

Article

Application of Harmony Search Algorithm to Slope Stability Analysis

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Abstract: Slope stability analysis is undoubtedly one of the most complex problems in geotechnical engineering and its study plays a paramount role in mitigating the risk associated with the occurrence of a landslide. This problem is commonly tackled by using limit equilibrium methods or advanced numerical techniques to assess the slope safety factor or, sometimes, even the displacement field of the slope. In this study, as an alternative approach, an attempt to assess the stability condition of homogeneous slopes was made using a machine learning (ML) technique. Specifically, a meta-heuristic algorithm (Harmony Search (HS) algorithm) and K-means algorithm were employed to perform a clustering analysis by considering two different classes, depending on whether a slope was unstable or stable. To achieve the purpose of this study, a database made up of 19 case studies with 6 model inputs including unit weight, intercept cohesion, angle of shearing resistance, slope angle, slope height and pore pressure ratio and one output (i.e., the slope safety factor) was established. Referring to this database, 17 out of 19 slopes were categorized correctly. Moreover, the obtained results showed that, referring to the considered database, the intercept cohesion was the most significant parameter in defining the class of each slope, whereas the unit weight had the smallest influence. Finally, the obtained results showed that the Harmony Search algorithm is an efficient approach for training K-means algorithms.

Keywords: machine learning; K-means algorithm; harmony search; clustering analysis; slope stability



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1. Introduction

Landslides are recognized all over the world as one of the most dangerous natural hazards [1–4]. Deformation and failure processes of slopes are commonly schematized in four different stages [5]: pre-failure, failure, post-failure and reactivation. Thanks to the availability of powerful calculation tools, as well as advanced numerical techniques, all these stages can be properly studied nowadays [6–13]. However, analysis of the whole deformation process is often limited in the research field, whereas the assessment of the factor of safety (FS) of slopes is still the most common procedure in engineering practice when dealing with slope stability. The factor of safety is commonly evaluated by using the well-known limit equilibrium methods [14–21], the finite element method [22–25], and physically-based [26–28] and statistically-based [29] methods.

In another study, Michalowski (2002) introduced stability charts for uniform slopes which, according to the kinematic method of limit analysis, lead to a strict lower bound on stability number $c/\gamma H$ or an upper bound on the safety factor. In the proposed charts, pore water pressure and seismic forces were considered for slopes [30]. However, some applications in the field of slope stability have also been recently presented using artificial

intelligence approaches [31–35]. Mishra et al. (2019) conducted slope stability assessment based on several recent meta-heuristic techniques. They compared the results with the antlion optimizer and made some recommendations for slope stability analysis [36]. Yuan and Moayed (2019) investigated slope stability analysis and failure recognition using several artificial intelligence techniques. They found that the combination of multilayer perceptron with biogeography-based optimization can provide a higher performance capacity in the stability assessment compared to other intelligence methods [37]. In another study, Mishra et al. (2020) carried out a slope stability assessment using the multiverse optimization (MVO) algorithm. The obtained results indicated that this algorithm can be applied as a powerful tool for investigating the factor of safety [38].

In the present study, machine learning (ML) was employed as an alternative approach for a preliminary evaluation of the stability condition of slopes. In particular, the harmony search (HS) algorithm was employed. Traditional A.I. techniques, such as linear programming, non-linear programming, and dynamic programming, have been commonly employed for analyzing optimization problems. These techniques can guarantee global optima in simple and ideal models. However, they present some drawbacks: in linear programming, considerable losses occur when a linear ideal model from a non-linear real world problem is developed; in dynamic programming, an increase in the number of variables would exponentially increase the number of evaluations of the recursive functions and tax the core memory; in non-linear programming, if the functions used in computation are not differentiable, the solving algorithm may not find the optimum. Careful attention is also required in the selection of the initial values in order to guarantee convergence to the global optimum and not to the local optima. The HS algorithm is employed in the present study for the purpose of overcoming the above mentioned limitations. It should be mentioned that the utilization of the clustering method for the evaluation of slope stability using the HS algorithm has not been used in previous studies. Additionally, in this study, the HS algorithm was used due to the smaller number of control parameters and also because it is easier to perform than many optimization algorithms. Even though this approach cannot overtake the capabilities of the methods commonly employed in geotechnical engineering (such as limit equilibrium methods or the strength reduction method), it could be taken into account to provide a primary evaluation of the slope safety factor.

The remainder of this paper is organized as follows: Section 2 describes the harmony search (HS) algorithm. Section 3 explains the reasons for selecting the main geotechnical parameters and geometrical characteristics for this analysis and introduces Case study. In Section 4, the necessary analyses are performed to achieve the optimal model and the efficiency of the model is discussed. Section 5 concludes the paper and recommends future studies.

2. The Harmony Search (HS) Algorithm

In the field of artificial intelligence, there is a wide range of optimization algorithms and techniques used in various fields of industry and academia [39–42]. In computational science, meta-heuristic algorithms play a key role in solving complex problems [43–47]. The harmony search (HS) algorithm is a meta-heuristic algorithm and was introduced by Geem [48–50]. The HS algorithm is an efficient algorithm in unpredicted and uncertain systems that can provide a higher performance capacity for dealing with linear and nonlinear data sets. In addition, the HS algorithm has been successfully applied in the engineering and academic sectors [51–57]. The working principles of optimization of HS algorithm are similar to the composition of songs by musicians using various notes. In fact, this algorithm is inspired by the process of producing a new song. In this process, the composer produces a new song by combining pieces and music notes, and in the next song, according to the composer's limited memory, the best notes and pieces are considered [58]. The basic HS algorithm comprises four main steps, as follows:

Step 1: Initialize the harmony search memory (*HM*). A number of randomly generated solutions is produced according to the conditions of the problem. If there is an *n*-dimension

problem based on Equation (1), a *HM* with a size equal to the *HMS* can be introduced as follows:

$$HM = \begin{bmatrix} x_1^1, x_2^1, \dots, x_n^1 \\ x_1^2, x_2^2, \dots, x_n^2 \\ x_1^3, x_2^3, \dots, x_n^3 \\ \vdots \\ x_1^{HMS}, x_2^{HMS}, \dots, x_n^{HMS} \end{bmatrix} \quad (1)$$

where $[x_1^i, x_2^i, \dots, x_n^i]$ ($i = 1, 2, 3, \dots, HMS$) is considered an answer candidate.

Step 2: Create a new solution from the *HM*.

Step 3: The new solution from the last step is evaluated and then the *HM* is updated. If the new solution is better than the worst solution stored in the *HM*, the two solutions are swapped.

