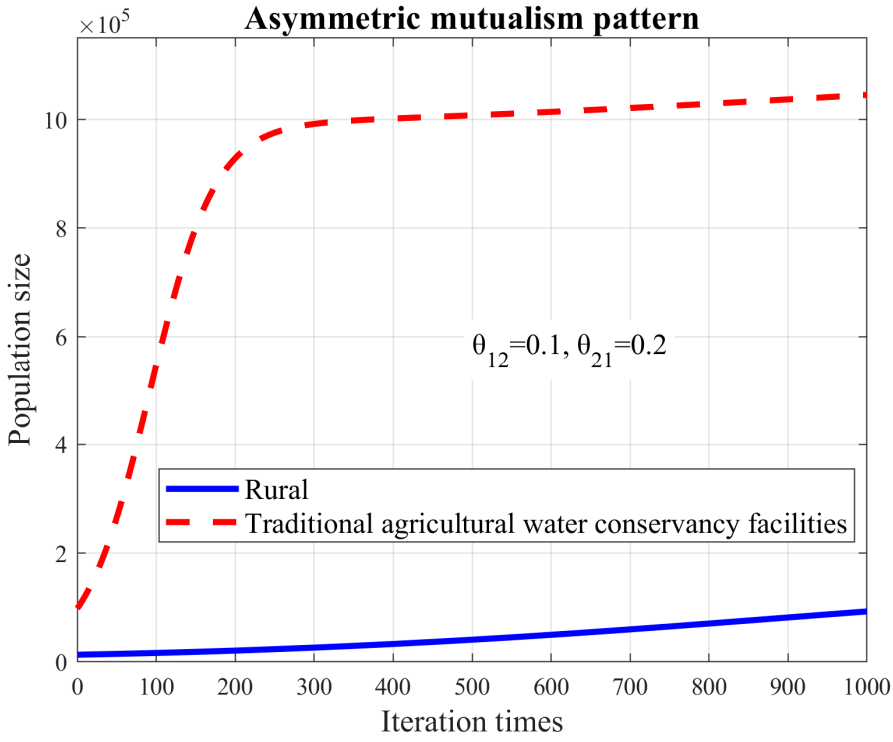


Figure 11. Asymmetric mutualism pattern. Changes in the size of the two populations after 1000 iterations (θ_{12} = 0.1 and θ_{21} = 0.2).

3.7. Results Comparison of Five Symbiotic Patterns

Table 3 presents three key characteristic indicators for rural areas and agricultural water conservancy facilities under five symbiotic patterns, including steady-state values, relative growth rates, and the number of iteration steps at inflection points.

Table 3. Characteristics of rural population and irrigation systems under five symbiotic patterns.

Symbiotic Pattern	Rural Population Steady State (Persons)	Rural Population Growth (%)	Rural Population Inflection Step	Irrigation Area Steady State (mu)	Irrigation Area Growth (%)	Irrigation Area Inflection Step
independent	69,741	425.4	957	980,000	900.0	93
commensalism	80,329	505.1	921	980,000	900.0	93
parasitic	91,034	585.8	880	916,278	835.0	93
competitive	52,041	292.0	1000	943,157	862.4	93
mutualism	92,874	599.6	896	1,045,359	966.7	93

The results indicate that significant differences exist among the various symbiotic models. Among them, the mutualistic symbiosis model demonstrates optimal performance, with a steady state population of 92,874 in rural areas and an inflection point occurring at 896 iterations. Parasitic coexistence pattern and commensalism pattern follow, achieving steady-state populations of 91,034 and 80,329, respectively, while the independent symbiosis model yields a steady-state population of 69,741. Additionally, the competitive symbiosis model yielded the lowest value at 52,041, the overall ranking of population sizes is as follows: asymmetric mutualism pattern > parasitic coexistence pattern > commensalism pattern > independent coexistence pattern > competitive coexistence pattern (Figure 12). Regarding the irrigated area benefiting from agricultural water conservancy facilities, the reciprocal symbiosis model achieved the highest value at 1,045,359, while

the parasitic symbiosis model yielded the lowest at 916,278. The ranking of symbiosis patterns by irrigated area was asymmetric mutualism pattern > independent coexistence pattern > commensalism pattern > competitive coexistence pattern > parasitic coexistence pattern (Figure 13). The results indicate that the symbiosis coefficient and the symbiotic evolution pathways between villages and traditional agricultural water conservancy facilities exert a determinant influence.

