

## Article

# Agricultural Water Productivity Oriented Water Resources Allocation Based on the Coordination of Multiple Factors

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**Abstract:** Agricultural water productivity (AWP), which is associated with multiple factors, is an important index for measuring the effectiveness of agricultural water management. The purpose of this study is to promote AWP through optimally allocating limited agricultural water resources with the coordination of related elements. Firstly, the coordination effects of multiple factors related to AWP are quantified as relative optimum membership degrees based on the fuzzy optimum selecting theory. Secondly, based on the relative optimum membership degrees for various crops, a linear fractional programming model is established to maximize AWP in agricultural water resources allocation. Thirdly, the impacts of the allocation schemes on the development of social-economy and ecological environment are discussed using the multi-dimensional regulation theory. The developed integrated system has advantages in increasing agricultural water productivity through the coordination of multiple factors with aspects of economy, society and resources. Moreover, the system is capable of screening schemes considering harmonious development of resources, economy, society and ecology based on optimization results, providing decision makers with more sustainable schemes for irrigation water allocation. The integrated system including the aforementioned three parts is applied to a real-world case study in China to demonstrate its feasibility and applicability. Different water allocation schemes for various crops under different scenarios were obtained. The average value of AWP is  $1.85 \text{ kg/m}^3$ , which is  $0.31 \text{ kg/m}^3$  higher than the current value of AWP. An optimum scheme with  $1.1405 \times 10^8 \text{ m}^3$  of water being allocated was also selected due to its highest level of coordination for resources, economy, society and ecology. The developed system can provide an effective method for AWP promotion. The obtained results can help local decision makers adjust water resources allocation schemes.

**Keywords:** agricultural water productivity; water resources allocation; coordination of multiple factors; linear fractional programming; multi-dimensional regulation

## 1. Introduction

The rapid socio-economic development and population growth, coupled with conflicts between limited water resources and increased water demands, have emphasized the need to reasonably and effectively allocate water resources. In many countries, agriculture is the biggest water use sector. For example, in China, agriculture is the largest water consumption sector, accounting for about 63.5% of the national total water consumption (MWRPRC, 2016) [1]. The low efficiency of water use has aggravated the shortage of irrigation water resources (Du et al., 2014) [2]. By 2030, the population of China is forecasted to reach 1.6 billion, consuming 640 billion kg of food per year (Feng et al., 2014) [3]. Agricultural water shortage will then reach 90 billion  $\text{m}^3$  (Kang et al., 2017) [4]. To alleviate agricultural water crisis, the increase of agricultural water productivity (AWP) is necessary to produce more food

with less water (Kijne, 2003) [5]. Increasing AWP is critical for water-saving agriculture (Birendra et al., 2011; Du et al., 2015) [6,7]. Optimization techniques for water resources allocation can be an important measure for increasing AWP, which is important to improve efficient agricultural water utilization and enhance food security.

Various optimization techniques have been used to generate optimal water allocation patterns in recent years (Singh, 2014) [8], such as linear programming which was extensively used by researchers for irrigation management because of its easy formulation and application (Anwar and Clarke, 2001; Hsu et al., 2008; Singh and Panda, 2012) [9–11], nonlinear programming which is capable for handling nonlinear problems (Garg and Dadhich 2014; Zarghami et al., 2015) [12,13], dynamic programming with inherent advantages (Jin et al., 2012; Davidsen et al., 2015) [14,15], and stochastic programming which can reflect random characteristic (Azaiez et al., 2005; Fu et al., 2016) [16,17]. The objectives of most of these optimization techniques were to maximize net benefits/yields or minimize cost/labor force. Few of them considered the optimization of AWP. AWP can be regarded as a quantitative presentation of water use efficiency (Lankford, 2006) [18], and it can be defined as the ratio of food output to the amount of water consumed (Molden et al., 2010) [19]. It is essentially an efficiency problem which falls within the capability of fractional programming. In recent years, some researchers attempted to optimize AWP using linear fractional programming (Guo et al., 2014; Li et al., 2015; Zhou et al., 2016) [20–22], with the numerator part expressing total crop yield and the denominator part expressing total agricultural water consumption or use. For arid areas, water resources scarcity is serious and surface water supply alone cannot meet the water demand of crop growth. This leads to the necessity to conjunctively use surface water and groundwater. The conjunctive use of surface water and groundwater has been successfully conducted in many optimization studies (Singh, 2014) [23]. For example, Safavi and Esmikhani (2013) [24] presented a simulation-optimization model for conjunctive use of surface water and groundwater at a basin-wide scale. This conjunctive use model coupled numerical simulation with nonlinear optimization to minimize shortages of irrigation water for four irrigation systems. An-Vo et al. (2015) [25] proposed an innovative nonlinear programming model for the optimization of conjunctive water use options in an irrigation command area based on maximized objectives with crop water production and profit functions. Wu et al. (2016) [26] implemented physically-based, fully integrated surface water and groundwater modeling in the optimization of water management practices. However, few researches with the aim of increasing AWP within the framework of fractional programming considered the conjunctive use of both surface water and groundwater. The increase of AWP is the process of the coordination of multiple factors (Molden et al., 2010; Gutiérrez-Martín et al., 2017) [19,27]. From the perspective of crops, AWP is related to factors such as crop types, crop yields and irrigation quota. From the perspective of management, AWP is related to factors such as irrigation methods, irrigation costs, economic benefits and social values of crop planting. For example, higher crop yields will lead to a higher AWP under the same water consumption. Water-saving measures such as trickle irrigation, sprinkling irrigation, and tube-well irrigation will also increase AWP through saving water and increasing water utilization efficiency. The changes in any of these influence factors can result in different food production and water allocation plans, leading to the variations in AWP. The problem becomes more complex when considering the combined effects of these varying factors. However, few studies that focused on the optimization of AWP using fractional programming considered the coordination of multiple factors.

AWP can potentially be improved through decreasing the water use amount. However, the decrease of irrigation water amount will also lead to a drop in crop production, threatening food security and economic development. With the widely perceived theory of sustainable development, water resources management should be directed towards the coordinated development of economy, society, and environment. Moreover, the multi-dimensional attributes of water resources require managing water resources through equally weighing social, economic, ecological and environmental aspects. Therefore, how to quantitatively evaluate the efficiency of water resources

management, and how to analyze the impacts of water allocation schemes on the social-economic and ecological environment are issues that deserve research (Jiang et al., 2015; Li et al., 2017) [28,29]. This will help decision makers for agricultural water resources management choose appropriate allocation plans according to practical situations and further improve the agricultural sustainability. However, few studies comprehensively evaluated agricultural water resources schemes obtained from optimization models.

Therefore, the aim of this study is to establish a linear fractional programming model for optimal agricultural water resources allocation through the coordination of multiple factors. The study entails several elements. First, relative optimum membership degrees of crops which can be considered as the degree of multi-factor coordination will be calculated. Second, a linear fractional programming model with maximized AWP will be established based on the relative optimum membership degrees. Third, the optimum water allocation schemes from the model will be analyzed, evaluated and then screened based on multi-dimensional regulation theory. This work will provide novel methods for efficient and systematical management of agricultural water resources.

## 2. Methodology

Increasing AWP is important for water resources management. AWP can be described as crop yield per unit water use amount. How to quantify the coordination of multiple factors that affect AWP, how to reveal the relationship between crop yield and agricultural water use amount under different scenarios, and how to scientifically evaluate the efficiency of agricultural water allocation schemes are particularly important (Zuo et al., 2007; Wang et al., 2017) [30,31]. Based on the fuzzy optimum selecting theory, this paper established an agricultural water resources allocation model within the framework of linear fractional programming to increase AWP based on the coordination of multiple factors. The results of water distribution were evaluated and further optimized through analyzing the impacts of these results on the social, economic and ecological development on the basis of multi-dimensional regulation theory.

