

Article

Watershed Variability in Streambank Erodibility and Implications for Erosion Prediction

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Abstract: Two fluvial erosion models are commonly used to simulate the erosion rate of cohesive soils: the empirical excess shear stress model and the mechanistic Wilson model. Both models include two soil parameters, the critical shear stress (τ_c) and the erodibility coefficient (k_d) for the excess shear stress model and b_0 and b_1 for the Wilson model. Jet erosion tests (JETs) allow for in-situ determination of these parameters. JETs were completed at numerous sites along two streams in each the Illinois River and Fort Cobb Reservoir watersheds. The objectives were to use JET results from these streambank tests to investigate variability of erodibility parameters on the watershed scale and investigate longitudinal trends in streambank erodibility. The research also determined the impact of this variability on lateral retreat predicted by a process-based model using both the excess shear stress model and the Wilson model. Parameters derived from JETs were incorporated into a one-dimensional process-based model to simulate bank retreat for one stream in each watershed. Erodibility parameters varied by two to five and one to two orders of magnitude in the Illinois River watershed and Fort Cobb Reservoir watershed, respectively. Less variation was observed in predicted retreat by a process-based model compared to the input erodibility parameters. Uncalibrated erodibility parameters and simplified applied shear stress estimates failed to match observed lateral retreats suggesting the need for model calibration and/or advanced flow modeling.

Keywords: ERODIBILITY parameters; jet erosion test; variability; streambank retreat

1. Introduction

Excess sediment continues to be a major pollutant of surface waters in the United States, with streambank erosion being a primary contributor [1,2]. Streambank erosion is a complex process that involves three primary mechanisms (subaerial processes, fluvial erosion, and mass wasting) and is driven by several soil properties that are spatially variable. Subaerial processes include wetting/drying cycles, freeze/thaw cycles, and other processes that weaken the streambank soil [3]. Mass wasting or geotechnical failure occurs when there is an imbalance between the forces resisting erosion and the gravitational forces acting on the streambank. Fluvial erosion is a continual process in which soil particles are detached by the hydraulic forces from streamflow when the applied shear stress exceeds a critical shear stress for the soil. Many streambank erosion models simulate both fluvial erosion and mass wasting processes.

Several particle detachment models are used to predict fluvial erosion for cohesive sediments with the most common being the linear excess shear stress model [4–6]:

$$\varepsilon_r = k_d(\tau - \tau_c)^a \quad (1)$$

where ε_r is the erosion rate (cm s^{-1}), k_d is the erodibility coefficient ($\text{cm}^3 \text{N}^{-1} \text{s}^{-1}$), τ is the average hydraulic boundary shear stress (Pa), τ_c is the critical shear stress (Pa), and a is an empirical exponent that is assumed to be one. Once the τ exerted by the water in a stream exceeds the τ_c of the soil, erosion begins at a rate of k_d . The two erodibility parameters, τ_c and k_d , are soil dependent. Models such as the Bank Stability and Toe Erosion Model (BSTEM), Conservational Channel Evolution and Pollutant Transport System (CONCEPTS), and HEC-RAS with BSTEM use the linear excess shear–stress equation to predict fluvial erosion and require τ_c and k_d as input [7–9]. The Soil and Water Assessment Tool (SWAT) either allows the user to input τ_c and k_d , or calculates the parameters based on soil characteristics and empirical relationships [10].

A nonlinear mechanistic detachment model was developed by Wilson [11,12] based on a two-dimensional representation of soil particles to predict fluvial erosion of soil particles and aggregates:

$$\varepsilon_r = b_0 \sqrt{\tau} \left[1 - \exp \left\{ - \exp \left(3 - \frac{b_1}{\tau} \right) \right\} \right] \quad (2)$$

where b_0 ($\text{g m}^{-1} \text{s}^{-1} \text{N}^{-0.5}$) and b_1 (Pa) are the mechanistically derived parameters of the model. Physically, b_0 is similar to k_d and b_1 is similar to τ_c [13,14] in Equation (1). The Wilson model parameters, b_0 and b_1 , must be currently measured and cannot be estimated a priori from soil properties. The benefit of the Wilson model is that it simulates fluvial erosion as a nonlinear process which may be more representative of actual erosion processes at higher applied τ than typically used in erosion testing.

Various techniques can be used to measure the excess shear stress parameters, τ_c and k_d , as well as the Wilson model parameters, b_0 and b_1 , such as flume studies, hole erosion tests, and submerged jets. While flume studies and hole erosion tests can be used to measure parameters in laboratory settings, a submerged jet test, known as the Jet Erosion Test (JET), was developed to measure erodibility parameters in situ [6]. The JET impinges a small jet of water into the streambank at a constant pressure to create a scour hole. Scour depth is measured over time to determine the erosion rate. Field JETs rely on the use of a constant head tank or a pressure gauge and water pumped from a nearby stream. Several solver techniques (Blaisdell, scour depth, and iterative solutions) can be used to fit the measured data and iteratively solve for τ_c and k_d based on the measurements from the JET [15–17]. The Wilson model parameters can also be determined from the JET using the analysis described by Al-Madhhachi et al. [17].

Physical, geochemical, and biological properties of soil are thought to influence fluvial erodibility [18]. Soil particle size is an important factor when considering soil erodibility. For cohesive soils, the higher amount of clay-sized particles causes higher levels of cohesion and more resistance to erosion while higher amounts of sand-sized particles cause less resistance to erosion [19]. Particle sizes of the streambed and banks tend to exhibit longitudinal trends, which may contribute to longitudinal trends in soil erodibility. Bed particle size tends to decrease downstream [20–23], as the larger particles settle out more quickly and the finer particles can be transported further downstream.

