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Two-Dimensional Transient Flow in a Confined Aquifer with a Cut-Off Curtain Due to Dewatering

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Abstract: Long, narrow, deep excavations commonly encountered in practice, such as those for subway stations, require effective groundwater management to prevent disasters in water-rich areas. To achieve this, a cut-off curtain and pumping well are typically employed during long, deep foundation pit dewatering. The unsteady groundwater flow behavior in the confined aquifer must consider the influence of the cut-off curtain during dewatering. This paper establishes a two-dimensional analytical model to describe transient groundwater flow in a confined aquifer with a cut-off curtain. Both the dewatering well pumped at a steady discharge inside the pit and the cut-off curtain are partially penetrating in the anisotropic confined aquifer. With the help of the Laplace and Fourier cosine transformations, the semi-analytical drawdown solution for the model is derived and validated against numerical solution and unsteady pumping test data. It is shown that the inserted cut-off curtain depth and the structural parameters of the pumping well significantly affect the drawdown inside the pit. Sensitivity analysis reveals that, regardless of whether the observation is made inside or outside the curtain, the drawdown is very sensitive to the change in pumping rate, aquifer thickness, storage coefficient, and horizontal hydraulic conductivity. Additionally, drawdown near the cut-off curtain outside the pit is sensitive to the vertical hydraulic conductivity of the aquifer, the width of the pit, and the interception depth of the cut-off curtain, while drawdown far from the curtain outside the pit is not sensitive to the location and length of the well screen.

Keywords: two-dimensional transient flow; semi-analytical solution; confined aquifer; cut-off curtain; sensitivity analysis



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1. Introduction

The classic analytical solutions for unsteady flow in a confined aquifer due to pumping were first developed by Theis [1] for a full penetration well and by Hantush [2] for a partial penetration well. After that, the majority of well hydraulic literature has extensively examined transient flow behavior in different aquifer systems (e.g., Neuman and Witherspoon [3]; Moench [4]; Bear [5]; Perina and Lee [6]; Lin et al. [7]; Feng and Zhan [8]). The theories proposed by these studies are available in the design of a dewatering scheme for a foundation pit without a cut-off curtain in the aquifer. However, the cut-off curtain is commonly applied in deep foundation pits to enhance environmental safety and mitigate groundwater-related issues, including subsidence, sand flow, or others [9–14]. The influence of the cut-off curtain on flow behavior in aquifers during dewatering has been extensively investigated in many studies (e.g., Wu et al. [15]; Xu et al. [16]; Pujades

et al. [17]; Zhang et al. [18]; Wang et al. [19]; Xu et al. [20]; Li et al. [21]; Zeng et al. [22,23]; Zhang et al. [24]). These studies demonstrate that the presence of a cut-off curtain must be considered in the analysis of flow behavior in aquifers, particularly for understanding the seepage field in the vicinity of the curtain.

Many scholars have explored analytical methods for studying flow behavior in aquifer systems with cut-off curtains, providing direct and effective tools for dewatering design (e.g., Shen et al. [25]; Feng et al. [26]; Yang et al. [27]; Feng and Lin [28]). Wu et al. [29] considered the impacts of wellbore storage and well partial penetration and gave an analytical approach to analyze the characteristics of drawdown in confined aquifers with a cut-off curtain. Shen et al. [25] developed a simplified equation to compute the head difference on both sides of the curtain but failed to analyze the flow field in the entire confined aquifer. Lyu et al. [30] derived the head solutions for steady flow inside and outside the partial penetration cut-off curtain in a confined aquifer caused by a constant discharge or drawdown pumping within the pit. Feng and Lin [28] proposed an analytical approach for describing transient groundwater flow to a fully penetrating pumping well in a confined aquifer with a circular cut-off curtain. It is important to note that most of the available analytical solutions were developed on the assumption of a circular or rectangular cut-off curtain. In practical engineering, however, a long, narrow cut-off curtain is commonly constructed, especially for subway stations, deep foundation pits, and open cut tunnels in water-rich areas due to the development of urban underground spaces [31]. Under such circumstances, the flow in this long, deep excavation can be simplified into a two-dimensional flow problem in the vertical plane due to the pit's length significantly exceeding its width. However, analytical studies for dealing with two-dimensional flow in aquifers with a cut-off curtain are very limited. Tan et al. [32] proposed an analytical solution to examine steady-state groundwater flow in a confined aquifer with a cut-off curtain during the dewatering of a long-strip foundation pit. However, their study did not address the transient flow behavior in the aquifer.

To our knowledge, the analytical study on the two-dimensional transient flow due to dewatering in a confined aquifer with a partially penetrated cut-off curtain has not been investigated so far. Therefore, this study seeks to propose a semi-analytical model to describe two-dimensional transient flow in a long, deep excavation with a cut-off curtain due to dewatering in a confined aquifer. The seepage field in the whole aquifer is partitioned into two zones based on the cut-off curtain, and corresponding equations, including the governing equation and initial and boundary conditions, are established. By the use of Laplace transformation, finite Fourier cosine transformation, and discretization methods, the corresponding drawdown solution for the model is derived and subsequently validated by comparing it with both finite element method results and field pumping data. Finally, the transient drawdown response inside and outside the cut-off curtain is explored in this paper, and sensitivity analysis is performed to examine the key parameters. The proposed analytical solution provides a relatively simple, straightforward, and reliable tool to assist in the design of dewatering schemes for long foundation pit projects. It is particularly useful for the determination of some key design parameters, including the insertion depth of the cut-off curtain, as well as the number and structural configuration of dewatering wells. Additionally, the solution allows for the estimation of hydraulic parameters in the confined aquifer under the influence of the cut-off curtain.

2. Methods

2.1. Mathematical Model

Figure 1 displays a conceptual diagram of a two-dimensional flow in the vertical plane in an infinite confined aquifer with a partially penetrating well and a cut-off curtain. Note

that the pumping well inside the pit is simplified as an equivalent well located at the center of the excavation. The well operates at a constant discharge and has a screen length of $(l - d)$. Additionally, the cut-off curtain partially penetrates the confined aquifer of constant thickness B , creating an open interval of length B_a . Because of the presence of the curtain, groundwater flow in the entire aquifer can be categorized into two distinct zones: Zone 1 (where $0 < x \leq x_0$) and Zone 2 (where $x_0 < x$).