Step 4: The algorithm process and calculations are repeated. In fact, the loop of steps 2 and 3 repeats until the stop conditions are achieved, such as the desired precision level or maximum iteration [58].

The flowchart of the HS algorithm is shown in Figure 1 [49].

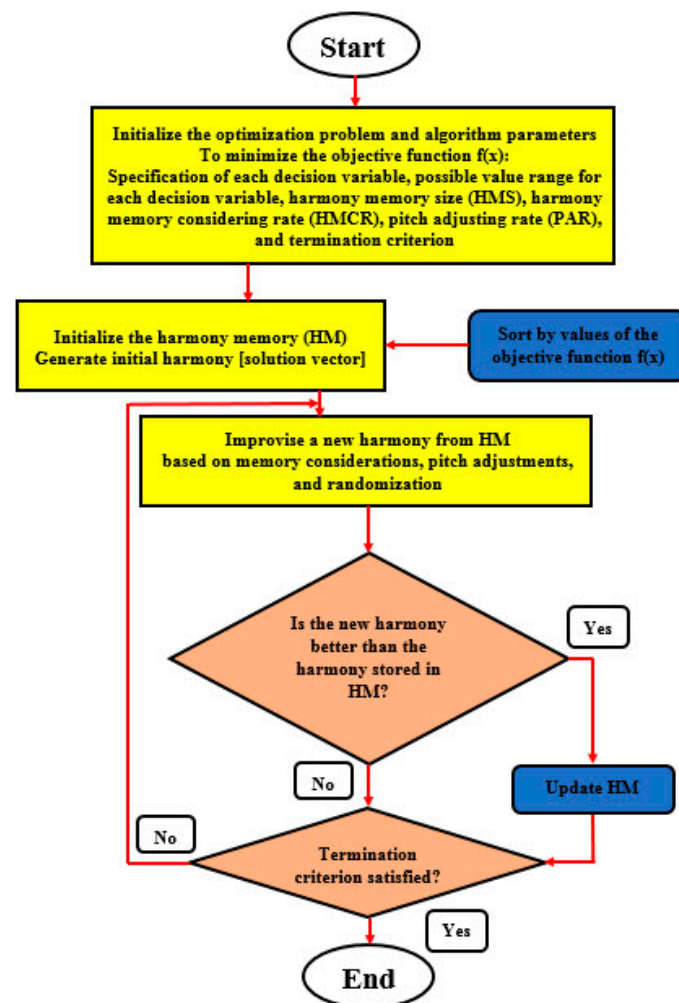


Figure 1. Flowchart of the HS.

3. Case Study

In this study, the described HS algorithm was applied in the field of slope stability. The novelty of this research work consisted of considering the nature of complex and uncertain

systems and their full compliance with meta-heuristic algorithm concepts. Various parameters have been used by many researchers to analyze the stability of slopes in artificial intelligence methods. Table 1 shows some of the most effective research works in this area. Figure 2 demonstrates the frequency of the parameters affecting the stability of the slope used in these research works. Hence, according to the importance of the parameters used in Figure 1, slope height (H), slope angle (α), cohesion (c'), angle of shearing resistance (φ'), unit weight (γ), and pore pressure ratio (r_u) were considered for modelling in this study.

Table 1. Literature review of machine learning techniques applied to slope stability problems.

References	Geotechnical Properties and Geometry						Slip Surface		Number of Cases
	H	α	c'	φ'	γ	r_u	PP	PGA	
Feng [59]	•	•	•	•	•	•		Circular	82 cases
Lu and Rosenbaum [60]	•	•	•	•	•	•		Circular	32 cases
Li [61]	•	•	•	•	•			Circular	59 cases
Huang et al. [62]	•	•	•	•	•	•		Circular	64 cases
Sakellariou and Ferentinou [63]	•	•	•	•	•	•		Circular	46 cases
Wang et al. [64]	•	•	•	•	•			Circular	27 cases
Samui et al. [65]	•	•	•	•	•	•		Circular	46 cases
Zhao [66]			•	•	•			Circular	10 cases
Das et al. [67]	•	•	•	•	•	•		Circular	46 cases
Erzin and Cetin [68]	•	•	•	•	•			Circular	675 modeled cases
Liu et al. [69]	•	•	•	•	•	•		Circular	97 cases
Gordan et al. [70]	•	•	•	•				• Circular	699 modeled cases
Hoang and Pham [71]	•	•	•	•	•	•		Circular	168 cases
Suman et al. [72]	•	•	•	•	•	•		Circular	103 cases
Verma et al. [73]		•	•	•			•	Circular	100 modeled cases
Fattahi [74]	•	•	•	•	•	•		Circular	67 cases
Rukhaiyar et al. [75]	•	•	•	•	•	•		Circular	83 cases
Xue [76]	•	•	•	•	•	•		Circular	46 cases
Chakraborty and Goswami [77]	•	•	•	•	•	•		Circular	200 cases
Feng et al. [78]	•	•	•	•	•	•		Circular	69 cases
Salmasi and Jafari [79]	•	•	•	•	•	•		Circular	250 cases

Slope height (H), Slope angle (α), Cohesion (c'), Angle of shearing resistance (φ'), Unit weight (γ), Pore pressure ratio (r_u), Pore pressure (PP), Peak ground acceleration (PGA). Bullets: model inputs used in the mentioned studies.

Referring to the slope whose scheme is reported in Figure 3, the following input data were considered in the present study: slope height (H), slope angle (α), intercept cohesion (c'), angle of shearing resistance (φ'), soil unit weight (γ) and pore pressure ratio (r_u). A dataset consisting of 19 case studies was considered randomly, referring to the circular slip surface. For the considered slopes, the above-mentioned parameters were available, as well as the factor of safety (Table 2) [80].

In order to classify slope stability cases, the k-means algorithm (Lloyd's algorithm) was used as it is one of the most common and popular clustering methods optimized by the HS algorithm. Equation (2) shows the k-means algorithm [81]:

$$Obj.Function = \sum_{i=1}^n \min_{1 \leq j \leq k} d(x_i, m_j) \quad (2)$$

where x_i represents a member of a cluster with $i = 1, 2, 3, \dots, n$. In addition, m_j and j are the center of each cluster and the number of clusters, respectively, with $j = 1, 2, 3, \dots, k$. Moreover, d expresses the Euclidean distance between each member of a cluster (x_i) and the center of the same cluster (m_j). In fact, the fitness function can have two main purposes: maximizing the distance between the centers of the clusters and minimizing the distance between the members of a cluster. Usually, when the goal of optimization is minimization, the fitness function is also called cost function. Therefore, in this study, the purpose of

optimization was to minimize the distances between each member of each cluster, and the value of the cost function was actually the minimum distance between each member and the center of each cluster. Generally, when the values of the cost function are close together across two consecutive iterations, it can indicate convergence of the algorithm, and when these values are equal, it means that the error rate is zero and the algorithm has reached convergence.