Streambank soil type can be highly variable throughout a watershed and along the streambanks, but bank material also tends to become finer downstream [24]. This can be attributed to the historical deposition of fine sediments in floodplains, which are often areas of sediment storage within a watershed [25]. Historically, sediment was deposited in floodplains which led to channelization, reduction of floodplain storage capacity, and the acceleration of channel erosion in downstream reaches [26,27]. Similarly, higher cohesion of streambank material due to downstream fining has also been observed [24] and could also contribute to an increased resistance to erosion downstream. Konsoer et al. [28] measured soil particle size, cohesion, and τ_c of streambank soils around two meander bends, each approximately 5 km in length, of the Wabash River in Illinois. Bank materials, cohesion and the τ_c varied between the two river bends and within each bend. Percentage of fines in the soil increased downstream on the first bend and was more uniform in the downstream bend. The authors concluded that the variation in particle size was most likely due to the variability of riparian vegetation and floodplain development due to deposition. However, no significant change in τ_c or k_d was observed along the river.

Wynn et al. [29] performed JETs at six sites along Stroubles Creek in Virginia and observed four orders of magnitude variation in k_d , but only one order of magnitude variation in τ_c . The same soil was tested in a laboratory setting where it was packed to a consistent bulk density and moisture content. The remolded samples exhibited less variability in erodibility parameters than the field JETs, suggesting that variations in bulk density (BD) and moisture content may also account for some of the variability in the field.

Typically, multiple JETs are performed at a site and average τ_c and k_d (or b_0 and b_1) are used in predictive modeling. Only a few studies have investigated how parameters vary on the watershed scale [13] and single values of τ_c and k_d are still widely used for an entire watershed. While in situ testing with the JET is recommended to determine erodibility parameters [9], running multiple tests at multiple sites within a watershed or stream system becomes time consuming, as it takes at least an hour to run a single JET. The amount of tests needed to adequately characterize the erodibility parameters for each site of interest on an entire stream reach or watershed may be very high and access to certain locations may be limited. Ideally, soil properties could be measured at a few sites and the values extrapolated to other sites within the stream system. However, this could potentially be a major source of uncertainty due to the highly variable nature of streambank soil and bed sediment properties [30]. Therefore, an understanding of how the parameters vary within the specific stream or at the watershed scale is important to validate such an extrapolation. If a longitudinal trend in erodibility is present, this may allow for the results from the JETs to be extended up and downstream of the test locations.

Therefore, the objectives of this study were: (i) to investigate variability of streambank fluvial erodibility parameters for both the excess shear stress model and the Wilson model obtained from the JETs on the watershed scale, (ii) to investigate longitudinal trends in fluvial erodibility parameters obtained from the JET within two contrasting watersheds; and (iii) to determine the impact of this variability on predicted lateral retreat by a process-based streambank erosion and failure model when using both the excess shear stress model and Wilson model for simulating fluvial erosion.

2. Materials and Methods

2.1. Watershed Description

The Fort Cobb Reservoir watershed (Figure 1), which is located in western Oklahoma and the Central Great Plains ecoregion, has been selected for this study. The Fort Cobb Reservoir, which provides public water supply, recreation, and wildlife habitat, is on the Oklahoma 303(d) list for impairment by nutrients, sediments, and siltation [31], as well as its four main tributaries. The watershed has a drainage area of 878 km², a 270 km-long stream network and elevations ranging from 387 to 564 m [32]. Generally, sandy-textured soils are found in the central and eastern parts of the watershed, and silty loams are found in the western part and about 20% of the watershed is overlain by highly erosive soils [33]. The watershed is predominately agricultural, with cropland and grazing land accounting for 56% and 34% of the watershed area, respectively, while roads and urban areas account for 5% of the watershed and water less than 2% [34]. Numerous upland and riparian conservation practices (reduced or no-till cropland, conversion of cropland to pastureland, terracing, riparian buffers, cattle exclusion from streams, etc.) and various structural and water management practices to reduce sediment loading were implemented in the Fort Cobb Reservoir watershed as part of the Conservation Effects Assessment Project, CEAP [35]. However, the reservoir still fails to meet water quality standards based on sediment concentrations. Using radionuclide tracers, it was determined that 50% of the suspended sediment in Fort Cobb Reservoir originated from unstable tributary streambanks [1]. Reservoir tributaries are narrow (<10 m), shallow (<2 m) and sandy streams. Streambanks in the watershed consist of either single sand or sandy loam layer, while others exhibit layering with sand or sandy loam layers and more cohesive layers with higher clay content.