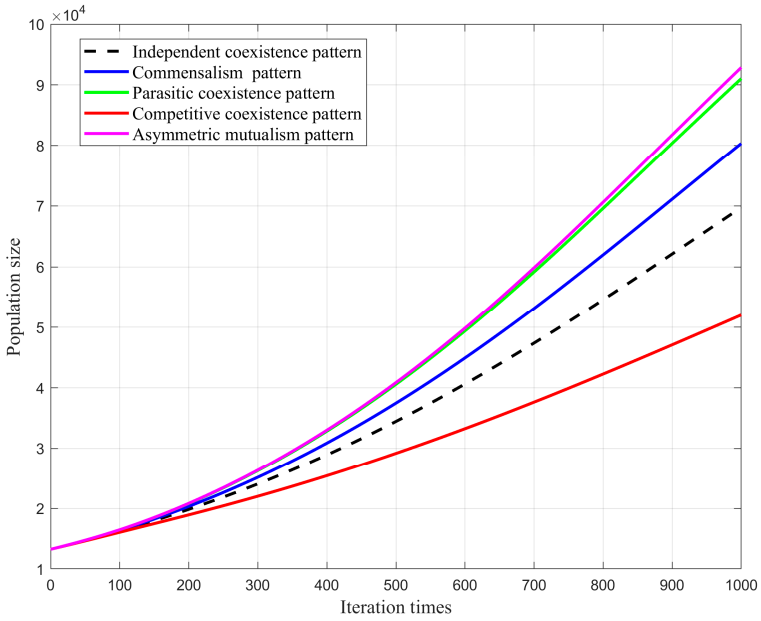


Figure 12. Comparative analysis of rural population development under five symbiotic models.

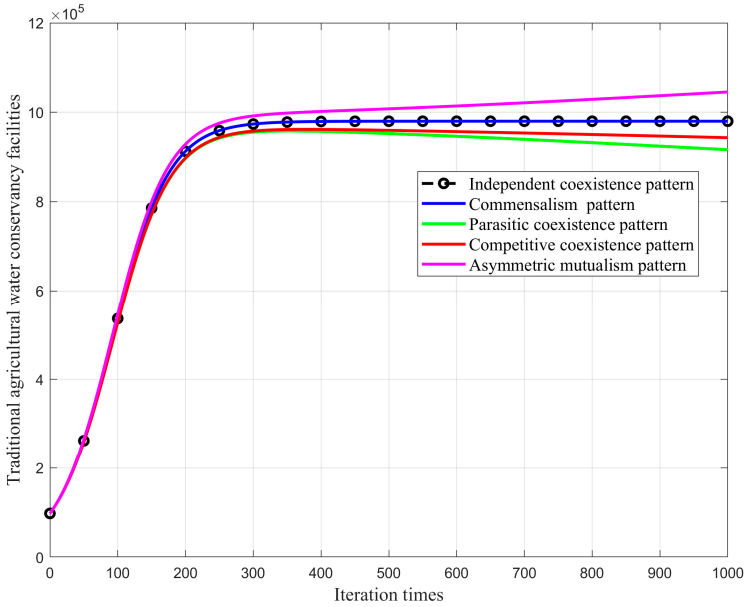


Figure 13. Development diagram of agricultural water conservancy facilities in five symbiotic models.

According to the normalized indicator data in Figure 14, the mutualistic symbiosis model exhibits the highest normalized values across all indicators. The parasitic symbiosis model ranks second in population steady-state values, with a value of 0.955. The population normalization value for the asymmetric mutualism pattern is 0.658, while the irrigation area is approximately 0.494. The competition pattern exhibits the lowest normalization value at 0. The steady-state value for the irrigation area is 0.208. The ranking of natural growth rates for rural populations was asymmetric mutualism pattern (1.000) > parasitic co-

existence pattern (0.955) > commensalism pattern (0.693) > independent coexistence pattern (0.433) > competitive coexistence pattern (0.000). Analysis of the five coexistence patterns indicates that the mutualistic pattern is optimal for sustaining long-term stable coexistence between rural communities and water management systems. Therefore, enhancing rural investment in and management of water infrastructure, while promoting the establishment of positive feedback mechanisms, will enable the long-term sustainable development of both rural communities and traditional agricultural water facilities (Figure 14).