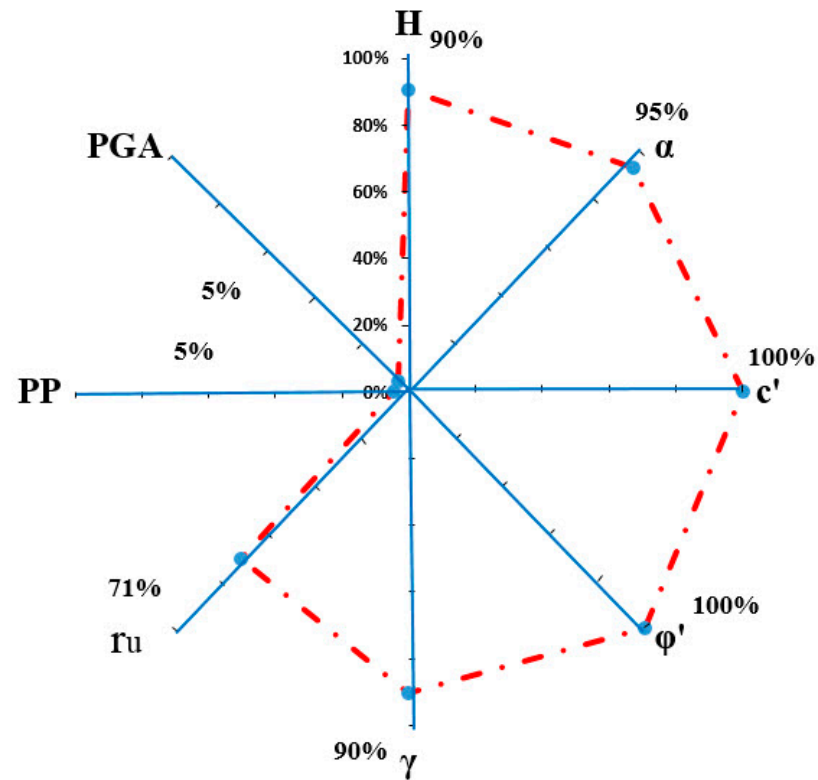


Figure 2. Frequency of geotechnical and geometrical parameters studied in the slope stability analysis research works documented in Table 1.

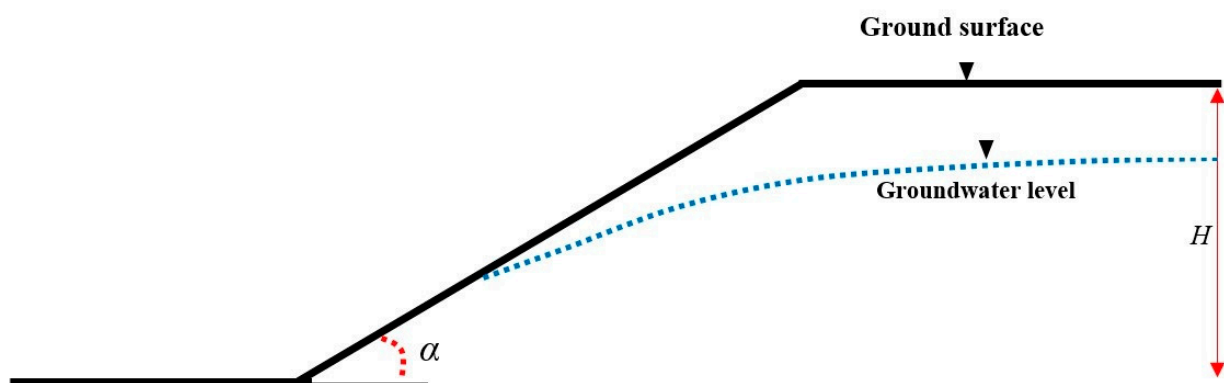


Figure 3. Schematic of the considered slopes.

Table 2. Dataset considered in the present study (Sah et al. [80]).

Case No.	Location	Geotechnical Properties and Geometry						FS
		γ (kN/m ³)	c' (kPa)	ϕ' (°)	α (°)	H (m)	r_u	
1	Seven Sisters Landslide, Uk	20.41	24.9	13	22	10.67	0.35	1.4
2	Case 1: The Northolt Slide, UK	19.63	11.97	20	22	12.19	0.405	1.35
3	Selset Landslide, Yorkshire, Uk	21.82	8.62	32	28	12.8	0.49	1.03
4	Saskatchewan Dam, Canada	20.41	33.52	11	16	45.72	0.2	1.28
5	Case 2: The Northolt Slide, UK	18.84	15.32	30	25	10.67	0.38	1.63
6	River Bank Side, Alberta, Canada	19.06	11.71	28	35	21	0.11	1.09
7	Unknown	18.84	14.36	25	20	30.5	0.45	1.11
8	Case 2: Open-Pit Iron Ore Mine, Goa, India	14	11.97	26	30	88	0.45	0.625
9	Athens Slope, Greece	18	24	30.15	45	20	0.12	1.12
10	Case 1: Open-Pit Coal Mine, Alberta, Canada	22.4	100	45	45	15	0.25	1.8
11	Case 2: Open-Pit Coal Mine, Alberta, Canada	22.4	10	35	45	10	0.4	0.9
12	Case 3: Open-Pit Coal Mine, Newcastle Coalfield, Australia	20	20	36	45	50	0.25	0.96
13	Case 4: Open-Pit Coal Mine, Newcastle Coalfield, Australia	20	20	36	45	50	0.5	0.83
14	Case 5: Open-Pit Coal Mine, Newcastle Coalfield, Australia	20	0	36	45	50	0.25	0.79
15	Case 6: Open-Pit Coal Mine, Newcastle Coalfield, Australia	20	0	36	45	50	0.5	0.67
16	Case 1: Harbour Slope, Newcastle, Australia	22	0	40	33	8	0.35	1.45
17	Case 2: Harbour Slope, Newcastle, Australia	24	0	40	33	8	0.3	1.58
18	Case 3: Harbour Slope, Newcastle, Australia	20	0	24.5	20	8	0.35	1.37
19	Case 4: Harbour Slope, Newcastle, Australia	18	5	30	20	8	0.3	2.05

4. Modelling by the HS Algorithm

The current study focused on the applicability of a combination the HS algorithm with the K-means algorithm as an unsupervised clustering technique in order to cluster and evaluate slope stability according to compiled datasets from case studies of the circular critical failure mechanism.