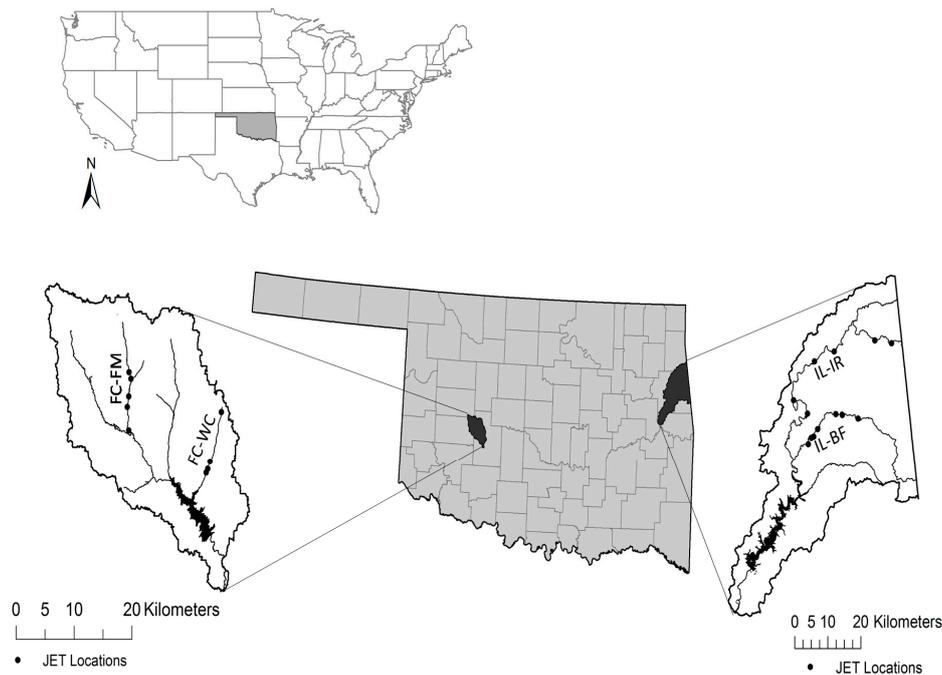


Figure 1. Selected field data collection sites along Fivemile (FC-FM) and Willow Creeks (FC-WC) in the Fort Cobb (FC) Reservoir Watershed and the Illinois River (IL-IR) and Barren Fork Creek (IL-BF) in the Illinois River (IR) Watershed.

This research used an additional set of JET data from the Illinois River watershed (Figure 1). The watershed has a drainage area of 4330 km² with approximately 54% of the watershed located in Oklahoma and the remaining portion in Arkansas [13]. The stream network length is 1000 km and elevations range from 136 to 601 m. The majority of the Oklahoma portion of the watershed is in the Ozark Highland Ecoregion. Soils in the Oklahoma portion of the watershed are gravelly or stony loams. This part of the watershed is forested, with most of the remaining land used for hay production or pasture, and includes Tenkiller Ferry Lake, which provides drinking water to a large portion of the region. Streams are typically clear, high gradient, riffle and pool type with coarse gravel, cobble, boulder, and bedrock substrates. Many have been designated scenic rivers and have created a recreational and tourism industry for the area [36,37]. Streambanks in the watershed are comprised of a cohesive silty loam top layer above an unconsolidated gravel layer [8,13].

2.2. Jet Erosion Tests

For each watershed, two streams reaches were selected and JETs were carried out at several sites along these streams using the “mini”-JET device (Table 1). Within the Fort Cobb (FC) Reservoir watershed, eight sites were selected along two of the main tributaries to the reservoir, Fivemile Creek (FC-FM) and Willow Creek (FC-WC) based on accessibility (Figure 1). These tributaries are located on opposite sides of the watershed and the sites were selected to be representative of the entire watershed. JETs were conducted at four sites along a 10.25-km reach of FC-FM and four sites along a 10.1-km reach of FC-WC between March and September 2014 using the “mini”-JET device [17]. Since the clay layer was not exposed at all sites, JET results from only the sand layer will be used in this study. Within the Illinois River (IL) basin, “mini”-JETs were conducted at seven sites along a 25.5-km reach of Barren Fork Creek (IL-BF) and six sites along a 69.1-km reach of the Illinois River (IL-IR) between October 2011 and April 2012. JETs were only conducted in the silty loam layer. At least two JETs were performed at each site. One or two additional tests were performed if time allowed.

Table 1. Characteristics of streams selected for jet erosion tests.

Stream *	Length (km)	Bank Description	# of Sites	JET Testing Period
FC-FM	10.25	Homogeneous sandy loam, or composite	4	March–September 2014
FC-WC	10.1	sandy loam and clay layers	4	March–September 2014
IL-BF	25.5	Composite, silty loam top layer with an	7	October 2011–April 2012
IL-IR	69.1	unconsolidated gravel toe	6	October 2011–April 2012

Notes: * FC-FM = Fivemile Creek; FC-WC = Willow Creek; IL-BF = Barren Fork Creek; IL-IR = Illinois River.

The operation of the JETs followed previously described protocols for the “mini”-JET [13,14,17]. Heads ranged from 31 to 46 cm in the sand layer for FC-FM and FC-WC and 57 to 345 cm for IL-BF and IL-IR. At least one 5-cm diameter by 5-cm long cylindrical soil core sample was taken from the streambank at each site. The cylindrical soil core sample was used to determine bulk density and moisture content for each site. At least one soil sample taken at each site was analyzed for particle size using a hydrometer and sieve analysis according to ASTM Standards D421 [38] and D422 [39].

The scour depth solution, developed by Daly et al. [40], was used to derive erodibility parameters from recorded scour depths, time, and constant head setting. This technique minimizes the sum of squared errors (SSE) between measured scour and predicted scour from the excess shear stress model by using an initial guess and solver routine to determine τ_c and k_d . Wilson model parameters were also derived from observed data using a similar technique as the scour depth approach following Al-Madhhachi et al. [17]. The approach minimizes the error between the measured data and the functional solutions using the solver routine in Microsoft Excel, which utilized the generalized reduced gradient method. Constraints were used within the Excel solver routine to limit potential solutions of the Wilson model parameters (b_0 and b_1), as recommended by Wilson [11,12] and Al-Madhhachi et al. [17].

Statistical analyses were performed using Mini-Tab 17 (Mini-Tab, Inc., State College, PA, USA) and Sigma-Plot 12.5 (Systat Software, Inc., San Jose, CA, USA). Average erodibility parameters (τ_c , k_d , b_0 , and b_1) and soil physical properties were determined from the JETs for each of the streams. Additionally, a coefficient of variation (CV) was determined for each parameter. The CV is a measure of relative standard deviation and is calculated as the ratio of the standard deviation to the mean. This results in a dimensionless parameter which allows for the comparison of variation between parameters with different units and among parameters with large and small values. A regression analysis was conducted in Mini-Tab 17 for the erodibility parameters and soil properties versus distance for FC-FM, FC-WC, IL-BF and IL-IR. Distance was measured in km upstream from the reservoir or the confluence for FC-WC and FC-FM, respectively. Distance was measured in km upstream from the most downstream site on IL-IR and IL-BF. Finally, a Kruskal–Wallis test was performed to determine if significant differences in erodibility parameters existed between sites within each stream. The Kruskal–Wallis test is the non-parametric version of an ANOVA and can be used for data with small sample sizes, skewed data, or non-normal data [41]. An $\alpha = 0.05$ was used for all statistical analyses.