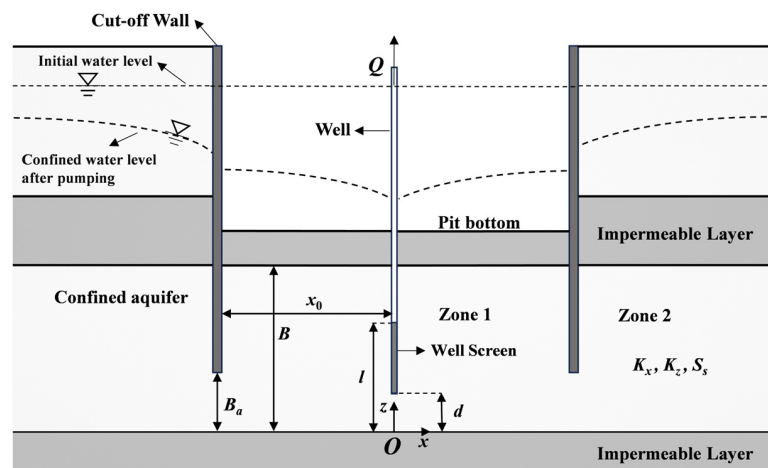


Figure 1. Sketch of the two-dimensional flow in the vertical plane in an infinite confined aquifer with a partially penetrating well and a cut-off curtain.

The assumptions are used in this study as follows (Hantush [2]; Bear [5]; Feng and Lin [28]). (1) The thickness of the cut-off curtain is negligible. (2) The pumping well has an infinitesimally small radius, and wellbore storage can be neglected. (3) The flow adheres to Darcy's law. (4) The confined aquifer is homogeneous and anisotropic.

The governing equation, along with the boundary and initial conditions for flow in Zone 1, can be written as follows:

$$K_x \frac{\partial^2 s_1(x, z, t)}{\partial x^2} + K_z \frac{\partial^2 s_1(x, z, t)}{\partial z^2} = S_s \frac{\partial s_1(x, z, t)}{\partial t}, 0 \leq x \leq x_0, 0 \leq z \leq B \quad (1)$$

$$s_1(x, z, 0) = 0 \quad (2)$$

$$\lim_{x \rightarrow 0} \frac{\partial s_1(x, z, t)}{\partial x} = \begin{cases} 0, & 0 \leq z \leq d \\ \frac{-Q}{2K_x(l-d)}, & d \leq z \leq l \\ 0, & l \leq z \leq B \end{cases} \quad (3)$$

$$\frac{\partial s_1(x, 0, t)}{\partial z} = \frac{\partial s_1(x, B, t)}{\partial z} = 0 \quad (4)$$

$$K_x \frac{\partial s_1(x_0, z, t)}{\partial x} = \begin{cases} q(z, t), & 0 \leq z \leq B_a \\ 0, & B_a \leq z \leq B \end{cases} \quad (5)$$

where t denotes the pumping time, Q refers to the equivalent pumping rate, K_x and K_z represent the horizontal and vertical hydraulic conductivities, respectively, and $q(z, t)$ denotes the specific flux across the open interval. This function is continuous with respect to both z and t ; s is the drawdown. Notably, Equation (3) describes the discharge condition at the well face and indicates that the flux is uniform along the well screen [2,5,26].

Similarly, the governing equation, initial and boundary conditions for flow in Zone 2 are given by:

$$K_x \frac{\partial^2 s_2(x, z, t)}{\partial x^2} + K_z \frac{\partial^2 s_2(x, z, t)}{\partial z^2} = S_s \frac{\partial s_2(x, z, t)}{\partial t}, x_0 \leq x \leq \infty, 0 \leq z \leq B \quad (6)$$

$$s_2(x, z, 0) = 0 \quad (7)$$

$$s_2(\infty, z, t) = 0 \quad (8)$$

$$\frac{\partial s_2(x, 0, t)}{\partial z} = \frac{\partial s_2(x, B, t)}{\partial z} = 0 \quad (9)$$

$$K_x \frac{\partial s_2(x_0, z, t)}{\partial x} = \begin{cases} q(z, t), & 0 \leq z \leq B_a \\ 0, & B_a \leq z \leq B \end{cases} \quad (10)$$

where the subscripts 1 and 2 represent Zone 1 and Zone 2, respectively. Notably, Equations (5) and (10) show the flux continuity at $x = x_0$, $0 \leq z \leq B_a$, and additionally, the continuity of drawdown at the two zone interfaces yields:

$$s_1(x_0, z, t) = s_2(x_0, z, t), 0 \leq z \leq B_a \quad (11)$$

2.2. Solution

By first taking the Laplace transform to Equations (1)–(11) and then applying the finite Fourier cosine transform to Equations (1)–(10), the transient drawdown solutions for Zone 1 and Zone 2 can be derived using the procedure outlined in Appendix A. The final solution for Zone 1 is

$$\bar{s}_1(x, z, p) = \frac{2}{B} \sum_{n=0}^{\infty} \left\{ \frac{\cosh[\eta(x_0 - x)]}{\eta \sinh[\eta x_0]} \frac{Q}{2K_x(l - d)p} \delta + \frac{\cosh(\eta x)}{\eta \sinh(\eta x_0)} \frac{1}{K_x} \sum_{j=1}^M \bar{q}_j(p) F_j \right\} \cos\left(\frac{n\pi z}{B}\right) \quad (12)$$

where $\eta = [(K_z(n\pi/B)^2 + S_z p)/K_x]^{1/2}$, $\delta = B[\sin(n\pi l/B) - \sin(n\pi l/B)]/(n\pi)$. Notably, parameters in the Laplace domain are denoted with an overbar. p is the Laplace transform variable. Furthermore, I_0 and K_0 represent the modified Bessel functions of zeroth order for the first and second kinds, respectively, and I_1 and K_1 denote the modified Bessel functions of the first order for the first and second kinds, respectively. M signifies the quantity of discretization segments within the open interval. $\bar{q}_j(p)$ and F_j can be obtained by Equation (A22) and Equation (A23), respectively.

The solution for Zone 2 can be expressed as

$$\bar{s}_2(x, z, p) = \frac{2}{B} \sum_{n=0}^{\infty} \left\{ \frac{e^{\eta(x_0 - x)}}{-\eta} \frac{1}{K_x} \sum_{j=1}^M \bar{q}_j(p) F_j \right\} \cos\left(\frac{n\pi z}{B}\right) \quad (13)$$

Notably, the term for $n = 0$ in Equations (12) and (13) must be adjusted by multiplying it by $1/2$.