### 2.1. Fuzzy Optimum Selecting Theory

To improve AWP, it is necessary to consider the coordination of multiple factors associated with the aspects of economy, society, resources, etc. Such an issue can be considered as a multi-objective problem. Multi-objective problem is a complex and highly nonlinear system. The most commonly used method to solve multi-objective problems is to endow different weights for different objectives or factors, which will involve the subjective desires or preferences of decision makers (Chen, 1990; Chen, 1994) [32,33]. However, there are no definite boundaries in the processes of preference identification and weights definition, leaving fuzzy characteristics when addressing multi-objective problems of water resources allocation. Fuzzy optimum selecting theory is an efficient tool to handle multi-objective weights by integrating multiple objectives into a comprehensive coefficient (Chen, 1990) [32]. The key of the optimum selecting theory is the fuzzy optimum selecting model. Because the solving process of the optimum selecting theory is easy and uncomplicated with strong operability, it has been applied efficiently in various research fields. The derivation of the fuzzy optimum selecting model which can be expressed as a comprehensive coefficient of relative optimum membership degree ( $u_{bj}$ ) are expressed as follows (Li et al., 2016) [34].

Let  $j$  represent schemes and there are  $n$  schemes in total. All the schemes form a set that can be expressed as  $x = \{x_1, x_2, \dots, x_n\}$ ; Let  $i$  represent properties/objectives, and the total number of

properties is  $m$ , so  $x_j = (x_{1j}, x_{2j}, \dots, x_{mj})^T$ . Thus, the index set of schemes can be expressed as a matrix with  $m \times n$  dimensions as follows:

$$X = \begin{pmatrix} x_{11} & x_{12} & \cdots & x_{1n} \\ x_{21} & x_{22} & \cdots & x_{2n} \\ \vdots & \vdots & & \vdots \\ x_{m1} & x_{m2} & \cdots & x_{mn} \end{pmatrix} = (x_{ij}) \quad (1a)$$

where  $x_{ij}$  is the value of property  $i$  ( $i = 1, 2, \dots, m$ ) of scheme  $j$  ( $j = 1, 2, \dots, n$ ).

(1) Calculate the relative membership matrix

Indicators associated with schemes are usually divided into two types: “the bigger the better” and “the smaller the better”. The relative membership degrees are described as follows:

“The bigger the better”

$$r_{ij} = x_{ij} / \max x_j \quad (1b)$$

“The smaller the better”

$$r_{ij} = \min x_j / x_{ij} \quad (1c)$$

where  $r_{ij}$  is the relative membership degree of scheme  $j$  with property  $i$ ; and  $\max x_j$  and  $\min x_j$  represent the maximum and minimum values of property  $i$  among the  $n$  schemes, respectively.

Then, the relative membership degree matrix can be described as

$$R = \begin{pmatrix} r_{11} & r_{12} & \cdots & r_{1n} \\ r_{21} & r_{22} & \cdots & r_{2n} \\ \vdots & \vdots & & \vdots \\ r_{m1} & r_{m2} & \cdots & r_{mn} \end{pmatrix} = (r_{ij})_{m \times n} \quad (1d)$$

where  $R$  is the relative membership degree matrix with  $m \times n$  dimensions.

(2) Determination of superior and inferior membership degrees

With the maximum characteristic value of each row of Equation (1d), a new matrix  $r_b$  will be formed indicating the ideal superior scheme.  $r_b$  can be expressed as:

$$\begin{aligned} r_b &= (r_{b1}, r_{b2}, \dots, r_{bm}) \\ &= (\max r_{1j}, \max r_{2j}, \dots, \max r_{mj}) = (1, 1, \dots, 1) \end{aligned} \quad (1e)$$

Similarly, if the minimum value of each row of Equation (1d) is extracted, a new matrix  $r_w$  will be formed indicating the ideal inferior scheme.  $r_w$  can be expressed as:

$$\begin{aligned} r_w &= (r_{w1}, r_{w2}, \dots, r_{wm}) \\ &= (\min r_{1j}, \min r_{2j}, \dots, \min r_{mj}) = (0, 0, \dots, 0) \end{aligned} \quad (1f)$$

Each scheme will attach to the ideal superior scheme ( $r_b$ ) and the ideal inferior scheme ( $r_w$ ) with a certain membership degrees of  $u_{bj}$  and  $u_{wj}$ , respectively. The  $u_{bj}$  and  $u_{wj}$  are, respectively, named as superior and inferior membership degrees, which have: (1)  $0 \leq u_{bj} \leq 1$ ; (2)  $0 \leq u_{wj} \leq 1$ ; (3)  $u_{bj} + u_{wj} = 1$ .

### (3) Derivation of the relative optimum membership degree

Assuming  $S_{bj}$  and  $S_{wj}$  to be the weighted superior Euclidean distance and weighted inferior Euclidean distance of scheme  $j$ ,  $S_{bj}$  and  $S_{wj}$  can be expressed as:

$$S_{bj} = u_{bj} \sqrt{\sum_{i=1}^m [\omega_i (r_{bj} - r_{ij})]^2} \quad (1g)$$

$$S_{wj} = u_{wj} \sqrt{\sum_{i=1}^m [\omega_i (r_{ij} - r_{wj})]^2} \quad (1h)$$

where  $u_{wj} = 1 - u_{bj}$ .

In order to calculate the optimal value of relative membership degree ( $u_{bj}$ ), the following equation is established:

$$\min \left\{ u_{bj}^2 \sum_{i=1}^m [\omega_i (r_{ij} - r_{bj})]^2 + (1 - u_{bj})^2 \sum_{i=1}^m [\omega_i (r_{ij} - r_{wj})]^2 \right\} \quad (1i)$$

where  $\omega_i$  is the weighted vector and  $\sum_{i=1}^m \omega_i = 1$ .

Take the derivative with respect to  $u_{bj}$  of Equation (1i) and let it equal 0, then

$$u_{bj} = \frac{1}{1 + \left\{ \sum_{i=1}^m [\omega_i (r_{ij} - 1)]^2 / \sum_{i=1}^m (\omega_i r_{ij})^2 \right\}} \quad (1j)$$

Equation (1j) is the fuzzy optimum selecting model and  $u_{bj}$  is the relative optimum membership degree, which will be one of the inputs of the optimization models.

In this study, the “schemes” were typical crops in the studied irrigation district. The “properties” associated with various crops were net benefit per unit area, commodity proportion, per capital guaranteed rate of grain and irrigation quota, which indicated the indicators from economy, society and resources.

## 2.2. Fractional Programming Model for Agricultural Water Allocation

Limited agricultural water resources should be rationally allocated under limited water supply considering the coordination of multiple factors to enhance AWP, especially for arid and semi-arid regions. The relative optimum membership degree ( $u_{bj}$ ) is the comprehensive embodiment of the elements of resources, economy and society that affect AWP, which can be embedded into a fractional programming model framework. The objective of the developed optimization model is to allocate limited water resources to different crops considering the coordination of multiple factors. The objective function is to maximize AWP and it is expressed as a fractional form, with total crop yield in the numerator and total water use amount in the denominator. Among which, the crop yield can be expressed as the linear form of the water production function during the whole growth period. Because of the shortage of surface water supply, this study used the technique of conjunctive use of surface water and groundwater. The constraints of the developed model mainly included water supply constraints for surface water and groundwater, crop water requirement constraint and policy constraint. The developed model is described as follows:

Objective Function:

$$\max AWP = \frac{\sum_j^n u_{bj} \left[ a_j \sum_{t=1}^T (SW_{jt} + GW_{jt} + EP_{jt}) + b_j \right]}{\sum_{j=1}^n \sum_{t=1}^T (SW_{jt} + GW_{jt})} \quad (2a)$$

where  $j$  is the crop;  $t$  is the study period, in this study, it represents each month during the whole growth period;  $SW_{jt}$  and  $GW_{jt}$  are the net water allocation amount of surface water and groundwater for crop  $j$  in period  $t$ , respectively, which are decision variables ( $\text{m}^3/\text{ha}$ );  $EP_{jt}$  is the effective precipitation for crop  $j$  in period  $t$  ( $\text{m}^3/\text{ha}$ );  $a_j \sum_{t=1}^T (SW_{jt} + GW_{jt} + EP_{jt}) + b_j$  is the linear water production function for crop  $j$ ; and  $a_j$  and  $b_j$  are the regression coefficients of the linear water production function for crop  $j$ .

Constraints:

(1) Surface water supply constraints.

