

Article

Risk Analysis Approach to Rainwater Harvesting Systems

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Abstract: Urban rainwater reuse preserves water resources and promotes sustainable development in rapidly growing urban areas. The efficiency of a large number of urban water reuse systems, operating under different climate and demand conditions, is evaluated here on the base of a new risk analysis approach. Results obtained by probability analysis (PA) indicate that maximum efficiency in low demanding scenarios is above 0.5 and a threshold, distinguishing low from high demanding scenarios, indicates that in low demanding scenarios no significant improvement in performance may be attained by increasing the storage capacity of rainwater harvesting tanks. Threshold behaviour is displayed when tank storage capacity is designed to match both the average collected volume and the average reuse volume. The low demand limit cannot be achieved under climate and operating conditions characterized by a disproportion between harvesting and demand volume.

Keywords: risk analysis; efficiency; rain water harvesting

1. Introduction

Sustainable water resource management is an urgent task in evolving land where the population grows in rapidly expanding urban areas [1,2]. Increased runoff, impeded groundwater recharge, poor water quality of surface water bodies and increasing water demand represent a threat to population health and development [3]. Urban water reuse is a key process of urban hydrology, since it closes the water cycle [4] in environments where soil and atmosphere are disconnected by large and highly populated impervious areas [5–7].

Rainwater harvesting tanks are expected to provide reuse-water according to demand with limited overflow, thus avoiding the loss of water resources. The tank size affects the cost of the RWH system and the optimum storage capacity is achieved when no significant improvement of performance may be attained by increasing size and costs. Efficiency of rain water harvesting (RWH) systems is commonly evaluated by numerical continuous simulation (CS) of daily, weekly or monthly water balance within behavioural models. Abdulla and Al-Shareef [8] evaluate the efficiency of RWH cisterns for various domestic uses in Jordan based on averaged monthly precipitation data. Ghisi et al. [9] evaluate the efficiency of a RWH system for washing vehicles in Brazilia (Brazil) based on 30 years of daily rain observed data. Mwenge Kahinda et al. [10] evaluate RWH systems located in four climatic regions of South Africa, for household water requirements, by using rainfall data forecasted by six global circulation models based on 10 to 20 years of observation. Mehrabadi et al. [11] estimate the Reliability of RWH systems for residential buildings in three Iranian cities located in very different climatic regions with mean annual rainfall ranging from 130 to 1300 mm·yr^{−1}. Daily rainfall statistics are derived from 50 years of observation. Palla et al. [12] analyze the performance of RWH systems for residential use in Italy based on daily rainfall recorded over 30 or 100 years. Zhang et al. [13] estimate RWH system efficiency in four Australian cities belonging to different climatic regions, based on monthly averaged from 80 years of observed daily rainfall. A literature review [14] highlights

a threshold between low and high demanding applications. Whether there is a relation between threshold demand and optimum tank size needs further investigation. In low demanding applications, efficiency reaches a maximum which does not depend on the water demand. Whether this is a general result or a specific feature of the subset of tanks analyzed by Sanches-Fernandes et al. [14] and referred here, is not clear, neither literature data clarify what is the impact of climate and operating conditions on the demand threshold.

Efficiency is an increasingly good estimate of the probability that a RWH system performs its task (Reliability), as the number of years of observation increases. Risk of failure is the complement to Reliability. Alternatively to CS, probability analysis (PA) [15] provides closed form expressions for hydraulic Risk of failure, under the assumption that both rain depth and inter-storm interval are exponentially distributed. This assumption is supported by statistical analysis of rainfall series under very different climate conditions [15–17] and was previously successfully applied to risk analysis of different kind of Best Management Practice [18–21].

Data collected by Sanches-Fernandes et al. [14] are re-analyzed here. Literature results based on statistically significant rainfall series, obtained by CS and new results obtained by PA are compared. Maximum Efficiency and the demand threshold separating low and high demanding applications [14] are re-evaluated and confirmed here in light of a new PA-based risk analysis approach. Risk analysis supports previous findings, envisions their limit of application, and leads to broader statements on optimal RWH tank size.

2. Model

A schematic RWH system (Figure 1), includes (1) the contributing area (S) of the catchment with runoff coefficient ϕ which provides the rainfall volume ϕSh to the RWH tank; (2) the tank which has maximum storage capacity V_s (3) and overflow discharge to final destination. Rainwater is stored in the tank during rainfall up to the maximum storage capacity, and excess rainfall is lost. During the inter-event time t water is reused at rate Q_0 .

