

Article

Influence of the Semicircular Cycle in a Labyrinth Weir on the Discharge Coefficient

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Abstract

The labyrinth weir is an effective hydraulic structure, offering high discharge efficiency and economic advantages, making it a suitable option for dam construction or rehabilitation projects. Owing to its complex geometry, significant research efforts have been dedicated to enhancing its hydraulic performance. Since the beginning of this century, Computational Fluid Dynamics (CFD) has emerged as a vital approach, complementing traditional methods in the design of hydraulic structures. This study employs CFD ANSYS FLUENT to examine the discharge coefficient of a semicircular labyrinth weir, featuring a cyclic arrangement and a half-round crest profile. The numerical models and simulations address two-phase flow (air and water) under incompressible and free-surface conditions. The CFD ANSYS FLUENT approach used is multiphase flow modeling using the Volume of Fluid method to track the free water surface. For turbulence effects, it is complemented with the standard $k-\epsilon$ model and the Semi-Implicit Method for Pressure Linked Equations algorithm for pressure–velocity coupling. In addition, for boundary conditions, the flow velocity was defined as the inlet to the channel and atmospheric pressure as the outlet, and the walls of the channel and weir are considered solid, stationary, and non-sliding walls. The model was validated with experimental data reported in the literature. The results indicate that the semicircular labyrinth weir achieves greater discharge capacity when the headwater ratio H_T/P increases for $H_T/P \leq 0.25$. A regression analysis mathematical model was also developed, using the H_T/P ratio, to predict the discharge coefficient for $0.05 \leq H_T/P \leq 1$. Relative to other geometrical configurations, the semicircular labyrinth weir demonstrated a discharge capacity that was up to 88% higher than that of the trapezoidal labyrinth weir. Both weir and cycle efficiency were assessed, and maximum weir efficiency was observed when $H_T/P \leq 0.1$, while cycle efficiency peaked at $H_T/P \leq 0.25$. The geometric configuration under analysis demonstrated greater economic efficiency by providing a reduced total length and enhanced discharge capacity relative to trapezoidal designs, especially when the sidewall angle α is considered as $\alpha \leq 12^\circ$. The study concludes by presenting a design sequence detailing the required concrete volume for construction, which is subsequently compared to the specifications of a trapezoidal labyrinth weir.



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1. Introduction

Weirs are hydraulic structures designed to regulate water levels and mitigate flooding within reservoirs, thereby supporting dam safety and stability. Additionally, these installations facilitate flow measurement, water diversion, and sediment transport management within alluvial channels [1]. However, the discharge capacity of weirs is frequently constrained by factors such as topography, channel width, and weir length. To enhance hydraulic performance, the use of labyrinth weirs has been adopted. These polygonal hydraulic structures offer substantial advantages over conventional designs, including greater discharge capacity and reduced hydraulic head. The geometric features of a labyrinth weir are depicted in Figure 1.

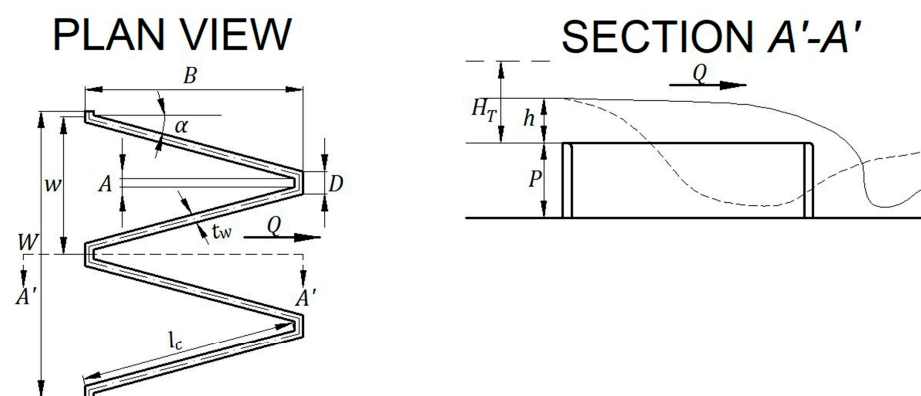


Figure 1. Geometric parameters of a trapezoidal labyrinth weir. Where P is the height of the weir [m], B length of apron [m], t_w is the wall width [m], D outside width of apex [m], A inside width at apex [m], l_c is the length of the side wall [m], α is the angle of the side wall [$^\circ$], W is the channel width [m], w is the cycle width [m], H_T is the total head [m], h is the piezometric head [m] and Q is the flow rate [$\text{m}^3 \text{s}^{-1}$] [2,3].