Additionally, the Stehfest [33] algorithm is employed to perform numerical Laplace inversion on the developed solutions, yielding their corresponding time-domain results.

3. Verification

3.1. Comparison with Numerical Solution

To test the validity and accuracy of the obtained solutions, we first compare the results obtained from solutions shown in Equations (12) and (13) with numerical solutions generated using the partial differential equation (PDE) module in COMSOL Multiphysics 6.0. It is important to note that the numerical scheme employed by the PDE module utilizes

the finite element method rather than the finite difference method. The values used for the parameters are the following: $B = 20$ m, $B_a = 10$ m, $x_0 = 20$ m, $l = 20$ m, $d = 12$ m, $Q = 2$ m²/d, $K_x = 1$ m/d, $K_z = 0.5$ m/d, $S_s = 0.0005$ /m. Figure 2 shows the mesh division diagram of the numerical model with COMSOL. The model consists of 6445 free triangular elements, characterized by a maximum element size of 740 and a minimum element size of 0.0019. The simulation duration is set to 10 days, with exponentially increasing time steps resulting in a total of 1001 steps.

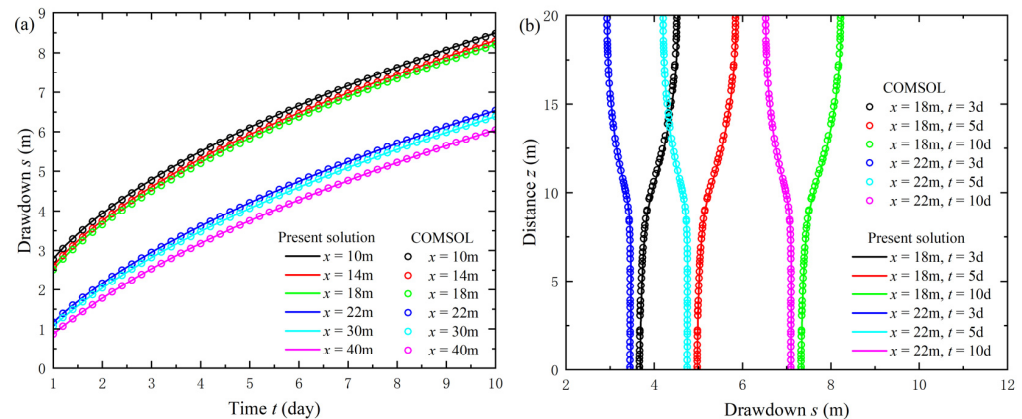


Figure 2. Comparison of drawdown obtained by the present solution (solid curve) and the numerical solution (circle symbols): (a) s versus t for Zone 1 ($x = 10$ m, 14 m and 18 m) and Zone 2 ($x = 22$ m, 30 m and 40 m), where $z = 18$ m; (b) z versus s at $t = 3$ d, 5 d and 10 d, where $x = 18$ m and 22 m.

Figure 2a displays the drawdown values in Zone 1 ($x = 10$ m, 14 m, 18 m) and Zone 2 ($x = 22$ m, 30 m, 40 m) at $z = 18$ m over the entire pumping period. Figure 2b illustrates the vertical distribution curves of drawdown for Zone 1 ($x = 18$ m) and Zone 2 ($x = 22$ m) at various pumping times ($t = 3$ d, 5 d, 10 d). These figures demonstrate a high degree of consistency in drawdown results between numerical simulations and analytical calculations using the developed solution, confirming the reliability and correctness of the new semi-analytical approach.

3.2. Comparison with Field Test

The developed solution can be further validated with field data of a pumping test performed in a long, deep excavation in the study of Shen et al. [27]. The flow in this pit, with dimensions of roughly 25 m in width and 152 m in length, can be simplified into a two-dimensional problem, and the detailed geological conditions and information in this site can refer to Shen et al. [27]. There are three pumping wells (Y1 to Y3) inside the pit to lower the confined water, with constant discharges of 5.42 m³/h, 4.65 m³/h, and 5.11 m³/h, respectively. The equivalent total pumping discharge can be obtained as 0.1 m²/h. The other values used in the calculations are the following [27]: $B = 11$ m, $B_a = 0.2$ m, $x_0 = 12.5$ m, $l = 11$ m, $d = 4$ m, $K_x = 5.06$ m/d, $K_z = 0.51$ m/d, $S_s = 3 \times 10^{-5}$ /m. Figure 3 shows the drawdowns in the observation well G2 outside the pit, calculated using the developed solution and field test data. Note that the sketch at the bottom of Figure 3 illustrates the relative position of observation well G2 in relation to the long foundation pit. It can be observed from Figure 3 that the current solution is applicable for evaluating transient drawdown, particularly for the maximum drawdown that is of interest to engineers.

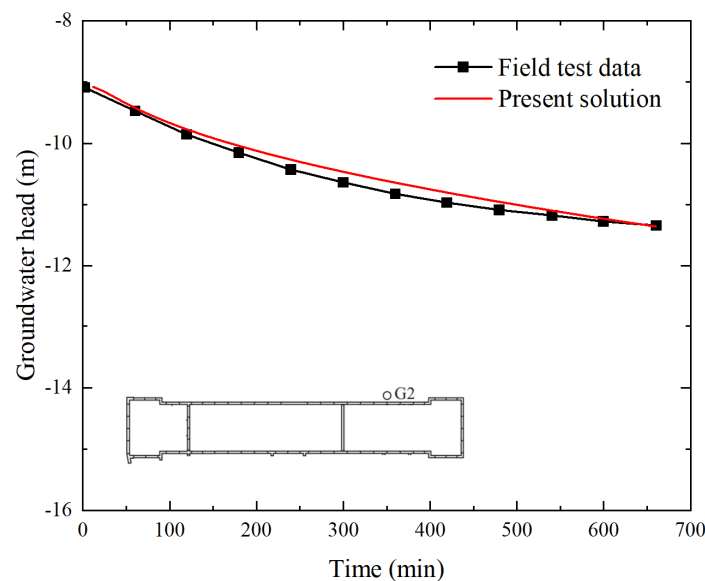


Figure 3. Groundwater head: Comparison between observed data and calculated results.