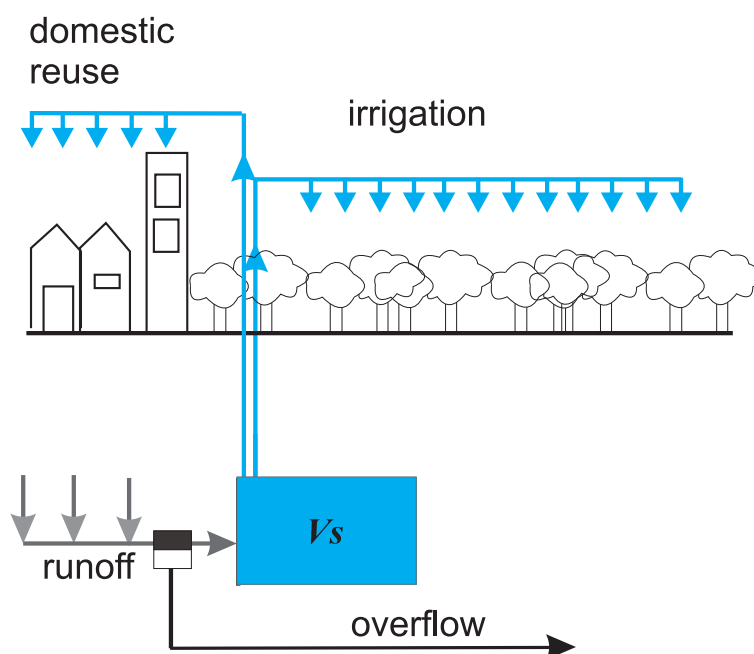


Figure 1. Conceptual model. Schematic representation of catchment, rain water harvesting (RWH) and reuse system.

The meteorological input to the catchment is represented by the probability density functions of rainfall depth (h) and inter-storm interval (t) Following the approach proposed by

Adams and Papa [15] h and t are modelled as random variables with exponential probability density functions.

$$f_h = \zeta e^{-\zeta h} \quad (1)$$

$$f_t = \lambda e^{-\lambda t} \quad (2)$$

where ζ is the inverse of expected value of rainfall depth and λ the inverse of expected inter-storm interval.

The water volume stored in the tank at the end of the first of two consecutive rain events is available for reuse thus allowing the RWH system to perform its task throughout the whole or a part of the following dry inter-storm interval. Yet, a residual volume in the tank increases the probability of overflow when the second rain event occurs. The Risk of failure is the probability that the tank does not perform its task because it cannot provide sufficient reuse-water when required.

Closed form solutions for the Risk of water scarcity and Reliability of a reuse tank are derived in the following Section. Efficiency is assimilated to tank Reliability. The closed form solution for the Risk of overflow is derived, as it could clarify whether low Efficiency is caused by insufficient storage volume or unbalanced demand and climate conditions.

3. Materials and Methods

With regard to the Risk of water scarcity, the most and least conservative assumptions are that the tank is empty and full, respectively, at the end of the first of two consecutive rain events.

Under the most conservative assumption, the Risk of water scarcity R_i is the probability that rainfall volume is $\phi Sh < V_s$ and the time to the next rainfall event is greater than $\phi Sh / Q_0$, or that rainfall volume is $\phi Sh \geq V_s$ and the next rainfall event occurs after complete tank draw-down, i.e., V_s / Q_0 .

$$R_i = P[h < V_s / (\phi S)] \cdot P[t > \phi Sh / Q_0] + P[h \geq V_s / (\phi S)] \cdot P[t > V_s / Q_0] \quad (3)$$

Combining Equations (1)–(3) and integrating, the Risk of water scarcity results in the following expression

$$R_i = \frac{b}{a+b} e^{-(a+b)} + \frac{a}{a+b} \quad (4)$$

where a and b are dimensionless parameters accounting for the climate, catchment characteristics and RWH system size and management. Namely: $a = \frac{\zeta V_s}{\phi S}$ and $b = \frac{\lambda V_s}{Q_0}$.

Under the least conservative assumption, Equation (4) reduces to

$$R'_i = P[t > V_s / Q_0] = e^{-b} \quad (5)$$

With regard to the Risk of overflow during the second of two conservative rain events, the most and least conservative assumptions are that the tank is full at the end of the first and empty at the beginning of the second of two consecutive rain events, respectively.