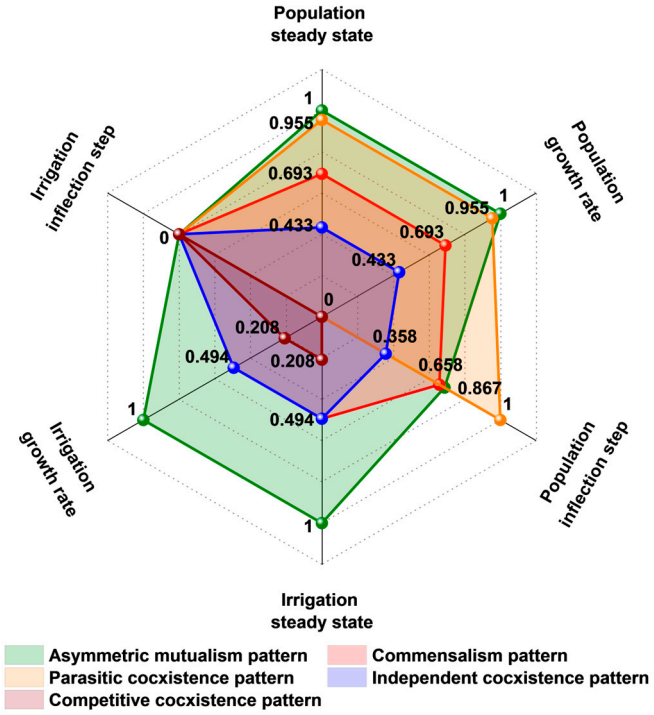


Figure 14. Development diagram of agricultural water conservancy facilities in five symbiotic models.

4. Discussion

This study aims to address how rural areas and ancient agricultural water conservancy facilities can achieve sustainable and healthy development to maximize their benefits. Therefore, it adopts a symbiotic perspective and integrates relevant theories from ecology and management to explore the mechanisms underlying the long-term evolution of this relationship. First, a theoretical framework for the symbiosis between rural areas and ancient agricultural water conservancy facilities is established. Historical maps, chronicles, and other historical documents are analyzed to examine the evolutionary process of rural areas and agricultural water conservancy facilities, which reveals the mechanisms underlying the evolution of their relationship. Second, a symbiotic evolutionary model of rural areas and ancient agricultural water conservancy facilities is constructed on the basis of the Lotka–Volterra framework. Finally, MATLAB software is used to perform simulation analysis, which validates the scientific validity and feasibility of their symbiotic relationship. This study holds practical significance for the rational utilization and maintenance of traditional agricultural water conservancy facilities, improving the efficiency of agricultural production, enhancing local agricultural productivity and the ecological environment, and promoting the sustainable development of rural areas. The simulation results of this study indicate that the symbiotic development model between rural communities and traditional agricultural water conservancy facilities determines their long-term stability and sustainability. Among the symbiotic system models, the mutualistic symbiosis model demonstrated the most favorable outcomes, with a 599.6% increase in rural population

and a 966.7% increase in the irrigated area benefiting from agricultural water conservancy facilities. This indicates a positive feedback loop between rural communities and agricultural water conservancy facilities. In the parasitic symbiosis model, the steady-state rural population reached 91,034 people, while the irrigated area covered by agricultural water conservancy facilities reached 916,278 mu, both lower than in the reciprocal symbiosis model. This suggests that when rural systems become overly dependent on water conservancy facilities, the functionality of these facilities gradually deteriorates, leading to short-term benefits but long-term decline. In the parasitic symbiosis model, the steady-state rural population is 80,329 people, and the steady-state irrigated area is 980,000 mu. This model indicates that while the rural system benefits from water conservancy facilities, these facilities lack adequate protection and investment, resulting in a one-way benefit scenario. In the competitive symbiosis model, the steady-state rural population is only 52,041 people, and the steady-state irrigated area is 943,157 mu. This model exhibits a pronounced negative coupling relationship. Under this scenario, the functional degradation of water conservancy facilities and the growth of the rural population are suppressed, leading to the decline of the symbiotic system.