4. Results and Discussion

The drawdown characteristics inside and outside the pit are mainly investigated, and the adopted default parameters thereafter remain consistent with those employed in Section 3.1.

4.1. Effect of Curtain Insertion Depth

Figure 4 illustrates the drawdown behavior influenced by the width of the open interval B_a , ranging from 2 m to 20 m. A smaller B_a indicates a larger insertion depth of the cut-off curtain. The three subfigures in Figure 4 illustrate that variations in B_a significantly affect drawdown levels in Zone 1 and Zone 2. Specifically, Figure 4a shows that a smaller B_a leads to greater drawdown inside the curtain and reduced drawdown near the curtain outside the pit during pumping. Figure 4b demonstrates that a smaller B_a results in increased drawdown differences between the two sides of the curtain within the confined aquifer thickness. Furthermore, Figure 4c reveals that drawdown curves versus radial distance exhibit more pronounced discontinuities with greater insertion depths of the cut-off curtain. Therefore, engineers can effectively regulate groundwater levels outside the pit by adjusting the curtain insertion depth.

4.2. Effect of Well Screen Location

Figure 5 demonstrates the drawdown characteristics influenced by the well screen location, with a constant $B_a = 10$ m and $l - d = 8$ m. It can be observed from Figure 5a,b that the position of the well screen within the confined aquifer significantly affects the drawdown in Zone 1, while it has a negligible impact on the drawdown in the vicinity of the roof of the confined aquifer in Zone 2. A larger value of l signifies that the well screen is positioned closer to the top of the confined aquifer. This closer proximity results in a more pronounced drawdown in Zone 1. Consequently, the obvious difference in the hydraulic head on the two sides of the cut-off curtain can be observed, as illustrated in Figure 5c. Based on the analysis above, it is recommended that the optimal location for the well screen is as close as possible to the upper boundary of the confined aquifer when designing relief wells for dewatering deep excavations inside pits. Engineers can effectively control groundwater levels within the pit by adjusting the depth at which the well screen is installed.

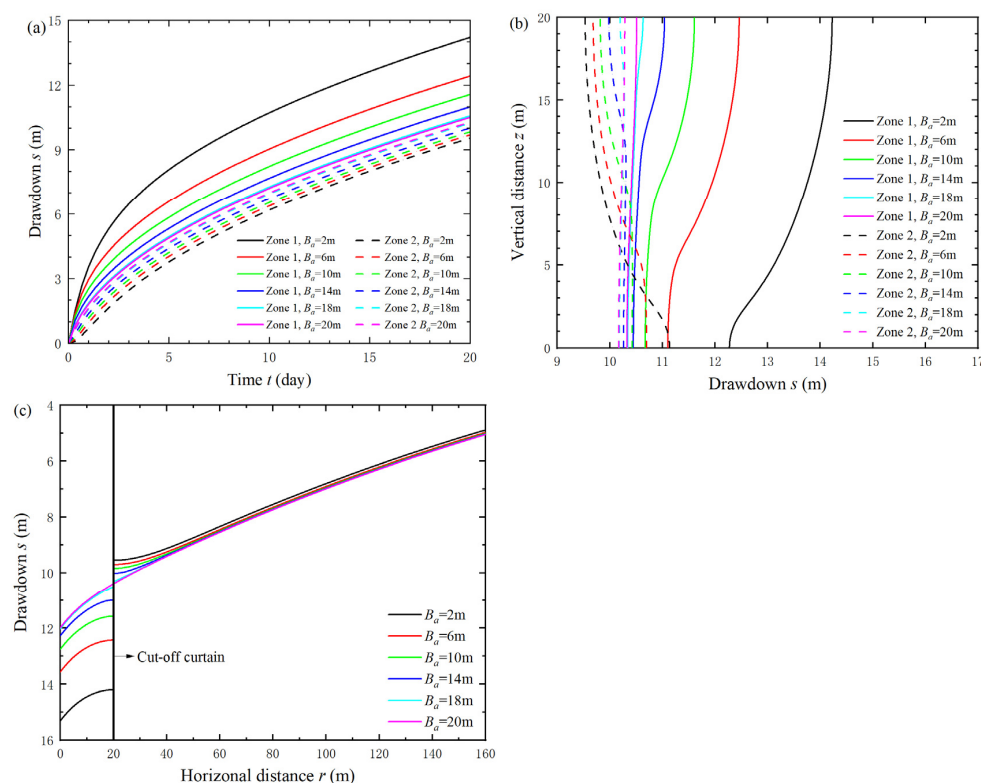


Figure 4. Curves of drawdown for various B_d (2 m, 6 m, 10 m, 14 m, 18 m, 20 m): (a) drawdown versus time at $z = 18$ m with $x = 18$ m for the Zone 1 and $x = 22$ m for Zone 2; (b) drawdown versus vertical distance at $t = 20$ d with $x = 18$ m for Zone 1 and $x = 22$ m for Zone 2; (c) drawdown versus horizontal distance at $t = 20$ d and $z = 18$ m.

