

Article

Dynamic Analysis of Submarine-Buried Pipelines Considering Fluid–Structure Interaction

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Abstract: Buried pipes are widely used for submarine water transportation, but the complex operating conditions in the seabed pose challenges for the modeling of buried pipes. In order to more accurately capture the dynamic behavior of the buried pipes in the seabed, in this study, considering the pipeline and soil as a systematic structure is proposed, improving the fluid–structure interaction four-equation model to make it applicable for the calculation of buried pipe system modes. After verifying the practicality of the model, considering the external seawater as uniform pressure, the coupling at the joints, and the Poisson coupling of submarine pipelines during transient processes are discussed, revealing that structural vibrations under both forms of coupling will cause greater hydraulic oscillations. The impact of soil elastic modulus on the system’s response is further discussed, revealing that increasing the modulus from 0 to 10^{15} Pa raises the wave speed from 498 m/s to 1483 m/s, causing a 40% increase in the amplitude of pressure oscillations. Finally, the vibration modes of the combined structure of pipe wall and soil are discussed, revealing that the vibration modes are mainly dominated by water hammer pressure, with the superposition of pipeline stress waves and soil stress waves. In this study, the dynamic behavior of submarine pipelines is elucidated, providing a robust foundation for regulating and mitigating fatigue failures in such systems.



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Keywords: fluid–structure interaction; pipe flow; buried pipeline; water hammer

1. Introduction

Submarine pipelines are an important ocean engineering technology spanning multiple fields, including marine engineering, material science, and mechanical engineering. This technology involves the transportation of substances such as oil and gas, water, and electricity to the deep sea or overseas through submarine pipelines, achieving the effective development and utilization of resources [1]. Due to the difficulty in directly measuring the vibration characteristics of submarine pipelines [2,3], theoretical studies on submarine pipelines have become a prominent research focus in recent years. When the soil and the pipe wall are considered as an integral structure, and the structure is subjected to multiple excitations from inside and outside, pressure pulsations will occur, which can induce vibrations in the pipeline structure and the fluid within the pipe. These vibrations of the pipeline structure and the fluid couple with each other, resulting in fluid–structure interaction (FSI) vibrations.

Different supports in water transport systems lead to different FSIs, so FSI response analysis must be considered on a case-by-case basis according to the different boundary conditions [4]. Regarding the constraint problem in water transport pipelines, Li et al. [5] proposed a comprehensive model of fluid boundary conditions, simplified into fluid impedance and elastic supports with different stiffnesses and positioned in different directions. Li et al. [6] used six models to analyze the elastic support conditions and found that the elastic six-spring model simulates complex boundary conditions, extending the application of this method. Rooholla et al. [7] analyzed the FSI phenomenon under four axial support conditions in a typical case study. They concluded that suitable support significantly reduces displacement and stress in the pipeline. Ahmadi et al. [8] extended the application of this method. This indicates that most complex boundary conditions in water transport pipelines can be simplified into support conditions.

Different support stiffnesses significantly impact the FSI response when complex boundary conditions are simplified into support conditions in different directions. Heinsbroek et al. [9] studied the dynamic characteristics of fluid pressure and pipe wall stress in pipeline systems with different support stiffnesses. They found that in pipeline water transport systems, the fewer the number of the supports or the smaller the support stiffness, the stronger the system's FSI response. In addition, the higher the number of pipeline supports or the more significant the support stiffness, the weaker the system's FSI response, and the closer it approached that predicted in classical water hammer theory. Based on experimental analysis, Adam et al. [10] found that a greater support stiffness can enhance the damping effect, but it will produce high-frequency pressure fluctuations. Tijsseling et al. [11] detailed seven pipeline structures studied at the University of Dundee over 20 years, including a straight pipe with simple support, providing high-quality experimental data for strong FSI responses.

Although there have been significant achievements in FSI research in ocean engineering, some challenges remain. The complex operating conditions of the seabed environment lead to higher requirements for models used to predict the dynamic behavior of submarine-buried pipelines. Numerous scholars have conducted FSI response analyses on water transmission via pipelines with different layouts, but they only considered the external constraints of the pipeline as discrete forms, with the influences of these constraints on the FSI response treated as boundary conditions. Simplifying continuous support into boundary conditions for each unit indeed introduces approximations that may lead to iteration errors. When the system's continuous constraints cannot be treated as boundary conditions, existing studies often refer to multi-span pipeline models [12–15] and elastic constraint models to derive a mathematical model, and thus the accuracy of the numerical calculation results cannot be guaranteed. Buried water pipelines in elastic soil represent a typical example of continuous constraints.

However, few research studies have considered coupling external continuous boundary conditions in order to improve buried-pipeline models. To expand the application of FSIs, it is necessary to consider the internal and external coupling of the pipe wall and derive a refined theoretical model to obtain accurate data on the actual dynamic behavior of buried pipelines. In this study, pipeline–elastic-soil coupling is introduced and a continuous constraint is derived from an FSI mathematical model of a water transmission system. The proposed model more accurately captures the dynamic behavior of buried pipelines under fluid flow, simulating their vibration characteristics, stress distribution, and deformation. This approach facilitates a deeper understanding of the dynamic responses of submarine pipelines within complex marine environments. Moreover, the findings provide critical insights into mitigating vibration amplitudes and reducing fatigue-induced damage, thereby enhancing the longevity and operational safety of the pipelines. In this

study, a scientific foundation is established for the design, operation, and maintenance of submarine pipeline systems. Through the integration of numerical simulations and experimental validations, the dynamic response characteristics of pipelines under diverse operational conditions are thoroughly analyzed, delivering precise and reliable data to inform and optimize pipeline design.

2. Method

2.1. Mathematical Model

By exploring the flow state of fluids within pipes and its impact on the pipe structure, the essence of FSI effects can be revealed. To uncover the system model of submarine pipelines considering FSI, it is necessary to consider the soil and pipe wall as an integral structure to derive a four-equation model suitable for buried pipes [16]. This includes the fluid motion equation, fluid continuity equation, structural constitutive equation, and structural motion equation.

In this study, we assume that the sea water pressure is a fixed value, the seabed soil is elastic and homogeneous, and the model is simplified to represent the entire pipeline as being subjected to a uniform elastic force from the soil around it, as shown in Figure 1. The axial FSI of the pipeline water delivery system can be analyzed separately, and a mathematical model for calculating FSI in buried pipelines can be derived [17,18].

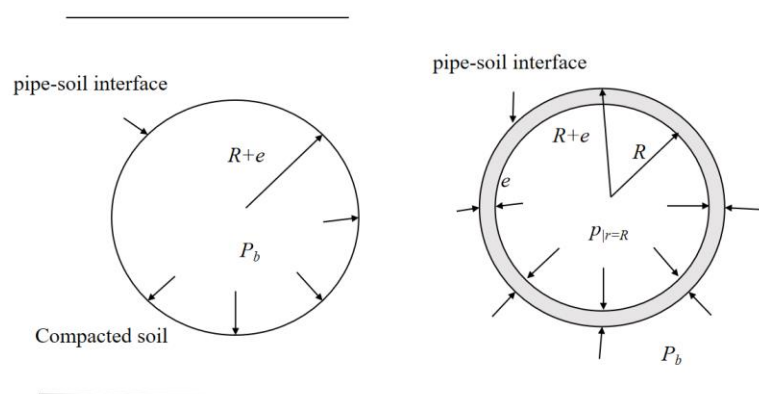


Figure 1. Simplified diagram of a pipeline in surrounding soil.