The water allocated to a variety of crops should not exceed the water supply of each period. This constraint can be expressed as follows:

$$\sum_{j=1}^n SW_{jt} \times A_j \leq S_t + \alpha \times T_t + Q_{t-1} \quad \forall j, t \quad (2b)$$

where  $A_j$  is the crop planting area for crop  $j$  (ha);  $S_t$  is the water supply from reservoir in period  $t$  ( $\text{m}^3$ );  $T_t$  is the water transfer amount in period  $t$  ( $\text{m}^3$ );  $\alpha$  is the proportion coefficient of water transfer; and  $Q_{t-1}$  is the residual water in period  $t - 1$ .

(2) Water balance constraint.

The surplus water of a certain time period equals the sum of water that runs into the study area minus allocated water and the surplus water of the last time period. This constraint can be expressed as follows:

$$Q_{t-1} = Q_{t-2} + (S_{t-1} + \alpha \times T_{t-1}) - \sum_{j=1}^n SW_{j(t-1)} \times A_j \quad Q_0 = 0 \quad \forall j, t \quad (2c)$$

where  $Q_{t-2}$  is the residual water in period  $t - 2$ ;  $S_{t-1}$  is the water supply from reservoir in period  $t - 1$  ( $\text{m}^3$ );  $T_{t-1}$  is the water transfer amount in period  $t - 1$  ( $\text{m}^3$ ); and  $SW_{j(t-1)}$  is the surface water allocation for crop  $i$  in period  $t - 1$  ( $\text{m}^3/\text{ha}$ ).

(3) Groundwater supply constraint.

The amount of groundwater allocated to a variety of crops should not exceed the supply of groundwater in each period. This constraint can be expressed as follows:

$$\sum_{j=1}^n GW_{jt} \times A_j \leq G_t \quad \forall t \quad (2d)$$

where  $G_t$  is the available groundwater exploitation amount in period  $t$  ( $\text{m}^3$ ).

(4) Crop water requirement constraint.

The amount of water allocated to a variety of crops should not exceed the maximum water requirement at any time, and not be less than the minimum amount of water requirement. This constraint can be expressed as follows:

$$SW_{jt} + GW_{jt} + EP_{jt} \geq \beta \times IR_{jt} \quad \forall j, t \quad (2e)$$

$$SW_{jt} + GW_{jt} + EP_{jt} \leq IR_{jt} \quad \forall j, t \quad (2f)$$

where  $IR_{jt}$  is the maximum water requirement for crop  $j$  in period  $t$  ( $\text{m}^3/\text{ha}$ ); and  $\beta$  is the minimum water requirement coefficient.

(5) Ecological health constraint.

The residual water amount for the lower reaches after irrigating crops in the relative upper reaches should ensure the ecological safety of the lower reaches. This constraint can be expressed as follows:

$$\sum_{t=1}^T (S_t + \alpha \times T_t) - \sum_{j=1}^n \sum_{t=1}^T SW_{jt} \times A_j \geq EWL \times \eta_1 \quad (2g)$$

where  $EWL$  is the ecological water requirement of downstream ( $\text{m}^3$ ); and  $\eta_1$  is the water use efficacy of surface water.

(6) Nonnegative constraint.

The water amount allocated to each crop in each time period should not be negative.

$$SW_{jt} \geq 0, GW_{jt} \geq 0 \quad \forall j, t \quad (2h)$$

### 2.3. Multi-Dimensional Regulation Theory

Increasing AWP is a complex system issue, and the developed optimization model mainly optimizes water resources allocation to achieve maximum AWP. However, in order to learn the effects of the optimal water allocation schemes on society, economy and ecological environment, it is necessary to comprehensively evaluate and analyze the optimal schemes (Mei et al., 2013) [35]. Water resources system is a complex giant system that is composed of macroeconomic, water resources, ecology and environment. Multi-dimensional regulation is capable of systematically analyzing multiple conflicting objectives, including resources, economy, society, ecology and environment. Moreover, multi-dimensional regulation is an efficient method to deal with the coordination and continuous evolution of the water resources system based on the optimal water allocation schemes, which will promote the sustainable utilization of water resources. Therefore, the multi-dimensional regulation theory was adopted to comprehensively evaluate, analyze and screen the optimized water allocation schemes obtained from the linear fractional programming model.

When using the multi-dimensional regulation theory, the selection of the attribute dimension and corresponding characteristic indexes is important. Multi-dimensional attributes mainly include resource, economic, social, ecological and environmental dimensions. Due to the conflicts and competitions, multi-dimensional regulation of water resources should obey the theories of compound system and synergy. According to the regulation target and principles of the attributes of each dimension, the resource dimension mainly considers the utilization efficiency of water supply. The economic dimension mainly considers the economic profit. The social dimension takes the security fairness as the core, reflecting the overall effectiveness of water resources utilization. The ecological dimension mainly maintains system sustainability and ensures the stability and restoration of key ecosystems. The environmental dimension mainly refers to the impact on water quality. For areas with water shortage problems, the coordination equilibrium of social-economy and ecological environment, as well as the stability and sustainability of water resource should be considered. Hence, evaluation indexes were selected for the sake of multi-dimensional regulation, including the proportion of surface water and groundwater allocation proportion for the resource dimension, revenue per unit water allocation amount for the economic dimension, AWP and grain guaranteed rate per capital for the social dimension, and the proportion of groundwater exploitation for the ecological dimension, as the evaluated indexes for multi-dimensional regulation. Secondly, the multi-dimensional normalized

objective function can be established based on the selected indexes. Because the selected indexes have different metrics and units, these indexes should be transformed into non-dimensional values when calculating the general multi-dimensional goal, through using the sum of scalar. The normalized objective function can be written as follows:

$$\max = f\{\max Wres(t, d), \max Econ(t, d), \max Soc(t, d), \max Ecol(t, d)\} \quad (3a)$$

where  $\max Wres(t, d)$  is the index value of resource dimension;  $\max Econ(t, d)$  is the index value of economic dimension;  $\max Soc(t, d)$  is the index value of social dimension; and  $\max Ecol(t, d)$  is the index value of ecological dimension.

The multi-dimensional normalized objective function will be solved using synergetic theory. The synergetic theory is intended to study how a system evolves from a non-equilibrium state to an ordered state through the synergy among different sub-systems. Order parameter can determine the systems evolutionary direction, which represents and measures the degree of coordination or competition among different subsystem. The five indexes selected above corresponding to different dimensions were considered as order parameters. The goal of the multi-dimensional regulation is to coordinate the subsystems of resources, economy, society and ecology, which requires the coordination of order parameters. The concept of order degree can be used to evaluate the overall status of the system. Generally, if the status of the system is more chaotic, the order degree is smaller. On the contrary, if the system is more orderly, the order degree is greater (Chang et al., 2002) [36]. The order degree of each subsystem can be calculated using the following equations:

$$OD_s(e_s) = \frac{e_s - \beta_s}{\alpha_s - \beta_s} \quad (3b)$$

$$OD_s(e_s) = \frac{\alpha_s - e_s}{\alpha_s - \beta_s} \quad (3c)$$

where  $s$  represents the subsystem,  $s = 1, 2, \dots, S$ ;  $OD_s(e_s)$  is the order degree of each subsystem;  $e_s$  is the order parameter of subsystem  $s$ ; and  $\alpha_s$  and  $\beta_s$  representatively represent the maximum and minimum value of order parameter. Equation (3b) represents the order parameter with the characteristic of “the smaller, the better” while Equation (3c) represents the order parameter with the characteristic of “the bigger, the better”. Based on the order degree, the coordination degree, which reflects the harmonious degree of the interrelated subsystems, can be obtained. The larger the coordination degree, the more harmonious and the more orderly the system is. There are many ways to calculate the coordination degree, such as the geometric mean method, the weighted average method and the variance method. The principles of these evaluations are roughly the same. Among them, the geometric mean method is relatively simple with more intuitive results, which does not require weight of the comprehensive indicators when calculating the coordination degree. Thus, this study adopted the geometric mean method to calculate the coordination degree, which can be expressed as:

$$H = (\min(OD_s) / |\min(OD_s)|)^J \sqrt[S]{\prod_{s=1}^S OD_s} \quad (3d)$$

where  $H$  is the coordination degree.

