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Evaluation of the Performance of a Finite Volume Physics-Based Model for Soil Erosion Simulation

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Abstract

Having reliable tools for characterizing rainfall-induced soil erosion is fundamental to the effective management of agroforestry systems in order to increase resilience against climate change. Physics-based models provide a robust, comprehensive and widely applicable framework to quantify runoff generation and soil erosion during intense rainfall events in agroforestry catchments. In this work, we propose a novel hydro-erosive model to simulate hydrodynamical flow and bed mobilization, movement and deposition. This hydro-erosive model solves the two-dimensional shallow water equations (SWE-2D) with hydrological source terms for runoff generation, coupled with the 2D depth-averaged solid transport and the soil surface evolution equations. The partial differential system is solved using a finite volume method. Alternative Integral/Differential Bed Slope and explicit upwind/implicit pointwise friction term discretization options can be used to improve performance in terms of numerical stability and conservation. The behavior of different discretization options in this hydro-erosive model is evaluated through an analytical hillslope verification, a benchmark V-catchment rainfall–runoff test and a laboratory dam-break experiment over an erodible bed. The results show that the Differential Bed Slope formulation combined with the upwind friction discretization provides the most accurate and conservative predictions. Also, an Upwind Bed Updating method for integrating soil surface elevation change is compared with the cell-centered integration of the bed change term by suppressing non-physical oscillations without compromising computational efficiency. Overall, the proposed open-source hydro-erosive model provides a reliable and computationally efficient framework for high-resolution simulations of rainfall-induced soil erosion and represents a valuable tool for environmental and agroforestry applications, but appropriate calibration and mesh resolution are required to ensure reliable predictions.

Keywords: water soil erosion; physics-based distributed models; sediment transport; friction integration; numerical methods



Academic Editor: Hongliang Kang

Received: 2 July 2026

Revised: 26 July 2026

Accepted: 30 July 2026

Published: 1 August 2026

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1. Introduction

Soil erosion by water has intensified worldwide because of both the expansion of aggressive farming practices and climate change [1]. Characterizing and quantifying soil loss processes due to water erosion is fundamental to increasing resilience to climate change. Rainfall-induced erosion on hillslopes not only reduces the fertile soil layer available for the growth of the protective vegetation canopy, decreasing land productivity, but also affects infiltration and subsequent runoff generation. Consequently, intensive erosion processes can modify the hydrological response of the catchment during heavy-rainfall

events and increase flooding risk downstream. Therefore, reliable tools for estimating and analyzing the spatio-temporal distribution of erosion are crucial to the smart management of agroforestry systems. In this regard, computational soil erosion models are a low-cost and useful option to support agroforestry planning with quantitative data.

Available models applied for assessing catchment-scale erosion can be classified in three main groups, depending on the background approach used to represent the involved environmental processes: (a) empirical models, (b) simplified process-based models and (c) distributed physics-based models. Empirical models (a) are based on parametric formulations that enable to estimate aggregated quantities at hillslope and catchment scales. They are easy to use and do not require a high computational effort but need a careful calibration adapted to each specific scenario. The most widely used empirical model is RUSLE, i.e., Revised Universal Soil Loss Equation [2]. RUSLE has been calibrated for predicting annually averaged soil loss rates at the catchment scale in different agricultural and forestry landscapes [3–5]. Advanced parametric models based on the RUSLE approach, such as WaTEM/SEDEM [6,7] and G2-ESDAC [8,9], have also been successfully calibrated for characterizing spatially distributed soil loss rates at catchment and regional scales.

Among these advanced empirical models, the Soil and Water Assessment Tool (SWAT) [10] is one of the most extensively applied watershed models. SWAT operates on a daily time step and uses a modified version of RUSLE to estimate runoff and sediment yield under different land-use and management scenarios [11]. Nevertheless, as SWAT was primarily developed to evaluate the long-term impacts of agricultural management on runoff and sediment yield [10], it is less suitable for applications involving detailed event-based flood propagation and sediment transport in forestry areas.

To better represent erosion processes, simplified process-based models (b) have also been extensively used for quantifying soil loss and sediment yield at hillslope and catchment scales. The WEPP model [12,13] deploys a simplified semi-analytical solution of the kinematic wave equation to transform rain into overland flow at the hillslope scale and the steady-state sediment continuity equation for estimating erosion. Similarly, the EUROSEM model [14,15] also solves the runoff mass conservation equation using the kinematic wave assumption at the hillslope and channel scale, together with the 1D dynamic balance equation for the soil mass over converging surfaces [16]. The Pan-European Soil Erosion Risk Assessment (PESERA) model [17] uses a process-based approach to estimate long-term averaged erosion rates in 1 km resolution coarse grids. Despite their simplified approach, process-based models are more sophisticated than parametric models and require a careful estimation of some semi-empirical parameters.

One step further in complexity are distributed physics-based models (c), which solve the depth-averaged conservation equations of momentum and mass (shallow water equations, SWE-2D) to transform rainfall into the temporal and spatial distribution of runoff. The erosion process is included by solving the 2D sediment transport equation coupled with the bed surface evolution equation for modeling the mobilization, movement and deposition of the soil particles [18,19]. Hence, these models allow for the dynamic and distributed exchange of soil material between the erodible soil layer and the overland flow, controlled by rainfall energy and the shear stress exerted by the runoff on the erodible stratum. In contrast to RUSLE-based or process-based models, physics-based computational models might offer suitable predictions of the local erosion features only by calibrating values of the erosion parameters in the equations within “reasonable” physical ranges, allowing for a more scientific and intuitive calibration process.

The main drawback of physics-based hydro-erosive models is the high computational cost required when combined with fine computational grids. This limitation has hampered

their application to estimate erosion processes at large scale with sufficiently high resolution to capture these local erosion features. Application of these models to soil erosion at catchment scale has been attempted previously [20–22] but always with coarse grid resolutions, hindering the potential of these models. In recent years, the emergence of intensive parallelization within GPU devices has fostered the development of distributed computational models for geophysical surface flows to increase greatly their potential as analytical and predictive tools [23–26]. Nevertheless, the numerical discretization implemented for the main terms in equations can also affect model performance when dealing with large-scale high-resolutions applications. Several alternatives have been proposed in the literature for the numerical integration of the bed slope [27], friction dissipation [28,29] and bed erosion terms in surface flows, but their combination within erosive models has not been addressed.

In this work, we assess a physics-based hydro-erosive model (SERGHEI model-SE module [30]) for simulating surface runoff and soil redistribution rates applicable to large domains with high space/time resolution. As a preliminary stage before ambitious and large-scale applications, it is convenient to evaluate the sensitivity of the proposed model to parameters and the different discretization options available using controlled test cases. This is the main purpose of the present work. This work is structured as follows: Section 2 is devoted to introduce the governing equations for the runoff movement and soil mobilization, transport and deposition processes; in Section 3, the numerical method is detailed, focusing on the new integration procedures; the results of the hydro-erosive model for analytical and experimental tests are reported and analyzed in Section 4; and finally the main conclusions are drawn in Section 5.

2. Governing Equations

The hydro-morphodynamic framework is composed of the 2D shallow water equations (SWE-2D) for overland flow, the 2D advective transport equation for the multigrain sediment mass suspended in the flow and the soil surface elevation equation [21]. The equations forming the system can be written in expanded component form as:

$$\frac{\partial h}{\partial t} + \frac{\partial(hu)}{\partial x} + \frac{\partial(hv)}{\partial y} = r_0 - r_f + \xi N_b \quad (1)$$

$$\frac{\partial(hu)}{\partial t} + \frac{\partial}{\partial x} \left(hu^2 + \frac{1}{2}gh^2 \right) + \frac{\partial}{\partial y} (huv) = -gh \frac{\partial z_b}{\partial x} - ghC_f |\mathbf{u}|u - u \left(\frac{\rho_b}{\rho} - 1 \right) \xi N_b \quad (2)$$

$$\frac{\partial(hv)}{\partial t} + \frac{\partial}{\partial x} (huv) + \frac{\partial}{\partial y} \left(hv^2 + \frac{1}{2}gh^2 \right) = -gh \frac{\partial z_b}{\partial y} - ghC_f |\mathbf{u}|v - v \left(\frac{\rho_b}{\rho} - 1 \right) \xi N_b \quad (3)$$

$$\frac{\partial(h\phi)}{\partial t} + \frac{\partial(hu\phi)}{\partial x} + \frac{\partial(hv\phi)}{\partial y} = N_b \quad (4)$$

$$\frac{\partial z_b}{\partial t} = -\xi N_b \quad (5)$$

where h [m] is the flow depth; $\mathbf{u} = (u, v)$ [m/s] is the depth-averaged velocity vector; z_b [m] is the bed-layer elevation; g [m/s²] is the gravity acceleration; r_0 [m/s] is the rainfall rate; r_f [m/s] is the infiltration rate in the underlying porous soil; $\rho = \rho_w (1 - \phi) + \rho_s \phi$ [kg/m³] is the depth-averaged bulk density of the flow, with ρ_w [kg/m³] the water density and ρ_s [kg/m³] the sediment particle density; ϕ [-] denotes the depth-averaged volumetric concentration of the sediment in the flow; and N_b [m/s] is the total bed-material exchange rate.

The last term, N_b , participates in the momentum exchange between the overland flow and the underlying stratum [31]. According to that, net entrainment rates $N_b > 0$ involve a momentum reduction in the flow, whereas net deposition rates $N_b < 0$ lead to an increment in the flow momentum. This is one of the fundamental principles in fluvial dynamics, usually referred to as the principle of self-adaptation [32]. This principle states that the flow becomes weaker when entraining sediment and the soil surface degrades, whereas deposition of the transported sediment load leads to a stronger flow.

Besides that, in this work the effective rainfall is obtained directly from the hourly rainfall intensity Ir_{60} [mm/h]. Moreover, the infiltration rate is estimated using a constant infiltration value over the entire domain.

The boundary shear stress at the soil surface τ_b [Pa] is here estimated using the quadratic pure-turbulent relation as [33]:

$$\tau_b = (\tau_{bx}, \tau_{by}) = \rho g h C_f |\mathbf{u}| \mathbf{u} \quad (6)$$

where $C_f = n_b^2 h^{-4/3}$ [s²/m²] is the turbulent friction coefficient, with n_b [s/m^{1/3}] the Manning roughness parameter of the terrain.

The last source term in the momentum equations accounts for the bulk mass and momentum exchange between the flow and the erodible soil layer [32] and is expressed in terms of the void factor ξ [-]:

$$\xi = \frac{1}{1 - p_b} \quad \rho'_b = \rho_s (1 - p_b) \quad \rho_b = \rho'_b + \rho_w p_b \quad (7)$$

where p_b [-] is the bulk porosity of the soil layer. The property ρ'_b [kg/m³] is the dry apparent density of the solid aggregate, and ρ_b [kg/m³] accounts for the density of the saturated soil layer.

Bed-Material Exchange in Channels (BMEC) Model

Following the sediment transport theory in rivers [34], estuaries and shore environments, the total net exchange rate N_b can be expressed as:

$$N_b = E_b - D_b \quad (8)$$

where E_b [m/s] and D_b [m/s] are the specific entrainment and deposition rates, generalized here as [31]:

$$E_b = \alpha_e \omega_s \frac{q_s^*}{\sqrt{q_x^2 + q_y^2}} \quad (9)$$

$$D_b = \alpha_d \omega_s \phi$$

where α_e [-] and α_d [-] are two empirical fitting parameters for the entrainment and the deposition rates respectively, ω_s [m/s] is the specific settling velocity and q_s^* [m²/s] is the specific solid transport capacity of the flow for the j -sediment.