In the first step of clustering, all data were normalized. For this purpose, all data belonging to each category were divided by the largest-value data of the same category [82]. Then, determining of control parameters of the algorithm was necessary. The control parameters of algorithms contributed greatly to finding the optimum results, and increased the convergent speed of the algorithm. There are often no special equations and rules, so, based on a trial-and-error procedure, expert opinions and data sets are considered [83]. After the initial analysis, the control parameters were determined, including maximum iterations equal to 300, and a pitch adjustment rate (PAR) equal to 0.1, i.e., the neighboring values were selected with a 10% probability. In addition, based on previous studies, the value of the harmony memory size (HMS) is normally considered to be within the range of 50 to 100. Specifically, a value of 60 was used in this application following a trial-and-error procedure [58,83–85]. Two classes were considered: the class with the label “1” included unstable slopes ($FS \leq 1$), whereas stable slopes ($FS > 1$) belonged to the class with the label “2”. The process of optimization based on iterations, and the results of clustering for the 19 cases with two classes, are shown in Figure 4 and Table 3, respectively.

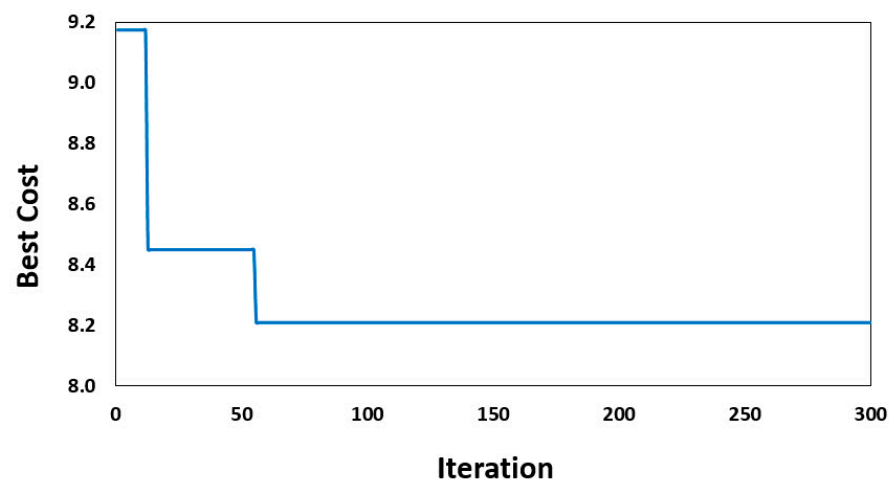


Figure 4. Best cost-per-iteration provided by the HS algorithm.

Table 3. Optimization and clustering of the considered dataset by the HS algorithm.

No. of Case	Euclidean Distance from		Defined Class	FS	Verification
	First Class	Second Class			
1	0.659	0.268	2	1.4	Satisfied
2	0.549	0.129	2	1.35	Satisfied
3	0.474	0.352	2	1.03	Satisfied
4	0.848	0.592	2	1.28	Satisfied
5	0.445	0.253	2	1.63	Satisfied
6	0.699	0.543	2	1.09	Satisfied
7	0.529	0.169	2	1.11	Satisfied
8	0.742	0.843	1	0.625	Satisfied
9	0.822	0.537	2	1.12	Satisfied
10	1.053	1.158	1	1.8	Not Satisfied
11	0.637	0.340	2	0.9	Not Satisfied
12	0.377	0.777	1	0.96	Satisfied
13	0.411	0.729	1	0.83	Satisfied
14	0.357	0.798	1	0.79	Satisfied
15	0.392	0.751	1	0.67	Satisfied
16	0.559	0.427	2	1.45	Satisfied
17	0.605	0.481	2	1.58	Satisfied
18	0.570	0.271	2	1.37	Satisfied
19	0.559	0.362	2	2.05	Satisfied

According to Figure 4, the best cost for the clustering process was reached after 55 iterations and equaled 8.209. Since the best cost represents the performance of the algorithm, Figure 4 indicates an appropriate convergence speed of the HS algorithm. It is worth mentioning that the best value is a dimensionless number, and that if the difference between the results of two consecutive iterations is less than the minimum acceptance precision, the process of optimization will be fixed.

In the clustering analysis, whose results are summarized in Table 3, all cases were classified into two classes based on the results of the optimum partition. The optimum partition, determined the Euclidean distance of each case study from the center of each class and then the class of each case study, is indicated in the fourth column (Defined class). If the class of sample conformed to the initial assumption, the clustering was verified as “Satisfied”, otherwise it was not verified, and labelled as “Not Satisfied”. For instance, Case 3 had Euclidean distances equal to 0.474 from Class 1 and 0.352 from Class 2; hence, this case belonged to Class 2. Besides this, Case 15 had Euclidean distances equal to 0.392 from Class 1 and 0.751 from Class 2; hence, this case belonged to Class 1. In the fifth column, the values of the factor of safety for the considered slopes are indicated, and a comparison was made between the fourth and fifth columns. For instance, in Case 10, the Euclidean

distances were calculated as 1.053 and 1.158 from Class 1 and Class 2, respectively; hence, Case 10 should belong to Class 1, while it belonged to the Class 2 according to the value of the FS—therefore, it could be concluded that this case was wrongly classified. Afterwards, the accuracy of the results provided by the HS algorithm were verified, as shown in the last column. As can be seen, it can be concluded that the model was able to correctly classify 17 of 19 case studies, with a model accuracy equal to 89%.

5. Results and Discussion

In this study, the 19 case studies were classified into two classes by a combination of the HS algorithm and the K-means algorithm. As mentioned earlier, the accuracy of this modelling was 89%, with only two cases that were classified incorrectly. Consequently, it was found that the combination of HS algorithm and K-means can provide high performance capacity in clustering and evaluating slope stability. Figure 5 shows the result of clustering.