The optimization of hydraulic performance in labyrinth weir design presents significant challenges due to the variability introduced by geometric parameters and the three-dimensional nature of flow discharge. The study on the optimization of its hydraulic performance has been challenged for different researchers [4–6]. As noted by Bilhan et al. [7], key geometric characteristics influencing labyrinth weir optimization include the cycle configuration, crest shape, and weir orientation. Research suggests that half-rounded crest profiles generally offer greater efficiency compared to sharp or quarter-rounded crests [2,8]. Several studies have demonstrated that the configuration of the apex within weir cycles significantly influences weir efficiency. Crookston [3] found that a smooth transition at the upstream apexes during discharge markedly improves efficiency compared to the abrupt transitions seen in trapezoidal labyrinth weirs. Similarly, Tullis and Young [9], observing hydraulic operations at the Brazos Dam (Waco, TX, USA), reported enhanced discharge efficiency attributable to the smooth flow over the weir crest when circular vertices are incorporated at the apexes of trapezoidal weir cycles. Bilhan et al. [10] conducted research on semicircular cycle weirs, achieving positive outcomes by employing nappe breakers to regulate sub-atmospheric pressures downstream of the weir. Furthermore, Dabling et al. [11] examined the hydraulic performance of labyrinth weirs with multiple crest elevations—termed staged systems—and concluded that discharge rates are partic-

ularly sensitive to the position of the lower stage weir segment. Consequently, owing to the benefits provided by this form of hydraulic infrastructure, numerous studies have addressed design methodologies for labyrinth weirs with the objective of enhancing their hydraulic performance [6,12–20]. Based on the above, and due to the complex geometry of the labyrinth weir, the literature shows that a weir is hydraulically more efficient if the flow discharge over the weir has a less abrupt transition, suggesting that a labyrinth weir composed of semicircular cycles and with a semi-rounded crest profile may be more efficient than labyrinth weirs composed of straight walls.

A notable approach in enhancing the conceptual design of infrastructure of this nature involves applying Computational Fluid Dynamics (CFD) as a versatile numerical approach. CFD facilitates the numerical resolution of fluid flow equations, specifically the Navier–Stokes equations. However, challenges persist in numerical modeling, notably regarding the selection of turbulence models capable of accurately predicting flow behavior and the precision required for free surface tracking. Consequently, rigorous validation against experimental data is essential to ensure the reliability of numerical results. Furthermore, achieving spatial discretization that yields outputs comparable to physical experiments often necessitates significant computational resources. Recent advances in high-performance computing have proven dependable for resolving intricate issues related to hydraulic structures, especially about the analysis of labyrinth weirs [21–25]. For example, Idrees et al. [26] employed CFD to simulate the hydraulic behavior of a labyrinth weir with notches in the crests of the side walls. The study demonstrated that, under elevated hydraulic heads, the hydraulic performance exhibits similarities to oblique and linear weirs. Furthermore, the findings confirmed the formation of air cavities and bubbles behind the nappe, as well as the occurrence of negative pressure during aerated flow conditions. Yildiz et al. [27] employed ANSYS FLUENT CFD software to simulate triangular labyrinth weirs with different heights and cycle numbers. The study concluded that weir height does not have a significant effect on discharge capacity, while an increase in the number of weir cycles results in a reduction in the discharge coefficient. Carrillo et al. [28] conducted a numerical analysis using CFD to examine how submerged and free-flow conditions influence the discharge coefficient in labyrinth weirs. On the other hand, the monitoring of free-surface water flow over weirs has attracted considerable attention from researchers. Indeed, an example of this is the study by Torres et al. [29], who, using ANSYS FLUENT version 17.2 and OpenFOAM software as CFD approaches, modeled the water flow over a labyrinth weir. Their results revealed that the use of the Volume of Fluid (VOF) algorithm combined with the standard $k-\epsilon$ turbulence model is capable of accurately predicting the behavior and complex characteristics of the flow, including velocities, depths, and the definition of the free surface. Similarly, Rezazadeh et al. [30] simulated water flow over different sharp-crested weir geometries using OpenFOAM and investigated the influence of the weir's geometric configuration on the hydraulic characteristics of the flow. It is noteworthy that these authors also employed the $k-\epsilon$ turbulence model together with the VOF algorithm for free-surface tracking. Their results demonstrated acceptable agreement between the predicted and experimental values of the free-surface profile and discharge coefficient. Additionally, Hargreaves et al. [31] validated the use of CFD for modeling the free surface over broad-crested weirs employing the VOF algorithm. Finally, the use of CFD has demonstrated its reliability for studying labyrinth weirs at a low cost and time. It has the ability to perform detailed studies without the need for physical models. Once validated with experimental results, it enables the optimization of weir designs with multiple geometric configurations that can enhance the hydraulic efficiency of the weir, thereby overcoming the limitations of studies conducted in physical models.