2.3. Streambank Erosion Prediction

The erodibility parameters from JETs along FC-FM and IL-BF were input into the CONservational Channel Evolution and Pollutant Transport System (CONCEPTS) to determine the impact of erodibility parameter variability on lateral retreat prediction. CONCEPTS is a one-dimensional, process-based model that simulates sediment transport and streambank erosion processes (fluvial erosion and mass-wasting) at a user-defined number of cross sections along a stream reach, and allows for vertical bed adjustment along the entire reach [7,42]. CONCEPTS requires very detailed information on channel and floodplain geometry, soil properties, soil layering, sediment properties, sediment layering, and channel and floodplain roughness for each cross-section and water and sediment discharge information at the upstream boundary. Streambank soil parameter inputs (Table 2) included the effective cohesion, c' , effective internal angle of friction ϕ' , and erodibility parameters. Fluvial erosion

is typically predicted in CONCEPTS using the linear excess shear stress model (Equation (1)) with τ_c and k_d as input. For this research, the excess shear stress model was replaced by the Wilson model (Equation (2)) in a second set of simulations with b_0 and b_1 as inputs to the model.

CONCEPTS simulations for IL-BF and FC-FM were used for this study; more details about model implementation can be found in Daly [43] and Enlow [44], respectively. Simulation periods extended from 2007–2011 for IL-BF and from 2008–2013 for FC-FM. For the IL-BF simulations, only the erodibility parameters for the silt layer were adjusted. Wilson model parameters for the gravel layer were determined by Khanal et al. [45]. For simulations on FC-FM, only the erodibility parameters for the sand layer were adjusted.

The sensitivity of erosion predictions to the site-scale and stream reach-scale variability in JET derived erodibility parameters was investigated. A single cross-section was selected for each stream reach. For IL-BF, the cross section experiencing the highest streambank retreat was selected for the analysis. For FC-FM, the cross-section experiencing the highest lateral retreat was the final cross-section in the model. However, Daly [43] reported that simulation results for the most upstream and downstream cross-sections were sensitive to the user-defined boundary conditions. Therefore, to limit the effect of boundary conditions at the selected cross-section, an internal cross-section, corresponding to the section experiencing the second highest lateral retreat, was selected for FC-FM. Several model runs were performed for each site using different values of erodibility parameters. In a first set, erodibility parameters from each individual JET completed at the selected cross-section as well as the mean and median values were used. In a second set, erodibility parameters derived from individual JETs performed along the entire stream reach, and the mean and median values at the reach scale were applied.

The bank retreat simulated at the selected cross-section was compared to the observed retreat determined using aerial images from the National Agricultural Imagery Program (NAIP) between 2008 and 2013 and 2008 and 2010 for FC-FM and IL-BF, respectively. Each image was georeferenced using ArcMap (v10.0) and streambanks were digitized at the site. Average distances between polylines were used as observed retreat [46]. The estimated error owing to georeferencing and bank identification was estimated at approximately 1 m based on similar studies that have utilized the same approach [46,47].

Table 2. Summary statistics for soil parameters measured along Fivemile Creek (FC-FM), Willow Creek (FC-WC), Barren Fork Creek (IL-BF), and Illinois River (IL-IR).

Stream	Statistic	Critical Shear Stress	Erodibility Coefficient	Wilson Model Parameters		% Sand	% Silt	% Clay	Bulk Density	Median Particle Size
		τ_c	k_d	b_0	b_1				BD	d_{50}
		(Pa)	($\text{cm}^3 \text{N}^{-1} \text{s}^{-1}$)	($\text{g m}^{-1} \text{s}^{-1} \text{N}^{0.5}$)	(Pa)			(g cm^{-3})	(mm)	
FC-FM	Mean	0.8	159.3	95.6	7.1	72	19.3	8.7	1.5	0.1
	Median	0.7	120.4	84.3	4.8	75.7	15.7	9.1	1.6	0.1
	Std. dev	0.5	113.6	74.9	6.2	12.8	9.8	4.9	0.2	0.02
	CV	0.64	0.71	0.78	0.88	0.18	0.51	0.56	0.11	0.17
	Count	12	12	12	11	11	12	12	12	12
FC-WC	Mean	0.7	255.7	257.5	3.6	77	15.7	7.3	1.3	0.21
	Median	0.7	203.4	315.1	3.6	79.5	13.1	7.5	1.3	0.11
	Std. dev	0.3	196.9	149.4	1.2	8.3	5.6	2.8	0.2	0.32
	CV	0.45	0.77	0.58	0.34	0.11	0.36	0.39	0.11	1.53
	Count	12	12	12	12	9	9	9	9	9
IL-BF	Mean	3.3	54.6	202	24.8	32.8	50	15	1.2	0.13
	Median	2.2	36.6	98.9	16.7	25.5	54.8	15.7	1.3	0.04
	Std. dev	3.8	78.3	379	28.3	17.4	15.5	3.8	0.1	0.18
	CV	1.13	1.43	1.88	1.14	0.53	0.31	0.25	0.07	1.39
	Count	18	18	18	18	11	11	11	7	11
IL-IR	Mean	3.3	35.7	112.3	23.5	17.2	61.9	17.9	1.2	0.04
	Median	3	20	55.6	20.4	10.7	69.2	19.5	1.3	0.03
	Std. dev	4	51	144.1	21.5	14.8	16.6	3.4	0.04	0.02
	CV	1.21	1.43	1.28	0.92	0.86	0.27	0.19	0.03	0.58
	Count	18	18	18	18	6	6	6	6	6

3. Results and Discussion

3.1. Variability of Erodibility Parameters

Similar average values of τ_c were observed for FC-FM and FC-WC and similarly for IL-IR and IL-BF (Table 2). Higher τ_c and lower k_d were observed within the Illinois River watershed (IL-BF and IL-IR) when compared to the Fort Cobb Reservoir watershed (FC-FM and FC-WC). This suggested the soils within the Illinois River watershed were less erodible. This could be related to the higher clay content in IL-BF and IL-IR soils, predominately silt with a clay content around 20%, while soils from FC-FM and FC-WC consisted of 79–97% sand with less than 12% clay (Figure 2).