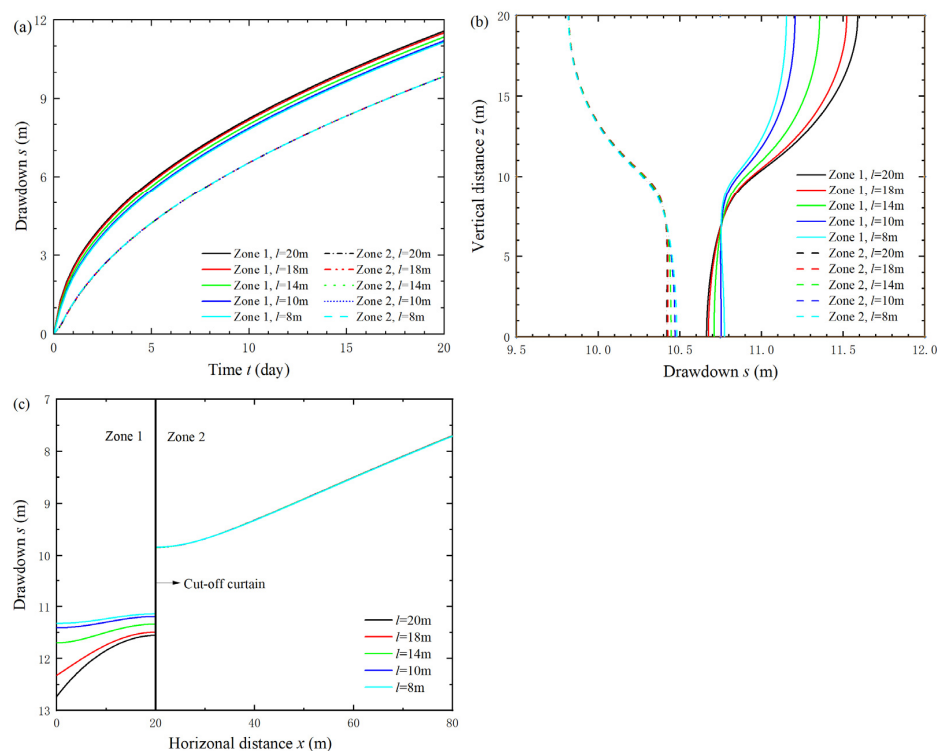


Figure 5. Curves of drawdown for various l (20 m, 18 m, 14 m, 10 m, 8 m) with $B_d = 10$ m: (a) drawdown versus time at $z = 18$ m with $x = 18$ m for Zone 1 and $x = 22$ m for Zone 2; (b) drawdown versus vertical distance at $t = 20$ d with $x = 18$ m for Zone 1 and $x = 22$ m for Zone 2; (c) drawdown versus horizontal distance at $t = 20$ d and $z = 18$ m.

4.3. Effect of Well Screen Length

Figure 6 demonstrates the drawdown responses in two zones influenced by different well screen lengths, with a fixed value of $l = 20$ m. Figure 6a,b shows that changes in both the length and location of the well screen have similar impacts on drawdown, particularly emphasizing a great effect on drawdown in Zone 1 compared to that in Zone 2, while the impact of the well screen length is relatively minor. A longer well screen leads to a smaller drawdown in Zone 1, indicating that it is a better choice to use a partially penetrating well in controlling the water level during dewatering design for excavation engineering. Figure 6c illustrates that variations in well screen length do not impact the horizontal distribution of drawdown outside the pit. However, there is an observable effect on the head difference on the two sides of the cut-off curtain. Specifically, increasing the length of the well screen results in a reduction in this head difference. Furthermore, for cases where $d = 15$ m and 10 m, implying that the well screen length $l - d \leq B_a = 10$ m, a more pronounced drawdown within the pit is evident when $l - d \leq B_a$ compared to situations when $l - d > B_a$. This observation suggests that enhanced dewatering effectiveness within the pit can be achieved when the length of the well screen does not exceed the depth of the cut-off curtain insertion.

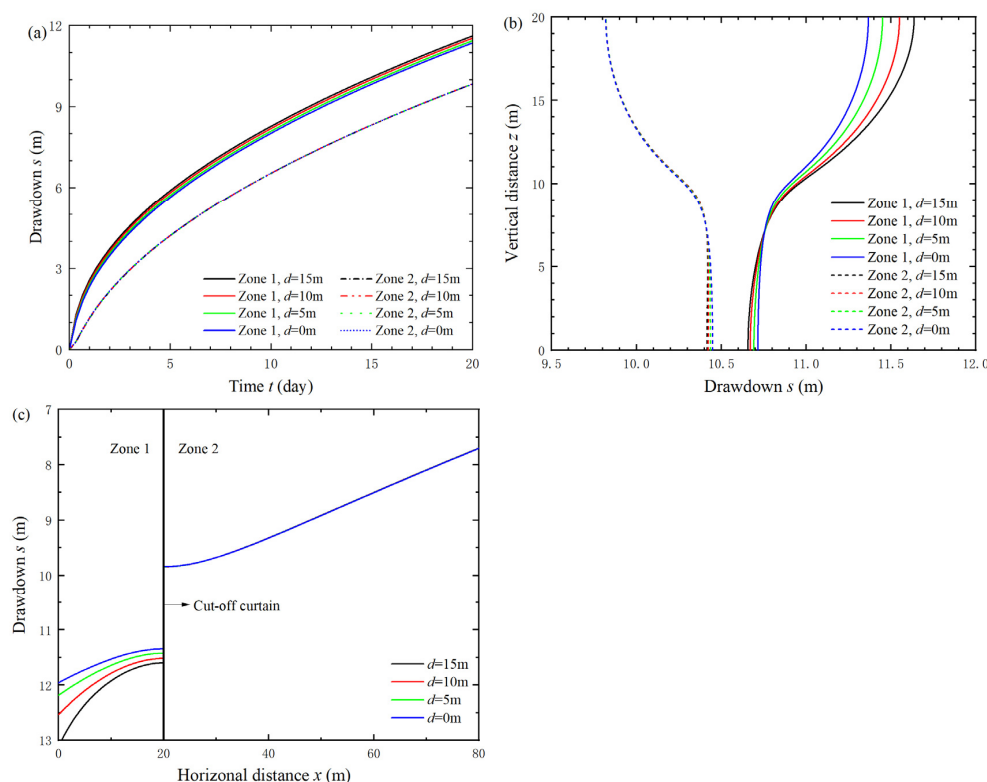


Figure 6. Curves of drawdown for various d (15 m, 10 m, 5 m, 0 m) with $B_a = l = 10$ m: (a) drawdown versus time at $z = 18$ m with $x = 18$ m for Zone 1 and $x = 22$ m for Zone 2; (b) drawdown versus vertical distance at $t = 20$ days with $x = 18$ m for Zone 1 and $x = 22$ m for Zone 2; (c) drawdown versus horizontal distance at $t = 20$ d and $z = 18$ m.

4.4. Sensitivity Analysis

Sensitivity analysis is commonly employed to evaluate the influencing degree of different hydrogeological parameters on computational outcomes [34,35]. This study adopts the sensitivity analysis method introduced by Huang and Yeh [36], which is defined as follows:

$$X_{i,j} = P_j \frac{\partial O_i}{\partial P_j} = P_j \frac{O_i(P_j + \Delta P_j) - O_i(P_j)}{\Delta P_j} \quad (14)$$

where $X_{i,j}$ refers to the normalized sensitivity coefficient (NSC) of parameter (P_j) at the j -th time step. The NSC value indicates the influence magnitude of the input parameter on the output quantity. O_i represents the dependent variable, which is the drawdown herein. ΔP_j denotes a minute increment, usually set at $10^{-2} \times P_j$. With the help of the obtained solution, one can carry out the sensitivity analysis including the following parameters: hydraulic conductivities K_x and K_z , specific storage S_s , well structure parameters l and d , open interval width B_a , and pumping discharge Q .