The innovations of this study are as follows. (1) This study addresses a relatively unexplored area in research on the symbiotic relationship between rural areas and agricultural water conservancy facilities. To our knowledge, it is among the first to adopt an ecological–symbiosis perspective and to apply the Lotka–Volterra framework to construct a dynamic symbiosis model linking a Anfengtang with the surrounding rural system. (2) The study integrates a historical cognition approach with model-based simulation. By using historical documents, local chronicles, and stone inscriptions to identify actual interaction stages, and then testing these stages through symbiosis theory and MATLAB simulations, the study demonstrates the scientific validity and feasibility of interpreting historical rural water relations as symbiotic evolution and identifies the conditions under which this interaction is optimal. By utilizing symbiotic theory and simulation analysis, this study validates the scientific validity and feasibility of their relationship and identifies their optimal interaction. In addition, this study has certain limitations. Owing to the difficulty of obtaining samples from rural areas and ancient agricultural water conservancy facilities, conducting an empirical analysis of the simulation results is challenging. Moreover, the study did not account for the impact of external factors such as climate change, social policies, and economic conditions on the evolution of symbiotic systems. In future research, we will utilize multi-source remote sensing data and real-time test data, incorporate external variable indicators including policy interventions, climate risks, and institutional arrangements, thereby enhancing the accuracy of model predictions and the practicality of policy recommendations.

5. Conclusions

Agricultural water conservancy facilities form a critical foundation for agricultural development. The continued development and maintenance of well-functioning agricultural water conservancy facilities contribute to the healthy and sustainable growth of the agricultural sector and safeguard the successful implementation of national food security strategies. This study applies symbiotic theory and employs the Lotka-Volterra model to investigate the evolutionary patterns and processes of agricultural water conservancy facilities and rural areas from ancient times to the present. To explore the mechanisms of sustainable development between rural areas and agricultural water conservancy facilities, numerical simulations were conducted. The study reveals that when all symbiotic coefficients are greater than zero, the symmetric symbiosis model is the optimal mode of coexistence between rural areas and agricultural water conservancy facilities.

The main conclusions are as follows. Firstly, the relationship between Anfengtang and rural areas has evolved through four stages and modes: commensalism, parasitism, competitive symbiosis, and asymmetric mutualism. Currently, Anfengtang still plays a vital role in agricultural irrigation and offers valuable references and successful experiences for agricultural production and sustainable rural development in China. When Anfengtang was first constructed, it pioneered water conservancy in the Jianghuai region. The extensive irrigation provided by this ancient agricultural water conservancy facility facilitated the rapid development of rural agriculture in the Shouchun area and met the military grain demands of the Chu State. This contributed to the strength of Chu and enabled it to rival the Qin State for more than a decade. Successive rulers placed great importance on the maintenance and reconstruction of water conservancy projects. However, owing to factors such as climate change, warfare, and human activities, the relationship between Anfengtang and rural areas gradually weakened over time. After the 1950s, the local government resumed the management of Anfengtang and revitalized its relationship with rural areas. This event highlights the critical importance of government intervention and governance in the development of Anfengtang and the surrounding rural areas.

Secondly, when all symbiotic coefficients are greater than zero, the symmetric symbiosis model is identified as the optimal mode of coexistence between rural areas and agricultural water conservancy facilities. This model is most conducive to the sustainable development of rural areas. In this context, this study successfully established a theoretical framework for the relationship between rural areas and ancient agricultural water conservancy facilities and developed a symbiotic model using the Lotka-Volterra framework. Moreover, this study presents a symbiotic development mechanism on the basis of the interdependence between rural areas and ancient agricultural water conservancy facilities, which provides valuable insights for the harmonious symbiotic development of rural areas and major agricultural production projects in the future. Notably, Anfengtang has endured for more than a millennium and remains in use today. It serves rural agricultural production and has become a valuable part of the global irrigation and agricultural heritage, providing crucial water conservancy support for local agricultural production and ecosystems.

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