In the case where the pipeline is subjected to a uniform compressive stress from the surrounding soil, it can be assumed that its stress is only a function of the radial coordinate r since the pipeline has an axisymmetric structure [19]. In this special case, the stress components can be simplified as follows:

$$\sigma_r = \frac{1}{r} \frac{d\varphi}{dr}, \quad \sigma_\theta = \frac{d^2\varphi}{dr^2}, \quad \tau_{r\theta} = \tau_{\theta r} = 0 \quad (1)$$

where σ_r and σ_θ are the radial and circumferential stresses of the pipe wall, respectively; φ is the flux through the cross-section; and $\tau_{r\theta}$ and $\tau_{\theta r}$ are shear stresses which are both zero. Thus, the force along the circumference of the pipe is uniform. From Equation (1), the compatibility equation can be simplified as follows:

$$\left(\frac{d^2}{dr^2} + \frac{1}{r} \frac{d}{dr}\right)^2 \varphi = 0 \quad (2)$$

Solving Equation (2), the general solution form of the compatible equation is as follows:

$$\varphi = A \ln r + B r^2 \ln r + C r^2 + D \quad (3)$$

where A , B , C , and D are coefficients and are only related to the pressures acting on the inside and outside of the pipe and the inherent properties of the pipe itself.

By substituting the general solution Equation (3) into Equation (1), the equations for the radial and circumferential stresses are obtained as follows:

$$\sigma_r = \frac{A}{r^2} + B(1 + 2\ln r) + C \quad (4)$$

$$\sigma_\theta = -\frac{A}{r^2} + B(3 + 2\ln r) + C \quad (5)$$

When the pipe wall is subjected to symmetric stress, the circumferential displacement component is as follows [20]:

$$u_r = \frac{4Br\theta}{E} + Hr - I\sin\theta + K\sin\theta \quad (6)$$

in which $4Br\theta/E$ is a multivalued term and (r, θ) and $(r, \theta + 2\pi)$ represent the same point in the pipe wall; hence, the circumferential displacement should be equal. Therefore, it must be the case that $B = 0$. According to the boundary conditions for the force on an infinitesimal pipe section, the following is true [21]:

$$\sigma_r|_{r=R} = P, \quad \sigma_r|_{r=R+e} = -P_b \quad (7)$$

where P is the internal pressure of the pipe, and P_b is the external pressure of the pipe. Substituting $B = 0$ and Equation (7) into the circumferential ring term expression, we obtain the following:

$$A = \frac{R^2(R+e)^2(P-P_b)}{(R+e)^2 - R^2}, \quad C = \frac{R^2P - (R+e)^2P_b}{(R+e)^2 - R^2} \quad (8)$$

We substitute A and C into the equation to obtain the Lamé solution form for the stress in the buried pipe wall [22]:

$$\sigma_r = \frac{\frac{(R+e)^2}{r^2} - 1}{\frac{(R+e)^2}{R^2} - 1} P - \frac{1 - \frac{R^2}{r^2}}{1 - \frac{R^2}{(R+e)^2}} P_b \quad (9)$$

$$\sigma_\theta = \frac{\frac{(R+e)^2}{r^2} + 1}{\frac{(R+e)^2}{R^2} - 1} P - \frac{1 + \frac{R^2}{r^2}}{1 - \frac{R^2}{(R+e)^2}} P_b \quad (10)$$

where R is the pipe's internal diameter, and e is the thickness of the pipe wall.

According to Hooke's Law, the relationship between circumferential strain and stress in a pipe is as follows:

$$\frac{w_r}{R} = \frac{1}{E} [\sigma_\theta|_{r=R+e} - \nu(\sigma_r|_{r=R+e} + \sigma_z)] \quad (11)$$

where w_r is the radial displacement of the pipe; ν is the Poisson ratio; and σ_z is the axial stress. Assuming that the soil is homogeneous and purely elastic, the external pressure on the water pipe wall is proportional to the soil strain ε . This means that the following is true:

$$P_b = E_s \times \varepsilon \quad (12)$$

where E_s is the elastic modulus of the soil. Substituting Equation (12) into Equation (11), we can obtain the following:

$$\frac{P_b}{E_s} = \frac{R}{Ee}(P - P_b) - \frac{\nu}{E}(-P_b + \sigma_z) \quad (13)$$

The water supply pipeline and the axial stress of the pipe can represent the pressure on the outer wall of the pipe, as follows:

$$P_b = \frac{R}{Ee/E_s + R + \nu e}P - \frac{\nu e}{Ee/E_s + R + \nu e}\sigma_z \quad (14)$$

where, for ease of calculation, Equation (14) can be simplified as follows:

$$P_b = aP + b\sigma_z \quad (15)$$

where $a = \frac{R}{Ee/E_s + R + \nu e}$ and $b = -\frac{\nu e}{Ee/E_s + R + \nu e}$.

2.1.1. Balanced Force Equation for a Fluid Element

The forces acting on a differential element of a straight pipe section are shown in Figure 2 [23].

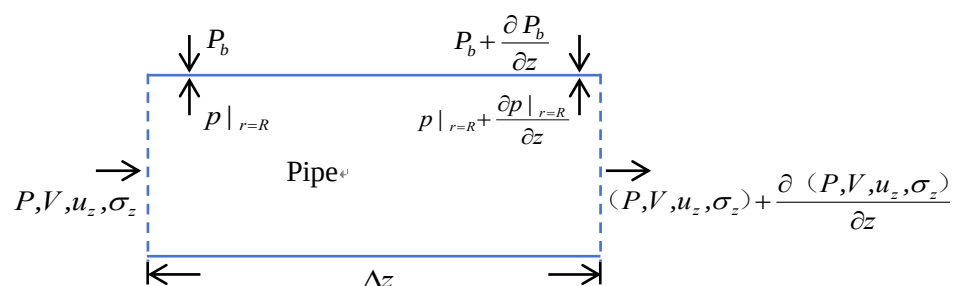


Figure 2. System differential force end model for a straight pipeline section.

According to Newton's second law, the balance equation for a fluid element is as follows:

$$\rho_f A_f dz \frac{\partial V}{\partial t} = \left[P - \left(P + \frac{\partial P}{\partial z} dz \right) \right] A_f \quad (16)$$

where ρ_f is the fluid density; A_f is the internal area of the pipe; z is the axial distance along the pipe; V is the average cross-sectional flow velocity; and t represents the time.

The axial motion equation for fluids is as follows:

$$\frac{\partial V}{\partial t} + \frac{1}{\rho_f} \frac{\partial P}{\partial z} = 0 \quad (17)$$

2.1.2. Balanced Force Equation for a Structural Element

According to Figure 2, the axial force balance equation is as follows:

$$\rho_t A_t dz \frac{\partial u_z}{\partial t} = A_t \left[\left(\frac{\partial \sigma_z}{\partial z} dz + \sigma_z \right) - \sigma_z \right] - \mu 2\pi R P_b dz \quad (18)$$

We can simplify this as follows:

$$\rho_t e \frac{\partial u_z}{\partial t} - e \frac{\partial \sigma_z}{\partial z} = -\mu P_b \quad (19)$$

And substitute Equation (14) into Equation (19), as follows:

$$\rho_t e \frac{\partial u_z}{\partial t} - e \frac{\partial \sigma_z}{\partial z} = -\mu a P - \mu b \sigma_z \quad (20)$$

where u_z is the average value of the axial vibration velocity at a certain cross-section of the pipe, and the axial force acting on the pipe from the soil body is the frictional force $f = \mu P_b A_t$. μ is the dry friction coefficient between the pipe and the soil body. When the pipe is relatively stationary concerning the soil body, μ is the static friction coefficient; when the pipe moves relative to the soil body, μ is the dynamic friction coefficient [17].