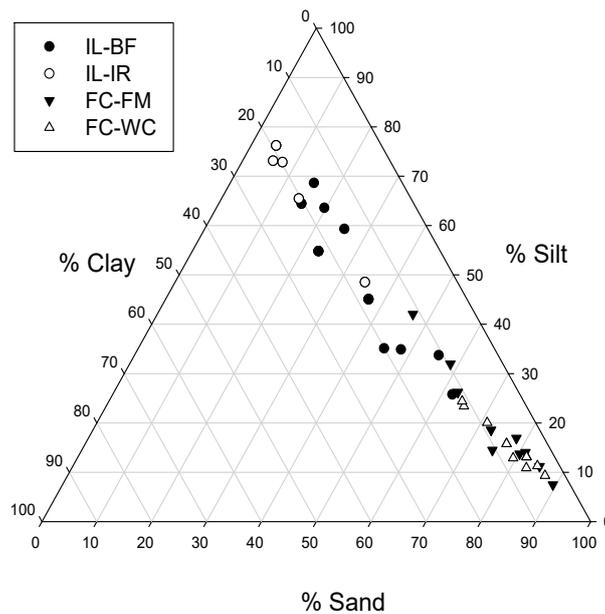


Figure 2. Soil texture of streambank soil samples collected at field data collection sites along Barron Fork Creek (IL-BF), Illinois River (IL-IR), Fivemile Creek (FC-FM) and Willow Creek (FC-WC).

A higher degree of variability in erodibility parameters was observed for the Illinois River watershed than the Fort Cobb Reservoir watershed (Figure 3). Although FC-FM and FC-WC are on opposite sides of the watershed, similar variability was observed between the two creeks. For FC-FM and FC-WC, the τ_c varied by less than one order of magnitude and CVs were 0.64 and 0.45 for FC-FM and FC-WC, respectively. This indicates a lower standard deviation relative to the mean when compared to IL-IR and IL-BF. The highest variability in τ_c (five orders of magnitude) was observed at IL-IR. Three orders of magnitude of variation was observed for k_d along IL-IR and IL-BF, and only two orders of magnitude of variation was observed for FC-FM and FC-WC. The CVs were greater than one for τ_c and k_d along IL-BF and IL-IR. The k_d was more variable for all four streams compared to τ_c based upon CVs. Such results are consistent with the amount of variability in soil textures observed within both watersheds (Figure 2).

As suggested by previous research [13,29,48–50], variability observed in erodibility parameters can be attributed to soil heterogeneity and subaerial processes. When compared to other studies [13,29,48–50], less variability in erodibility parameters was observed along FC-FM and FC-WC. In addition, soil physical properties, percent sand, silt and clay, d_{50} , and BD within the Fort Cobb watershed, exhibited less variation than soil properties in most of the other studies. The standard deviations were generally small when compared to the means for all properties for both soil layers, with the exception being d_{50} for FC-WC. A similar degree of watershed-scale variability in erodibility parameters and soil physical properties to that of IL-IR and IL-BF was observed by Thoman and Niezgodna [50] in the Powder River Basin of Wyoming.

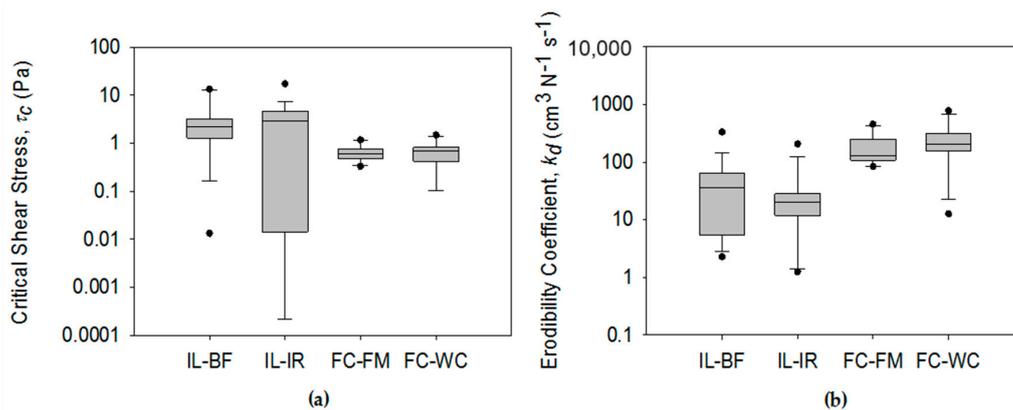


Figure 3. Variation of excess shear stress parameters measured using JETs along Barren Fork Creek (IL-BF), Illinois River (IL-IR), Fivemile Creek (FC-FM) and Willow Creek (FC-WC): (a) τ_c ; and (b) k_d .