Under the assumption that the first event completely fills the tank, overflow occurs with probability R_f

$$R_f = P[t \geq V_s / Q_0] \cdot P[\phi Sh > V_s] + P[t < V_s / Q_0] \cdot P[\phi Sh > Q_0 t] \quad (6)$$

Combining Equations (1), (2) and (6) and integrating,

$$R_f = \frac{a}{a+b} e^{-(a+b)} + \frac{b}{a+b} \quad (7)$$

Under the assumption that the tank is empty at the beginning of the second rain event, Equation (7) reduces to

$$R'_f = P[\phi S h > V_s] = e^{-a} \quad (8)$$

When either $V_s \gg \phi S \zeta^{-1}$ or $V_s \gg Q_0 \lambda^{-1}$, thus a or $b \gg 1$, meaning that the tank storage capacity is much larger than the average runoff volume $\phi S \zeta^{-1}$ or reuse volume $Q_0 \lambda^{-1}$, Equations (4) and (7) reduce to

$$R''_f = \frac{b}{a+b} \quad (9)$$

and

$$R''_i = \frac{a}{a+b} \quad (10)$$

When a or $b \gg 1$, $R''_f + R''_i = 1$.

Efficiency is the capacity of the RWH system to satisfy the water demand. Thus, based on PA, Efficiency may be estimated as the probability that RWH tank provides water when needed, which is the complementary probability of the Risk of water scarcity. Under the assumption that the reservoir is empty at the beginning of the first of two rain events, Efficiency E is:

$$E = 1 - \left[\frac{b}{a+b} e^{-(a+b)} + \frac{a}{a+b} \right] = \frac{b}{a+b} \left[1 - e^{-(a+b)} \right] \quad (11)$$

and under the assumption that the tank is empty at the beginning of any rain event, Efficiency becomes:

$$E' = 1 - e^{-b} \quad (12)$$

The Demand Ratio is defined as the ratio between the average inter-storm demand and the average stored rainwater at the end of a rain event. The average collected rainwater corresponds to the non-overflowing rain water diverted to the tank. If the tank is empty at the beginning of the second of two consecutive rain events, it is the minimum between $\phi S h$ and V_s .

With regard to the Demand Ratio, the most conservative assumption is that the tank is full at the end of the first of two consecutive rainfall events. When the second of two consecutive rain events occurs in a time t shorter than the draw down time $t < V_s/Q_0$, the collected rainwater volume is the minimum between $\phi S h$ and $Q_0 \cdot t$; when $t \geq V_s/Q_0$, the collected volume is the minimum between $\phi S h$ and V_s . Thus, the average collected volume V_c may be estimated as follows:

$$V_c = \int_0^{\frac{V_s}{Q_0}} \lambda e^{-\lambda t} \left(\int_0^{\frac{Q_0 t}{\phi S}} \phi S h \zeta e^{-\zeta h} dh + Q_0 t \int_{\frac{Q_0 t}{\phi S}}^{\infty} \zeta e^{-\zeta h} dh \right) dt \\ + \int_{\frac{V_s}{Q_0}}^{\infty} \lambda e^{-\lambda t} \left(\int_0^{\frac{V_s}{\phi S}} \phi S h \zeta e^{-\zeta h} dh + \frac{V_s}{\phi S} \int_{\frac{V_s}{\phi S}}^{\infty} \zeta e^{-\zeta h} dh \right) dt \quad (13)$$

By integrating, Equation (13)

$$V_c = \phi S \zeta^{-1} \frac{a}{a+b} \left[1 - e^{-(a+b)} \right] = \frac{V_s}{a+b} \left[1 - e^{-(a+b)} \right] \quad (14)$$

The least conservative assumption is that the tank is empty at the beginning of any rain events. In this case, the collected rainwater volume is the minimum between $\phi S h$ and V_s , and the average collected volume V'_c is

$$V'_c = \int_0^{\frac{V_s}{\phi S}} \phi S h \zeta e^{-\zeta h} dh \int_{\frac{V_s}{\phi S}}^{\infty} \zeta e^{-\zeta h} dh = \frac{V_s}{a} (1 - e^{-a}) \quad (15)$$

As the average inter-storm demand is $Q_0 \lambda^{-1}$, the Demand Ratio, depending on the modelling assumption concerning the initial condition, ranges between

$$DR = \frac{Q_0}{V_c \lambda} = \frac{(a+b)}{b a} \left[1 - e^{-(a+b)} \right]^{-1} \quad (16)$$

and

$$DR' = \frac{Q_0}{V'_c \lambda} = \frac{a}{b} (1 - e^{-a})^{-1} \quad (17)$$

4. Results

Risk of water scarcity, Risk of overflow, Efficiency and Demand Ratio are estimated based on PA for a large number of RWH systems designed to perform their task under different climate and operating conditions. Figure 2 shows R_i versus R_f estimated under the most conservative assumptions (red symbols), and R'_i versus R'_f estimated under the least conservative assumptions (green symbols).