Multiple empirical relationships exist to estimate the specific solid transport capacity q_s^* , usually derived by fitting transport data observed in natural rivers. Particularly, the formula from [34] for suspended sediment capacity is widely used due to its adaptability to different sediment sizes:

$$\frac{q_s^*}{F_b \sqrt{(\rho_s / \rho_w) g d_s^3}} = 2.62 \cdot 10^{-5} \left(\frac{|\tau_b|}{\tau_c} \frac{|\mathbf{u}|}{\omega_s} \right)^{1.74} \quad (10)$$

where $\tau_{cj} = \theta_c (\rho_s - \rho_w) g d_{sj}$ [Pa] is the threshold of turbulent shear stress for the detachment of the j -sediment, with θ_c [-] the critical dimensionless Shields stress for the incipient motion. The specific settling velocity ω_{sj} is here estimated using the relationship from [35], derived from empirical measurements for a wide range of sediment sizes in natural flows:

$$\omega_s = \left[\left(13.95 \frac{\nu}{d_s} \right)^2 + 1.09 \left(\frac{\rho_s - \rho_w}{\rho_w} \right) g d_s \right]^{1/2} - 13.95 \frac{\nu}{d_s} \quad (11)$$

where ν [m²/s] is the kinematic viscosity of water. It is worth mentioning that the coefficients α_e and α_d might be calibrated for realistic applications depending on the specific features of the case.

3. Numerical Method

The governing equations are solved using a finite volume (FV) method. The spatial domain is discretized into the orthogonal mesh defined by the Digital Terrain Model (DTM), and the equations are integrated in each square computational i -cell. The implementation of the hydrodynamic flux computation is described in [30] and referenced therein. Therefore, the present work is focused on the novel parts that are detailed next.

3.1. Updating the Runoff and Sediment Transport

Considering the overland components of the system, the piecewise initial variables at the i -cell are updated from time t^n to an intermediate step, denoted by superindex *, following:

$$h_i^* = h_i^n - \frac{\Delta t}{\Delta l} \sum_{k=1}^4 (q_n^{\downarrow-})_k^n + \Delta t (r_0 - r_f)_i^n \quad (12)$$

$$(hu)_i^* = (hu)_i^n - \frac{\Delta t}{\Delta l} \sum_{k=1}^4 (m_x^{\downarrow-} + s_x^{\downarrow-})_k^n \quad (13)$$

$$(hv)_i^* = (hv)_i^n - \frac{\Delta t}{\Delta l} \sum_{k=1}^4 (m_y^{\downarrow-} + s_y^{\downarrow-})_k^n \quad (14)$$

$$(h\phi)_i^* = (h\phi)_i^n - \frac{\Delta t}{\Delta l} \sum_{k=1}^4 (q_n^{\downarrow-} \phi^{\downarrow-})_k^n \quad (15)$$

where Δl is the spatial resolution of the DTM, $\Delta t = t^{n+1} - t^n$ is the time step, and $q_n^{\downarrow-}$ is the hydrodynamic normal mass flux, with $q_n = h \vec{u} \cdot \hat{n}$ and $(m_x^{\downarrow-}, m_y^{\downarrow-})_k$ being the upwind x - and y -momentum fluxes, respectively, at the k -edge. These upwind fluxes at the cell edges are computed using the eigenvalues and eigenvectors of the Jacobian matrix of the hydrodynamic system and also include source strengths accounting for the bed pressure contribution. The bed pressure source appears explicitly in the momentum equations as $-gh \frac{\partial z_b}{\partial x}$ and $-gh \frac{\partial z_b}{\partial y}$. Additionally, $(s_x^{\downarrow-}, s_y^{\downarrow-})_k$ denote the upwind edge-based discretization of the frictional momentum source, which is associated with the bed shear-stress vector τ_b , defined in Equation (6).

3.1.1. Upwind and Explicit Friction Term Discretization

Following [36], the integrated friction contribution at edge k , evaluated at time t^n , can be written as:

$$\left(\frac{\tau_b}{\rho_w} \right)_k^n = g (\tilde{h} S_f)_k^n d_n \quad S_{f,k}^n = \left(\frac{n_b^2 (\tilde{\mathbf{u}} \cdot \mathbf{n}) |\tilde{\mathbf{u}}|}{\tilde{h}^{4/3}} \right)_k^n \quad (16)$$

where d_n is the distance between neighboring cell centers in the normal direction to the edge, $\tilde{\mathbf{u}}$ is the averaged velocity vector at the edge, and $\mathbf{n}$ is the outward unit normal vector to the edge. The term S_f is the Manning friction slope, which is related to the friction coefficient $C_f = n_b^2 h^{-4/3}$ by $S_{f,x} = C_f |\mathbf{u}| u$ in the x -direction and $S_{f,y} = C_f |\mathbf{u}| v$ in the y -direction. Thus, this formulation treats friction as an edge-based source contribution, coupled with the explicit upwind flux computation. It will be referred to as Upwind Friction Slope (UFS).

3.1.2. Pointwise and Implicit Friction Term Discretization

As an alternative, the updated momentum components $(hu)_i^*$ and $(hv)_i^*$ can be obtained through an implicit pointwise centered treatment. This approach follows the idea of applying the resistance term as a local correction of the frictionless momentum, as discussed by [28] for visco-plastic friction dissipation. In this case, the hydrodynamic flux update is first computed without the friction source contribution:

$$\begin{aligned} M_x &= (hu)_i^n - \frac{\Delta t}{\Delta l} \sum_{k=1}^4 (m_x^{\downarrow-})_k^n \\ M_y &= (hv)_i^n - \frac{\Delta t}{\Delta l} \sum_{k=1}^4 (m_y^{\downarrow-})_k^n \end{aligned} \quad (17)$$

Here, M_x and M_y are auxiliary cell-centered quantities representing the momentum components after the hydrodynamic flux update, including bed slope terms and before the local friction correction. The corrected cell-centered momentum components after the pointwise friction treatment can be written as:

$$\begin{aligned} (hu)_i^* &= M_x - \Delta t \left(\frac{\tau_{b,x}}{\rho} \right)_i^* \\ (hv)_i^* &= M_y - \Delta t \left(\frac{\tau_{b,y}}{\rho} \right)_i^* \end{aligned} \quad (18)$$

Using τ_{bx} and τ_{by} as defined in (6), $C_f = n_b^2 h^{-4/3}$, $u = (hu)_i^* / h_i^*$, and $v = (hv)_i^* / h_i^*$, Equation (18) can be rewritten as:

$$\begin{aligned} (hu)_i^* &= M_x - \Delta t g n_b^2 \frac{\sqrt{[(hu)_i^*]^2 + [(hv)_i^*]^2}}{(h_i^*)^{7/3}} (hu)_i^* \\ (hv)_i^* &= M_y - \Delta t g n_b^2 \frac{\sqrt{[(hu)_i^*]^2 + [(hv)_i^*]^2}}{(h_i^*)^{7/3}} (hv)_i^* \end{aligned} \quad (19)$$

Since the pointwise friction correction reduces the magnitude of the momentum vector while preserving its direction, the corrected momentum vector $((hu)_i^*, (hv)_i^*)$ is proportional to the frictionless momentum vector (M_x, M_y) . Therefore, $\frac{(hu)_i^*}{(hv)_i^*} = \frac{M_x}{M_y}$. Using this relation, one component can be written as a function of the other, which leads to:

$$\begin{aligned} (hu)_i^* &= T_i M_x \\ (hv)_i^* &= T_i M_y \end{aligned} \quad (20)$$

where

$$T_i = \frac{-1 + \sqrt{1 + 4\Delta t g n_b^2 (h_i^*)^{-7/3} \sqrt{M_x^2 + M_y^2}}}{2\Delta t g n_b^2 (h_i^*)^{-7/3} \sqrt{M_x^2 + M_y^2}} \quad (21)$$

This procedure will be referred to as Pointwise Friction Slope (PFS).

3.1.3. Bed Slope Term Discretization

After the treatment of the frictional momentum dissipation, the bed pressure contribution can be detailed. Following the source term integration proposed by [36], the bed pressure contribution can also be numerically integrated using either a differential or an integral approximation. The differential approximation is based on the assumption that the variables vary smoothly between two neighboring cells, denoted by i and $i + 1$. In this case, the bed pressure term at edge k in the x -direction is approximated by:

$$\left(-gh \frac{\partial z_b}{\partial x}\right)_k \approx -g \left(\tilde{h} \frac{\Delta z_b}{\Delta x}\right)_k \quad \tilde{h} = \frac{h_i + h_{i+1}}{2} \quad \Delta z_b = (z_b)_{i+1} - (z_b)_i \quad (22)$$

where $\tilde{h}$ is the mean water depth between neighboring cells i and $i + 1$, and Δz_b is the difference between their bed-layer elevations. This option will be referred to as Differential Bed Slope (DBS).

In the integral approximation, the bed variation is not represented as a smooth slope between neighboring cells. Instead, it is treated as a bed step located at the cell interface. Consequently, the bed pressure contribution is computed from the hydrostatic thrust acting on the submerged portion of this step. This approximation is written as:

$$\left(-gh \frac{\partial z_b}{\partial x}\right)_k \approx -g \left(h_j - \frac{|\Delta z'_b|}{2}\right) \Delta z'_b \quad j = \begin{cases} i & \text{if } \Delta z_b \geq 0, \\ i + 1 & \text{if } \Delta z_b < 0, \end{cases} \quad (23)$$

$$\Delta z'_b = \begin{cases} h_i & \text{if } \Delta z_b \geq 0 \text{ and } d_i < (z_b)_{i+1}, \\ h_{i+1} & \text{if } \Delta z_b < 0 \text{ and } d_{i+1} < (z_b)_i, \\ \Delta z_b & \text{otherwise.} \end{cases} \quad d = h + z_b$$

where h_j is the water depth on the reference side of the bed step, $\Delta z'_b$ is the effective submerged part of the bed elevation difference, and d is the free-surface elevation. When the bed step is fully submerged, the whole bed elevation difference contributes to the hydrostatic thrust; therefore $\Delta z'_b = \Delta z_b$. Otherwise, $\Delta z'_b$ is limited by the local water depth, so that the hydrostatic force is only applied over the submerged portion of the bed step that is actually covered by water. This option will be referred to as Integral Bed Slope (IBS).

3.1.4. Updating the Solid Transport Concentration

For updating the solid transport equation, the upwind value of the convective solid mass flux for the sediment at the k -edge is computed using the hydrodynamic normal mass flux $q_n^{\downarrow-}$ and the upwind solid concentration at the edge $\phi^{\downarrow-}$, calculated as:

$$\phi^{\downarrow-} = \frac{(\phi)_i^n + (\phi)_{ik}^n}{2} - \text{sgn}(q_n^{\downarrow-}) \frac{(\phi)_{ik}^n - (\phi)_i^n}{2} \quad (24)$$

where $(\phi)_i^n$ is the sediment concentration in the i -cell and $(\phi)_{ik}^n$ is the sediment concentration in the k -neighboring cell, both at the current time t^n . Note that the upwind behavior is given here by the direction of the hydrodynamic normal mass flux $\text{sgn}(q_n^{\downarrow-})$, where sgn denotes the sign of the corresponding quantity.