Research indicates that the intricate geometry of labyrinth weirs, along with the three-dimensional nature of flow discharge, significantly affects the operational efficiency of the weir and its associated discharge coefficient. Table 1 compares previous studies related to the geometry of labyrinth weirs to evaluate their hydraulic efficiency. However, there are no studies on labyrinth weirs with semicircular cycles and a half-rounded crest profile. This study suggests an important contribution to improving the hydraulic efficiency of labyrinth weirs. The objective of this study is to examine the influence on efficiency and discharge coefficients of a labyrinth weir composed of semicircular cycles oriented opposite to the flow direction and incorporating a half-rounded crest for free flow for ratios of $0.05 \leq H_T/P \leq 1$. The analysis was carried out through hydrodynamic simulations performed with the commercial CFD software ANSYS FLUENT (version 14.5), used as the numerical study approach.

Table 1. Previous studies related to the geometry of the labyrinth weir.

Authors	Cycle Geometry	Apex Geometry	Crest Profile
Present study	Semicircular	Semicircular	HR
Mattos-Villarroel et al. [20]	Trapezoidal	Semicircular	HR
Idrees et al. [26]	Trapezoidal	Straight	Flat
Samadi et al. [4]	Trapezoidal, rectangular, triangular	Straight	QR
Yildiz et al. [27]		–	Flat
Carrillo et al. [28]	Trapezoidal	Straight	QR
Bilhan et al. [10]	Semicircular	Semicircular (upstream)- straight (downstream)	Sharp
Shaghaghian et al. [22]	Triangular	–	Sharp
Dabling et al. [11]	Trapezoidal	Straight	QR
Crookston and Tullis [19]	Trapezoidal	Straight	QR-HR
Tullis and Young [9]	Trapezoidal	Semicircular	QR

Note: QR: Quarter-round; HR: Half-round.

2. Materials and Methods

2.1. Physical Model

The evaluation of numerical modeling in CFD was based on the experimental study conducted by Crookston and Tullis [19]. The geometric characteristics of the physical model are summarized in Table 2 and consist of a trapezoidal labyrinth weir (Figure 1) constructed of high-density polyethylene and located inside a channel.

Table 2. Geometry and dimensions of the trapezoidal labyrinth weir (adapted from Crookston and Tullis [19]).

α (°)	N	L (m)	A (m)	w (m)	P (m)	W (m)	Crest Profile
15	2	4	0.038	0.617	0.305	1.235	Quarter-round

To improve the accuracy of the experimental results, Crookston and Tullis [19] took care to minimize random and systematic errors, collected approximately 85 hydraulic head measurements for each test, and established a control system in which at least 10% of the data were repeated to ensure accuracy and assess the repeatability of the measurements. Furthermore, the geometric size of the prototype was selected to minimize potential scale effects that could influence the discharge capacity relative to the physical model's dimensions.

2.2. Numerical Study

2.2.1. Computational Fluid Dynamics

The numerical simulations were carried out using ANSYS FLUENT version 14.5, a general-purpose CFD code based on the finite volume method for the spatial discretization of the computational domain and the numerical solution of the general transport or conservation equations, commonly known as the Navier–Stokes equations. Its application to the simulation of free-surface flows, such as those occurring over weirs, has been validated with satisfactory results [2,20,29].