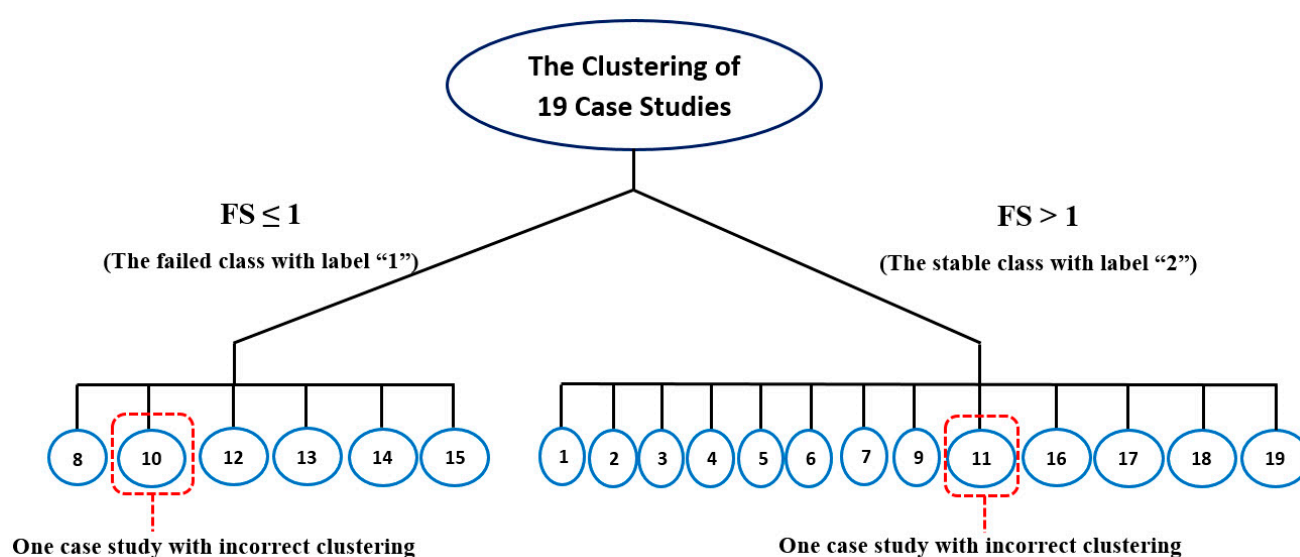


Figure 5. Results of clustering with two classes by the HS algorithm.

In order to evaluate the effect of each parameter on slope stability, referring to the dataset considered in this study, the Euclidean distance of each effective parameter from the centre of each class was determined and measured. The results are shown in Table 4.

Table 4. Euclidean distance of each parameter from the centre of the concerning class.

Clusters	γ	c'	φ'	α	H	r_u
Class 1 (Failed class)	0.753	0.063	0.922	0.812	0.736	0.714
Class 2 (Stable class)	0.844	0.001	0.484	0.462	0.237	0.702

According to Table 4, for all cases in Class 1 (failed class), the cohesion (c') was the most significant parameter, with an Euclidean distance equal to 0.063, followed by the pore pressure ratio (r_u), slope height (H), unit weight (γ) and slope angle (α), with Euclidean distances equal to, respectively, 0.714, 0.736, 0.753 and 0.812—contributing greatly to the existence of instability in the slopes. Finally, the angle of shearing resistance (φ') had the lowest impact on the stability of the analyzed cases. It is worth pointing out that this latter statement holds only for the considered dataset and does not have general validity. In Class 2 (the stable class), the cohesion (c') had the most significant effect on all the cases. It is worth mentioning that the cohesion (c') played a key role in the stability of slopes in both classes, including the failed and stable classes. In addition, the unit weight (γ) had the lowest impact on all cases in the second class, with a Euclidean distance equal to 0.844.

On the one hand, one of the most important strengths of clustering techniques as unsupervised methods is that it is possible to analyze problems using a limited amount of data. On other hand, in some geotechnical problems, it is difficult to prepare laboratory and field data; therefore, based on the results of this study, it can be concluded that a combination of the HS and k-means algorithms as a clustering approach can be used as a reliable system for modelling and evaluating some problems involved in geotechnical engineering. In addition, it should be noted that this model is unique, and it cannot be used directly in other case studies.

6. Conclusions

This paper presents an application of the machine learning technique to slope stability analysis by using a combination of the HS and K-means algorithms. For this propose, 19 case studies documented in the literature were considered, for which the most relevant geotechnical parameters and geometrical characteristics were collected. Specifically, these were the slope height (H), slope angle (α), intercept cohesion (c'), angle of shearing resistance (ϕ'), soil unit weight (γ) and pore pressure ratio (r_u). Besides this, all the considered slopes were characterized by a circular slip surface. A clustering analysis was conducted by defining two classes: the first class included unstable slopes ($FS \leq 1$), whereas the second class concerned stable slopes ($FS > 1$). Then, a comparison was made between the results of the clustering analysis and the actual FS. The figures show that 17 of the 19 case studies were classified correctly into the two classes, with a model accuracy equal to 89%. In addition, for case studies considered in this work, the intercept cohesion (c') was the parameter that exerted the greatest influence on slope stability for both classes (failed and stable). The purpose of this manuscript was to demonstrate the capability of the HS algorithm of dealing with problems of slope stability. Even though this approach cannot outperform the methods currently commonly employed in geotechnical engineering (such as limit equilibrium methods or the strength reduction method), it could be taken into account for providing a primary evaluation of slope stability conditions. From a general point of view, the HS algorithm should be employed by using a much larger dataset to 'learn' how the factor of safety is affected by input parameters (geotechnical and geometrical). Afterwards, the algorithm can be used to perform a preliminary evaluation of the stability conditions of a slope that is not already included in the database. In this context, the dataset could be built referring to a set of slopes included in a particular geographical region.