Various water allocation schemes can be obtained through solving the optimization model established above. Different water allocation schemes correspond to different values of coordination degree. Then, optimal water allocation schemes can be obtained based on the maximum coordination degree.

With the consideration of the coordination of multiple factors and the impact of the optimal water allocation schemes on the entire system, a systematic method for increasing AWP through irrigation water resources allocation can be developed. The framework of such an integrated system can be seen in Figure 1.



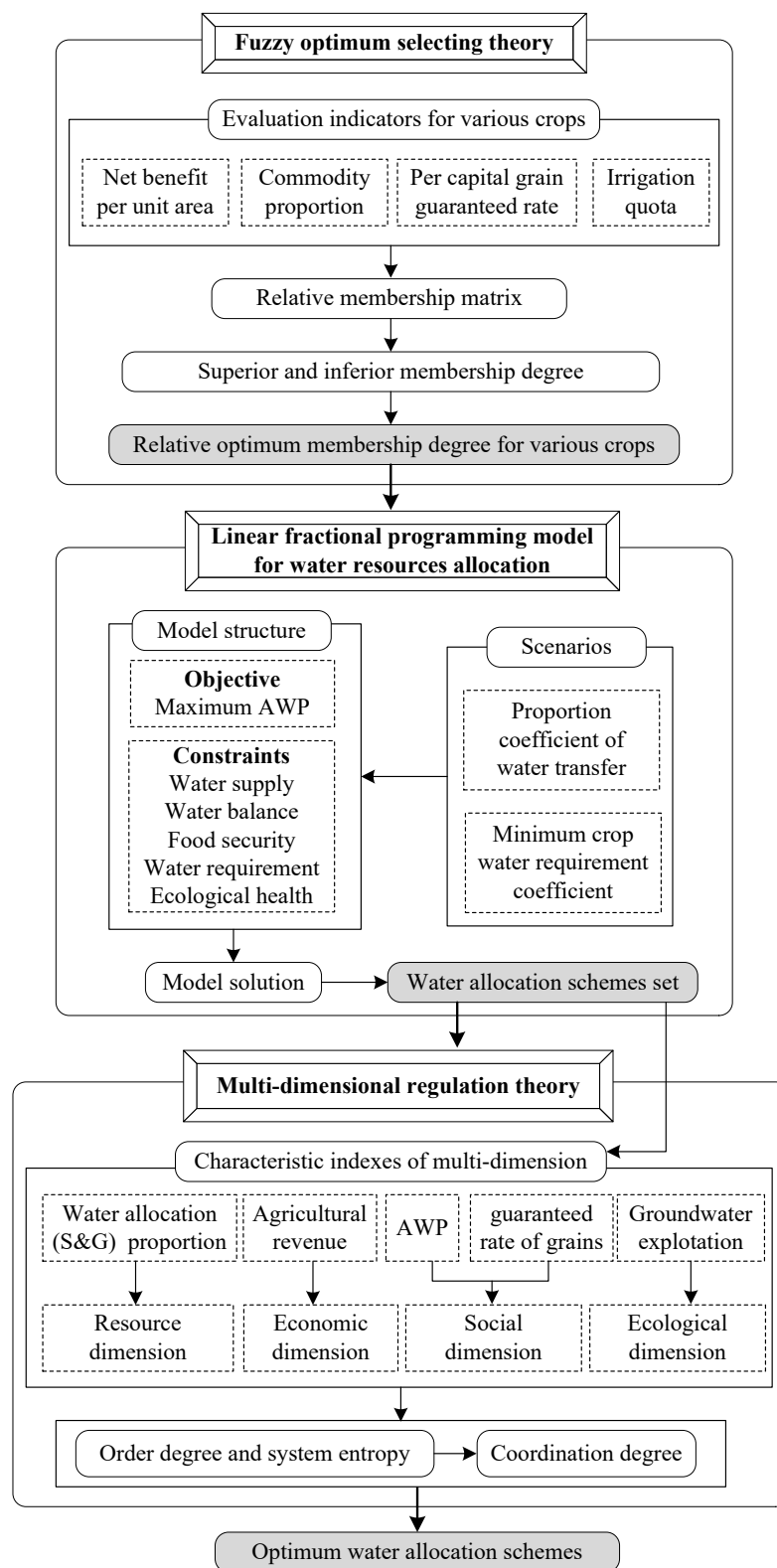


Figure 1. Study framework.

### 3. Case Study

#### 3.1. Study Area

The developed framework is applied to a real case study to demonstrate its feasibility and applicability. The study area is located in the Hongyashan irrigation district (ID) of Minqin County, Gansu Province in China (see Figure 2). The annual average precipitation is 113 mm, while the annual evapotranspiration reaches 2644 mm. Agriculture is the major water consumer of Hongyashan ID where agricultural water use accounts for nearly 90% of the total water use Li et al., 2016 [37]. Agricultural water supply mainly comes from the runoff from the upper reaches of Shiyang River Basin, diverted water from areas with relatively abundant water resources, and groundwater exploitation. The development of economy and the growth of population led to the difficulties in meeting agricultural water demand (Gonzalez et al., 2016) [38]. In addition, most of the irrigation methods in the Hongyashan ID rely on surface, especially the border irrigation method. Low irrigation water use efficiency in the study area will exacerbate the contradiction of agricultural water use and deterioration of ecological environment (Yu et al., 2015) [39]. Although the local government has implemented diversion measures to alleviate water contradictions, agricultural water supply still cannot meet water demand of Hongyashan ID. Besides, uneven distribution of runoff and precipitation further exacerbates the complexity in agricultural water allocation. Therefore, effectively managing available water resources and increasing AWP through optimization models is of strategic importance for the sustainable development of agricultural water resources in Hongyashan ID.

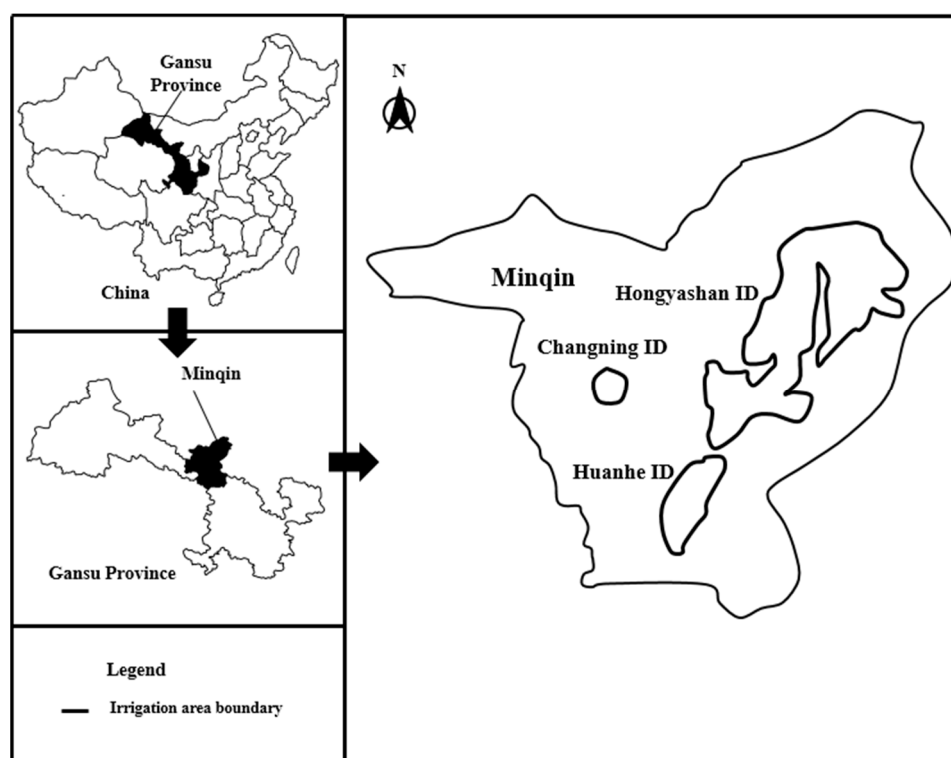


Figure 2. Study area.