The VOF scheme is included in the ANSYS FLUENT version 14.5 software to perform the evolution of the interface between two or more immiscible fluids. The numerical simulations were performed using a two-phase flow model (air–water) under incompressible conditions and with a free-surface interface. As the flow was considered both two-phase and free-surface, the VOF multiphase model was employed to enhance the accuracy of tracking the air–water interface.

The standard k - ϵ model was used to study the effects of turbulence. The standard k - ϵ turbulence model is a robust model based on the modeling of two transport equations with Reynolds stresses for turbulent kinetic energy (k) and its dissipation rate (ϵ). However, the k - ϵ model is inaccurate in modeling flows near solid walls, something that can be overcome with the k - ω model [32,33]. In addition, the model shows limited accuracy in recirculating zones and rotating flows. Although this model has limitations in reproducing recirculation, studies by Koldewyn [34] and Idrees et al. [26] have demonstrated that the standard k - ϵ model can successfully predict nappe aeration gradients in good agreement with experimental results. The realizable k - ϵ model or RNG k - ϵ model can improve the prediction of flow recirculation, and the k - ω SST model is more accurate for simulating rotating flows [32,33]. On the other hand, several researchers have demonstrated the reliability and effectiveness of the k - ϵ model over other turbulence models for the simulation of both confined and free-surface flows [10,35–38]. Compared with the mixing-length and Spalart–Allmaras models, the k - ϵ model offers greater accuracy for complex flow conditions and within boundary layers, providing a more comprehensive description of turbulence. In comparison with the k - ω model, the k - ϵ model is more robust and exhibits higher stability and accuracy in the free-surface region of the flow, as well as improved numerical stability. For more complex turbulence models, such as LES and RSM, the k - ϵ model provides superior numerical stability, faster convergence, and significantly lower computational cost [32,33,39].

The Semi-Implicit Method for Pressure Linked Equations (SIMPLE) algorithm is used for pressure–velocity coupling [40]. This algorithm is based on the reformulation of pressure and approximates convergence through a series of intermediate pressure and velocity fields that satisfy continuity [39]. On the other hand, the second-order Upwind spatial discretization system was used to ensure stable schemes by minimizing the numerical diffusion errors that the first-order Upwind scheme can generate [39].

The CFD solution process comprises three basic steps: (i) Pre-processing, where the geometry to be modeled is defined, the spatial discretization of the conceptual model, the identification of physical phenomena, the definition of fluid properties and initial and boundary conditions; (ii) Process, is the step where the equations that define the phenomenon to be simulated are solved iteratively; (iii) Post-process, which consists of analyzing the results in graphical or scalar form, evaluating specific metrics, and visualization of contour maps of the different variables.

The conceptual models were created in the Geometry module of ANSYS FLUENT. A coordinate system (x , y , z) was established with its origin at the lower right corner upstream of the channel. The x -coordinate is in the direction of flow, the y -coordinate is perpendicular

to the flow, and the z -coordinate is in the positive direction of the water depth in the channel. The spatial discretization of the conceptual models was carried out in the Meshing module, employing predominantly structured hexahedral meshes. This type of mesh improves the convergence of the solution, as well as reducing the number of cells [41]. The mesh was refined in the vicinity of the crest of the weir, where turbulence is dissipated and a greater gradient in the fluid variables is inferred (Figure 2B).