Under the least conservative assumptions the Risk of overflow R'_f (Equation (8)) vanishes for $R'_i > 0.2$ (Figure 2, green symbols). Meaning that the tank storage capacity V_s is much larger than the average runoff volume $\phi S \zeta^{-1}$ diverted to the tank. As a consequence, under the most conservative assumptions R_f and R_i (Equations (4) and (7)) approach the limit values R''_f and R''_i (Equations (3) and (10)) and fulfil the relation $R_f \approx 1 - R_i$.

The Risk of failure of the system lays between R'_i and R_i and it increases with decreasing Risk of overflow, for the real tanks re-examined here. Results shown in Figure 2 suggest that small tanks with high Risk of overflow operate in low demanding scenarios and those with low Risk of overflow operate in high demanding scenarios.

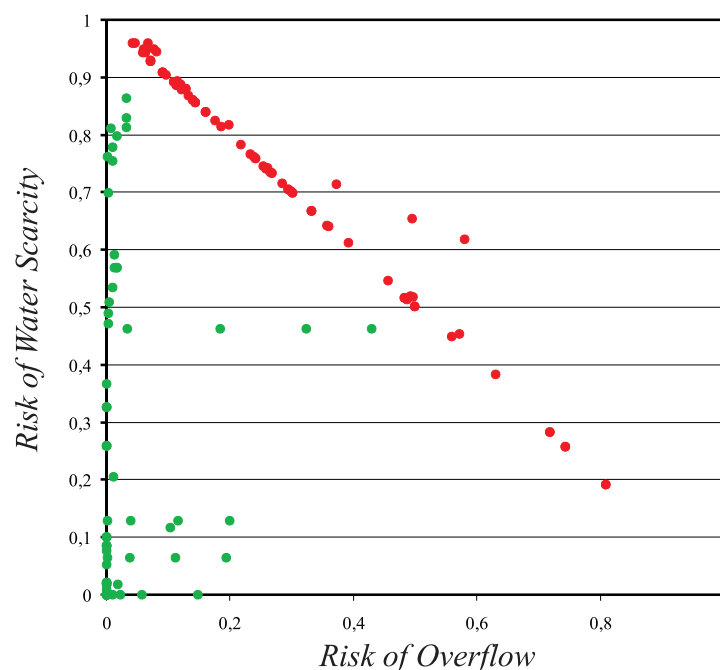


Figure 2. Risk of water scarcity vs. Risk of overflow: Red: Risk of overflow R_f and Risk of water scarcity R_i are estimated under the most conservative assumptions. Green: Risk of overflow R'_f and Risk of water scarcity R'_i are estimated under the least conservative assumptions. The estimates are based on the characteristics of the RWH systems referred by Sanches-Fernandes et al. [14].

Figure 3 shows Efficiency vs. Demand Ratio, evaluated by CS as a function of Demand Ratio [14] (blue symbols) and the upper and lower limits of Efficiency (E and E' respectively), estimated by PA

under the most conservative assumption (red symbols) and under the least conservative assumption (green symbols).

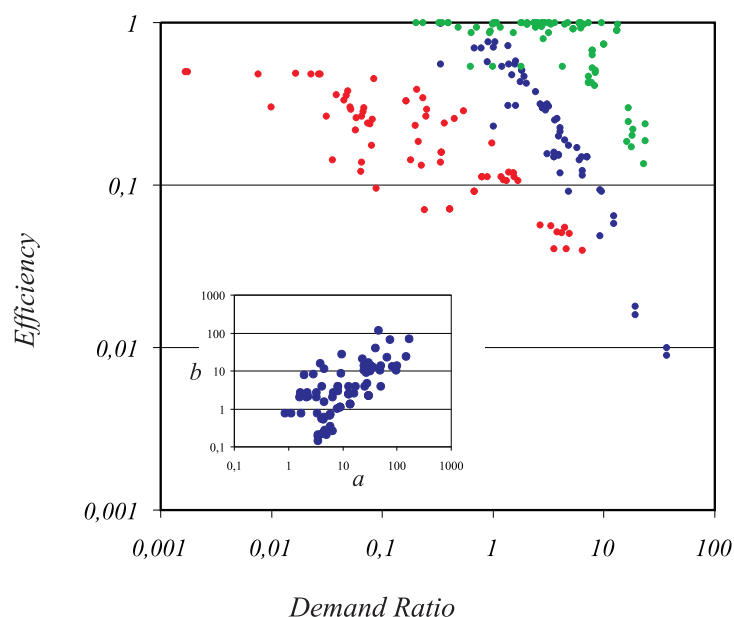


Figure 3. Efficiency vs- Demand Ratio. Blue: data taken from literature [14]. Red: Efficiency E and Demand Ratio DR are estimated under the most conservative assumptions. Green: Efficiency E' and Demand Ratio DR' are estimated under the least conservative assumptions. Inset: corresponding a and b , estimated from literature data.