2.1.3. Equation of Pipe Wall Geometry

According to Hooke's law, the expression for the relationship between axial strain and stress in the pipe is as follows:

$$\varepsilon_z = \frac{1}{E} [\sigma_z - \nu(\sigma_r + \sigma_\theta)] \quad (21)$$

where the axial strain $\varepsilon_z = \partial w_z / \partial z$, z is the distance of the axial element and w_z is the change in the distance of the axial element of the pipe, where $u_z = \frac{dw_z}{dt}$; when $e \ll R$, the equation is simplified as follows:

$$\bar{\sigma}_r + \bar{\sigma}_\theta = \frac{1}{2\pi(R + e/2)e} \int_R^{R+e} 2\pi r(\sigma_r + \sigma_\theta) dr = \frac{R}{e} (P - P_b) \quad (22)$$

$$\frac{\partial u_z}{\partial z} - \frac{1}{E} \frac{\partial \sigma_z}{\partial t} + \frac{\nu R}{Ee} \frac{\partial (P - P_b)}{\partial t} = 0 \quad (23)$$

$$\frac{\partial u_z}{\partial z} - \left(\frac{1}{E} + \frac{\nu Rb}{Ee} \right) \frac{\partial \sigma_z}{\partial t} + \frac{\nu R(1-a)}{Ee} \frac{\partial P}{\partial t} = 0 \quad (24)$$

2.1.4. Continuity Equation for Incompressible Fluids

Ignoring the circumferential vibration velocity of the pipe, the continuity equation for symmetrical flow is as follows:

$$\frac{\partial \rho_f}{\partial t} + \frac{1}{r} \frac{\partial}{\partial r} (\rho_f V_r r) + \frac{\partial}{\partial z} (\rho_f V) = 0 \quad (25)$$

where V_r is the fluid's radial flow velocity, which is equal to the radial vibration velocity u_r of the pipe. According to the definition of wave velocity, $\sqrt{\partial P / \partial A_f} = \sqrt{K / \rho_f}$, where K is the bulk modulus of the fluid, Equation (25) can be represented as follows:

$$\frac{1}{K} \frac{\partial P_a}{\partial t} + \frac{\rho_f}{r} \frac{\partial (V_r r)}{\partial r} + \frac{\partial V}{\partial z} = 0 \quad (26)$$

In [5], a one-dimensional flow continuity equation was obtained by taking the average of the cross-section of the two-dimensional flow, as follows:

$$\frac{1}{\pi R^2} \int_0^R 2\pi r \left(\frac{1}{K} \frac{\partial P_a}{\partial t} + \frac{\rho_f}{r} \frac{\partial (V_r r)}{\partial r} + \frac{\partial V}{\partial z} \right) dr = 0 \quad (27)$$

We simplify Equation (27) as follows:

$$\frac{1}{K} \frac{\partial P}{\partial t} + \frac{2V_r}{R} + \frac{\partial V}{\partial z} = 0 \quad (28)$$

The radial flow velocity at the junction of the pipe and the fluid is equal to the coupled radial vibration velocity of the pipe. Hence, the following equation is true:

$$\frac{V_r}{R} = \frac{u_r}{R} = \frac{1}{E} \frac{\partial [\sigma_\theta|_{r=R} - \nu(\sigma_z|_{r=R} + \sigma_r|_{r=R})]}{\partial t} \quad (29)$$

We substitute Equation (25) into Equation (24), as follows:

$$\frac{\partial V}{\partial z} + \left[\frac{1}{K} + \frac{2R}{Ee} \right] \frac{\partial P}{\partial t} - \frac{2R}{Ee} \frac{\partial P_b}{\partial t} - \frac{2\nu}{E} \frac{\partial \sigma_z}{\partial t} = 0 \quad (30)$$

And substitute Equation (15) into Equation (30), as follows:

$$\frac{\partial V}{\partial z} + \left[\frac{1}{K} + \frac{2R(1-a)}{Ee} \right] \frac{\partial P}{\partial t} - \left(\frac{2Rb}{Ee} + \frac{2\nu}{E} \right) \frac{\partial \sigma_z}{\partial t} = 0 \quad (31)$$

Considering the pipe wall and soil as an integrated structure, the fluid motion equation, fluid continuity equation, structural motion equation, and structural constitutive equation were derived, as shown in Equation (17), Equation (20), Equation (24), and Equation (31), respectively. An FSI four-equation model suitable for submarine pipelines is established.

2.2. Numerical Solution

2.2.1. Finite Volume Method

In this study, we considered the continuous constraint pressure of the soil body on the water delivery system to be variable [24]. The pressure on the outer wall of the pipe can be linearly represented by the internal pressure of the pipe and the pipe stress, meaning that the buried-pipe FSI model can be derived based on the open-channel FSI model. When solving this problem using the finite volume method (FVM), each term in the integral equation has a clear physical meaning since the integral solution is performed for each control volume. In this study, we adopted the FVM to calculate FSIs in a pipe. In the FVM, the control volume is integrated to discretize the equations, and the continuity equation is integrated from time t to $t + \Delta t$ within the control volume [20].

Considering the highly complex operating environment of submarine pipelines, characterized by variable geometries and stringent boundary conditions, the Crank–Nicolson finite volume method demonstrates exceptional adaptability. This method effectively addresses these challenges by accurately capturing the intricate interactions between fluid flow and pipeline structures, thereby delivering precise and reliable results for submarine pipeline simulations. Based on the Crank–Nicolson implicit difference method [22], the corresponding difference equation is derived as follows:

$$\int_{\Delta t} \int_{\Delta V} \frac{\partial V}{\partial t} dV dt = \int_{\Delta V} (V_i^{n+1} - V_i^n) dV = (V_i^{n+1} - V_i^n) \Delta V \quad (32)$$

$$\int_{\Delta t} \int_{\Delta V} \frac{1}{\rho_f} \frac{\partial P}{\partial z} dV dt = \frac{A}{\rho_f} \int_{\Delta t} \int_{\Delta z} \frac{\partial P}{\partial z} dz dt = \frac{A \Delta t}{2 \rho_f} [(P_{i+1}^{n+1} - P_i^{n+1}) + (P_{i+1}^n - P_i^n)] \quad (33)$$