3.1.5. Final Updating Step

Then, the bed-material exchange term is included for updating the flow, and the sediment transport equations to the next time t^{n+1} is given by

$$h_i^{n+1} = h_i^* + \Delta t (\xi N_b)_i^n \quad (25)$$

$$(hu)_i^{n+1} = (hu)_i^* - \Delta t \left[u \left(\frac{\rho_b}{\rho} - 1 \right) \xi N_b \right]_i^n \quad (26)$$

$$(hv)_i^{n+1} = (hv)_i^* - \Delta t \left[v \left(\frac{\rho_b}{\rho} - 1 \right) \xi N_b \right]_i^n \quad (27)$$

$$(h\phi)_i^{n+1} = (h\phi)_i^* + \Delta t (N_b)_i^n \quad (28)$$

where $(N_b)_i^n$ is the total bed-material exchange at the i -cell for the current time; hence:

$$N_b = (E_b - D_b)_i^n \quad (29)$$

Stability is enforced by limiting the global time step by using the CFL condition. The local time step allowed at the i -cell is estimated using the fastest wave celerity depending on the local flow features and the spatial resolution of the computational grid. Then, the global time step Δt is limited by minimum of the local time steps as:

$$\Delta t = \text{CFL} \min \left(\frac{\Delta l}{\max(|u| + \sqrt{gh}, |v| + \sqrt{gh})} \right)_i^n \quad (30)$$

with $\text{CFL} < 0.5$ for square-orthogonal grids [37]. Further details on this numerical scheme and additional considerations such as entropy fixes, source terms limitations and wet–dry treatment can be found in [36].

3.2. Updating the Soil Surface Elevation

This section is dedicated to the integration and updating of the soil surface level, where the net exchange of sediment with the runoff contributes to the soil surface elevation evolution. In this work, two different numerical approaches are proposed for updating the soil surface elevation due to the vertical solid exchange flux.

3.2.1. Centered Bed Updating (CBU)

The classical approach to update the bed level [32,34] consists of explicitly integrating the vertical exchange flux within each computational cell,

$$\frac{d}{dt} \int_{\Omega_i} z_b d\Omega = -\xi \int_{\Omega_i} N_b(t = t^n) d\Omega \quad (31)$$

leading to the simple bed elevation updating formula:

$$(z_b)_i^{n+1} = (z_b)_i^n - \Delta t \xi (N_b)_i^n \quad (32)$$

where the total exchange rate $(N_b)_i^n$ at computational cell i for time t^n is hence computed using (29) depending on the erosion model.

3.2.2. Upwind Bed Updating (UWBU)

A novel approach is herein proposed based on splitting the integral of the vertical exchange flux into solid load contributions associated to each intercell edge. Hence, the bed change equation can be expressed as:

$$\frac{d}{dt} \int_{\Omega_i} z_b d\Omega = -\xi \sum_{k=1}^4 \int_{\Omega_k} N_b(t = t^n) d\Omega \quad (33)$$

where Ω_k denotes the area associated to the k -edge, and the total exchange rate N_b is computed using (8). Assuming the exchange of sediment between the erodible soil layer and the overland flow constant during a time step, the edge integral of the vertical net flux represents an increment on the solid discharge transported by the overland flow throughout the k -edge; hence:

$$\xi \int_{\Omega_k} N_b(t = t^n) d\Omega = \xi (\tilde{N}_b)_k^n \frac{\Delta l^2}{2} = \xi \Delta q_s^k \Delta l \quad (34)$$

where $(\tilde{N}_b)_k^n = \frac{(N_b)_i^n + (N_b)_{ik}^n}{2}$ is the arithmetic average of the vertical exchange flux at the k -edge for the current time, $(N_b)_{ik}^n$ is the vertical flux in the k -neighboring cell to the i -cell at the current time, and $\Delta q_s^k = \frac{\Delta l}{2} (\tilde{N}_b)_k^n$ is the resulting increment in the solid transport discharge due to the vertical exchange.

Hence, by replacing (34) into (33), the soil surface evolution equation can be transformed into the following upwind updating expression for the i -cell soil surface elevation:

$$(z_b)_i^{n+1} = (z_b)_i^n - \frac{\Delta t}{\Delta l} \sum_{k=1}^4 \xi \Delta q_s^{\leftarrow k} \quad (35)$$

where $\Delta q_s^{\leftarrow k}$ denotes the upwind contribution to the solid discharge increment due to soil material exchange at the k -edge. Note that the downwind contribution to the solid discharge $\Delta q_s^{k \rightarrow}$ at the k -edge, which contributes to the soil surface change in the opposite neighboring cell, must satisfy $\Delta q_s^{\leftarrow k} + \Delta q_s^{k \rightarrow} = \Delta q_s^k$, ensuring mass conservation for the solid material. The reconstruction of these upwind and downwind vertical contributions are detailed below.

The computation of these upwind and downwind solid discharge increments, $\Delta q_s^{\leftarrow k}$ and $\Delta q_s^{k \rightarrow}$ respectively, is based on the weak solution of the local Riemann problem for the bed-level evolution at cell edges, originally developed for bedload transport in rivers [19,38,39]. A detailed explanation on this procedure can be found in [40]. This discrete approach derives a local virtual wave celerity $\tilde{\lambda}_b^k$ for the soil surface elevation due to the vertical exchange flux at the k -edge, expressed as:

$$\tilde{\lambda}_b^k = \xi \frac{(\tilde{N}_b)_k^n}{2 S_b^k} \quad (36)$$

with $S_b^k = \Delta z_b / \Delta l$ the discrete local terrain slope at the cell edge. Depending on both the vertical exchange flux and the local terrain slope, the following soil surface evolution configurations arise.

Forward step configuration: It is characterized by positive values of the local bed change celerity $\tilde{\lambda}_b^k > 0$, leading to the forward propagation of terrain level steps, i.e., along

the direction of the outward unit normal vector at the edge. The solid discharge increments at the cells on both sides of the k -edge are computed as:

$$\Delta q_s^{\leftarrow k} = 0 \quad \Delta q_s^{k \rightarrow} = \frac{\Delta l}{2} (\tilde{N}_b)_k^n \quad (37)$$

Two combinations cause this configuration (see Figure 1): net erosion $(\tilde{N}_b)_k^n > 0$ in upward slopes $S_b^k > 0$ and net deposition $(\tilde{N}_b)_k^n < 0$ in downward slopes $S_b^k < 0$.

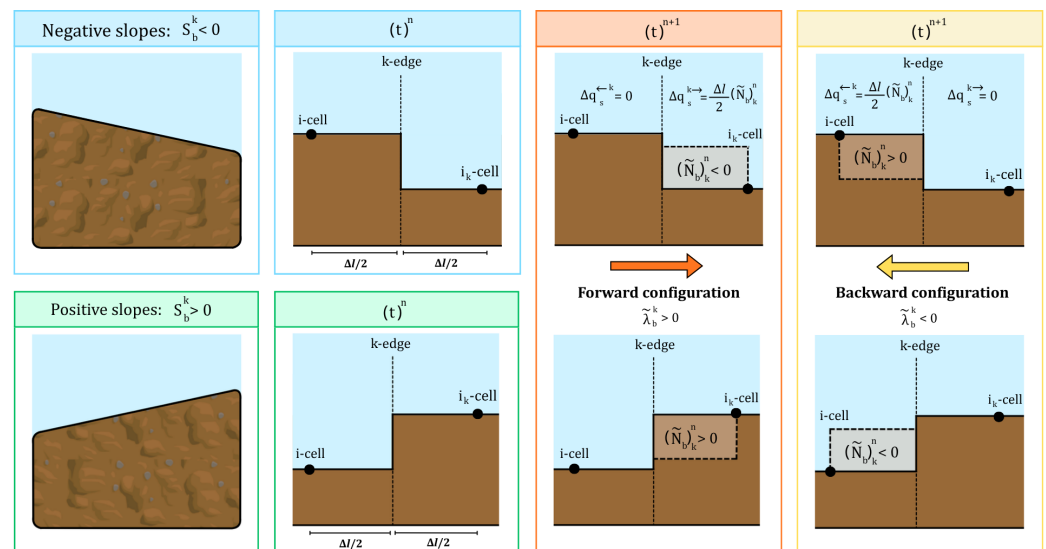


Figure 1. Local bed-update configurations for negative and positive slopes.

Backward step configuration: It is characterized by negative values of the local bed change celerity $\tilde{\lambda}_b^k < 0$, leading to the backward propagation of terrain level steps, i.e., opposite to the direction of the outward unit normal vector at the edge. The solid discharge increments at both sides of the k -edge are computed as:

$$\Delta q_s^{\leftarrow k} = \frac{\Delta l}{2} (\tilde{N}_b)_k^n \quad \Delta q_s^{k \rightarrow} = 0 \quad (38)$$

Two scenarios can be distinguished in this configuration (see Figure 1): net erosion $(\tilde{N}_b)_k^n > 0$ in downward slopes $S_b^k < 0$ and net deposition $(\tilde{N}_b)_k^n < 0$ in upward slopes $S_b^k > 0$.

Flat bed configuration: The local bed change celerity $\tilde{\lambda}_b^k$ is undefined. The balance between the vertical exchange flux at the left and right cells of the k -edge at the current time, computed as $(\Delta N_b)_k^n = (N_b)_{ik}^n - (N_b)_i^n$, determines the direction of the emergent slope. The solid discharge increments at both sides of the edge are computed as:

$$\Delta q_s^{\leftarrow k} = \frac{\Delta l}{4} (N_b)_i^n \quad \Delta q_s^{k \rightarrow} = \frac{\Delta l}{4} (N_b)_{ik}^n \quad (39)$$

Two scenarios can appear in this configuration (see Figure 2). First, positive flux balance $(\Delta N_b)_k^n > 0$ leads to the appearance of downward slopes for both net erosion and deposition scenarios. Second, negative flux balance $(\Delta N_b)_k^n < 0$ leads to the appearance of upward slopes, regardless of whether erosion or deposition predominates.

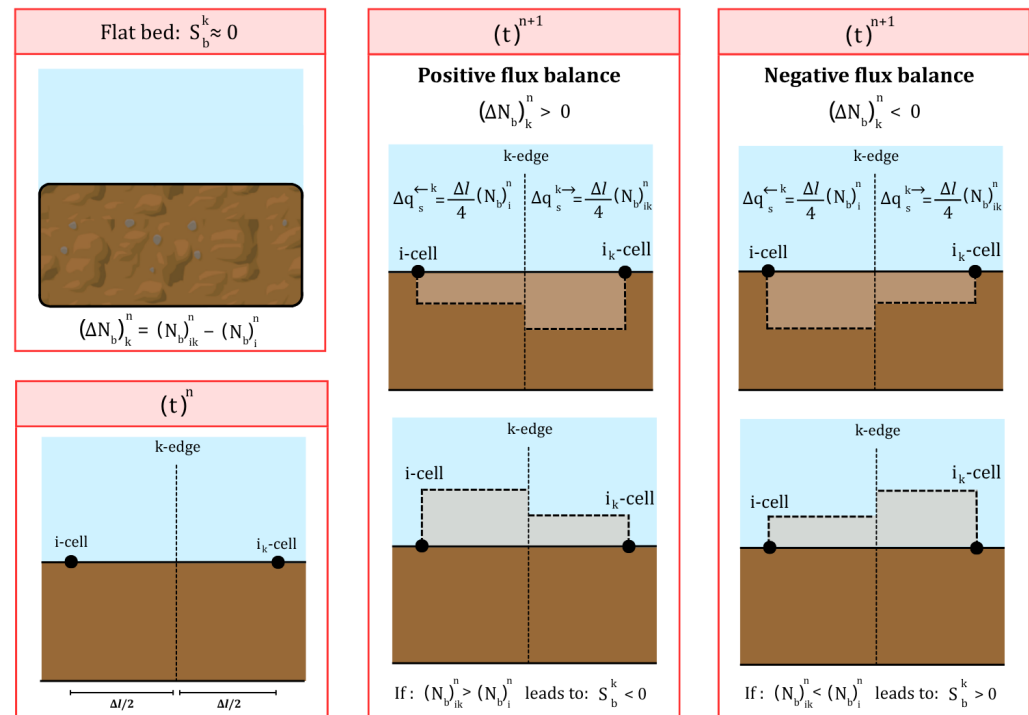


Figure 2. Flat bed configuration.