Figure 7 depicts the drawdown at $x = 18$ m, $z = 18$ m within the cut-off curtain to parameters B_a , K_x , K_z , S_s , B , l , d , x_0 , and Q . It can be observed that the drawdown within the cut-off curtain to the pumping rate Q and well structure parameters l and d exhibits a positive correlation, and other parameters demonstrate a negative correlation, suggesting that the change of these parameters results in smaller drawdown inside the pit. Figure 7 illustrates that the drawdown within the cut-off curtain is highly responsive to Q , K_x , S_s , and B during the entire pumping process (within 20 days), and their sensitivities gradually increase with the increased pumping time, with the largest NSC for Q . Moreover, the drawdown in the pit is sensitive to K_z , x_0 , and B_a and reaches a constant NSC at $t \geq 3$ d. Additionally, it is worth noting that the sensitivity of well structure parameters l , d to drawdown is primarily evident at the early stage of pumping and tends to stabilize as pumping progresses. Compared to the other parameters, the sensitivity of drawdown to l and d appears relatively weak, indicating that the impact of well structure parameters on the drawdown in the pit is not significant.

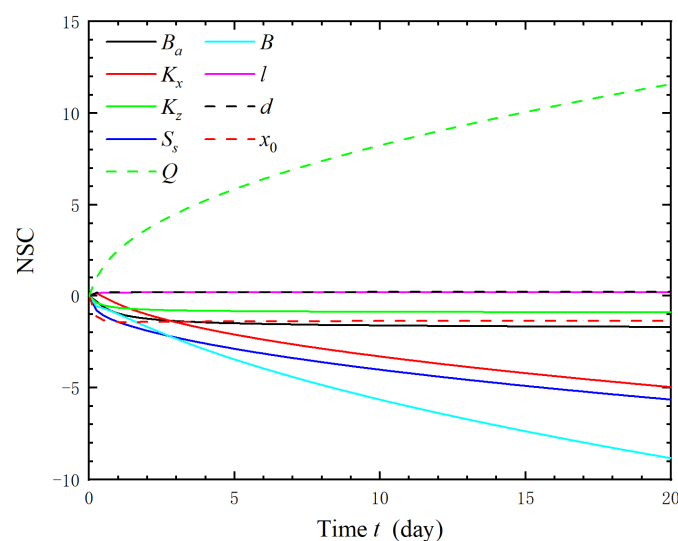


Figure 7. The NSC analysis of drawdown for the parameters B_a , K_x , K_z , S_s , B , l , d , x_0 , and Q inside the cut-off curtain at $x = 18$ m, $z = 18$ m.

Figure 8 illustrates the curves of the NSC for drawdown in Zone 2 to the change in parameter B_a , K_x , K_z , S_s , B , l , d , x_0 , and Q observed at $z = 18$ m, $x = 22$ m near the cut-off curtain for Figure 8a and $x = 40$ m away from the curtain for Figure 8b. It can be found that the pumping rate Q , vertical hydraulic conductivity K_z , and the width of the open interval B_a exhibit a positive influence on the drawdown outside the curtain, suggesting that the increase in these parameters results in a larger drawdown outside the pit. However, it is important to note that K_z has a slight influence on the drawdown, while the other parameters demonstrate a negative influence. Both Figure 8a,b demonstrate that the drawdown at any location outside the curtain is very sensitive to four parameters, Q , B , S_s , and K_x ; the observation is the same as that for the drawdown in Zone 1. An important observation is that the width of the open interval B_a produces a positive influence on drawdown in Zone 2 and a negligible influence on drawdown in Zone 1. This suggests that

a greater curtain insertion depth (i.e., a smaller B_a) results in a reduced drawdown outside the pit and improves the blocking effect of the curtain. Furthermore, one can observe from Figure 8a that the drawdown near the cut-off curtain outside the pit is sensitive to K_z , x_0 , and B_a , while the change of these parameters has negligible influence on drawdown far away from the cut-off curtain, as displayed in Figure 8b. Finally, it can be found from Figure 8 that the change of well structure parameters l and d has no influence on drawdown outside the pit.

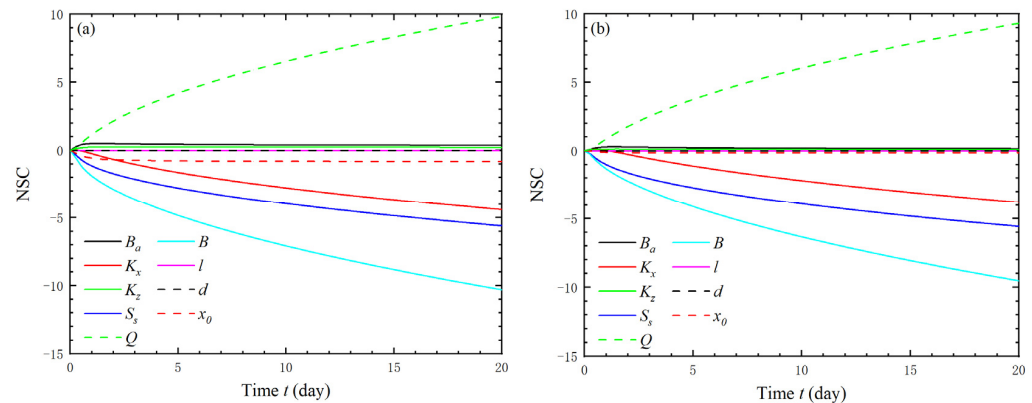


Figure 8. The NSC of drawdown outside the cut-off curtain for the parameters B_a , K_x , K_z , S_s , B , l , d , x_0 , and Q (a) at $x = 22$ m, $z = 18$ m; (b) at $x = 40$ m, $z = 18$ m.