The differential form of Equation (33) is obtained as follows:

$$\Delta z V_i^{n+1} + \frac{\Delta t}{2 \rho_f} (P_{i+1}^{n+1} - P_i^{n+1}) = V_i^n \Delta z - \frac{\Delta t}{2 \rho_f} (P_{i+1}^n - P_i^n) = f_1 \quad (34)$$

$$\begin{aligned} \frac{1}{2 \Delta z} (V_i^{n+1} - V_{i-1}^{n+1}) - \frac{\nu b R + e}{E e \Delta t} (\sigma_z)_{i-1}^{n+1} + \frac{\nu(1-a)R}{E e \Delta t} P_{i-1}^n \\ = -\frac{1}{2 \Delta z} (V_i^n - V_{i-1}^n) - \frac{\nu b R + e}{E e \Delta t} (\sigma_z)_{i-1}^n + \frac{\nu(1-a)R}{E e \Delta t} P_{i-1}^n = f_2 \end{aligned} \quad (35)$$

$$\begin{aligned} & \frac{\rho_i e}{\Delta t} (u_z)_i^{n+1} - \frac{e}{2\Delta z} \left[(\sigma_z)_{i+1}^{n+1} - (\sigma_z)_i^{n+1} \right] + \frac{\mu a}{2} P_i^{n+1} + \frac{\mu b}{2} (\sigma_z)_i^{n+1} \\ &= \frac{\rho_i e}{\Delta t} (u_z)_i^n + \frac{e}{2\Delta z} \left[(\sigma_z)_{i+1}^n - (\sigma_z)_i^n \right] - \frac{\mu a}{2} P_i^n - \frac{\mu b}{2} (\sigma_z)_i^n = f_3 \end{aligned} \quad (36)$$

$$\begin{aligned} & \frac{1}{2\Delta z} (V_i^{n+1} - V_{i-1}^{n+1}) + \frac{1}{\Delta t} \left[\frac{1}{K} + \frac{2(1-a)R}{Ee} \right] P_{i-1}^{n+1} - \frac{2bR+2v}{Ee\Delta t} (\sigma_z)_{i-1}^{n+1} \\ &= -\frac{1}{2\Delta z} [V_i^n - V_{i-1}^n] + \frac{1}{\Delta t} \left[\frac{1}{K} + \frac{2(1-a)R}{Ee} \right] P_{i-1}^n \\ &\quad - \frac{2bR+2v}{Ee\Delta t} (\sigma_z)_{i-1}^n = f_4 \end{aligned} \quad (37)$$

where superscript represents the period, and subscript represents the pipe section. Δt and Δz represent the duration of each period and the length of each pipe section, respectively. i represents the i th time step, and n represents the n th computing unit.

Therefore, the FVM is adopted to determine the FSIs in the piping system. The specific derivation of Equation (19) is shown as follows:

$$A \frac{\partial Q}{\partial t} + B \frac{\partial Q}{\partial z} = S \quad (38)$$

The solution process using MATLAB R2023b is as follows:

$$\int_{\Delta t} \int_{\Delta V_c} \left(A \frac{\partial Q}{\partial t} + B \frac{\partial Q}{\partial z} - SQ \right) dt dV_c = 0 \quad (39)$$

$$\int_{\Delta t} \int_{A_c \Delta z} \left(A \frac{\partial Q}{\partial t} + B \frac{\partial Q}{\partial z} - SQ \right) dt dA_c z = 0 \quad (40)$$

$$\int_{\Delta t} \int_{\Delta z} \left(A \frac{\partial Q}{\partial t} + B \frac{\partial Q}{\partial z} - SQ \right) dt dz = 0 \quad (41)$$

$$A \int_{\Delta z} \partial Q dz + B \int_{\Delta t} \partial Q dt - S \int_{\Delta z} \int_{\Delta t} Q dz dt = 0 \quad (42)$$

$$A(Q_i^{n+1} - Q_i^n) \Delta z + \frac{B}{2} [(Q_{i+1}^{n+1} - Q_i^{n+1}) + (Q_{i+1}^n - Q_i^n)] \Delta t = S \frac{Q_i^{n+1} + Q_i^n}{2} \Delta t \Delta z \quad (43)$$

$$A Q_i^{n+1} \Delta z + \frac{B}{2} (Q_{i+1}^{n+1} - Q_i^{n+1}) \Delta t - \frac{S}{2} Q_i^{n+1} \Delta t \Delta z = A Q_i^n \Delta z - \frac{B}{2} (Q_{i+1}^n - Q_i^n) \Delta t + \frac{S}{2} Q_i^n \Delta t \Delta z \quad (44)$$

The FVM is adopted to solve the following:

$$Q_i^n = [V \quad u_z \quad u_s \quad P \quad \sigma_z \quad \sigma_s]^T \quad (45)$$

$$C_i = A Q_i^n \Delta z - \frac{B}{2} (Q_{i+1}^n - Q_i^n) \Delta t + \frac{S}{2} Q_i^n \Delta t \Delta z \quad (46)$$

$$\begin{pmatrix} A_1 & D_1 & & & & \\ B_2 & A_2 & D_2 & & & \\ & B_3 & A_3 & D_3 & & \\ & & \ddots & \ddots & \ddots & \\ & & & B_{n-1} & A_{n-1} & D_{n-1} \\ & & & & B_n & a_n \end{pmatrix}_{4n \times 4n} \begin{pmatrix} Q_1 \\ Q_2 \\ Q_3 \\ \vdots \\ Q_{n-1} \\ Q_n \end{pmatrix}_{4n \times 1} = \begin{pmatrix} C_1 \\ C_2 \\ C_3 \\ \vdots \\ C_{n-1} \\ C_n \end{pmatrix}_{4n \times 1} \quad (47)$$

$$B_i = \frac{B}{2} \begin{pmatrix} I_{2 \times 2} & \\ & 0_{2 \times 2} \end{pmatrix} \quad (48)$$

$$D_i = \frac{B}{2} \begin{pmatrix} 0_{2 \times 2} & \\ & I_{2 \times 2} \end{pmatrix} \quad (49)$$

As seen in Equation (46), C_i relates to the values of Q_n at the i th and $(i + 1)$ th cells; C_i is known at the $(n + 1)$ th time step. Thus, Q_{i-1} at the $(n + 1)$ th time step can be solved using Equation (45). In these equations, $i = 1 \sim n$, where $i = 1$ and $n + 1$ refer to the inlet and outlet.

2.2.2. Boundary Conditions

Upstream Inlet

In this study, the water level remains constant during the transient flow process.

$$P = P_0 \quad (50)$$

The inlet pipe is ideally strongly constrained, with an axial vibration velocity of 0.

$$u_z = 0 \quad (51)$$

Downstream Outlet

The fluid flow rate at the downstream end of the pipe is the relative value of the fluid's flow velocity and the pipe's axial vibration velocity; hence, there is a relationship equation, as follows:

$$V - u_z = Q/A_f \quad (52)$$

The downstream end of the pipe is fixed, and the pipe is also rigidly connected.

The fluid flow rate at the downstream end of the pipe is the relative value of the fluid's velocity and the axial vibration velocity of the pipe, so Equation (37) still applies to the boundary equation at the free outlet. The downstream end of the pipe is a free outlet, considering the mass of the valve, the force balance at the outlet end is as follows [25]:

$$m_V \frac{\Delta u_z}{\Delta t} = A_f(1 - \tau)P - A_t \sigma_z \quad (53)$$

where m_V is the mass of the outlet valve; τ is the opening of the outlet valve; and A_t is the cross-sectional area of the pipe wall. When the outlet end of the pipe is fixed, $m_V = \infty$.

3. Results and Discussion

3.1. Model Verification

To verify the effectiveness and correctness of the FVM for solving FSI models, in this study, we consider joint coupling and friction coupling. When the radial foundation coefficient of the soil is 0 and both ends are free outlet, the system is equivalent to a free water delivery system. The layout of the water delivery system is shown in Figure 3. Considering that the pressure of the soil on the outer wall of the pipeline is 0, the FVM is used for the numerical solution. The numerical calculation results are consistent with the experimental results in the literature [26].

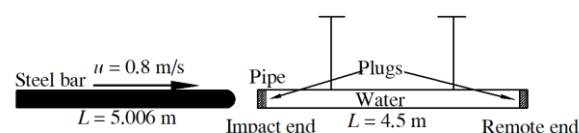


Figure 3. Experimental water delivery system described in [26].