4. Model Validation and Performance

In this section, we validate the model and discuss its performance. We use a series of numerical experiments to demonstrate the robustness, efficiency and performance of the proposed model.

4.1. Test 1: Hillslope Case Study

The first numerical case involves an idealized hillslope designed to provide a controlled verification of the hydrodynamic and morphodynamic response of the model. Owing to the 1D character of the flow and the uniform bed slope, the Differential Bed Slope (DBS) formulation was adopted in this test. The analysis therefore focuses only on the influence of the friction source term discretization by comparing the upwind (UFS) and the pointwise centered (PFS) formulations. The numerical results are compared against the analytical reference solution derived in Appendix A for both non-erosive and erosive conditions. In this way, the test evaluates the ability of the model to reproduce the expected runoff depth and velocity profiles, as well as the associated bed elevation changes.

The computational domain is rectangular, with a length of 1000 m and a width of 200 m, having a uniform negative longitudinal slope of 0.01. Figure 3 shows a representative illustration of the case study. The erodible soil layer is composed of a single sediment class with diameter $d_s = 0.1$ mm and dry apparent density $\rho'_b = 1406$ kg/m³. The hydraulic resistance is represented by a uniform Manning roughness coefficient of $n_b = 0.02$ s/m^{1/3}. The domain is initialized in dry conditions and subjected to a constant rainfall of 200 mm/h, applied only to the upper half of the slope, between $x = 0$ and $x = 500$ m, as illustrated by the blue rectangle in Figure 3. The domain was discretized with a spatial resolution of $\Delta l = 1$ m, and the simulation was run for 3600 s using a CFL = 0.5.

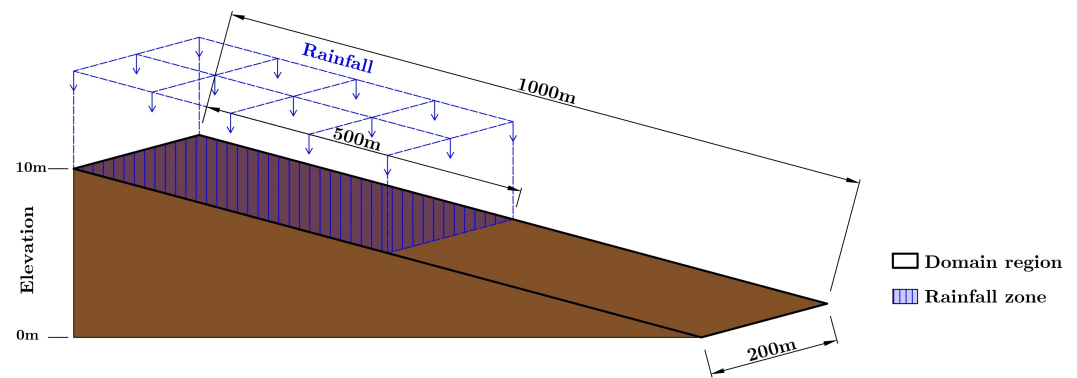


Figure 3. Test 1—Schematic representation of the hillslope domain and rainfall zone.

In order to establish a baseline scenario for the model hydrodynamic response, erosion, deposition and infiltration processes were not considered in the first set of simulations. Figure 4 shows the runoff depth h and the x -velocity u along the hillslope centerline at times $t = 180$ s, $t = 540$ s, and $t = 3600$ s after the rainfall starts, comparing the UFS and PFS formulations. Since the exact solution presented in Appendix A corresponds to a steady-state condition, it is included only for the final profile at $t = 3600$ s.

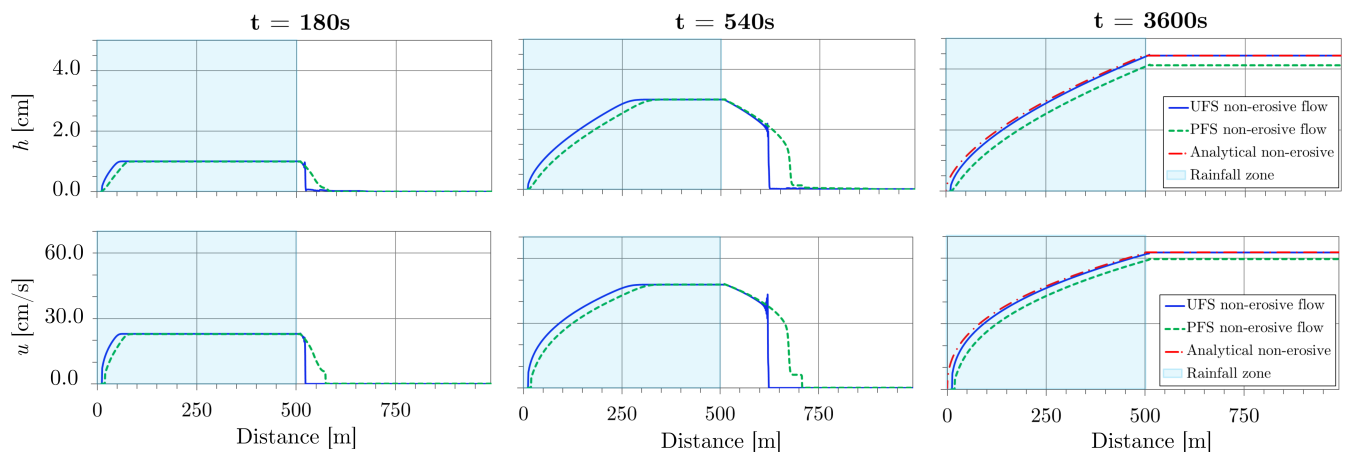


Figure 4. Test 1—Temporal evolution of the runoff depth and x -velocity along the plane centerline for the non-erosive soil condition.

The overland flow develops progressively along the rainfall zone. Between $x = 0$ and $x = 500$ m, the specific discharge increases downstream due to the continuous inflow of water from precipitation. After the rainfall zone ends ($x > 500$ m), no further water input is added, so that the runoff is a uniform regime along the downstream portion of the slope. At the end of the simulation ($t = 3600$ s), the flow reaches a fully steady state in both the upper-half and lower-half regions of the domain.

In the uniform downstream region, the exact solution yields a reference depth $h \approx 4.43$ cm and an x -velocity $u \approx 62.64$ cm/s. The numerical values obtained downstream of the rainfall zone for the UFS are the same as the analytical solution, i.e., $h_{\text{UFS}} \approx 4.43$ cm and $u_{\text{UFS}} \approx 62.64$ cm/s. Meanwhile, the values obtained for PFS are $h_{\text{PFS}} \approx 4.12$ cm and $u_{\text{PFS}} \approx 59.67$ cm/s. Although both formulations reproduce the expected steady-flow pattern, the UFS formulation provides closer agreement with the exact solution for this hillslope configuration.

Before evaluating soil erosion, the agreement between the analytical and numerical profiles was also assessed for $t = 3600$ s. The relative error of the spatial integral was computed for both water depth and averaged velocity over the domain, as shown in Table 1. The UFS scheme reproduced the analytical solution with relative errors below 1.3% for

both variables, whereas the PFS scheme yielded errors of approximately 10% for water depth and 7.5% for x -velocity.

Table 1. Test 1—Relative errors for the case without erosion.

Variable	Model	Integral Area	Relative Error (%)
h	UFS	3565.05	1.20
	PFS	3248.50	9.98
	Analytical	3608.44	—
u_x	UFS	52,896.83	1.05
	PFS	49,438.74	7.52
	Analytical	53,457.16	—

In order to compare the numerical model results with the exact solution for the idealized erosive case with infiltration, infiltration was simplified and assumed constant, and the erosion source term was adapted to match the same simplified expression assumed in Appendix A. Therefore, instead of using Equation (8), N_b was computed according to the simplified expression presented in Equation (A10).

Figure 5 shows the evolution of x -velocity u , the net bed exchange rate N_b , and the bed elevation change Δz_b along the steep plane for both UFS and PFS at times $t = 180$ s, $t = 540$ s, and $t = 3600$ s after the rainfall starts. The comparison with the exact solution is included only for the final profile at $t = 3600$ s.

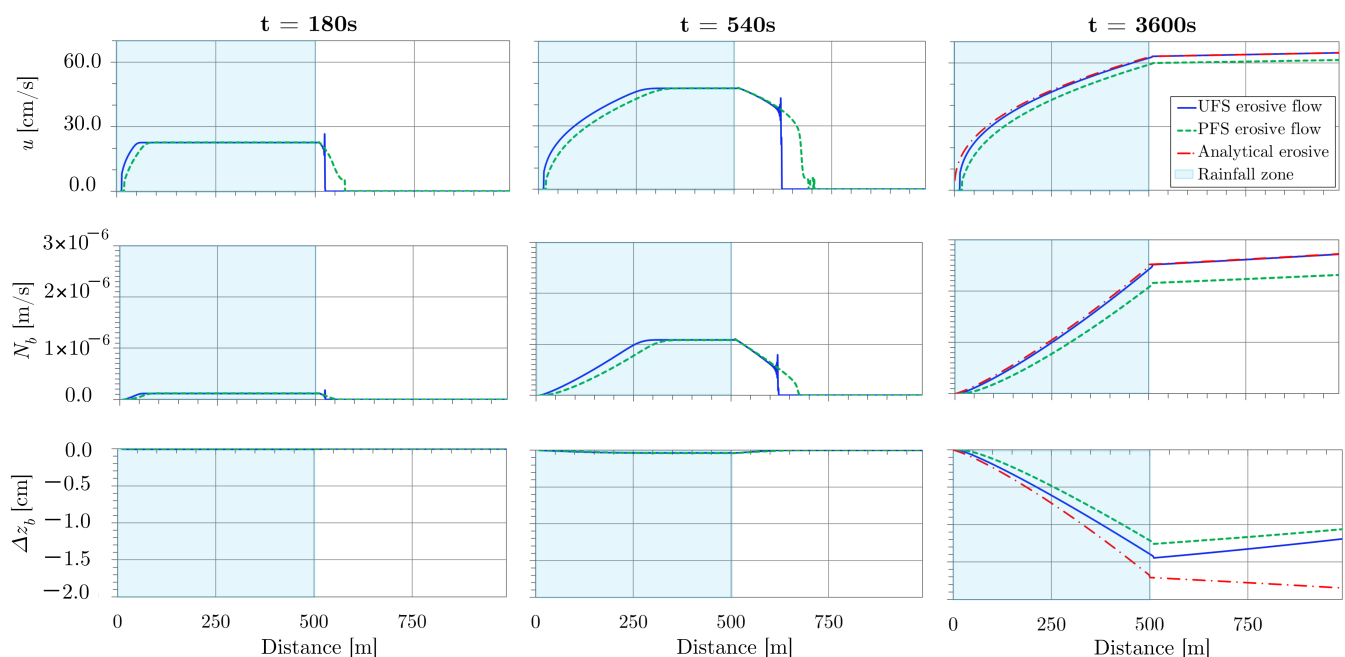


Figure 5. Test 1—Temporal evolution of x -velocity, net bed exchange rate, and bed elevation change along the plane centerline for the erosive soil condition.