#### 3.2. Parameter Determination

The major crops of Hongyashan ID include spring wheat, spring maize, oil flax, vegetables and melons. The linear fractional programming model for water resources allocation is based on crop water production functions (CWPFs). The CWPFs can be divided into two types, including CWPFs for crops' entire growth periods and CWPFs for each stage during the course of crop growth.

For the former, there are two types as well. The first type is called the absolute-value model which describes the relationship between crop's absolute yield and absolute evapotranspiration. For this type, linear model and quadratic models can be used. The second type is called the relative-value model which describes the relationship between crop's relative yield and relative evapotranspiration, mainly including Hiller–Clark, Hanks and DK models (Aljamal et al., 2000) [40]. As for CWPfS for each stage during crops' growth periods, additive models (such as Blank, Stewart and Singh models) and multiplicative models (such as Jensen, Minhas and Rao models) are included (Igbadun et al., 2007) [41]. The CWPfS for the crops' whole growth periods are usually used to study the impacts of the total input of water on crop yields, or used to optimally allocate water among different crops which is the aim of this study. Therefore, this study used the CWPfS for the crops' whole growth periods. For most cases, nonlinear CWPfS are more accurate than linear ones, but may lead to complex algorithms. Linear models are generally suitable for areas with insufficient water supply and lower management level for irrigation. For these considerations, linear water production functions were adopted for the studied crops. The basic data for fitting the crop water production functions (CWPfS) of the main crops can be found in Kang et al. (2009) [42].

The water availability of surface water and groundwater can be found in "Water-saving Reform Report of Hongyashan Irrigation District". The crop planting area and irrigation quota refer to the "Annual Report of Irrigation and Water Conservancy". Basic data are listed in Table 1, among which, crop agricultural water productivity is the ratio of the actual crop yield and water use amount. Irrigation water efficiency of surface water and groundwater are 0.55 and 0.85, respectively. Ecological water requirement of downstream is 76 million  $\text{m}^3$ . Water requirement, precipitation and water supply conditions are listed in Figure 3. Table 2 shows the data of the irrigation requirement and effective precipitation for each studied crop in Hongyashan ID. The market prices come from "Agricultural Product Price Information Network of China", and per capita demand of each food refers to Su et al. (2014) [43].

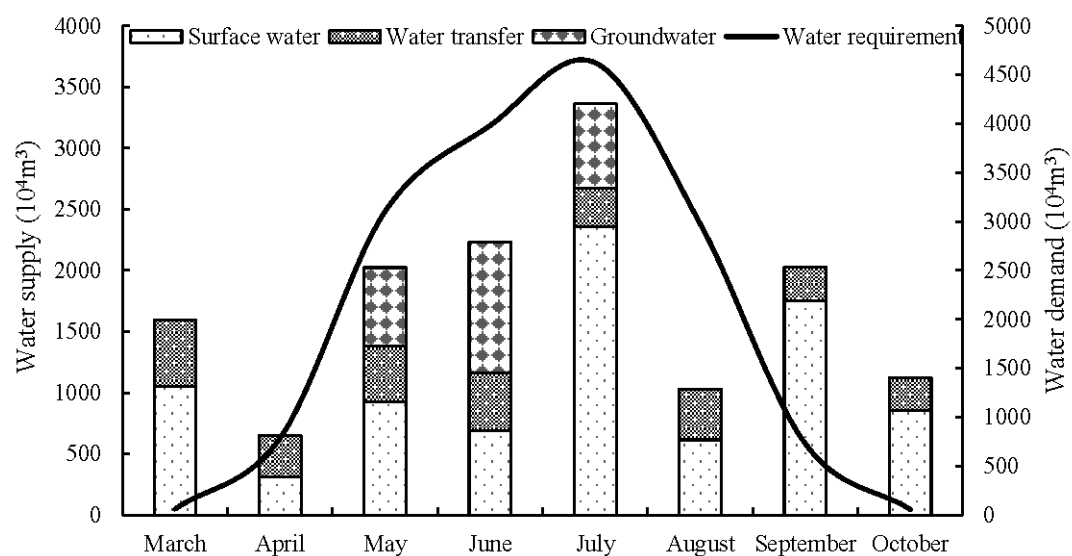


Figure 3. Monthly water supply and water requirement.

**Table 1.** Basic data for typical crops in Hongyashan irrigation district.

Parameter	Unit	Spring Wheat	Spring Maize	Oil Flax	Seed Watermelon	Cotton
ai	-	1.8148	2.4091	0.5133	0.6973	0.6753
bi	-	−507.96	−421.35	529.72	1478.61	−132.88
Irrigation quota	m <sup>3</sup> /ha	4100	4050	3500	2250	2100
Planting area	10 <sup>4</sup> ha	0.4026	0.8682	0.96	0.2088	0.6798
Water use productivity	kg/m <sup>3</sup>	1.79	2.5	0.76	1.45	0.77
Market price	Yuan/kg	2	1.7	6	7	12
Planting cost per unit area	Yuan/ha	3500	3500	3000	4500	3000
Labor cost per unit area	Yuan/ha	6000	8500	7000	7000	8000
Planting proportion	%	12.91	27.83	30.78	6.69	21.79
Output per unit area	kg/ha	7321.16	10,143.4	2652.38	3268.62	1621.67
Profit per unit area	Yuan/ha	5142.32	6258.12	8566.5	8111.72	8460.02
Per capita crop yield demand	kg/per	200	150	30	5	50

**Table 2.** Irrigation requirement (IR) and effective precipitation (EP) for different crops m<sup>3</sup>/ha.

Crop	Index	Growth Period	March	April	May	June	July	August	September	October	Total
Spring wheat	IR	3.21–7.16	154.35	1023.15	1755.04	1969.12	1145.25	0.00	0.00	0.00	6046.91
	EP		2.84	54.40	60.80	100.80	170.12	0.00	0.00	0.00	388.95
Spring maize	IR	4.14–9.13	0.00	195.30	908.86	1413.32	1731.43	1745.37	284.65	0.00	6278.92
	EP		0.00	29.01	60.80	100.80	329.60	287.20	33.97	0.00	841.39
Oil flax	IR	4.17–8.27	0.00	172.68	1347.62	1667.40	1647.38	754.19	0.00	0.00	5589.27
	EP		0.00	23.57	60.80	100.80	329.60	250.14	0.00	0.00	764.92
Seed watermelon	IR	5.1–9.17	0.00	0.00	579.79	794.00	1378.42	662.97	268.23	0.00	3683.41
	EP		0.00	0.00	60.80	100.80	329.60	287.20	44.43	0.00	822.83
Cotton	IR	4.21–10.22	0.00	58.16	282.06	285.84	1159.89	879.45	627.90	84.80	3378.09
	EP		0.00	16.32	60.80	100.80	329.60	287.20	78.40	70.40	943.52