The verification experiment was performed by Vardy and others at the Department of Civil Engineering at the University of Dundee, UK. The pipe used in the experiments was a 4.5 m long steel pipe, with both ends closed and capped. To truly simulate the free state of the pipeline, the pipeline was suspended. The transient load caused by the pipeline system was generated by the axial impact of a 5.006 m long solid steel rod at the closed

end of the pipeline, as shown in Figure 3. When the pipeline is free, it is equivalent to a pipeline buried in soil with an elastic modulus $E_s = 0$, so the results of the calculation model presented in this paper can be compared with the experimental data.

Figure 4 shows the sensitivity of the calculation to computational cell length Δx and time step size Δt . When $\Delta x > 0.1$ m, the numerical results for different Δx are significantly different and cannot be ignored. If $\Delta x < 0.1$, the differences are acceptable and negligible, but they consume more computational resources. Similarly, the smaller the Δt , the more computational resources are consumed. When a valve is quickly closed causing a water hammer, the FSI response frequency of the system is less than 200 Hz, so $\Delta t = 0.005$ s is chosen. After comprehensive consideration, the pipe spacing and computational time step are set to 0.1 m and 0.005 s, respectively, ensuring the stability and convergence of the calculation results.

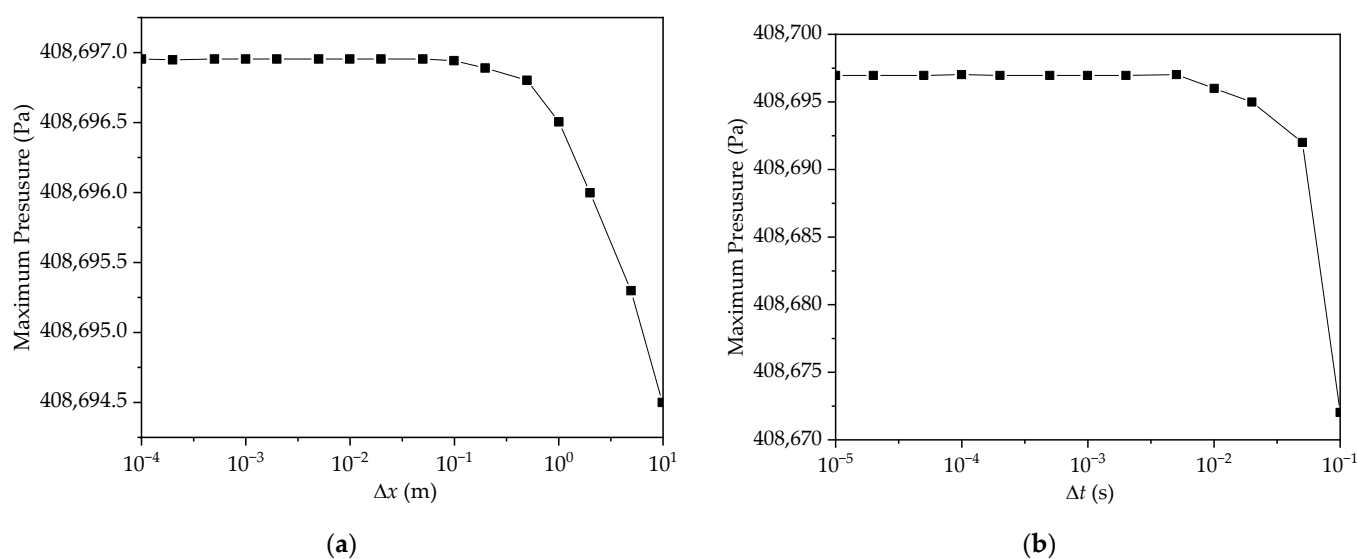


Figure 4. The variation curve of maximum pressure with different (a) computational cell length Δx and (b) time step size Δt .

The FVM was used to calculate the pressure pulsation at the outlet and inlet of the pipeline caused by the impact considering atmospheric pressure inside the pipeline, and the results are compared with the experimental results in the literature, as shown in Figure 5. Figure 5 shows the calculated values for pressure pulsation for a buried water delivery pipeline with both ends fixed and buried in soil with an elastic modulus of $E_s = 10^{10}$, and FSI were not considered to obtain the calculated values. The time step for numerical calculation is 10^{-4} s in order to reflect high-frequency pressure pulsation in the fluid. In contrast, the maximum pressure head sensor used in the experimental test is 80 Hz, ignoring the high-frequency pressure pulsation. This is reflected in the numerical results with many local mutations in the pressure pulsation, while the experimental results have fewer local mutations.

The error between the calculation and experimental results is small. This indicates that only the calculation of the axial coupling response and X–Y, Y–Z, and torsional coupling can be decoupled for separate analyses; it also indicates that using the FVM for FSI calculations produces accurate results. When the elastic modulus of the soil is very large and both ends of the pipeline are fixed, the vibration in the pipeline caused by the change in internal pressure can be ignored, which is equivalent to not considering the FSI response. The numerical results of the pressure pulsation match the experimental results in both cases, indicating that the buried-pipeline calculation model is correct.

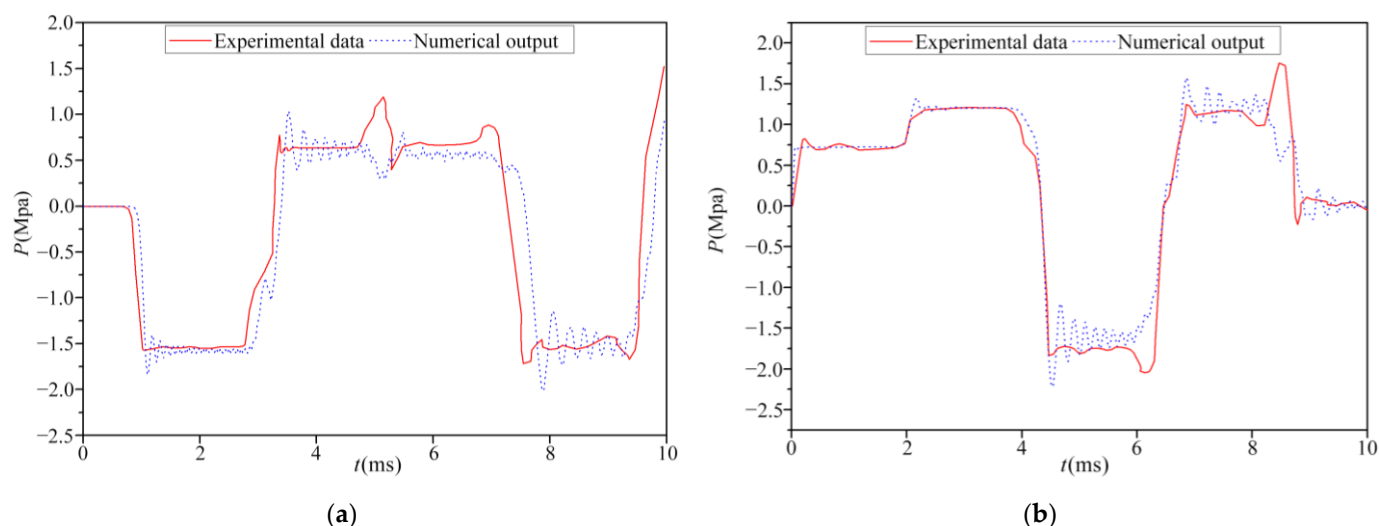


Figure 5. Pressure pulsation. (a) At the impact end of the pipeline and (b) at the far end of the pipeline.