The results show that the numerical model reproduces the expected temporal development of the erosive flow. At the early stage, $t = 180$ s, the runoff is still mainly confined to the rainfall area, and both numerical solutions predict relatively small values of u and N_b . As a consequence, the accumulated bed elevation change remains almost negligible. At $t = 540$ s, the runoff front has propagated downslope, producing larger velocities and, consequently, larger values of N_b . Since the imposed erosion formulation depends on the local velocity, the spatial distribution of N_b follows the evolution of the velocity field.

At the final time, $t = 3600$ s, the UFS solution shows very close agreement with the analytical profile for both the x -velocity and N_b . This agreement indicates that the numerical model correctly reproduces the final velocity field and that the modified expression used to compute N_b is being applied consistently. The PFS solution captures the same spatial pattern, although slightly lower velocities are obtained along the plane. Since the imposed erosion formulation is proportional to u^3 , the slightly lower velocities predicted by the PFS approach produce lower values of N_b than those obtained with the UFS approach and the exact solution.

The bed elevation change exhibits the expected erosive behavior, with negative values of Δz_b indicating degradation of the soil surface. The largest erosion values occur within the rainfall area, especially toward the downstream end of the rainfall zone. However, the numerical profiles of Δz_b do not exactly reproduce the analytical curve. This discrepancy is expected because the analytical bed variation is obtained from the final steady erosive state, whereas the numerical bed change is accumulated during the full transient evolution of the event, starting from initially dry conditions.

Using the CBU approach, the bed level is updated locally as presented in Equation (32), so that the final bed variation depends on the time integral of the instantaneous exchange rate $N_b(x, t)$. During the early stages of the simulation, the runoff is still developing and the local velocity is lower than its steady-state value. Since the imposed erosion formula depends strongly on the flow velocity, the accumulated numerical erosion differs from the erosion obtained by applying the final steady value of N_b over the whole simulation time.

In addition, the comparison of Δz_b should be interpreted as a consistency check of the spatial erosion pattern and magnitude, rather than as an exact reproduction of the analytical steady-state estimate. In contrast, the close agreement observed for u and N_b at $t = 3600$ s confirms that the model properly recovers the final steady hydrodynamic and erosive forcing fields. The parameters considered in this verification case are summarized in Table 2.

Table 2. Test 1—Parameters used for the erosion model.

Parameter	Value	Comments
Sediment class	1	
Sediment size d_s	0.1 mm	Fine sand [41]
Soil bulk porosity p_b	0.469	From Equation (7)
Soil dry density ρ'_b	1406 kg/m ³	From Equation (7)
Infiltration r_f	0.001 mm/s	Default value
Empirical coefficient a	10 ^{−5} s ² /m ²	Default value

Lastly, the agreement between the analytical and numerical profiles was also assessed for the erosive case using the relative error of the spatial integral over the domain (Table 3). The UFS scheme reproduced the analytical solution with relative errors below 2% for both u_x and N_b , whereas the PFS scheme yielded errors of 7.85% and 17.36%, respectively. Larger discrepancies were observed for Δz_b , with relative errors of 22.58% for UFS and 33.70% for PFS. As discussed previously, this difference is expected because the analytical bed variation corresponds to the final steady erosive state, whereas the numerical bed change is accumulated throughout the transient evolution of the event. The lower relative errors obtained with UFS indicate closer agreement with the analytical solution.

Table 3. Test 1—Relative errors for the case with erosion.

Variable	Model	Integral Area	Relative Error (%)
u	UFS	53,520.51	1.12
	PFS	49,878.76	7.85
	Analytical	54,126.62	–
N_b	UFS	1.83×10^{-3}	1.61
	PFS	1.54×10^{-3}	17.36
	Analytical	1.86×10^{-3}	–
Δz_b	UFS	−977.48	22.58
	PFS	−837.07	33.70
	Analytical	−1262.57	–

4.2. Test 2: V-Catchment Case Study

The second test consists of an idealized V-shaped catchment, originally proposed by [42], which is commonly used to evaluate the ability of rainfall–runoff models to reproduce lateral flow convergence from hillslopes into a central drainage channel. In the present work, the test is used to also assess the sensitivity of the numerical model to different treatments of the bed slope and the friction source terms. In particular, UFS is compared with PFS, considering both differential and integral evaluations of the bed slope contribution, denoted by DBS and IBS, respectively. The goal is to evaluate which approach provides a more consistent, stable and conservative prediction of hillslope and channelized discharge convergence.

The computational domain represents a symmetric V-shaped basin composed of two inclined lateral planes draining toward a straight central channel. The channel is 20 m wide and aligned with the longitudinal direction of the domain. The total domain size is 1000 m in the channel direction and 1620 m in the transverse direction, corresponding to two 800 m wide hillslopes separated by the central channel. The longitudinal slope of the channel is 0.02, whereas the transverse slope of the hillslopes is 0.05. The domain is initially dry and is subjected to a spatially uniform rainfall intensity of 10.80 mm/h during the first 90 min. The simulation is then continued until $t = 200$ min, allowing the complete recession limb of the hydrograph to be analyzed. A uniform resolution of $\Delta l = 1$ m is used, and $CFL = 0.2$. A Manning roughness coefficient of $n = 0.015 \text{ s/m}^{1/3}$ is imposed over both the hillslopes and the channel.

To characterize the hydrological response of the system, two discharge monitoring sections are defined. The first discharge, denoted by Q_{slope} , is at the downslope boundary of the hillslope, immediately upstream of the central channel. This discharge represents the runoff generated over the hillslope before entering the channel. The second discharge, denoted by $Q_{channel}$, is at the downstream outlet of the central channel and represents the total channelized outflow from the catchment. Q_{slope} provides insight into how each bed slope treatment represents overland flow generation over inclined terrain, whereas $Q_{channel}$ allows for the assessment of the integrated response after flow concentration within the central drainage. Figure 6a shows the plan-view DEM, highlighting the domain extent, rainfall zone, central channel, and discharge monitoring sections. Figure 6b provides a three-dimensional view of the same domain, emphasizing the V-shaped topography and the lateral convergence of runoff toward the central channel. The simulated discharge hydrographs were then compared against the analytical solution reported by [42].

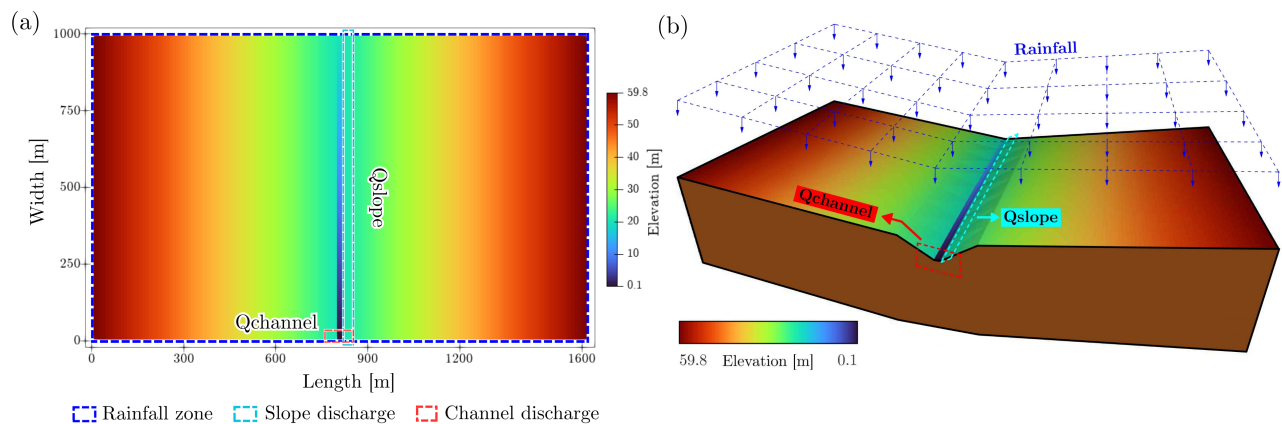


Figure 6. Test 2—V-Catchment case. (a) Plan-view representation of the computational domain. (b) V-catchment geometry.

It should be noted that the present setup is not strictly identical to the original configuration reported by [42]. In the classical benchmark, different Manning roughness coefficients are assigned to the hillslopes and to the channel, whereas here, a uniform value is used throughout the domain, as applied by [29]. Another difference concerns the spatial distribution of rainfall, which was applied over the whole domain, while in [42], precipitation was assumed not to affect the channel directly. These differences are expected to affect mainly the channelized response, since the lower channel roughness adopted in this work reduces flow resistance and may modify the timing and magnitude of the outlet hydrograph. For this reason, the analytical solution is used as a reference benchmark, while the interpretation focuses on the relative behavior of the different bed slope source term treatments.

The simulated hydrographs are shown in Figure 7. Since no infiltration is considered, the discharge values are controlled directly by the rainfall intensity and the contributing area. For the imposed rainfall intensity, the expected steady Q_{slope} is $2.40 \text{ m}^3/\text{s}$, while $Q_{channel}$ is $4.86 \text{ m}^3/\text{s}$. For the DBS, the channel discharge is well reproduced by both UFS and PFS. The two numerical hydrographs reach approximately the same quasi-steady discharge as the exact solution, with only small differences during the rising limb. The PFS formulation produces a slightly earlier increase in $Q_{channel}$, whereas the UFS formulation shows a smoother transition toward the plateau. After the rainfall stops at $t = 90 \text{ min}$, both UFS and PFS reproduce a similar recession behavior. However, the numerical hydrographs remain slightly above the analytical solution during the late recession stage, indicating slower drainage of the water stored within the computational domain.

The Q_{slope} obtained with DBS shows larger differences during the initial part of the event. The analytical solution reaches its steady value rapidly, whereas the numerical hydrographs present a smoother and delayed rising limb. Nevertheless, the UFS scheme reaches a plateau very close to the analytical value. In contrast, the PFS scheme slightly underestimates the hillslope steady discharge. During the recession stage, both numerical solutions decrease gradually than the analytical curve, again suggesting larger temporary storage of water over the hillslope.

The differences become more evident when the Integral Bed Slope (IBS) treatment is used. For $Q_{channel}$, both UFS and PFS produce a delayed response with respect to the exact solution. Although the peak discharge is close to the expected value, the rising limb is significantly slower, and the plateau is only reached close to the end of the rainfall period. This behavior indicates that the IBS treatment introduces additional temporal spreading in the channel response. The PFS reacts earlier than UFS, but both cases show a delayed recession and remain above the exact solution at late times. A similar pattern is observed for

Q_{slope} with IBS. The numerical hydrographs increase more slowly than the exact solution and reach their maximum values close to the end of the rainfall event. The UFS formulation reaches a peak discharge close to the exact plateau, whereas PFS remains slightly lower. After rainfall ceases, the recession limb is again smoother and more delayed than the reference, especially for UFS.