## 4. Results Analysis and Discussion

### 4.1. Results of Relative Optimum Membership Degree

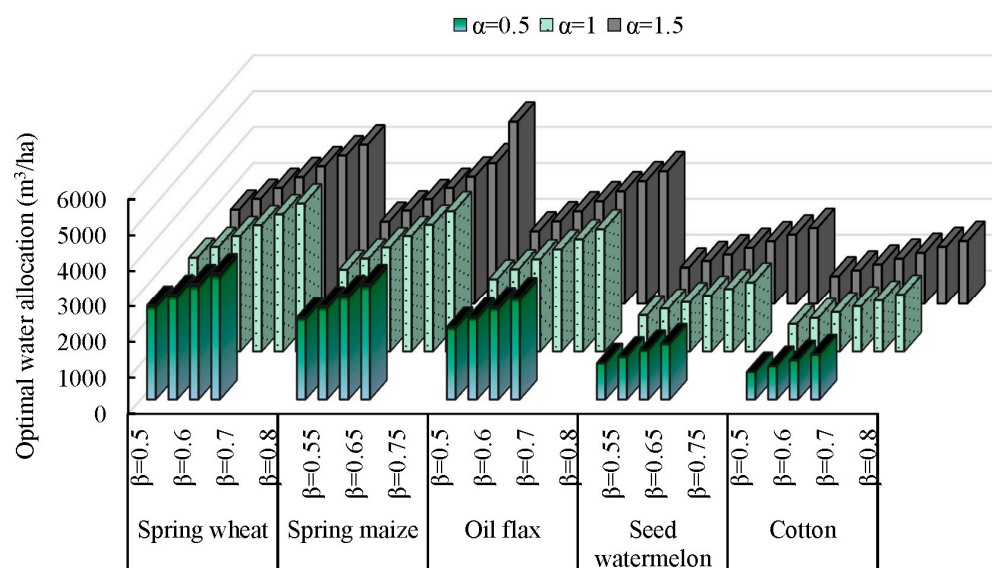
The relative optimum membership degree for different crops ( $u_{bj}$ ) is an important input for irrigation water resources allocation which reflects the coordination of multiple factors. The determination of  $u_{bj}$  is based on different evaluation indicators that have been introduced in Methodology Section 3.1, reflecting the characteristics of resources, economy and society. The values of the indicators and the calculated  $u_{bj}$  of different crops are listed in Table 3. Among which, net benefit per unit area is calculated using the crop output (the product of crop yield and market price) minus crop planting cost, commodity proportion is the ratio of the yield of each crop to the yield of all crops, and the value of per capital grain guarantee rate index was obtained by the ratio of the per capital share of grain to the corresponding average value of Gansu Province. Based on the fuzzy optimum selecting theory, the  $u_{bj}$  value for spring wheat, spring maize, oil flax, seed watermelon and cotton are 0.1919, 0.4374, 0.5730, 0.6897, and 0.5186, respectively. It is obvious that values of  $u_{bj}$  for economic crops are larger than that for grain crops, attributing to the larger irrigation quota and smaller economic benefit for food crops. For food crops, the value of  $u_{bj}$  for spring maize is larger than that for spring wheat. This is because the yield per unit area for spring maize is larger than that for spring wheat. In other words, the social benefit of spring maize is higher than that of spring wheat. When it comes to other crops, the sequence decreasingly for values of  $u_{bj}$  is seed watermelon, oil flax and cotton, with little difference.

**Table 3.** Main indicators for  $u_{bj}$  of different crops.

Crop	Net Benefit per Unit Area (Yuan/ha)	Commodity Proportion (%)	Irrigation Quota (m <sup>3</sup> /ha)	Per Capita Guarantee Rate	$u_{bj}$
Spring wheat	5142.32	18.32	4100	0.47	0.1919
Spring maize	6258.12	54.75	4050	1.86	0.4374
Oil flax	8566.50	15.83	3500	2.69	0.5730
Seed watermelon	8111.72	4.24	2250	4.33	0.6897
Cotton	8460.02	6.85	2100	0.70	0.5186

### 4.2. Water Resources Optimal Allocation Schemes

Based on the  $u_{bj}$  values and other basic data for different crops, the developed linear fractional programming model for water resources allocation was solved. Monthly water allocation of both surface water and groundwater under the scenarios of different  $\alpha$  values (the proportion coefficient of water transfer) and  $\beta$  values (the minimum water requirement coefficient) can be obtained (see Figure 4). As the figure shows, total water amount increases as  $\alpha$  value increases, which leads to the increase of the  $\beta$  value. This is because the study area is facing a serious water shortage problem, leading to the current water supply being unable to meet the water requirement, especially in April and August. The increase of water transfer can alleviate the condition of water shortage to some extent and thus larger  $\beta$  values can be adopted to ensure the optimization model has feasible solutions. For example, when the  $\alpha$  value equals 0.5, the maximum  $\beta$  value can be set 0.65 to ensure the model has feasible solutions. In other words, because of the inadequate of water supply, the minimum water requirement coefficient can only have four values, that is,  $\beta = 0.5$ ,  $\beta = 0.55$ ,  $\beta = 0.6$ , and  $\beta = 0.65$ . Similarly, when the  $\alpha$  value equals 1 and 1.5, the maximum  $\beta$  values can, respectively, reach 0.75 and 0.8. As  $\alpha$  value and  $\beta$  value increases, water allocation amounts increase in the premise of ensuring crop yield and improving agricultural economic benefits. Besides, as shown in Figure 4, water allocation showed an increasing trend as  $\beta$  value increases under different  $\alpha$  values.



**Figure 4.** Optimal water resources allocation amounts for different crops under different scenarios.

Taking  $\alpha = 1$  and  $\beta = 0.65$  as an example, the water allocation results can be seen in Figure 5. Water allocation amounts for spring wheat is  $1.426 \times 10^7 \text{ m}^3$ , for spring maize is  $2.813 \times 10^7 \text{ m}^3$ , for oil flax is  $2.753 \times 10^7 \text{ m}^3$ , for seed watermelon is  $0.328 \times 10^7 \text{ m}^3$  and for cotton is  $0.862 \times 10^7 \text{ m}^3$ . Total water allocation amount is  $8.812 \times 10^7 \text{ m}^3$ . It can be seen from the figure that the water resources amounts are allocated mainly from May to August, and water allocated during this period accounts for nearly 90% of the whole growth period. Because the value of water requirement is larger than the value of water supply, and water distribution is uneven, the deficit irrigation method should be adopted to irrigate the selected crops. Water shortage for grain crops such as spring wheat and spring maize is the largest, while the water shortage for economic crops such as cotton and seed watermelon is the smallest. This is because the relative optimum membership degree for different crops ( $u_{bj}$ ) of economic crops is larger than that of grain crops. For the developed optimization model, water resources amounts are given priority to the crops with higher  $u_{bj}$  value in order to maximize the comprehensive benefit in the premise of guaranteeing the minimum water requirement of each crop.

The yield of each crop can be obtained through inputting the optimized water (including both surface water and groundwater) amounts into the crop water product function during the whole growth period (see Figure 6). As the figure shows, the yield of each crop increases as the  $\beta$  value increases, and the crops corresponding to the increasing trend of yield from high to low are: spring maize, spring wheat, seed watermelon, cotton, and oil flax. Compared with the results of Figures 5 and 6, there is a linear relationship between crop yield and total water allocation amounts. Hence, the changing trend of yield and water allocation amounts is the same.

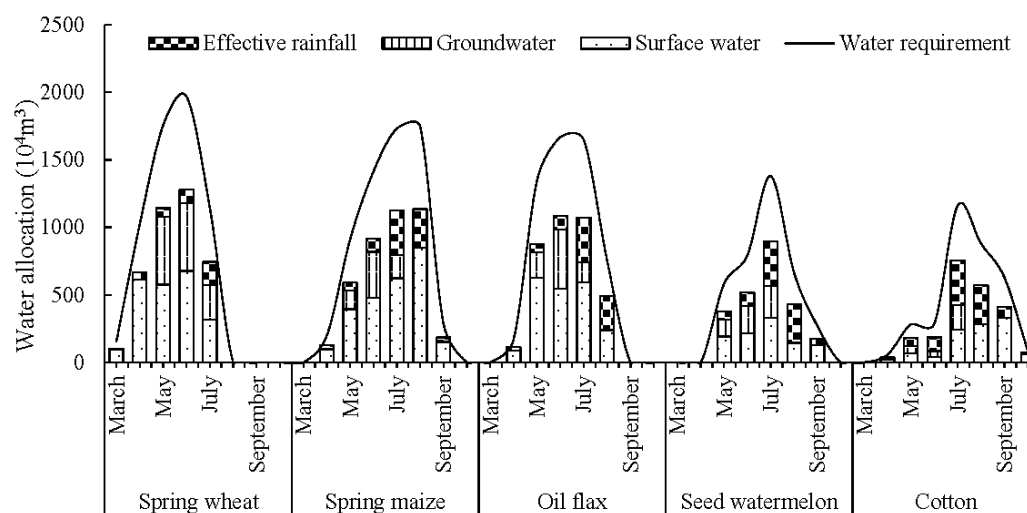


Figure 5. Optimal water allocation amount for typical crops in different months.