3.2. Model Results

3.2.1. The Impact of Different Coupling Forms on System Patterns

In this study, we analyzed a water supply system consisting of a submarine-buried pipeline, as shown in Figure 6. The effect of seawater on the structural vibration characteristics can be considered as a uniform pressure applied on the surface of the soil, while the influence of the fluctuation of seawater is neglected in this case [27]. When the system flow needs to be adjusted, the transient excitation maintaining the water delivery system is the closure of the valve. This causes intense pressure oscillations within the pipe, which in turn causes structural vibrations. Since the frictional coupling has a minor effect on the system modes [28], the coupling forms in the water delivery system are friction, mainly considering Poisson coupling, and joint coupling. Due to the small response from friction coupling to system coupling, in this study, we considered Poisson coupling and joint coupling. However, the friction coupling between the pipeline and the soil cannot be ignored. The soil is considered an ideal elastic model, and the external force on the pipeline is proportional to the radial displacement of the pipeline. In this study's calculation model, the total time step Δt and the length of the calculating cell are 0.0001 s and 0.1 m, respectively. The FSIs of the buried-pipeline water delivery system were analyzed.



Figure 6. Side view of the buried water delivery system.

Poisson coupling is an along-the-line effect that arises from the local interaction between the wall stress of a pipe and the liquid pressure, with the degree of coupling being closely related to the Poisson's ratio of the pipe structure. When fluid inside the pipe exerts pressure, it first causes radial deformation of the pipe. This radial deformation, through the unique Poisson's ratio characteristic of the pipe material, further triggers axial deformation of the pipe. The stress waves in the radial and axial directions caused by this deformation propagate along the pipe structure to the entire system, thereby having a profound impact on the hydraulic oscillation state of the entire system. To

visually demonstrate the effect of Poisson coupling on the system's hydraulic oscillation, Figure 7 compares the pressure oscillation at the valve under two different Poisson's ratio conditions. The case with a Poisson's ratio of 0.3 represents the effect of Poisson coupling being considered, while the case with a Poisson's ratio of 0 serves as a control. The latter theoretically represents a material that only undergoes radial deformation without axial deformation when under pressure, i.e., completely ignoring Poisson coupling. Through this comparison, we can clearly observe the importance of Poisson coupling in the pipe system and how it significantly affects the pressure oscillation characteristics at the valve.

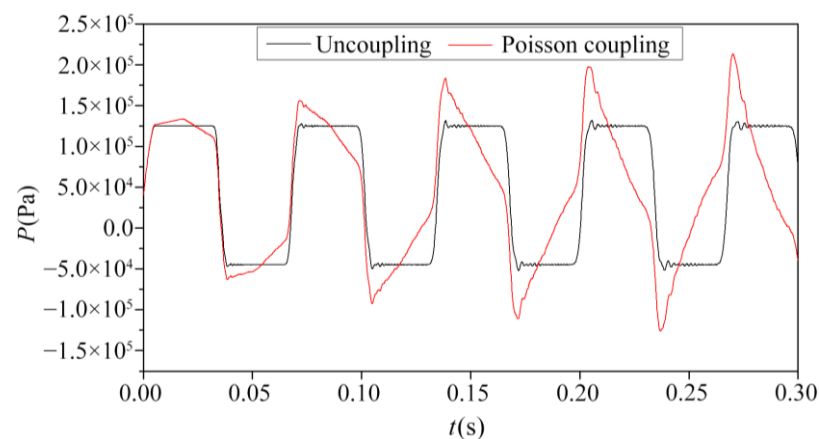


Figure 7. Comparison of the FSI response with and without Poisson coupling.

When fluid flows through the valve, due to the change in fluid momentum, it exerts a force on the structure. Correspondingly, the structure also causes a change in the state of motion of the fluid. A strong coupling effect occurs between the fluid and the structure at the junction, which is referred to as joint coupling. Joint coupling is closely related to the valve's motion state. To deeply explore the effect of joint coupling on the system's dynamic characteristics, Figure 8 visually compares the pressure oscillation under two different scenarios: one where the valve is axially freely movable, fully considering the effect of joint coupling; the other where the valve is fixed, equivalent to ignoring the effect of joint coupling. Through this comparison, we can clearly observe how the dynamic response of the valve significantly affects the system's pressure oscillation characteristics when joint coupling is considered, while ignoring joint coupling may lead to misunderstanding or underestimation of the system behavior. Such analysis is crucial for understanding complex fluid–structure interactions, optimizing valve design, and improving the performance of the entire pipeline system.

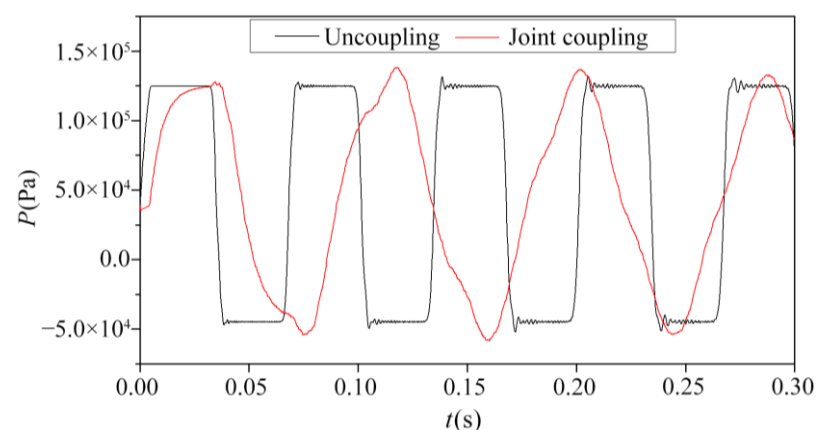


Figure 8. Comparison of the FSI response with and without joint coupling.

Under the synergistic effect of Poisson coupling and joint coupling, at the moment of valve closure, the composite structure formed by the pipe wall and the soil body exhibits a significant relative motion state. This phenomenon significantly delays the phase change in pressure pulsations within the pipeline, resulting in a relatively sharp characteristic of the fluid pressure pulsation waveform.

3.2.2. Effect of Different Soil Moduli on System Modes

On the one hand, Poisson coupling increases the amplitude of pressure pulsation in the pipeline, and at the same time, the radial expansion and contraction that it causes make the fluid pressure change irregularly; on the other hand, it causes rapid axial expansion and contraction in the pipeline, thereby advancing the phase of pressure pulsation in the pipeline, making the waveform sharper. When the soil stiffness increases, the Poisson coupling response weakens, significantly weakening the overall Poisson coupling response. The pulsation in the pipeline fluid mainly manifests as a strong fluctuation in pressure, primarily characterized by water hammer waves with superimposed pipe wall stress waves. In this paper, joint coupling was only considered at the valve; when the valve is closed, the pipeline undergoes relative motion, significantly delaying the phase of pipeline pressure pulsation. Due to the pipeline's relative motion, the fluid pressure pulsation waveform is relatively smooth [29].