4.5. Limitations of the Present Solution

The limitations of this study are addressed herein to enhance the application of the proposed solutions. Firstly, the aquifer system is assumed to have infinite horizontal extent, a common assumption in well hydraulic literature. However, if a river or fault is near the long, narrow foundation pit, the outer boundary is typically treated under constant head or no-flow conditions. In such cases, the developed solution may not accurately represent the flow behavior in the confined aquifer. Secondly, the slope of the confined aquifer has not been considered. Thirdly, this study focuses on investigating the two-dimensional flow response in a long, narrow foundation pit; thus, the current solution cannot be applied to other geometries exhibiting significant three-dimensional flow characteristics. Fourthly, certain general limitations of the physical problem, such as uncertainties in boundary conditions and hydraulic conductivity [37,38], are not taken into account. Additionally, the non-Darcy effect of flow should be taken into account in the future. Finally, the developed solutions cannot be used to investigate groundwater flow in a heterogeneous aquifer or within an irregular spatial domain.

5. Conclusions

The purpose of this paper is to explore the transient two-dimensional flow problem in a confined aquifer with a partial penetration cut-off curtain. An analytical model is developed by accounting for the effects of aquifer anisotropy and well partial penetration. With the help of integral transformation methods, a semi-analytical solution for the model is derived. Based on the obtained solution, the unsteady-state behavior of drawdown inside and outside the cut-off curtain is then investigated. Subsequently, sensitivity analyses are carried out to determine the controlling factors, resulting in the following findings:

- (1) The obtained solution is verified by making comparisons with the numerical solution using the finite element method and field pumping test data and shows its correctness and engineering applicability.

- (2) The influence of the cut-off curtain on drawdown within the pit is significant compared with that outside the pit, and the drawdown difference on two sides of the curtain increases as the insertion depth of the cut-off curtain increases.
- (3) To achieve better dewatering effects inside the pit, it is advisable to place the well screen near the top of the confined aquifer and maintain a screen length that is equal to or less than the insertion depth of the cut-off curtain.
- (4) Regardless of whether the observation is made inside or outside the curtain, the drawdown is very sensitive to changes in parameters Q , B , S_s , and K_x , and drawdowns inside the pit and near the cut-off curtain outside the pit are sensitive to K_z , x_0 , and B_a .
- (5) The parameter of the open interval in the aquifer produces a positive influence on the drawdown outside the pit and has a negative effect on the drawdown inside the pit. Additionally, the well structure parameters l and d have no impact on drawdown outside the cut-off curtain.

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Data Availability Statement: The original contributions presented in this study are included in the article. Further inquiries can be directed to the corresponding author.

Conflicts of Interest: The authors declare no conflict of interest.

Appendix A

Applying the Laplace transformation Equations (1)–(10) yields:

$$K_x \frac{\partial^2 \bar{s}_1(x, z, p)}{\partial x^2} + K_z \frac{\partial^2 \bar{s}_1(x, z, p)}{\partial z^2} = S_s p \bar{s}_1(x, z, p), 0 \leq x \leq x_0, 0 \leq z \leq B \quad (\text{A1})$$

$$\lim_{x \rightarrow 0} \frac{\partial \bar{s}_1(x, z, p)}{\partial x} = \begin{cases} 0, & 0 \leq z \leq d \\ \frac{-Q}{2K_x(l-d)}, & d \leq z \leq l \\ 0, & l \leq z \leq B \end{cases} \quad (\text{A2})$$

$$\frac{\partial \bar{s}_1(x, 0, p)}{\partial z} = \frac{\partial \bar{s}_1(x, B, p)}{\partial z} = 0 \quad (\text{A3})$$

$$K_x \frac{\partial \bar{s}_1(x_0, z, p)}{\partial x} = \begin{cases} \bar{q}(z, p), & 0 \leq z \leq B_a \\ 0, & B_a \leq z \leq B \end{cases} \quad (\text{A4})$$

$$K_x \frac{\partial^2 \bar{s}_2(x, z, p)}{\partial x^2} + K_z \frac{\partial^2 \bar{s}_2(x, z, p)}{\partial z^2} = S_s p \bar{s}_2(x, z, p), x_0 \leq x \leq \infty, 0 \leq z \leq B \quad (\text{A5})$$

$$\bar{s}_2(\infty, z, p) = 0 \quad (\text{A6})$$

$$\frac{\partial \bar{s}_2(x, 0, p)}{\partial z} = \frac{\partial \bar{s}_2(x, B, p)}{\partial z} = 0 \quad (\text{A7})$$

$$K_x \frac{\partial \bar{s}_2(x_0, z, p)}{\partial x} = \begin{cases} \bar{q}(z, p), & 0 \leq z \leq B_a \\ 0, & B_a \leq z \leq B \end{cases} \quad (\text{A8})$$

$$\bar{s}_1(x_0, z, p) = \bar{s}_2(x_0, z, p), 0 \leq z \leq B_a \quad (\text{A9})$$

Then, taking the finite Fourier cosine transformation to Equations (A1)–(A8) gives

$$\frac{\partial^2 \bar{s}_{1c}(x, n, p)}{\partial x^2} - \frac{1}{K_x} \left[\left(\frac{n\pi}{B} \right)^2 K_z + S_s p \right] \bar{s}_{1c}(x, n, p) = 0, \quad 0 \leq x \leq x_0 \quad (\text{A10})$$

$$\lim_{x \rightarrow 0} \frac{\partial \bar{s}_{1c}(x, n, p)}{\partial x} = \frac{-Q}{2K_x(l-d)} \delta, \quad d \leq z \leq l \quad (\text{A11})$$

$$K_x \frac{\partial \bar{s}_{1c}(x_0, n, p)}{\partial x} = \int_0^{B_a} \bar{q}(z, p) \cos\left(\frac{n\pi z}{B}\right) dz \quad (\text{A12})$$

$$\frac{\partial^2 \bar{s}_{2c}(x, n, p)}{\partial x^2} - \frac{1}{K_x} \left[\left(\frac{n\pi}{B} \right)^2 K_z + S_s p \right] \bar{s}_{2c}(x, n, p) = 0, \quad x_0 \leq x \leq \infty \quad (\text{A13})$$

$$\bar{s}_{2c}(\infty, n, p) = 0 \quad (\text{A14})$$

$$K_x \frac{\partial \bar{s}_{2c}(x_0, n, p)}{\partial x} = \int_0^{B_a} \bar{q}(z, p) \cos\left(\frac{n\pi z}{B}\right) dz \quad (\text{A15})$$