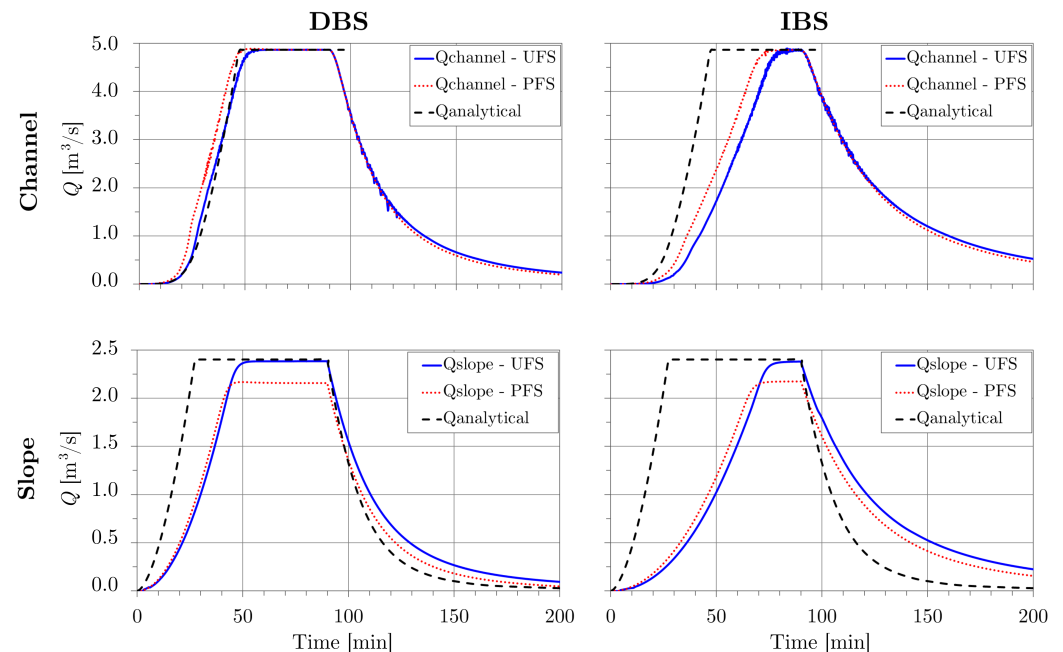


Figure 7. Test 2—Hydrographs obtained with UFS and PFS formulations using DBS (left column) and IBS (right column) for $\Delta l = 1$ m. The upper panels show the channel outlet discharge, while the lower panels show the hillslope discharge.

Overall, the DBS treatment provides the closest agreement with the benchmark, particularly for the channel outlet discharge. The IBS treatment produces a noticeable delay in both hillslope runoff generation and channel outflow, affecting mainly the timing of the rising limb and the recession. Regarding the comparison between source term formulations, UFS tends to better preserve the expected steady discharge magnitude, especially for Q_{slope} , whereas PFS generally produces an earlier response but slightly lower plateau values. Therefore, the main differences observed in this test are associated with the DBS–IBS treatment, while the differences between UFS and PFS are secondary but still relevant during transient stages.

To complement the hydrograph analysis, the total discharged volume was also compared for each numerical configuration. This volume was obtained from the area under each hydrograph over the common time interval between the exact and numerical time series.

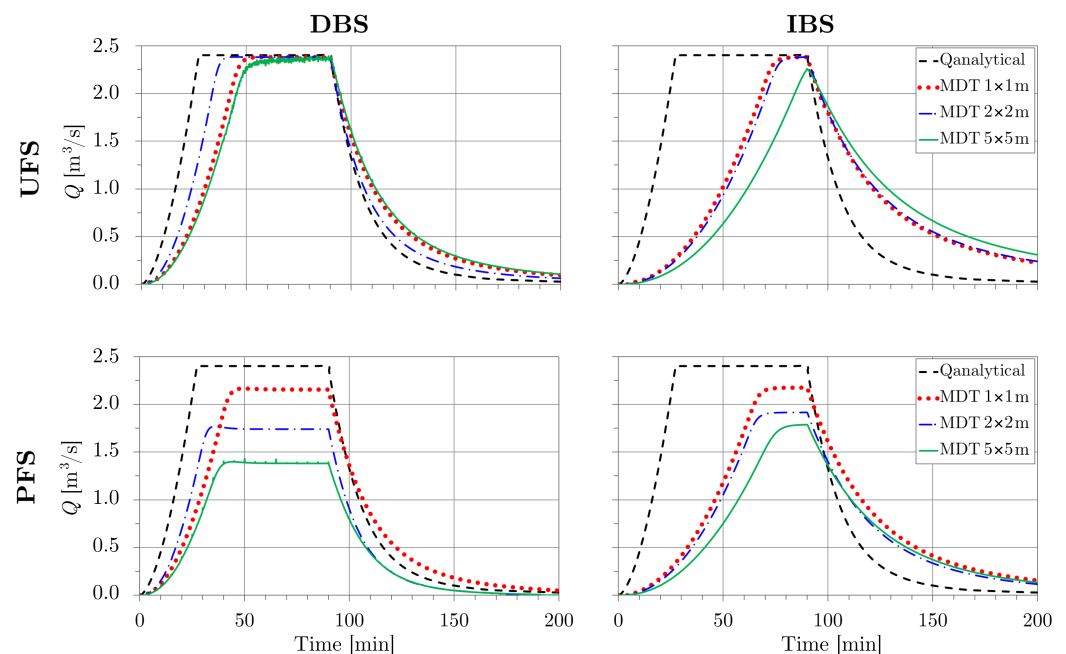
Table 4 shows that the DBS treatment provides the best agreement with the exact solution. For $Q_{channel}$, DBS–UFS gives the closest result, with a relative error of only -0.43% . DBS–PFS also performs reasonably well, although it slightly overestimates the discharged volume, with an error of 5.50% . In contrast, IBS shows larger differences. For $Q_{channel}$, both IBS–UFS and IBS–PFS underestimate the volume, with errors of -33.10% and -23.22% , respectively. This confirms that the delayed response observed in the IBS hydrographs has a direct effect on the total volume discharged at the outlet.

For Q_{slope} , all numerical configurations underestimate the volume. The smallest error is again obtained with DBS–UFS, while the largest difference is observed for IBS–PFS. This indicates that DBS represents the lateral runoff contribution from the hillslopes more accurately than IBS. Negative errors indicate underestimation by the numerical model.

Table 4. Test 2—Integrated discharge volume comparison over the common time interval between the exact and numerical hydrographs.

Case		Formulation	Volume (m ³)	Relative Error (%)
Channel	DBS	UFS	17,734.98	−0.43
		PFS	18,791.98	5.50
	IBS	UFS	11,915.28	−33.10
		PFS	13,675.05	−23.22
		Analytical	17,811.81	—
Slope	DBS	UFS	12,111.55	−7.29
		PFS	10,990.53	−15.87
	IBS	UFS	10,890.46	−16.64
		PFS	10,269.84	−21.39
		Analytical	13,063.73	—

Finally, to evaluate the influence of DTM resolution on Q_{slope} , a mesh sensitivity analysis was performed using three grid resolutions, $\Delta l = 1$ m, $\Delta l = 2$ m and $\Delta l = 5$ m, as shown in Figure 8. Overall, the 1 m and 2 m grids produce very similar hydrographs for both bed slope discretizations, indicating limited sensitivity to mesh refinement once a sufficiently fine resolution is adopted. In contrast, the 5 m grid introduces noticeable numerical diffusion, resulting in delayed rising limbs, smoother recession curves and, particularly for the PSF formulation, significant attenuation of the discharge peak. Among the evaluated combinations, the DBS generally preserves the characteristic hydrograph plateau more effectively than the IBS formulation, while the USF friction treatment exhibits greater robustness to mesh coarsening than PSF.

**Figure 8.** Test 2—Influence of three different resolutions on the outlet discharge hydrograph. In red $\Delta l = 1$ m, in blue $\Delta l = 2$ m, and in green $\Delta l = 5$ m.

4.3. Test 3: Dam-Break Experimental Case Study

The third case considered in this work corresponds to a dam-break experiment over an erodible bed, based on the laboratory data reported by [39,43]. This test was selected due to the strong interaction between the hydrodynamic and morphodynamic processes induced by an unsteady flow over a sudden flow contraction. The goal is to evaluate the

capability of the numerical model to reproduce the final bed morphology generated by the passage of the dam-break wave. In addition, the influence of the bed-update strategy is analyzed by comparing the Upwind Bed Updating (UWBU) formulation with the classical Centered Bed Updating (CBU) approach.

The experimental facility consists of a rectangular flume 3.25 m long and 0.24 m wide, connected to an upstream reservoir 1.57 m long and 0.81 m wide. The dam-break wave is generated by the opening of a pneumatic gate, releasing an initial water depth of 0.08 m over an initially dry bed. The movable-bed section consists of a 0.05 m thick layer of non-cohesive sand. The sediment bed is initially flat and level with the fixed channel bottom. The bed material is composed of two uniform sand classes with particle diameters of 0.7 and 1.3 mm, each representing 50% of the sediment mixture. The flow resistance was represented using Manning coefficients equal to $n_b = 0.012 \text{ s/m}^{1/3}$ in the fixed-bed regions and $n_b = 0.026 \text{ s/m}^{1/3}$ throughout the movable-bed region, following the numerical setup of the reference study.

The present work focuses on the Case 2 narrowing configuration [39], which consists of two rectangular obstacles with dimensions $11 \text{ cm} \times 5.5 \text{ cm}$ symmetrically arranged on the side walls of the flume. The obstacles generate a sharp contraction of the flow section, producing significant flow acceleration and increased sediment transport capacity in the narrow section. They were centered at 1.50 m downstream of the gate. A schematic representation of the experimental setup is shown in Figure 9, while the numerical parameters adopted in the simulations are summarized in Table 5.

The numerical simulations were carried out on a uniform mesh with spatial resolution $\Delta l = 0.005 \text{ m}$ and a final simulation time of $t = 30 \text{ s}$. The CFL was set to 0.4. The UFS and IBS formulations were used for the discretization of the friction and bed slope source terms, respectively. The calibrated entrainment and deposition factors were set to $\alpha_e = 140.0$ and $\alpha_d = 3.0$. Special attention is given to the erosion generated at the contraction and to the depositional pattern formed downstream of the obstacles, which constitute the main morphodynamic features observed in the experimental measurements.

Although this case has also been employed by [39] to validate bedload transport models, the present work does not aim to reproduce or compare those numerical results. Their formulation was specifically developed for bedload transport, whereas the present model considers suspended sediment transport. Therefore, the laboratory measurements reported by [43] constitute the reference dataset used to assess the influence of the UWBU and CBU formulations on the prediction of the final bed morphology.

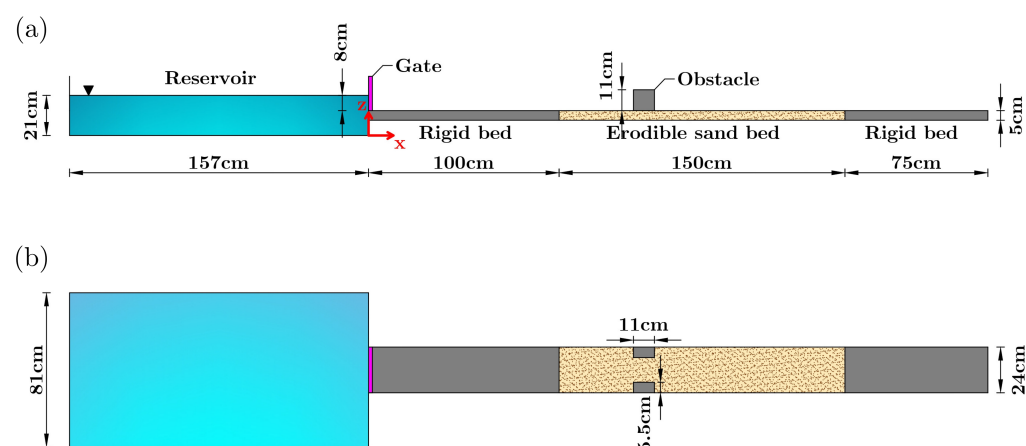


Figure 9. Test 3—Dam-break experimental configuration. (a) Side-view. (b) Top-view.

Table 5. Test 3—Parameters used for the numerical model.