When the water delivery pipeline is buried, the direction of the soil's constraint on the pipeline can be considered as the radial direction of the pipeline, and it can be considered that the soil's constraint on the pipeline is equivalent to increasing the thickness of the pipeline wall [30]. Due to the relative motion between the pipeline wall and the soil, the equivalent pipeline wall thickness increased by the soil constraint differs from the equivalent thickness of the pipeline with a buried coupling response. However, according to the formula in [31], the wave speeds are the same. From Figure 9, it can be seen that when the elastic modulus of the soil is $E_s = 10^{15}$, $c_f = 4L/Tc = 1483$ m/s, and when the elastic modulus of the soil is $E_s = 10^{10}$, $c_f = 4L/Tc = 571$ m/s. It can be seen that the elastic modulus of the soil has a significant influence on the water hammer wave speed, so it is necessary to analyze the relationship between the elastic modulus of the soil and the water hammer wave speed of the pipeline to determine the water hammer wave speed at different soil elastic moduli.

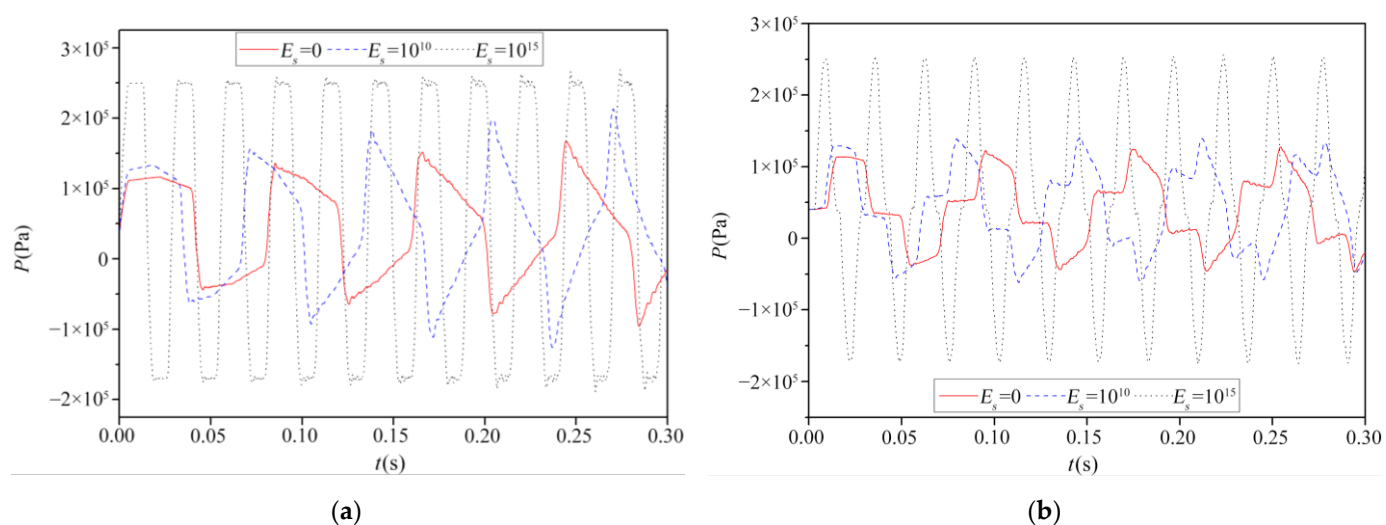


Figure 9. Pressure fluctuations in pipelines embedded in soil with different elastic moduli. (a) At the end and (b) at the midpoint of the pipeline.

To calculate the wave velocity in pipes for different soil elastic moduli, we first calculated the equivalent wall thickness of the buried pipe. The pipe wall and soil equilibrium equations are shown in Equation (13) and Equation (14), respectively.

$$P - P_b = \rho_t e a \quad (54)$$

When equivalent to an open pipe, the external pressure on the outer wall of the pipe is $P_b = 0$, and the radial acceleration of the pipe wall is equal to the acceleration of the soil.

$$P = \rho_t e_1 a \quad (55)$$

where $b \ll a$, $e \ll R$. We then substitute Equations (9) and (10) into Equation (55) to obtain the equivalent thickness of the pipe wall in the casing, as follows.

$$e_1 = \frac{P}{P - P_b} e = \frac{e}{1 - a} = \left(e + \frac{E_s}{E} R \right) \quad (56)$$

When calculating the wave velocity of the water hammer in buried pipes, the pipe wall thickness is taken as the equivalent pipe wall thickness. According to the formula for wave velocity in [32], when the soil's elastic modulus $E_s = \infty$, the equivalent pipe wall thickness $e = \infty$, and the wave velocity $c_f = 1483$ m/s. When the soil's elastic modulus is $E_s = 0$, the equivalent pipe wall thickness is $e = 0$ and the wave velocity is $c_f = 498$ m/s. The greater the soil's elastic modulus, the greater the wave velocity and the more severe the water hammer pressure fluctuations [33]. However, the greater the soil's elastic modulus, the greater the constraint on the pipe and the weaker the system coupling response, and the pressure fluctuation waveform is closer to that obtained without considering coupling. When the soil is rigid, the coupled pressure fluctuation of the buried pipe is consistent with the pressure fluctuation obtained without considering fluid–solid coupling, as shown in Figure 10. Increasing the equivalent thickness of the pipe wall causes the soil and the pipe to jointly bear the stress, reducing the stress on the pipe. For soils with different elastic moduli, different proportions of stress are shared by the pipe, consistent with the ratio of the soil-equivalent pipe wall thickness to the actual pipe wall thickness. The axial stress on the pipe under soils with different elastic moduli is shown in Figure 10.

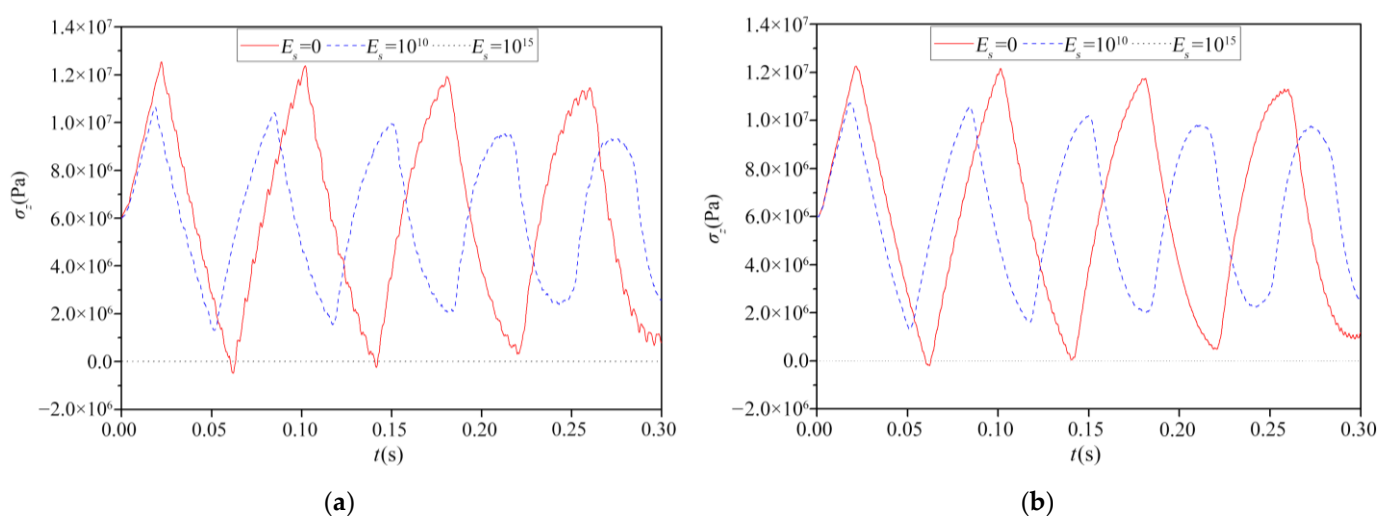


Figure 10. Axial stress at different soil moduli. (a) At the end and (b) at the midpoint of the pipeline.