The Wilson model's b_0 were similar across all four streams with FC-WC having slightly higher averages (Table 2). One to two orders of magnitude variation were observed for b_0 within the tests for each stream, with a slightly higher variation for IL-BF (Figure 4). Variability in b_0 (CV = 0.58–1.88) was similar to the variability observed in k_d (CV = 0.71–1.43) of the linear excess shear stress model for all four streams. The b_1 were lower for FC-FM and FC-WC when compared to IL-BF and IL-IR, which was expected due to the high correlation between b_1 and τ_c and the lower τ_c observed for FC-FM and FC-WC. Slightly more variation in b_1 was observed than τ_c from FC-FM and IL-BF and much less variation than τ_c for IL-IR. The similar amount of variability observed between excess shear stress model and Wilson model parameters can be attributed to the similar solver techniques [14].

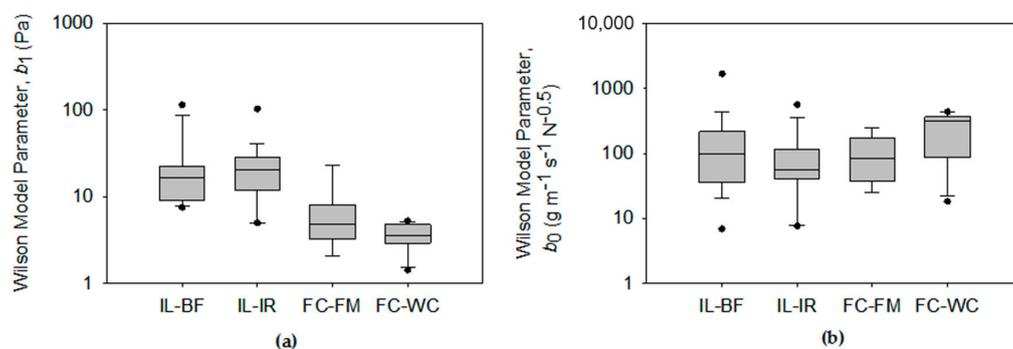


Figure 4. Variation of Wilson model parameters measure using JETs along Barren Fork Creek (IL-BF), Illinois River (IL-IR), Fivemile Creek (FC-FM) and Willow Creek (FC-WC): (a) b_0 ; and (b) b_1 .

3.2. Longitudinal Trends

No significant longitudinal trends were observed for the erodibility parameters (τ_c , k_d , b_0 or b_1) or soil physical properties for FC-FM, FC-WC, or IL-BF, with the exception of d_{50} along FC-WC (Table 3). The lack of longitudinal trends for erodibility parameters along FC-FM, FC-WC, and IL-BF could be attributed to the smaller amount of variability within these streams (Figure 5). Mean particle size of bank material decreased in the downstream direction for FC-WC and IL-IR, but not FC-FM or IL-BF. The downstream fining of bank material was expected, as discussed by Knighton [24] and shown by Konsoer et al. [28]. The lack of a trend for d_{50} along FC-FM may be attributed to the small amount of variability in mean particle size (d_{50} ranged from 0.06 to 0.11 mm) and soil heterogeneity within the stream system.

Three erodibility parameters (τ_c , k_d , and b_1) exhibited weak, but significant longitudinal trends along IL-IR ($r^2 = 0.30$ to 0.32). The τ_c decreased and the k_d increased in the upstream direction for IL-IR

(Figure 6). This may partially be due to the downstream fining of soil particle size that was observed along IL-IR. Fining of particles may therefore decrease erodibility in the downstream direction [24,28]. The significant trends may be attributed to the higher degree of variability within this stream and the larger spatial scale (69.1 km) in which erodibility was measured. Soil measurements were taken along 10.3, 10.1, and 25.5 km reaches of FC-FM, FC-WC, and IL-BF, respectively. A significant longitudinal trend may have been observed if measurements were conducted on longer reaches of these streams.

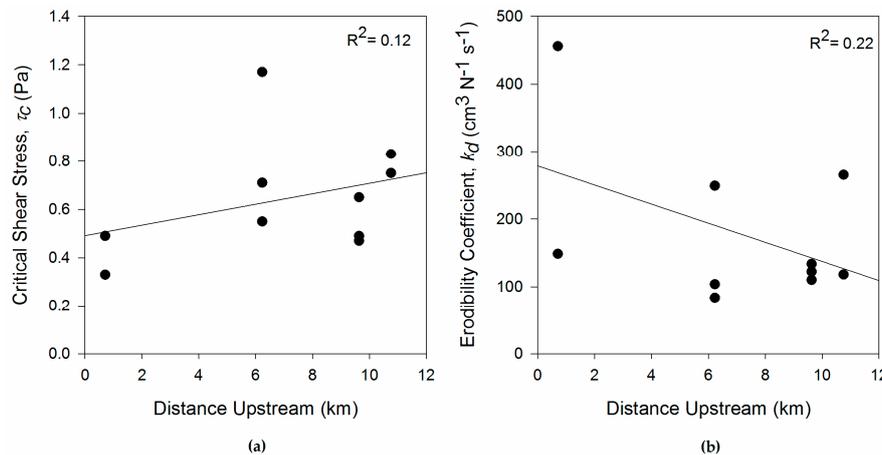


Figure 5. Regression for excess shear stress model parameters determined using JETs versus distance upstream for Fivemile Creek (FC-FM): (a) τ_c ; and (b) k_d .