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References

- Alonso, E.E.; Pinyol, N.M. Criteria for rapid sliding I. A review of Vaiont case. *Eng. Geol.* **2010**, *114*, 198–210. [\[CrossRef\]](#)
- Hungr, O.; Leroueil, S.; Picarelli, L. The Varnes classification of landslide type, an update. *Landslide* **2014**, *11*, 167–194. [\[CrossRef\]](#)
- Iverson, R.M.; George, D.L.; Allstadt, K.; Reid, M.E.; Collins, B.D.; Vallance, J.W.; Schilling, S.P.; Godt, J.W.; Cannon, C.M.; Magirl, C.S.; et al. Landslide mobility and hazards: Implications of the 2014 Oso disaster. *EPSL Earth Planet. Sci. Lett.* **2015**, *412*, 197–208. [\[CrossRef\]](#)
- Conte, E.; Donato, A.; Pugliese, L.; Troncone, A. Analysis of the Maierato landslide (Calabria, Southern Italy). *Landslides* **2018**, *15*, 1935–1950. [\[CrossRef\]](#)
- Leroueil, S. Natural slopes and cuts: Movement and failure mechanisms. *Géotechnique* **2001**, *51*, 197–243. [\[CrossRef\]](#)
- Pirulli, M.; Pastor, M. Numerical study on the entrainment of bed material into rapid landslides. *Géotechnique* **2012**, *62*, 959–972. [\[CrossRef\]](#)
- Yerro, A.; Alonso, E.E.; Pinyol, N.M. Run-out of landslides in brittle soils. *Comput. Geotech.* **2016**, *80*, 427–439. [\[CrossRef\]](#)
- Calvetti, F.; di Prisco, C.; Vairaktaris, E. DEM assessment of impact forces of dry granular masses on rigid barriers. *Acta Geotech.* **2017**, *12*, 129–144. [\[CrossRef\]](#)
- Conte, E.; Pugliese, L.; Troncone, A. Post-failure stage simulation of a landslide using the material point method. *Eng. Geol.* **2019**, *253*, 149–159. [\[CrossRef\]](#)
- Troncone, A.; Conte, E.; Pugliese, L. Analysis of the Slope Response to an Increase in Pore Water Pressure Using the Material Point Method. *Water* **2019**, *11*, 1446. [\[CrossRef\]](#)
- Yerro, A.; Soga, K.; Bray, J. Runout evaluation of Oso landslide with the material point method. *Can. Geotech. J.* **2019**, *56*, 1304–1317. [\[CrossRef\]](#)
- Conte, E.; Pugliese, L.; Troncone, A. Post-failure analysis of the Maierato landslide using the material point method. *Eng. Geol.* **2020**, *277*, 105788. [\[CrossRef\]](#)
- Troncone, A.; Pugliese, L.; Conte, E. Run-Out Simulation of a Landslide Triggered by an Increase in the Groundwater Level Using the Material Point Method. *Water* **2020**, *12*, 2817. [\[CrossRef\]](#)
- Choudhury, R.; Flentje, P.; Bhattacharya, G. *Geotechnical Slope Analysis*; CRC Press: Boca Raton, FL, USA, 2009.
- Conte, E.; Troncone, A. An analytical method for predicting the mobility of slow moving landslides owing to groundwater fluctuations. *J. Geotech. Geoenviron. Eng. ASCE* **2011**, *137*, 777–784. [\[CrossRef\]](#)
- Conte, E.; Troncone, A. Stability analysis of infinite clayey slopes subjected to pore pressure changes. *Géotechnique* **2012**, *62*, 87–91. [\[CrossRef\]](#)
- Conte, E.; Troncone, A. A method for the analysis of soil slips triggered by rainfall. *Géotechnique* **2012**, *62*, 187–192. [\[CrossRef\]](#)
- Conte, E.; Troncone, A. Simplified approach for the analysis of rainfall-induced landslides. *J. Geotech. Geoenviron. Eng. ASCE* **2012**, *138*, 398–406. [\[CrossRef\]](#)
- Duncan, J.M.; Wright, S.G.; Brandon, T.L. *Soil Strength and Slope Stability*, 2nd ed.; Wiley: Hoboken, NJ, USA, 2014.
- Troncone, A.; Pugliese, L.; Parise, A.; Conte, E. Prediction of Slow-Moving Landslide Mobility Due to Rainfall Using a Two-Wedges Model. *Water* **2021**, *13*, 2030. [\[CrossRef\]](#)
- Troncone, A.; Pugliese, L.; Lamanna, G.; Conte, E. Prediction of rainfall-induced landslide movements in the presence of stabilizing piles. *Eng. Geol.* **2021**, *288*, 106143. [\[CrossRef\]](#)
- Griffiths, D.V.; Lane, P.A. Slope stability analysis by finite elements. *Géotechnique* **1999**, *49*, 387–403. [\[CrossRef\]](#)
- Potts, D.M.; Zdravkovic, L. *Finite Element Analysis in Geotechnical Engineering: Application*; Thomas Telford: London, UK, 2001.
- Troncone, A. Numerical analysis of a landslide in soils with strain-softening behaviour. *Géotechnique* **2005**, *55*, 585–596. [\[CrossRef\]](#)
- Troncone, A.; Conte, E.; Donato, A. Two and three-dimensional numerical analysis of the progressive failure that occurred in an excavation-induced landslide. *Eng. Geol.* **2014**, *183*, 265–275. [\[CrossRef\]](#)
- Cislaghi, A.; Cohen, D.; Gasser, E.; Bischetti, G.B.; Schwarz, M. Field measurements of passive earth forces in steep, shallow, landslide-prone areas. *J. Geophys. Res. Earth Surf.* **2019**, *124*, 838–866. [\[CrossRef\]](#)
- Cohen, D.; Schwarz, M. Tree-root control of shallow landslides. *Earth Surf. Dyn.* **2017**, *5*, 451–477. [\[CrossRef\]](#)
- Rosso, R.; Rulli, M.C.; Vannucchi, G. A physically based model for the hydrologic control on shallow landsliding. *Water Resour. Res.* **2006**, *42*, W06410. [\[CrossRef\]](#)
- Bordoni, M.; Bonì, R.; Colombo, A.; Lanteri, L.; Meisina, C. A methodology for ground motion area detection (GMA-D) using A-DInSAR time series in landslide investigations. *CATENA* **2018**, *163*, 89–110. [\[CrossRef\]](#)
- Michalowski, R.L. Stability charts for uniform slopes. *J. Geotech. Geoenviron. Eng.* **2002**, *128*, 351–355. [\[CrossRef\]](#)
- Kang, F.; Li, J. Artificial bee colony algorithm optimized support vector regression for system reliability analysis of slopes. *J. Comput. Civ. Eng.* **2016**, *30*, 04015040. [\[CrossRef\]](#)
- Li, D.Q.; Yang, Z.Y.; Cao, Z.J.; Au, S.K.; Phoon, K.K. System reliability analysis of slope stability using generalized subset simulation. *Appl. Math. Model.* **2017**, *46*, 650–664. [\[CrossRef\]](#)
- Mahdiyar, A.; Hasanipanah, M.; Armaghani, D.J.; Gordan, B.; Abdullah, A.; Arab, H.; Majid, M.Z.A. A Monte Carlo technique in safety assessment of slope under seismic condition. *Eng. Comput.* **2017**, *33*, 807–817. [\[CrossRef\]](#)
- Wang, Y.; Cao, Z.; Au, S.K. Practical reliability analysis of slope stability by advanced Monte Carlo simulations in a spreadsheet. *Can. Geotech. J.* **2011**, *48*, 162–172. [\[CrossRef\]](#)
- Koopialipoor, M.; Armaghani, D.J.; Hedayat, A.; Marto, A.; Gordan, B. Applying various hybrid intelligent systems to evaluate and predict slope stability under static and dynamic conditions. *Soft Comput.* **2019**, *23*, 5913–5929. [\[CrossRef\]](#)
- Mishra, M.; Gunturi, V.R.; Miranda, T.F.D.S. Slope stability analysis using recent metaheuristic techniques: A comprehensive survey. *SN Appl. Sci.* **2019**, *1*, 1674. [\[CrossRef\]](#)