Parameter	Value	
Sediment classes	2	
Soil composition	Coarse sand	Fine sand
Soil fractions F_b	50%	50%
Sediment diameter d_s	0.7 mm	1.3 mm
Entrainment coefficient α_e	140.0	
Deposition coefficient α_d	3.0	
Manning roughness n_b	Movable-bed region	0.026 s/m ^{1/3}
	Fixed-bed regions	0.012 s/m ^{1/3}

Figure 10 compares the final experimental bed elevation after the passage of the dam-break wave with the numerical results obtained using the UWBU and CBU formulations. The same color scale, ranging from 11 to 15 cm, is adopted in all panels to facilitate a direct visual comparison of the predicted bed morphology.

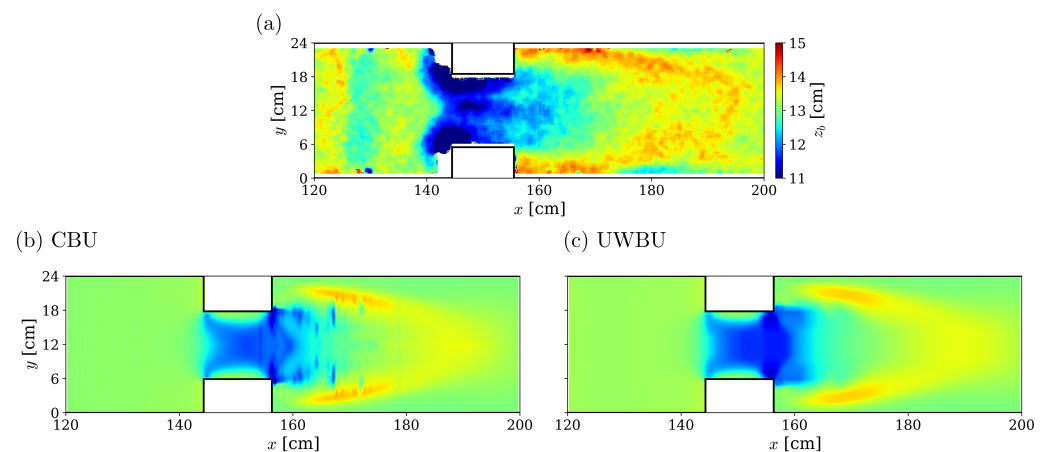


Figure 10. Test 3—Comparison between the experimental final bed elevation and the numerical results obtained with the UWBU and CBU formulations. (a) Experimental bed elevation after the first dam-break wave. (b) CBU numerical results. (c) UWBU numerical results.

The experimental measurements show a pronounced erosion region immediately downstream of the contraction, followed by the formation of a broad depositional region extending towards the downstream section of the channel. The erosion develops mainly along the channel centerline, whereas deposition occurs over a relatively wide area, indicating a significant lateral redistribution of the transported sediment after the contraction.

Both numerical models are able to reproduce the dominant morphological features observed in the experiment. In particular, the location of the erosion region and the subsequent depositional zone are well represented, demonstrating that the proposed suspended sediment transport model captures the overall bed response induced by the dam-break wave. However, the numerical solutions underestimate the lateral spreading of both the erosion and deposition regions. In the UWBU solution, the erosion remains more confined to the channel centerline, while the depositional region is narrower and more concentrated than that observed experimentally.

The difference between the two bed-update formulations becomes evident in the downstream region. The UWBU formulation produces a smooth and physically consistent bed evolution, preserving the symmetry of the erosion and deposition patterns and providing a stable prediction of the final bed morphology. On the other hand, the CBU formulation develops pronounced oscillatory structures immediately downstream of the contraction. These oscillations appear as alternating bands of erosion and deposition that

are absent from the experimental measurements, producing unrealistic local bed variations and fragmenting the depositional region. As a consequence, the agreement between the numerical and experimental bed morphologies is significantly reduced.

Figure 11 presents a quantitative comparison of the final bed elevation along the channel centerline. The profile was obtained by averaging the bed elevation across the central region of the channel ($y = 12 \pm 1$ cm), thereby reducing the influence of local measurement noise while preserving the dominant longitudinal morphological features.

The longitudinal profiles corroborate the observations obtained from the 2D bed elevation showed in Figure 10. Consistent with the previous comparison, both numerical formulations reproduce the overall erosion–deposition sequence induced by the flow contraction, capturing the onset of the main erosion region and the subsequent downstream recovery of bed elevation. However, both solutions underestimate the maximum erosion depth and predict a smoother longitudinal bed profile than observed experimentally. In particular, the transition from the initially flat bed to the eroded region is more gradual, resulting in a shallower bed depression than that measured in the laboratory.

Although the differences between the two bed-update strategies are relatively small in the longitudinal profile, the UWBU formulation exhibits closer agreement with the experimental measurements. Furthermore, the smoother longitudinal profile obtained with UWBU reflects its ability to suppress the non-physical oscillations observed in the CBU solution while preserving the overall erosion and deposition patterns observed experimentally.

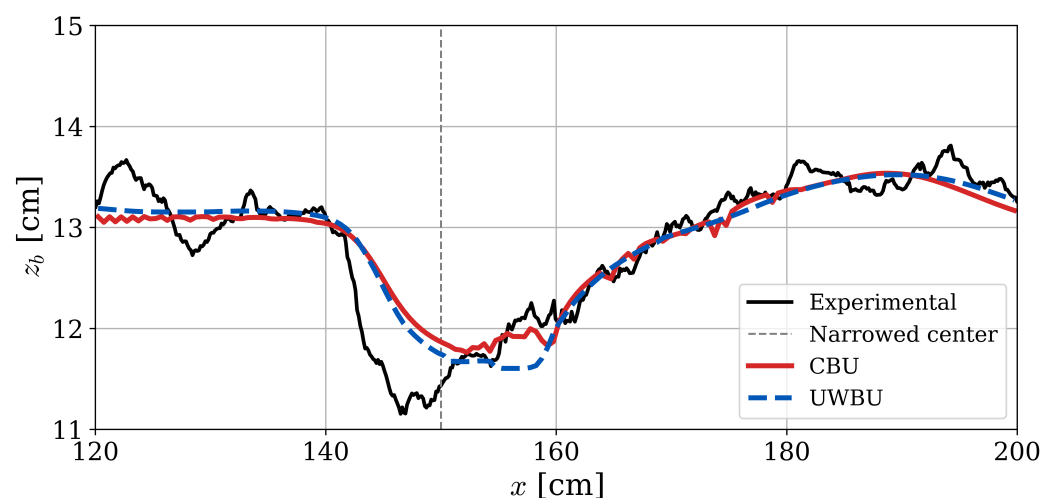


Figure 11. Longitudinal bed elevation profile based on the centerline of the channel. The entrainment and deposition factors were set to $\alpha_e = 140.0$ and $\alpha_d = 3.0$.

To complement the qualitative comparison of the longitudinal bed elevation profiles, a quantitative assessment was performed using the relative error based on the spatial integral of the bed elevation change, $\Delta z_b = z_b - z_{b0}$, where z_b and z_{b0} denote the final and initial bed elevations, respectively. This metric evaluates the ability of each sediment transport formulation to reproduce the overall balance between erosion and deposition over the analyzed longitudinal section.

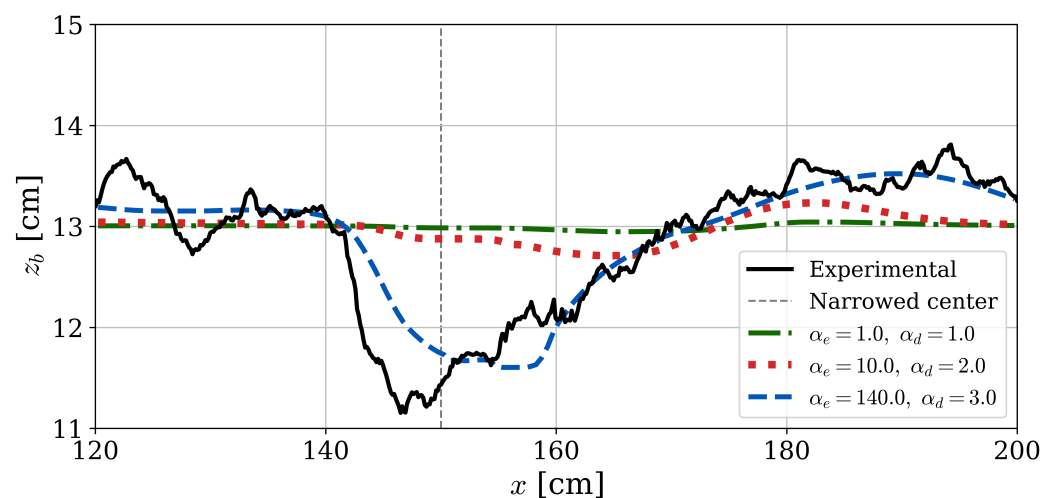
As shown in Table 6, the UWBU scheme achieved a relative error of 12.82%, compared with 21.23% for the CBU scheme, corresponding to an approximate 40% reduction in the error associated with the cumulative bed elevation change. This result indicates that the UWBU formulation better reproduces the overall sediment redistribution observed in the laboratory experiment.

Table 6. Test 3—Comparison of the cumulative bed elevation change along the analyzed longitudinal section.

Method	Integrated Bed Change (cm ²)	Relative Error (%)
CBU	−9.472	21.23
UWBU	−10.483	12.82
Experiment	−12.024	–

To further investigate the model performance under experimental conditions, a sensitivity-based calibration analysis was performed for UWBU. Three combinations of the entrainment and deposition coefficients (α_e and α_d) were evaluated: $\alpha_e = 1.0$ and $\alpha_d = 1.0$, $\alpha_e = 10.0$ and $\alpha_d = 2.0$, and $\alpha_e = 140.0$ and $\alpha_d = 3.0$. The objective was to investigate the influence of these parameters on the model accuracy and identify the parameter set providing the best agreement with the experimental observations.

Figure 12 compares the longitudinal bed elevation profiles obtained with each parameter set against the experimental measurements. Increasing the erosion and deposition coefficients led to a progressively better reproduction of the experimental bed profile, particularly in the region of maximum erosion and the downstream deposition area. Among the tested combinations, $\alpha_e = 140.0$ and $\alpha_d = 3.0$ provided the closest agreement with the experimental observations and was therefore adopted in the present study. Although some discrepancies remain, they are consistent with the limitations discussed previously, since the present model considers suspended sediment transport rather than the bedload transport.

**Figure 12.** Sensitivity of the experimental and simulated longitudinal bed elevation profiles for three calibration scenarios of UWBU, considering different values of α_e and α_d coefficients. In green $\alpha_e = 1.0$ and $\alpha_d = 1.0$, in red $\alpha_e = 10.0$ and $\alpha_d = 2.0$, and in blue $\alpha_e = 140.0$ and $\alpha_d = 3.0$.

Finally, Table 7 summarizes the relative errors obtained in all validation tests. Overall, UFS consistently reduced the relative errors with respect to PFS, whereas UWBU achieved lower errors than CBU in the experimental validation. In Test 1, the UFS formulation reduced the errors for both the hydraulic variables (h and u) and the sediment-related variables (N_b and Δz_b). In Test 2, the UFS formulation combined with the DBS treatment yielded the lowest discharge volume errors for both the channel and slope configurations. For Test 3, the UWBU formulation reduced the relative error associated with the spatially integrated bed elevation change from 21.23% to 12.82%, corresponding to an approximate 40% reduction. These results demonstrate that the proposed formulations improve the predictive capability of the model across both analytical and experimental cases.

Table 7. Summary of the relative errors obtained in the validation tests.