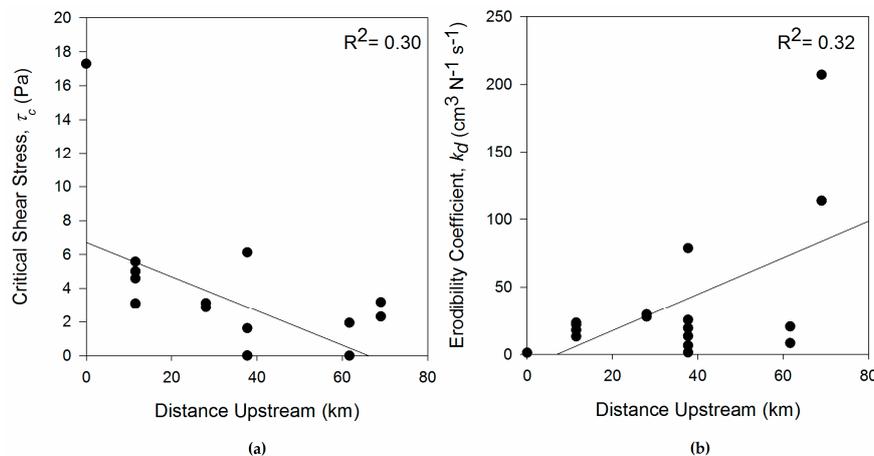


Figure 6. Regression for excess shear stress parameters determined using JETs: (a) τ_c ; and (b) k_d , versus distance upstream for Illinois River (IL-IR).

Since no longitudinal trend was observed, Kruskal–Wallis tests were also used to determine if a significant difference existed between sites. No significant differences were observed between sites for FC-FM or FC-WC for all erodibility parameters ($p = 0.238$ to 0.603). In addition, no significant difference between sites along IL-BF or IL-IR at $\alpha = 0.05$ was observed ($p = 0.084$ to 0.317). Previous studies have also shown no variations between sites along the same stream [28,29]. Ideally, a longitudinal trend could be used to extrapolate JET results to other sites where erodibility was not measured to minimize the number of JETs needed to adequately characterize the erodibility along an entire stream system. While a significant longitudinal trend was not present for either FC-FM or FC-WC, a significant difference between the sites was also not observed. Therefore, it would be expected that the erodibility at locations between sites could be approximated based on the values measured at one particular site using the JET.

Table 3. Coefficient of determination (R^2) for longitudinal regression for soil parameters versus distance upstream Fivemile Creek (FC-FM), Willow Creek (FC-WC), Barren Fork Creek (IL-BF), and Illinois River (IL-IR).

	IL-BF	IL-IR	FC-FM	FC-WC
Critical Shear Stress, τ_c (Pa)	0.04	0.30	0.12	0.03
Erodibility Coefficient, k_d ($\text{cm}^3 \text{N}^{-1} \text{s}^{-1}$)	0.08	0.32	0.22	0.22
Bulk Density, BD (g cm^{-3})	0.00	0.02	0.09	0.05
Median Particle Size, d_{50} (mm)	0.09	0.26	0.04	0.44
Sand (%)	0.04	0.00	0.07	0.00
Silt (%)	0.07	0.00	0.06	0.00
Clay (%)	0.04	0.00	0.07	0.01
Wilson Model Parameter, b_0 ($\text{g m}^{-1} \text{s}^{-1} \text{N}^{-0.5}$)	0.04	0.15	0.15	0.06
Wilson Model Parameter, b_1 (Pa)	0.05	0.29	0.09	0.01

Note: Bold indicates significance at $\alpha = 0.05$.

Understanding the degree to which erodibility parameters vary is crucial. In a watershed like Fort Cobb, where there was no statistical difference between sites and small variability, using average or median τ_c and k_d from a few locations to estimate the erodibility parameters at additional sites may provide acceptable results when utilized to model streambank erosion. Although a significant difference did not exist between sites along IL-BF or IL-IR, this approach would not be possible due to the high amount of variability in the JET results for these stream systems.

3.3. Implications for Lateral Retreat Prediction

Lateral retreat predicted by CONCEPTs based on JET site-scale measurements and JET stream-scale measurements were first compared for both fluvial erosion models (Table 4). Consistently, a slightly larger range of lateral retreat was predicted with the stream-scale as compared to the site-scale measurements. This was expected due to the larger range in erodibility parameters obtained from the JETs at the stream-scale. A higher range of lateral retreat was predicted along FC-FM for both models when compared to IL-BF. The CVs for input erodibility parameters were consistently larger than the CVs for predicted lateral retreat. For example, τ_c and k_d along FC-FM had a CV of 0.6 and 0.7, respectively, but resulted in a CV of 0.2 for the predicted lateral retreat. The CVs for τ_c and k_d along IL-BF were 1.1 and 1.9, respectively, but resulted in a CV of 0.5 for the predicted lateral retreat. The input variability was diminished due to the nonlinear influence between fluvial erodibility and mass wasting processes in the model. However, while the variation in predicted retreat was lower than the corresponding input variables, the large range in predicted retreat highlighted the uncertainty in using a single JET for simulating streambank erosion without calibration.

Table 4. Summary statistics for predicted lateral retreat (m) from CONCEPTs using JET results along Barren Fork Creek (IL-BF) and Fivemile Creek (FC-FM).

JET Data Source	Statistic	IL-BF		FC-FM	
		Excess Shear Stress Model	Wilson Model	Excess Shear Stress Model	Wilson Model
Site	Mean	12.3	31.1	34.1	37.6
	Std. dev. *	6.4	2.6	11.1	7.5
	CV *	0.52	0.08	0.32	0.20
	Range	12.6	5.3	22.1	14.1
Stream	Mean	12.1	30.6	40.4	29.1
	Std. dev.	6.0	2.0	9.6	13.4
	CV	0.50	0.06	0.24	0.46
	Range	15.9	6.6	31.6	38.5
Observed retreat		20.0		6.0	

Notes: * CV = coefficient of variation; Std. dev. = standard deviation.

For IL-BF, the excess shear stress model under predicted lateral retreat when compared to the observed retreat (Figure 7), while the Wilson model over predicted lateral retreat. The under prediction of lateral retreat by the excess shear stress model can be attributed to the increase in applied τ around the outside of the meander located at the IL-BF site that is not correctly accounted for in the model. Previous research has shown that the Wilson model predicted lower lateral retreat closer to the observed retreat than the excess shear stress model when integrated into the Bank Stability and Toe Erosion Model (BSTEM) [45]. This was not the case when the Wilson model was incorporated into CONCEPTS for IL-BF. However, the b_0 and b_1 from Khanal et al. [45] used for the gravel layer were estimated based on BSTEM simulations and were not directly measured. These values were also used in the CONCEPTS simulations. Direct measurement of b_0 and b_1 for the gravel layer may predict a lateral retreat closer to the observed retreat.