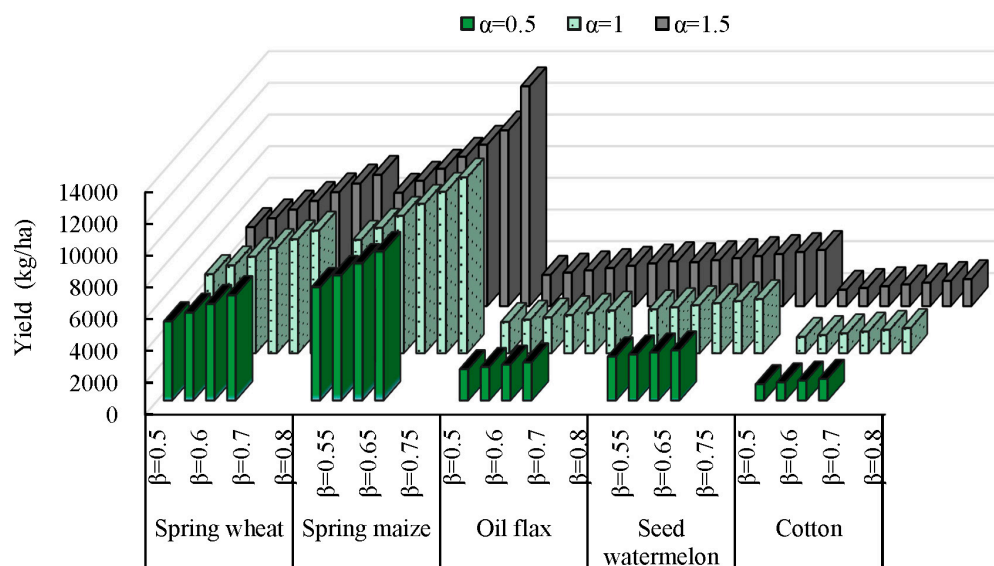
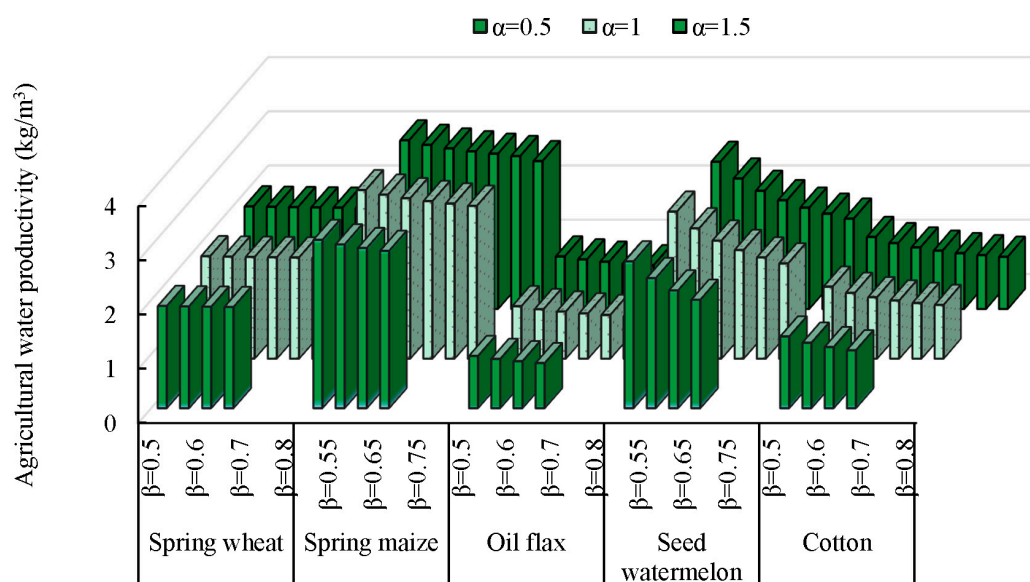


Figure 6. Yield for different crops under different scenarios.

As the quantitative expression of water transformation and water use efficiency, AWP refers to the crop yield or output per unit water use, and can be calculated by dividing the crop yield or revenue by water consumption. This study adopted the former expression, i.e., crop yield per unit water use. As shown in Figure 7, the AWP values for all the crops is decreasing as the value of  $\beta$  increases under  $\alpha = 1$ , among which, the decreasing range of seed watermelon is the largest, while the decreasing range of spring wheat is the lowest. There is a negative correlation between the decreasing range and the value of  $u_{bj}$ , that is, the larger the  $u_{bj}$  value, the smaller the decreasing range of AWP, and vice versa. The total AWP values fluctuate from 1.73 kg/m<sup>3</sup> to 1.98 kg/m<sup>3</sup>. The optimized AWP has improved compared with the current AWP (1.54 kg/m<sup>3</sup>). When  $\beta = 0.5$ , the maximum AWP for each crop can be obtained, i.e., 1.89 kg/m<sup>3</sup> for spring wheat, 3.11 kg/m<sup>3</sup> for spring maize, 0.87 kg/m<sup>3</sup> for oil flax, 2.71 kg/m<sup>3</sup> for seed watermelon and 1.33 kg/m<sup>3</sup> for cotton. When  $\alpha = 1.5$ , the minimum AWP for each crop was obtained, i.e., 1.85 kg/m<sup>3</sup> for spring wheat, 2.72 kg/m<sup>3</sup> for spring maize, 0.76 kg/m<sup>3</sup> for oil flax, 1.66 kg/m<sup>3</sup> for seed watermelon and 0.96 kg/m<sup>3</sup> for cotton. Moreover, the benefits per unit water use for different scenarios were also calculated and compared. With the increase of water allocation

amount, there is an obvious increasing trend for the benefit per unit water use. The increase of water allocation amount leads to the increase of crop yield, and thus results in the increase of benefit per unit water use, indicating that the increase of water allocation amount is beneficial for the increase of benefit per unit water use. When  $\beta = 0.75$ , the benefit per unit water use reaches its peak with the value of 6.03 Yuan/m<sup>3</sup>, and when  $\beta = 0.5$ , the benefit per unit water use falls to its lowest level with the value of 5.54 Yuan/m<sup>3</sup>. Generally, the benefit per unit water use appears to be rising as the value of  $\beta$  increases. However, when  $\beta = 0.8$ , the benefit per unit water use (5.99 Yuan/m<sup>3</sup>) is just lower than the corresponding value under  $\beta = 0.75$ . This results from the optimization model's structure, giving priority to allocate water for spring maize prior to other crops. Spring maize belongs to food crops, with lower benefit per unit water use compared with economic crops. The increased water supply that results from the increasing of  $\beta$  values was allocated to spring maize, leading to the lower benefit per unit water use under  $\beta = 0.8$  compared with that under  $\beta = 0.75$ . As the results show, if the decisions of managers are paid more attention, the water allocation for crops would be less, and then crop yield would decrease, but the AWP would increase. On the contrary, if the decisions of farmers are considered, the water allocation for crop would be more, and the crop yield and revenue would increase, but the AWP would decrease. Accordingly, how to balance the contradiction among water resources allocation, crop yield, revenue and AWP is an interesting topic that deserves further research.



**Figure 7.** Agricultural water productivity for different crops under different scenarios.

#### 4.3. Effects on the Multi-Dimensional System

In order to comprehensively analyze the effects of model results on the water resources utilization and agricultural sustainable development, multiple dimensions including resource, economy, society and ecology were selected to analyze the optimal water allocation results. The indicators for multi-dimensional regulation are listed in Table 4 under different  $\alpha$  and  $\beta$  values. Based on the multi-dimensional critical regulation theory, the order degree for each subsystem and the coordination degree of the various water allocation schemes obtained from different scenarios (different  $\alpha$  and  $\beta$  values) are listed in Table 5.



**Table 4.** Key indicators for multi-dimensional regulation under different scenarios.

$\alpha$	$\beta$	Resource Dimension	Economic Dimension	Social Dimension	Ecological Dimension
		Surface Water Allocation Proportion (%)	Revenue per Unit Water (Yuan/m <sup>3</sup> )	Comprehensive Agricultural Water Productivity (kg/ha)	Groundwater Exploitation (%)
0.5	0.5	42.65	0.79	1.98	59.86
	0.55	48.99	1.23	1.90	67.06
	0.6	56.07	1.56	1.85	71.11
	0.65	59.98	1.83	1.80	88.58
1	0.5	37.00	0.79	1.98	59.86
	0.55	42.50	1.23	1.90	67.06
	0.6	48.64	1.56	1.85	71.11
	0.65	52.03	1.83	1.80	88.58
	0.7	58.15	2.05	1.76	92.79
	0.75	64.11	2.25	1.73	100.00
1.5	0.5	43.54	0.79	1.98	59.86
	0.55	49.71	1.23	1.90	67.06
	0.6	55.87	1.56	1.85	71.11
	0.65	62.04	1.83	1.80	88.58
	0.7	68.20	2.05	1.76	92.79
	0.75	74.78	2.25	1.73	100
	0.8	86.47	2.51	1.75	100

Note:  $\alpha$  is the scaling factor of water transfer and  $\beta$  is the scaling factor of the minimum water requirement.