Test	Quantity	Formulation	Relative Error (%)
Test 1 (without erosion)	Water depth, h	UFS	1.20
		PFS	9.98
	Velocity, u_x	UFS	1.05
		PFS	7.52
Test 1 (with erosion)	Velocity, u_x	UFS	1.12
		PFS	7.85
	Sediment concentration, N_b	UFS	1.61
		PFS	17.36
	Bed elevation change, Δz_b	UFS	22.58
		PFS	33.70
Test 2	Channel discharge, Q_{channel} (DBS)	UFS	0.43
		PFS	5.50
	Channel discharge, Q_{channel} (IBS)	UFS	33.10
		PFS	23.22
	Slope discharge, Q_{slope} (DBS)	UFS	7.29
		PFS	15.87
	Slope discharge, Q_{slope} (IBS)	UFS	16.64
		PFS	21.39
Test 3	Bed elevation change, Δz_b	UWBU	12.82
		CBU	21.23

5. Conclusions

In this work we present a novel hydro-erosive model that solves the 2D shallow water equations (SWE-2D) for runoff generation and the 2D solid transport and the bed evolution equations for the mobilization, movement and deposition of erodible soil particles. Several novelties are included in this cutting-edge hydro-erosive model. The model is tested against synthetic and experimental cases, demonstrating its robustness, accuracy and performance. The following conclusions can be drawn:

1. We derived a novel Upwind Bed Updating (UWBU) method to integrate the soil erosion due to exchange of sediments between the runoff and the erodible soil layer. The proposed UWBU procedure is tested against the classical centered integration at cells, referred to as the CBU method. The UWBU method demonstrates enhanced robustness and non-oscillatory behavior compared with the CBU integration when dealing with high-resolution grids.
2. The hillslope verification tests demonstrate that the numerical model consistently reproduces the analytical behavior under both non-erosive and erosive conditions. The non-erosive case verifies the rainfall–runoff response and the hydrodynamic propagation over the steep plane, whereas the erosive case verifies the implementation of a simplified bed-exchange formulation. The close agreement obtained for u and N_b , particularly for the UFS approach at $t = 3600$ s, confirms that the imposed erosion law is correctly represented in the numerical model. The remaining discrepancies in Δz_b are expected, since the analytical solution is based on the final steady erosive state, whereas the numerical solution accounts for the transient evolution of the flow.
3. In the V-catchment test, the numerical model was able to reproduce the main rainfall–runoff response of the benchmark, including the lateral convergence from the hillslopes toward the central channel and the subsequent outlet discharge. The comparison between bed slope source term treatments showed that the DBS approach provided the closest agreement with the analytical hydrographs, especially at the channel outlet. Among the tested configurations, DBS–UFS gave the most consistent results, while the IBS treatment introduced a delayed hydrological response and a slower

recession. This indicates that the differential treatment of the bed slope source term is more suitable for representing the coupled hillslope–channel runoff dynamics in this idealized catchment.

4. In the experimental dam-break test over an erodible bed, the numerical model successfully reproduced the dominant morphodynamic response induced by a flow contraction, including the formation of the main erosion region and the downstream depositional pattern. Comparisons with laboratory measurements showed that both bed-update formulations captured the overall erosion–deposition sequence and the final bed morphology. However, the UWBU formulation produced smoother and more physically consistent solutions by suppressing the non-physical oscillations observed with the classical CBU approach. The longitudinal bed elevation profiles further confirmed these observations, showing that UWBU provided closer agreement with the experimental measurements while preserving the overall evolution of the bed.
5. Finally, among the numerical strategies investigated, the DBS formulation showed consistently better agreement with the reference solutions than the IBS, while the UFS generally outperformed the PFS approach, particularly under transient conditions. Appropriate parameter calibration and mesh resolution are required to ensure reliable predictions, particularly when PFS discretization is employed, whereas the UFS formulation exhibits greater robustness. In addition, the proposed UWBU method successfully reduced oscillations during bed evolution while maintaining computational efficiency, providing a robust alternative to the CBU procedure. It must be pointed out that the test cases shown are highly simplified and deviate substantially from real-world field conditions. Model applicability when considering vegetation coverage, multi-size sediment mixtures and large-scale field watersheds requires further effort. Future works should focus on extending the model to multiple sediment classes, vegetation effects and large-scale field applications in agroforestry catchments.

Author Contributions: Conceptualization, S.M.-A.; methodology, A.B., S.M.-A. and P.G.-N.; software, A.B. and S.M.-A.; validation, A.B.; formal analysis, A.B. and P.G.-N.; investigation, A.B. and P.G.-N.; data curation, A.B.; writing—original draft preparation, A.B. and P.G.-N.; writing—review and editing, A.B., S.M.-A. and P.G.-N.; visualization, A.B.; supervision, S.M.-A.; project administration, S.M.-A. and P.G.-N. All authors have read and agreed to the published version of the manuscript.

Funding: This work was supported by the Government of Aragón, through the research project PROY_T21_24 HIDROGIF. This work was also partially funded by project PID2022-137334NB-I00 funded by MCIN/AEI/10.13039/501100011033 and by “ERDF/EU”.

Data Availability Statement: The data that support the findings of this study are available from the corresponding author, S.M.A., upon reasonable request. The SERGHEI-SE model, available at <https://gitlab.com/serghei-model/serghei> (accessed on 29 July 2026) under a 3-clause BSD license, was used in this work.

Conflicts of Interest: The authors declare no conflicts of interest.

Appendix A. Analytical Solution–Test 1

A one-dimensional (1D) hillslope configuration is considered in order to obtain a reference solution for the hydrodynamic and morphodynamic variables. The hillslope has length L , constant bed slope S_0 , and width B . The flow is described in terms of the water depth $h(x)$, the unit discharge $q(x) = h(x)u(x)$, and the depth-averaged velocity $u(x)$. The x -coordinate is defined along the downslope direction, with $x = 0$ at the upstream end and $x = L$ at the downstream outlet.

Under steady-state conditions, assuming gradually varied flow and hydrostatic pressure distribution, the one-dimensional mass conservation equation for the water–sediment mixture can be written as:

$$\frac{dq}{dx} = r_0(x) - r_f + \xi N_b(x) \equiv C_1(x) \quad (\text{A1})$$

where $r_0(x)$ is the rainfall rate, r_f is the constant infiltration rate applied over the whole domain, $N_b(x)$ is the net bed exchange rate, and $\xi = \frac{1}{1-p_b}$ is the porosity correction factor, with p_b being the bed porosity.

The rainfall is prescribed over the upstream part of the hillslope, such that:

$$r_0(x) = \begin{cases} r, & 0 \leq x \leq L_r \\ 0, & L_r < x \leq L \end{cases} \quad (\text{A2})$$

where r is the rainfall intensity and L_r is the length of the rainfall area.

Appendix A.1. Non-Erosive Case

For the non-erosive reference case, no mass exchange between the bed and the flow is allowed. Therefore, $N_b(x) = 0$. The source term in the mass equation reduces to:

$$C_1(x) = r_0(x) - r_f \quad (\text{A3})$$

Thus, the unit discharge is obtained by direct integration of Equation (A1):

$$q(x) = q(0) + \int_0^x C_1(x') dx' \quad (\text{A4})$$

where x' is an intermediate position along the hillslope between 0 and x . Assuming zero inflow at the upstream boundary, $q(0) = 0$, constant rainfall over the rain zone, and constant infiltration over the whole domain, the unit discharge becomes:

$$q(x) = \begin{cases} (r - r_f)x, & 0 \leq x \leq L_r \\ rL_r - r_fx, & L_r < x \leq L \end{cases} \quad (\text{A5})$$

The water depth profile is obtained from the one-dimensional momentum equation. In gradually varied form, the equation is written as:

$$\frac{dh}{dx} = \frac{S_0 - S_f - \frac{C_1 x}{gh^2} \left(\frac{\rho_b}{\rho} - 1 \right) \xi N_b - \frac{2C_1^2 x}{gh^2}}{1 - Fr^2} \quad (\text{A6})$$

where g is the gravitational acceleration, ρ is the density of the water–sediment mixture, and ρ_b is the density of the saturated bed layer. The squared Froude number is $Fr^2 = \frac{q^2}{gh^3}$, and the friction slope is estimated using $S_f = \frac{n_b^2 q^2}{h^{10/3}}$, where n_b is the Manning roughness coefficient. For the non-erosive case, Equation (A6) simplifies to:

$$\frac{dh}{dx} = \frac{S_0 - S_f - \frac{2C_1^2 x}{gh^2}}{1 - Fr^2} \quad (\text{A7})$$

The downstream boundary condition for the water depth is imposed using the normal depth associated with the outlet discharge. For a wide rectangular section, this depth is approximated by:

$$h_n = \left(\frac{q_L n_b}{\sqrt{S_0}} \right)^{3/5} \quad (\text{A8})$$

where $q_L = q(L)$ is the unit discharge at the outlet. Since the flow is subcritical, the water depth equation is integrated from the downstream boundary towards the upstream direction using $h(L) = h_n$.

Appendix A.2. Erosive Case

In the erosive case, the flow is coupled with the bed through the net exchange rate $N_b(x)$. The mass conservation equation retains the general form given in Equation (A1), and the corresponding source term is calculated as:

$$C_1(x) = r_0(x) - r_f + \xi N_b(x) \quad (\text{A9})$$

Following the total net exchange rate formulation presented in the Section Bed-Material Exchange in Channels (BMEC) Model, N_b can be expressed as the difference between sediment entrainment and deposition rates, as defined in Equation (8). For the sake of simplicity, we adopt a simplified closure in which the net exchange rate is directly parameterized as a function of the local flow velocity. Therefore:

$$N_b(x) = au(x)^3 \quad (\text{A10})$$

where a is an empirical coefficient, set here to $a = 10^{-5} [\text{s}^2/\text{m}^2]$. The water depth profile is then obtained from the gradually varied flow equation, including the momentum exchange associated with the entrainment of bed material, as defined in Equation (A6). The depth-averaged suspended sediment concentration is obtained from the steady 1D sediment balance:

$$\frac{d(q\phi)}{dx} = N_b \quad (\text{A11})$$

where ϕ is the volumetric sediment concentration in the flow. Assuming that N_b and C_1 are locally constant, and using $dq/dx = C_1$, an approximation for the concentration is:

$$\phi(x) = \frac{N_b(x)}{C_1(x)} \quad (\text{A12})$$

Finally, the bed elevation evolves according to:

$$\frac{\partial z_b}{\partial t} = -\xi N_b \quad (\text{A13})$$

For a fixed erosion rate during the considered time interval, the bed elevation can be updated analytically as:

$$z_b(x, t) = z_{b,0}(x) - \xi N_b(x)t \quad (\text{A14})$$

where $z_{b,0}(x)$ is the initial bed elevation.

The resulting ordinary differential equations are also integrated in space from the downstream boundary towards the upstream direction. This choice is consistent with the imposed subcritical downstream boundary condition for the water depth. Since the

computational coordinate is stored from $x = L$ to $x = 0$, the explicit spatial updates are written as:

$$\begin{aligned} q_{i+1} &= q_i - C_{1,i} \Delta x \\ h_{i+1} &= h_i - \left(\frac{dh}{dx} \right)_i \Delta x \end{aligned} \quad (\text{A15})$$

At each spatial location, the local velocity is first computed as $u_i = \frac{q_i}{h_i}$. Then, the erosion rate $N_{b,i}$ and the source term $C_{1,i}$ are evaluated, and q and h are advanced to the next spatial point. This 1D solution provides a controlled reference case for assessing the hydrodynamic and morphodynamic response of a hillslope under rainfall, infiltration, and simplified erosion processes.

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