37. Yuan, C.; Moayed, H. The performance of six neural-evolutionary classification techniques combined with multi-layer perception in two-layered cohesive slope stability analysis and failure recognition. *Eng. Comput.* **2020**, *36*, 1705–1714. [\[CrossRef\]](#)
38. Mishra, M.; Ramana, G.V.; Maity, D. Multiverse optimisation algorithm for capturing the critical slip surface in slope stability analysis. *Geotech. Geol. Eng.* **2020**, *38*, 459–474. [\[CrossRef\]](#)
39. Mikaeil, R.; Haghshenas, S.S.; Hoseinie, S.H. Rock penetrability classification using artificial bee colony (ABC) algorithm and self-organizing map. *Geotech. Geol. Eng.* **2018**, *36*, 1309–1318. [\[CrossRef\]](#)
40. Salemi, A.; Mikaeil, R.; Haghshenas, S.S. Integration of finite difference method and genetic algorithm to seismic analysis of circular shallow tunnels (Case study: Tabriz urban railway tunnels). *KSCE J. Civ. Eng.* **2018**, *22*, 1978–1990. [\[CrossRef\]](#)
41. Haghshenas, S.S.; Faradonbeh, R.S.; Mikaeil, R.; Haghshenas, S.S.; Taheri, A.; Saghatforoush, A.; Dormishi, A. A new conventional criterion for the performance evaluation of gang saw machines. *Measurement* **2019**, *146*, 159–170. [\[CrossRef\]](#)
42. Hosseini, S.M.; Ataei, M.; Khalokakaei, R.; Mikaeil, R.; Haghshenas, S.S. Study of the effect of the cooling and lubricant fluid on the cutting performance of dimension stone through artificial intelligence models. *Int. J. Eng. Sci. Technol.* **2020**, *23*, 71–81. [\[CrossRef\]](#)
43. Mikaeil, R.; Bakhshinezhad, H.; Haghshenas, S.S.; Ataei, M. Stability analysis of tunnel support systems using numerical and intelligent simulations (case study: Kouhin Tunnel of Qazvin-Rasht Railway). *RGN Zbornik* **2019**, *34*, 1–10. [\[CrossRef\]](#)
44. Mikaeil, R.; Haghshenas, S.S.; Sedaghati, Z. Geotechnical risk evaluation of tunneling projects using optimization techniques (case study: The second part of Emamzade Hashem tunnel). *Nat. Hazards* **2019**, *97*, 1099–1113. [\[CrossRef\]](#)
45. Dormishi, A.; Ataei, M.; Mikaeil, R.; Khalokakaei, R.; Haghshenas, S.S. Evaluation of gang saws' performance in the carbonate rock cutting process using feasibility of intelligent approaches. *Int. J. Eng. Sci. Technol.* **2019**, *22*, 990–1000. [\[CrossRef\]](#)
46. Noori, A.M.; Mikaeil, R.; Mokhtarian, M.; Haghshenas, S.S.; Foroughi, M. Feasibility of intelligent models for prediction of utilization factor of TBM. *Geotech. Geol. Eng.* **2020**, *38*, 3125–3143. [\[CrossRef\]](#)
47. Keshtegar, B.; Correia, J.A.; Trung, N.T. Optimisation of nanocomposite pipes under internal fluid reinforced by FRP and CNTs under seismic load. *IJHM Int. J. Hydromechatron.* **2020**, *3*, 213–227. [\[CrossRef\]](#)
48. Geem, Z.W.; Kim, J.H.; Loganathan, G.V. A new heuristic optimization algorithm: Harmony search. *Simulation* **2001**, *76*, 60–68. [\[CrossRef\]](#)
49. Lee, K.S.; Geem, Z.W. A new structural optimization method based on the harmony search algorithm. *Comput Struct.* **2004**, *82*, 781–798. [\[CrossRef\]](#)
50. Lee, K.S.; Geem, Z.W. A new meta-heuristic algorithm for continuous engineering optimization: Harmony search theory and practice. *Comput. Methods Appl. Mech. Eng.* **2005**, *194*, 3902–3933. [\[CrossRef\]](#)
51. Geem, Z.W. Optimal cost design of water distribution networks using harmony search. *Eng. Optim.* **2006**, *38*, 259–277. [\[CrossRef\]](#)
52. Geem, Z.W. Harmony search optimisation to the pump-included water distribution network design. *Civ. Eng. Environ. Syst.* **2009**, *26*, 211–221. [\[CrossRef\]](#)
53. Geem, Z.W.; Cho, Y.H. Optimal design of water distribution networks using parameter-setting-free harmony search for two major parameters. *J. Water Resour. Plan. Manag.* **2011**, *137*, 377–380. [\[CrossRef\]](#)
54. Moayedikia, A.; Ong, K.L.; Boo, Y.L.; Yeoh, W.G.; Jensen, R. Feature selection for high dimensional imbalanced class data using harmony search. *Eng. Appl. Artif. Intell.* **2017**, *57*, 38–49. [\[CrossRef\]](#)
55. De Paola, F.; Galdiero, E.; Giugni, M. Location and setting of valves in water distribution networks using a harmony search approach. *J. Water Resour. Plan. Manag.* **2017**, *143*, 04017015. [\[CrossRef\]](#)
56. Gnawali, K.; Han, K.H.; Geem, Z.W.; Jun, K.S.; Yum, K.T. Economic Dispatch Optimization of Multi-Water Resources: A Case Study of an Island in South Korea. *Sustainability* **2019**, *11*, 5964. [\[CrossRef\]](#)
57. Nazari-Heris, M.; Mohammadi-Ivatloo, B.; Asadi, S.; Geem, Z.W. Large-scale combined heat and power economic dispatch using a novel multi-player harmony search method. *Appl. Therm. Eng.* **2019**, *154*, 493–504. [\[CrossRef\]](#)
58. Wang, X.; Gao, X.Z.; Zenger, K. *An Introduction to Harmony Search Optimization Method*; Springer International Publishing: New York, NY, USA, 2015; ISBN 978-3-319-08355-1. [\[CrossRef\]](#)
59. Feng, X. *Introduction of Intelligent Rock Mechanics*; Science Press: Beijing, China, 2000; pp. 239–241.
60. Lu, P.; Rosenbaum, M.S. Artificial neural networks and grey systems for the prediction of slope stability. *Nat. Hazards* **2003**, *30*, 383–398. [\[CrossRef\]](#)
61. Li, X. Comparative Studies of Artificial Neural Networks and Adaptive Neuro-Fuzzy Inference System-Based Approach for the Circular Sliding Slopes Stability Analysis. Master's Thesis, University of South China, Hengyang, China, 2004.
62. Huang, Z.; Cui, J.; Liu, H. Chaotic neural network method for slope stability prediction. *Chin. J. Rock Mech. Eng.* **2004**, *22*, 015.
63. Sakellariou, M.G.; Ferentinou, M.D. A study of slope stability prediction using neural networks. *Geotech. Geol. Eng.* **2005**, *23*, 419. [\[CrossRef\]](#)
64. Wang, H.B.; Xu, W.Y.; Xu, R.C. Slope stability evaluation using back propagation neural networks. *Eng. Geol.* **2005**, *80*, 302–315. [\[CrossRef\]](#)
65. Samui, P. Slope stability analysis: A support vector machine approach. *Environ. Geol.* **2008**, *56*, 255. [\[CrossRef\]](#)
66. Zhao, H.B. Slope reliability analysis using a support vector machine. *Comput. Geotech.* **2008**, *35*, 459–467. [\[CrossRef\]](#)
67. Das, S.K.; Biswal, R.K.; Sivakugan, N.; Das, B. Classification of slopes and prediction of factor of safety using differential evolution neural networks. *Environ. Earth Sci.* **2011**, *64*, 201–210. [\[CrossRef\]](#)
68. Erzin, Y.; Cetin, T. The prediction of the critical factor of safety of homogeneous finite slopes using neural networks and multiple regressions. *Comput. Geosci.* **2013**, *51*, 305–313. [\[CrossRef\]](#)
69. Liu, Z.; Shao, J.; Xu, W.; Chen, H.; Zhang, Y. An extreme learning machine approach for slope stability evaluation and prediction. *Nat. Hazards* **2014**, *73*, 787–804. [\[CrossRef\]](#)