PA and CS both demonstrate (Figure 3) that the interrelation between Efficiency and Demand Ratio provides a criterion to distinguish between tanks characterized by constant Efficiency (independent of water demand) and tanks with Efficiency decreasing with Demand Ratio. The threshold Demand Ratio indicates when no significant improvement of performance may be attained by increasing the storage capacity of the tank, thus providing information about its optimum size.

Efficiency evaluated by CS (Figure 3, blue symbols) is enveloped by the estimate obtained under the least and the most conservative assumptions (Figure 3, green and red symbols). The transition from constant efficiency to rapidly decreasing efficiency with increasing Demand Ratio is less sharp and switches toward lower Demand Ratio under the more conservative assumptions (Figure 3, red symbols). The threshold Demand Ratio distinguishing low demanding from high demanding scenarios lays between 0.4 and 4, according to PA results (Figure 3).

PA clarifies that the threshold behaviour shown in Figure 3 is typical of the subset of RWH systems for which V_s increases with average drainage volume and average reuse volume (a directly proportional to b , as shown in the inset). It may be easily demonstrated (by using Equations (11), (12), (16) and (17), not shown here) that when $a = b$ the threshold Demand Ratio is between 0.6 and 1, when $a = 10b$ it is between 2 and 10 and when $a = 0.1b$ it switches toward the lower Demand Ratio, namely between 0.2 and 1. Due to the fact that a and b are characteristics of operating tanks, they cluster around the diagonal $a = b$ which is no exact fit through the points. Point dispersion in the $a - b$ plane translates into non-sharp transition from low demanding to high demanding scenarios in the $E - DR$ plane. PA further demonstrates that when a and b (in a set of design alternatives) are inversely proportional, the low demand limit cannot be achieved (see Equations (11) and (16), not shown here).

5. Discussion and Conclusions

The setup of design criteria for sustainable urban water management and reuse becomes an urgent task in rapidly developing urban areas. Climate and land use change motivate the development of strategies for sustainable water management [3], which promote the diffusion of RWH systems. Furthermore, uncertainty and risk assessment, which are not systematically included in urban hydrology problem solving, need to be taken into account [22].

Design criteria based on rainfall time series which are not sufficiently long may not take into the due account the impact of long dry spells [14] which are expected to occur more and more often in climate change scenarios [10]. CS of tank performance and PA may both overcome this limitation, provided statistically significant rainfall data with adequate resolution. PA leads to closed form solutions for a preliminary analysis based on rainfall statistics. CS provides statistics of successful system performance based on the resolution of balance equations within behavioural models.

PA is developed here with focus on two consecutive rain events occurring at random inter-event time [21] and provides upper and lower limits of Risk of Failure, Reliability and Efficiency. By CS, intermediate conditions may be taken into account at the price of the analytical solution of the problem, with reference to a specific situation, and limited generality. PA is also useful for contextually assessing Risk of overflow, which needs to be considered in RWH tank design and management of water resources.

Temporal resolution of rainfall data, demand patterns, and the tank release rule algorithm of behavioural models may affect the performance of RWH systems simulated by CS [23–25]. Despite the simplicity of the risk model, it succeeds in enveloping CS results taken from literature. According to CS, a threshold Demand Ratio seems to separate low demanding and high demanding literature scenarios [14]. Upper and lower limits of Efficiency, estimated by PA, also demonstrate this threshold behaviour. Furthermore, PA clarifies that the threshold behaviour may be achieved if tank storage capacity is designed to match both: the average collected volume and the average reuse volume, whereas, the low demand limit cannot be achieved by increasing the tank size in systems characterized by a disproportion between harvesting and demand volume. PA is computationally inexpensive and provides a powerful instrument to discard inconvenient solutions in high demand scenarios, resulting in a useful instrument for quick cost benefit analysis and decision making.

Conflicts of Interest: The author declare no conflict of interest.

Abbreviations

The following abbreviations are used in this manuscript:

RWH: Rain Water Harvesting

CS: Continuous Simulation

PA: Probability Analysis

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