**Table 5.** Multi-dimensional regulation results under different scenarios.

$\alpha$	$\beta$	Order Degree				Coordination Degree
		Resource Dimension	Economic Dimension	Ecological Dimension	Social Dimension	
0.5	0.5	0.49	0.32	1.00	1.00	0.6282
	0.55	0.57	0.49	0.89	0.96	0.6976
	0.6	0.65	0.62	0.84	0.93	0.7505
	0.65	0.69	0.73	0.68	0.91	0.7467
1	0.5	0.43	0.32	1.00	1.00	0.6063
	0.55	0.49	0.49	0.89	0.96	0.6732
	0.6	0.56	0.62	0.84	0.93	0.7243
	0.65	0.60	0.73	0.68	0.91	0.7207
	0.7	0.67	0.82	0.65	0.89	0.7493
	0.75	0.74	0.90	0.60	0.87	0.7676
1.5	0.5	0.50	0.32	1.00	1.00	0.6314
	0.55	0.57	0.49	0.89	0.96	0.7001
	0.6	0.65	0.62	0.84	0.93	0.7499
	0.65	0.72	0.73	0.68	0.91	0.7531
	0.7	0.79	0.82	0.65	0.89	0.7798
	0.75	0.86	0.90	0.60	0.87	0.7977
	0.8	1.00	1.00	0.60	0.88	0.8529

Coordination degree is a measure of the harmonious degree among different elements, i.e., dimensions of resources, economy, society and ecology in this study; and the larger is the coordination degree, the closer it is towards to the equilibrium. The water allocation schemes corresponding to the largest coordination degree is the optimum. In Figure 5, the coordination degree of the system is increasing as both  $\alpha$  and  $\beta$  values increase, indicating the system is becoming increasingly coordinate when water allocation is increasing. From the results, the optimum scheme is obtained when  $\alpha = 1.5$  and  $\beta = 0.8$ . Under this scenario, the system entropy is 0.1269 and the coordination degree is 0.8529. The corresponding water allocation amounts for spring wheat, spring maize, oil flax, seed watermelon and cotton are  $1.791 \times 10^7 \text{ m}^3$ ,  $4.414 \times 10^7 \text{ m}^3$ ,  $3.558 \times 10^7 \text{ m}^3$ ,  $0.443 \times 10^7 \text{ m}^3$ ,  $1.197 \times 10^7 \text{ m}^3$ , respectively. Total water allocation amount is  $1.1405 \times 10^8 \text{ m}^3$ .

Considering the comprehensive impacts of the four dimensions including resource dimension, economic dimension, social dimension and ecological dimension, as water supply is increasing, the attribute value of economic dimension and resource dimension is increasing, while the attribute value of ecological dimension and social dimension is decreasing. The changing range of resource, economic, social and ecological dimensions are (0.49, 1), (0.32, 1), (0.6, 1), and (0.87, 1), respectively. The changes of the economic dimension is the most prominent while the changes of social dimension is the least significant, therefore, the indexes of the economic dimension is the main influence factor for the comprehensive analysis.

#### 4.4. Discussion

This study developed an integrated system for agricultural water resources optimal allocation within a linear fractional programming optimization model framework to maximize AWP. The advantage of this study is to consider the synergistic effects of the key factors that affect the AWP, which were quantified as coefficients  $u_{bj}$  to be inputted in the optimization model. Moreover, the optimal water allocation schemes under different scenarios obtained from the optimization model were screened considering the mutual development of different dimensions including resource, economy, society, and ecology to promote the sustainable development of the agricultural water management system. The core of the integrated system is the water resources optimal allocation model with the objective to increase AWP considering the constraints of water supply, water balance, food security, water requirement, and ecological health. All these constraints are associated with water resources. The changes in the irrigated land utilization, however, will also affect the water resources schemes and thus influences the AWP. This study did not consider the land condition because all the work of this study is based on a specific year (2014), and, for a specific year, the irrigated land utilization is stable. The sensitivity of the model toward land expansions and other elements will be analyzed in the future to improve the integrated framework.

For the developed system, the value of the relative optimum membership degree ( $u_{bj}$ ) is a very important input for the optimization model which will directly affect different water allocation schemes. For this study, the net benefit per unit area, commodity proportion, per capital guaranteed rate of grain and irrigation quota for various crops were considered, which indicate the indicators from economy, society and resources. Nevertheless, these indicators lead to the value of  $u_{bj}$  for economic crops is larger than that for food crops, and the value of  $u_{bj}$  for spring maize is larger than that for spring wheat. In such a condition, the water allocation to spring wheat is unfavorable. The selection of indicators need to be improved in order to more fully reflect actual conditions and involve more factors with the consideration of actual data and the applicability of indicators.

In addition, this study considered two scenarios by changing water transfer proportion and minimum water requirement. The smaller the value of the minimum water requirement, the value of AWP is larger, attributing to the model's structure. The objective function of the optimization model is to increase the comprehensive AWP for all studied crops, and the results generally tend to allocate the lower limit value of water allocation. When less water was allocated, the AWP value would increase, but crops yield and benefit would decrease. The values of  $\alpha$  and  $\beta$  affect each other. For example, when

the value of  $\alpha$  increases from 0.5 to 1.5, the value of  $\beta$  would increase from 0.65 to 0.8 on the premise of obtaining the feasible solutions of the developed optimization model. For Hongyashan irrigation area which located in semi-arid area with serious water shortage problems, increasing transferring water and thus increasing the minimum water allocation amount for each crop is necessary and beneficial to increasing the efficiency of water allocation.

## 5. Conclusions

An integrated system for increasing agricultural water productivity was developed. Such a system mainly included three parts: (1) the quantification of synergistic effects of multiple factors that are associated with AWP based on fuzzy optimum selecting theory; (2) the allocation of limited water supply (both surface water and groundwater) for agricultural use based on a linear fractional programming model to increase AWP; and (3) the evaluation and screenings of water allocation schemes based on multi-dimensional regulation theory. These three parts are closely connected, forming an integrated system that will promote sustainable agricultural water management. The developed integrated system has advantages in increasing agricultural water productivity through the coordination of multiple factors representing aspects of economy, society and resources by an innovative introduction of fuzzy optimum selecting theory into the fractional programming optimization framework. Moreover, the system is capable of screening schemes considering harmonious development of resources, economy, society and ecology based on optimization results, providing decision makers with more sustainable schemes for irrigation water allocation. This paper proposed a method for agricultural water resources management from a perspective of synergy and sustainability.

The developed system was applied to a real world study in Hongyashan irrigation district in Minqin County, northwest of China. Results of agricultural water resources allocation under different scenarios were generated. Decision makers for water resources management can choose water allocation scheme according to scenarios and actual conditions. Results show that the benefit per unit water use fluctuates from 5.54 Yuan/m<sup>3</sup> to 6.03 Yuan/m<sup>3</sup> and agricultural water productivity fluctuates from 1.73 kg/m<sup>3</sup> to 1.98 kg/m<sup>3</sup> under different combined scenarios of water transfer and crop water requirement, which interact with each other. Average agricultural water productivity increased by 0.31 kg/m<sup>3</sup> compared with actual conditions. Based on the multi-dimensional regulation theory, an optimum water allocation scheme was selected with the coordination degree being 0.8529, and total water allocation amount of  $1.1405 \times 10^8$  m<sup>3</sup>.

The developed system can also be used to optimize water or land resources at regional and other scales, for maximized system efficiency. This study does not consider uncertainties, which are ubiquitous in water resources management. This deserves further research in future studies.

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