-
70. Gordan, B.; Armaghani, D.J.; Hajihassani, M.; Monjezi, M. Prediction of seismic slope stability through combination of particle swarm optimization and neural network. *Eng. Comput.* **2016**, *32*, 85–97. [[CrossRef](#)]
 71. Hoang, N.D.; Pham, A.D. Hybrid artificial intelligence approach based on metaheuristic and machine learning for slope stability assessment: A multinational data analysis. *Expert Syst. Appl.* **2016**, *46*, 60–68. [[CrossRef](#)]
 72. Suman, S.; Khan, S.Z.; Das, S.K.; Chand, S.K. Slope stability analysis using artificial intelligence techniques. *Nat Hazards* **2016**, *84*, 727–748. [[CrossRef](#)]
 73. Verma, A.K.; Singh, T.N.; Chauhan, N.K.; Sarkar, K. A hybrid FEM–ANN approach for slope instability prediction. *J. Inst. Eng. India A* **2016**, *97*, 171–180. [[CrossRef](#)]
 74. Fattahi, H. Prediction of slope stability using adaptive neuro-fuzzy inference system based on clustering methods. *J. Min. Environ.* **2017**, *8*, 163–177.
 75. Rukhaiyar, S.; Alam, M.N.; Samadhiya, N.K. A PSO-ANN hybrid model for predicting factor of safety of slope. *J. Geotech. Eng.* **2018**, *12*, 556–566. [[CrossRef](#)]
 76. Xue, X. Prediction of slope stability based on hybrid PSO and LSSVM. *J. Comput. Civ. Eng.* **2017**, *31*, 04016041. [[CrossRef](#)]
 77. Chakraborty, A.; Goswami, D. Prediction of slope stability using multiple linear regression (MLR) and artificial neural network (ANN). *Arab. J. Geosci.* **2017**, *10*, 385. [[CrossRef](#)]
 78. Feng, X.; Li, S.; Yuan, C.; Zeng, P.; Sun, Y. Prediction of slope stability using naive Bayes classifier. *KSCE J. Civ. Eng.* **2018**, *22*, 941–950. [[CrossRef](#)]
 79. Salmasi, F.; Jafari, F. A simple direct method for prediction of safety factor of homogeneous finite slopes. *Geotech. Geol. Eng.* **2019**, *37*, 3949–3959. [[CrossRef](#)]
 80. Sah, N.K.; Sheorey, P.R.; Upadhyaya, L.N. Maximum likelihood estimation of slope stability. *Int. J. Rock Mech. Min.* **1994**, *31*, 47–53. [[CrossRef](#)]
 81. Lloyd, S.P. Least squares quantization in pcm. *IEEE Trans. Inf. Theory* **1982**, *28*, 129–137. [[CrossRef](#)]
 82. Guido, G.; Haghshenas, S.S.; Haghshenas, S.S.; Vitale, A.; Astarita, V.; Haghshenas, A.S. Feasibility of Stochastic Models for Evaluation of Potential Factors for Safety: A Case Study in Southern Italy. *Sustainability* **2020**, *12*, 7541. [[CrossRef](#)]
 83. Hosseini, S.M.; Ataei, M.; Khalokakaei, R.; Mikaeil, R.; Haghshenas, S.S. Investigating the role of coolant and lubricant fluids on the performance of cutting disks (case study: Hard rocks). *RGN Zbornik* **2019**, *34*, 13–24. [[CrossRef](#)]
 84. Mikaeil, R.; Haghshenas, S.S.; Shirvand, Y.; Hasanluy, M.V.; Roshanaei, V. Risk assessment of geological hazards in a tunneling project using harmony search algorithm (case study: Ardabil-Mianeh railway tunnel). *Civ. Eng. J.* **2016**, *2*, 546–554. [[CrossRef](#)]
 85. Mikaeil, R.; Ozcelik, Y.; Ataei, M.; Shaffiee Haghshenas, S. Application of harmony search algorithm to evaluate performance of diamond wire saw. *J. Min. Environ.* **2019**, *10*, 27–36.