The general solution for Equations (A10) and (A13) are given by

$$\bar{s}_{1c} = A_1 e^{-\eta x} + B_1 e^{\eta x} \quad 0 < x \leq x_0 \quad (\text{A16})$$

$$\bar{s}_{2c} = A_2 e^{-\eta x} + B_2 e^{\eta x} \quad x_0 \leq x < \infty \quad (\text{A17})$$

Substituting Equations (A16) and (A17) into the boundary condition in Equations (A11), (A12), (A14) and (A15), A_1 , B_1 , A_2 , and B_2 can be obtained as

$$\begin{cases} A_1 = \left[\frac{Q}{2K_x(l-d)p} \delta e^{\eta x_0} + \frac{1}{K_x} \int_0^{B_a} \bar{q}(z, p) \cos\left(\frac{n\pi z}{B}\right) dz \right] / [2\eta \sinh(\eta x_0)] \\ A_2 = \int_0^{B_a} \bar{q}(z, p) \cos\left(\frac{n\pi z}{B}\right) dz / (-K_x \eta e^{-\eta x_0}) \\ B_1 = \left[\frac{Q}{2K_x(l-d)p} \delta e^{-\eta x_0} + \frac{1}{K_x} \int_0^{B_a} \bar{q}(z, p) \cos\left(\frac{n\pi z}{B}\right) dz \right] / [2\eta \sinh(\eta x_0)] \\ B_2 = 0 \end{cases} \quad (\text{A18})$$

where $\eta = \{[(n\pi/B)^2 K_z + pS_s]/K_x\}^{1/2}$, $\delta = B[\sin(n\pi l/B) - \sin(n\pi d/B)]/(n\pi l/B)$.

Substitution of Equation (A18) into Equation (A16) and Equation (A17), respectively, and applying the inversion of the finite cos-Fourier transform leads to

$$\bar{s}_1(x, z, p) = \frac{2}{B} \sum_{n=0}^{\infty} \left\{ \frac{\cosh[\eta(x_0 - x)]}{\eta \sinh(\eta x_0)} \frac{Q}{2K_x(l-d)p} \delta + \frac{\cosh(\eta x)}{\eta \sinh(\eta x_0)} \frac{1}{K_x} \int_0^{B_a} \bar{q}(z, p) \cos\left(\frac{n\pi z}{B}\right) dz \right\} \cos\left(\frac{n\pi z}{B}\right) \quad (\text{A19})$$

$$\bar{s}_2(x, z, p) = \frac{2}{B} \sum_{n=0}^{\infty} \left\{ \frac{e^{\eta(x_0 - x)}}{-\eta} \frac{1}{K_x} \int_0^{B_a} \bar{q}(z, p) \cos\left(\frac{n\pi z}{B}\right) dz \right\} \cos\left(\frac{n\pi z}{B}\right) \quad (\text{A20})$$

Notably, the term for $n = 0$ in Equations (A19) and (A20) is multiplied by 1/2. The unknown flux $\bar{q}(z, p)$ shown in Equations (A19) and (A20) can be resolved by Equation (A9) with the aid of the discretization approach introduced by Chang and Chen [39] and Feng and Lin [28]. Specifically, the open interval B_a is put into M segments with the length of $\Delta \xi_j$ and uniform flux $\bar{q}_j$ for segment j . Replacing the integral in Equations (A19) and (A20) with summation results in:

$$\bar{s}_1(x, z, p) = \frac{2}{B} \sum_{n=0}^{\infty} \left\{ \frac{\cosh[\eta(x_0 - x)]}{\eta \sinh[\eta x_0]} \frac{Q}{2K_x(l-d)p} \delta + \frac{\cosh(\eta x)}{\eta \sinh[\eta x_0]} \frac{1}{K_x} \sum_{j=1}^M \bar{q}_j(p) F_j \right\} \cos\left(\frac{n\pi z}{B}\right) \quad (\text{A21})$$

$$\bar{s}_2(x, z, p) = \frac{2}{B} \sum_{n=0}^{\infty} \left\{ \frac{e^{\eta(x_0 - x)}}{-\eta} \frac{1}{K_x} \sum_{j=1}^M \bar{q}_j(p) F_j \right\} \cos\left(\frac{n\pi z}{B}\right) \quad (\text{A22})$$

in which

$$F_j = \frac{2B}{n\pi} \cos\left(\frac{n\pi \bar{\xi}_j}{B}\right) \sin\left(\frac{n\pi \Delta \xi_j}{2B}\right) \quad (\text{A23})$$

With the assumption of the average head continuity for each segment at $0 \leq z \leq B_a$, one can rewrite the boundary condition Equation (A9) as

$$\frac{1}{\bar{\xi}_{i+} - \bar{\xi}_{i-}} \int_{\bar{\xi}_{i-}}^{\bar{\xi}_{i+}} \bar{s}_1(x_0, z, p) dz = \frac{1}{\bar{\xi}_{i+} - \bar{\xi}_{i-}} \int_{\bar{\xi}_{i-}}^{\bar{\xi}_{i+}} \bar{s}_2(x_0, z, p) dz \quad (\text{A24})$$

where $\bar{\xi}_{j+} = \bar{\xi}_j + \Delta \bar{\xi}_j/2$, $\bar{\xi}_{j-} = \bar{\xi}_j - \Delta \bar{\xi}_j/2$ and $\Delta \bar{\xi}_j = \bar{\xi}_{j+} - \bar{\xi}_{j-}$.

Substituting Equations (A21) and (A22) into Equation (A24) leads to

$$\sum_{i=1}^M \bar{q}_j(p) X_{ij} = Y_i \quad (\text{A25})$$

where

$$X_{ij} = \frac{1}{K_x} \left[\frac{2}{B} \sum_{n=0}^{\infty} \frac{\cosh(\eta x_0)}{\eta \sinh(\eta x_0)} F_j F_i + \frac{2}{B} \sum_{n=0}^{\infty} \frac{1}{\eta} F_i F_j \right] \quad (\text{A26})$$

$$Y_i = -\frac{2}{B} \sum_{n=0}^{\infty} \left[\frac{1}{\eta \sinh(\eta x_0)} \frac{Q}{2K_x(l-d)p} \delta \right] F_i \quad (\text{A27})$$

$$F_i = \frac{2B}{n\pi} \cos\left(\frac{n\pi \bar{\xi}_i}{B}\right) \sin\left(\frac{n\pi \Delta \xi_i}{2B}\right) \quad (\text{A28})$$

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