The increase in the soil elastic modulus significantly accelerates the propagation rate of wave velocity, which in turn intensifies the oscillation intensity of internal pressure in

pipelines. The FSI effect plays a role in alleviating these oscillations to some extent, thereby leading to a reduction in wall stress.

3.2.3. The Effect of Different Soil Elastic Moduli on System Vibration

As shown in Figure 11, the excitation time of the midpoint of the pipeline is $t_1 = 0.0004$ s, and the stress wave propagates a distance of $l = 5$ m within the time period t_1 . Therefore, the stress wave velocity $c_s = l/t_1 = 1250$ m/s. The stress wave is faster than the pressure wave, so when the stress wave reaches the midpoint of the pipeline, the water hammer wave has not yet arrived. Hence, the axial vibration in the pipeline is solely caused by the stress wave and is linearly related to the magnitude of the stress. When the water hammer wave reaches the midpoint, the water hammer wave and the stress wave superimpose, causing the axial displacement of the pipe to decrease. Due to the quantitative relationship between the stress wave and the pressure wave, the axial vibration in the pipe is shown in Figure 11. The combined action of the fluid pressure pulsation and stress within the pipe leads to various vibration frequencies in the axial displacement in the pipe. The dry friction between the soil and the water pipe causes the vibration in the pipe to attenuate, as shown by the blue dashed line in Figure 11. The blue dotted line in Figure 11 represents the axial vibration of the buried pipe with the soil elastic modulus $E_s = 10^{10}$; the constraint effect of the soil on the pipe equivalently enhances the pipe stiffness, so the stress wave speed $c_s = (E/\rho)^{0.5}$ increases. The radial expansion or contraction of the pipe causes a vibration in the soil that propagates axially in the form of waves. The wave speed in the soil is $c_s = (E/\rho)^{0.5} = 2820$ m/s; therefore, the pipe and soil vibrations cause the pipe to produce higher-frequency vibrations. The constraints of the soil on the pipe significantly reduce the amplitude of the pipe vibrations, mainly dominated by the water hammer pressure, the superposition of the pipe stress wave, and the soil stress wave. The influence of the soil stress wave on the amplitude of pipe vibrations is small. Generally, the axial vibration of the buried pipe can be considered to be the result of the superposition of only the water hammer wave and the pipe stress wave, as shown by the blue dotted line in Figure 11.

Considering the combined structure of the pipe wall and soil, the structural vibration mode is mainly dominated by water hammer pressure, with the superposition of pipe stress waves and soil stress waves. The soil stress waves have a minor effect on the amplitude of pipe vibration, and generally, the axial vibration of buried pipes can be considered to be the result of the superposition of only water hammer waves and pipe stress waves.

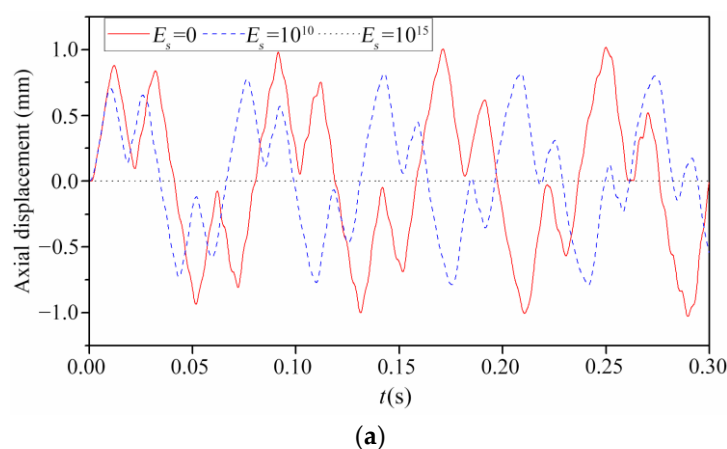


Figure 11. Cont.

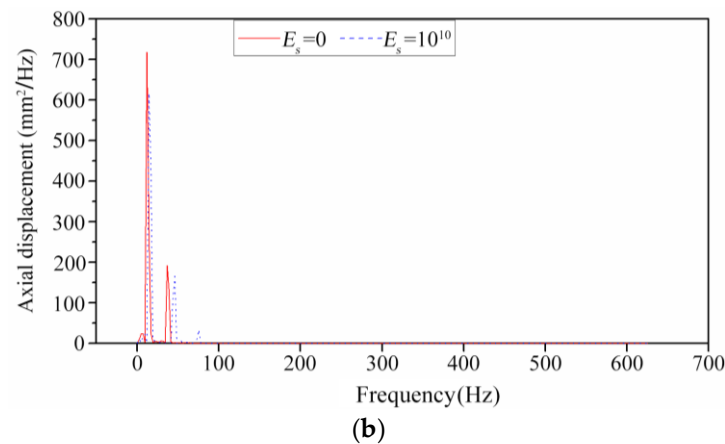


Figure 11. (a) Time-domain and (b) frequency-domain response of pipeline axial vibration.

4. Conclusions

In this study, we derive a refined model describing the dynamic response of a submarine-buried pipeline water transmission system, assuming that the seabed soil is elastic and homogeneous and considering the soil and the pipe wall as a combined body. A numerical solution was obtained using the finite volume method. Experimental results reported in the literature were used to verify the numerical solution, demonstrating the accuracy of the finite volume method in solving this model.

The Poisson coupling of the water transmission system is simulated, revealing that Poisson coupling increases the pressure inside the pipe. Considering the soil and the pipe wall as a combined body, the irregular radial expansion and contraction of the pipe wall cause the phase of pressure fluctuation inside the pipe to advance, specifically manifested as a sharper waveform. At the same time, since the combined body is considered the dynamic boundary of the internal fluid of the pipe, the coupling at the joint delays the phase of the pipeline pressure fluctuation, and the waveform of the fluid pressure fluctuation is relatively smooth. Considering the two coupling forms, the pressure fluctuation is mainly dominated by water hammer waves resulting from stress wave superposition.

Pipelines buried in soils with different elastic moduli have different equivalent pipe wall thicknesses, seriously affecting the magnitude of the water hammer wave speed. In this study, we derive the calculation formula for the equivalent thicknesses of different soil elastic moduli, and the pipe wall thickness derived from the calculation formula for the water hammer wave speed for buried pipes in soils with different elastic moduli is taken as the equivalent thickness. As the soil elastic modulus increases, the wave speed increases, and the pressure fluctuation inside the pipeline becomes more intense; however, the greater the soil elastic modulus, the weaker the pipeline coupling. When the soil is rigid, there is no coupling in the system. However, the greater the soil elastic modulus, the safer the water transmission system.

In this study, soil is modeled as a purely elastic continuum, thereby neglecting essential material properties such as plasticity, viscosity, and anisotropy, which play a pivotal role in governing soil mechanics and the dynamic behavior of pipelines. To address these limitations, future research should adopt more advanced soil models, such as elastoplastic and viscoelastic formulations, to provide a more comprehensive representation of soil behavior and its impact on pipeline dynamics.

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supervision, J.G.; project administration, J.G.; funding acquisition, J.G. All authors have read and agreed to the published version of the manuscript.

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