For FC-FM, both fluvial erosion models over predicted erosion (Figure 8). This was expected as soil erodibility parameters suggested highly erodible soil properties and also because the presence of heavy vegetation on the bank face, which can significantly decrease the applied τ reaching the detachable soil particles or aggregates [9,51–55]. This highlights the need to account for the impact of vegetation or meanders on applied τ during model setup and calibration. For both erosion models along FC-FM and IL-BF, using mean or median results from multiple JETs and adjusting parameters during calibration would likely result in a lateral bank retreat prediction closer to the measured historical retreat.

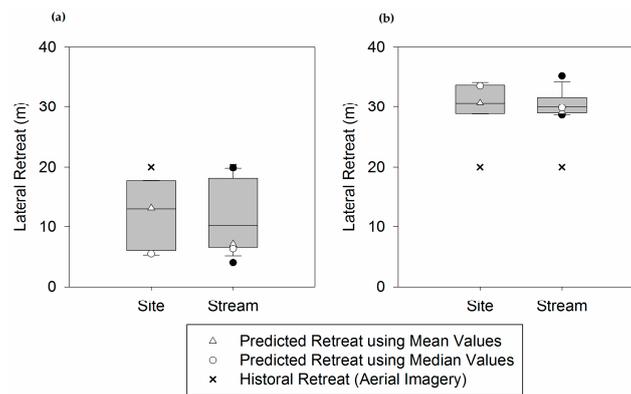


Figure 7. Boxplots of variation in CONCEPTS predicted lateral retreat at a site on Barren Fork Creek using JET results from the site and entire stream reach for: (a) excess shear stress model; and (b) Wilson model.

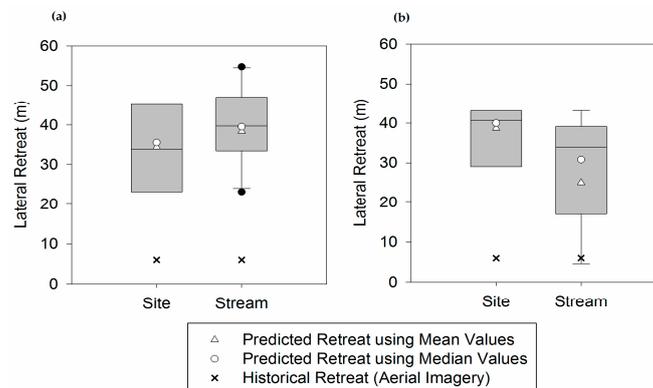


Figure 8. Boxplots of variation in CONCEPTS predicted lateral retreat at a site on Fivemile Creek using JET results from the site and entire stream reach for: (a) excess shear stress model; and (b) Wilson model.

3.4. Adjusting Erodibility Parameters during Model Calibration

Applied τ can be impacted by the presence of meanders (increased τ) or bank face vegetation roots and/or above-ground biomass (decreased τ) [28,51–55], which is not taken into account in the one-dimensional calculation of τ [9] in CONCEPTS. Because the model does not allow for the direct adjustment of τ , an ν -factor can be used to indirectly adjust τ by adjusting erodibility parameters as discussed in Langendoen et al. [42] and Daly et al. [37]:

$$\varepsilon_r = k_d(\nu\tau - \tau_c) = (\nu k_d)\left(\tau - \left(\frac{\tau_c}{\nu}\right)\right) \quad (3)$$

A similar method can be used to adjust τ in the Wilson model:

$$\varepsilon_r = b_0\sqrt{\nu\tau} \left[1 - \exp\left\{-\exp\left(3 - \frac{b_1}{\nu\tau}\right)\right\}\right] = (b_0\sqrt{\nu})\sqrt{\tau} \left[1 - \exp\left\{-\exp\left(3 - \frac{b_1}{\tau}\right)\right\}\right] \quad (4)$$

Based on model calibrations performed for IL-BF [43] and FC-FM [44] by comparing predicted retreat to observed retreat determined from NAIP aerial imagery for the time period simulated, an $\nu = 1.26$ was used for the IL-BF site to account for the increase in τ around meanders (sinuosity) and $\nu = 0.27$ was used for the FC-FM site to account for the decrease in τ due to heavy vegetative cover on the bank face. Note that these reported ν were based on use of the excess shear stress model. Combining these calibration factors with mean erodibility parameters measured at the site, the excess shear stress model resulted in a lateral retreat prediction of 18.7 m and 7.1 m for IL-BF and FC-FM, respectively.

4. Conclusions

Site and stream-reach variability in fluvial erodibility parameters may result in uncertainty when modeling particle detachment and fluvial erosion. Fluvial erodibility parameters corresponding to the linear excess shear stress model and Wilson model were less variable in watersheds with less cohesive soils. Changes in erodibility parameters in the longitudinal direction or differences between the sites were not observed for the shorter stream reaches; however, longitudinal trends were observed on longer stream reaches. Large degrees of variability may increase the error in using average or single-test values of erodibility parameters for a site, reach, or watershed. When JET results were incorporated into a streambank erosion and failure model, less variation was observed in lateral retreat prediction than input erodibility parameters regardless of the type of fluvial detachment model used. Uncalibrated erodibility parameters and simplified applied shear stress estimates failed to match observed lateral retreats. Factors such as vegetation on the streambank face and/or meandering need to be accounted for through model calibration or advanced two- or three-dimensional flow modeling.

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