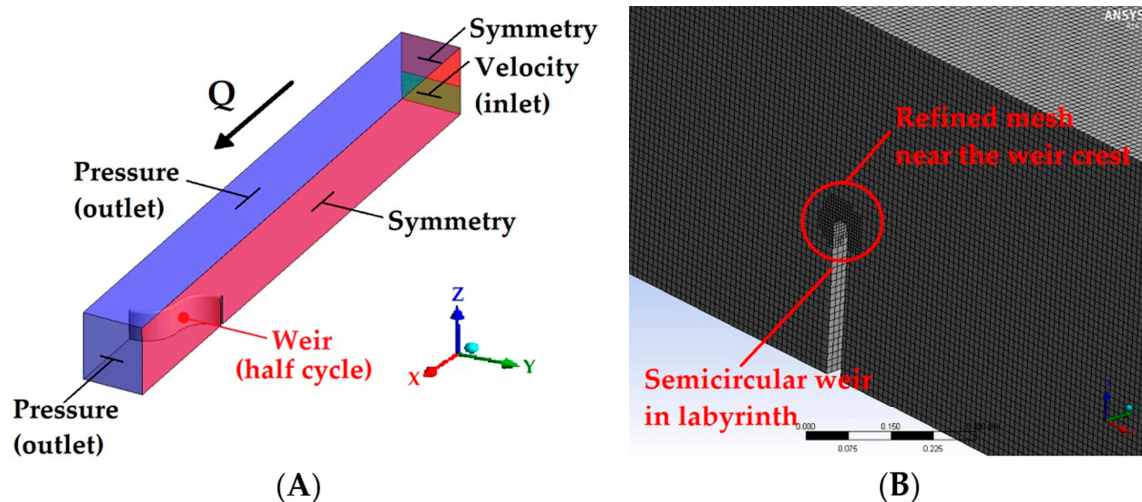


Figure 2. Conceptual model of the weir to be simulated. (A) Boundary conditions, (B) Refined mesh.

The boundary conditions were applied in the computational domain. The fluid velocity (upstream of the channel) as the inlet and atmospheric pressure (downstream and upper area of the channel) as the outlet [20]. The walls of the channel and weir are considered solid, stationary, and non-sliding walls [20]. Figure 2A illustrates the approach to the boundary conditions, and the definition of the fluid properties is summarized in Table 3.

Table 3. Definition of the fluid properties and the domain to be simulated.

Boundary and Initial Conditions	Solution Method
Inlet	Velocity
Outlet	Atmospheric pressure
Weir and channel	Solid, stationary, and non-slip.
Time	Steady-state
Viscosity model	k - ϵ standard
Multiphasic model	Volume of fluid (VOF)
Model fluid	Two-phase
Pressure–velocity coupling	SIMPLE
Spatial discretization scheme	Second-order Upwind

2.2.2. Grid Convergence Index (GCI)

This process is important for determining the degree of influence that the type and density of the mesh has on the simulation time and the accuracy of the results, and is known as mesh sensitivity.

The Richardson extrapolation method [42] allows the spatial convergence of meshes to be examined by applying the GCI, and has been widely used in various CFD studies [20,43–45]. The GCI indicates the deviation of the calculated value from the asymptotic numerical value; a small GCI value means that the solution is within the asymptotic range and that the numerical solution has reached stability [46]. The GCI indicates the

deviation of the calculated value from the asymptotic numerical value; a small GCI value means that the solution is within the asymptotic range and that the numerical solution has reached stability [46]. In CFD simulations, it is advisable to perform the study on at least three mesh densities to ensure that the solutions are within the asymptotic convergence range [41].

To obtain the CGI of a coarse and fine mesh, Equation (1) is used [47].

$$GCI_{fine} = \frac{F_s |\mathcal{E}_{i+1,1}|}{(r^p - 1)}; \quad GCI_{coarse} = \frac{F_s |\mathcal{E}_{i+1,1}| r^p}{(r^p - 1)} \quad (1)$$

where F_s is known as the safety factor, for comparisons of two meshes it is equal to 3 and 1.25 for the case of three or more meshes [47], r is the mesh refinement ratio, p is the order of convergence, and $\mathcal{E}$ is the error relative to the control variable f_i .

Equation (2) enables the calculation of the size of the representative cell (λ).

$$\lambda = \left[\frac{1}{N_C} \sum_{i=1}^N (\Delta V_i) \right]^{\frac{1}{3}} \quad (2)$$

where ΔV_i is the volume of the i -th cell and N_C is the number of cells. At least three different mesh sizes must be analyzed to determine the value of the control variables f_i . The mesh refinement factors were determined as $r_{21} = \frac{\lambda_2}{\lambda_1}$, $r_{32} = \frac{\lambda_3}{\lambda_2}$. The order of convergence p was calculated with Equation (3) [42].

$$p = \frac{\ln \frac{f_3 - f_2}{f_2 - f_1}}{\ln r} \quad (3)$$

The solutions must be within the asymptotic convergence range. Equation (4) is used to determine this [46].

$$\frac{GCI_{23}}{GCI_{12} r^p} \approx 1 \quad (4)$$

This work analyses two different grids, one to evaluate the computational model and the other to study the semicircular labyrinth weir. To carry out the CGI study, the hydraulic head h_i is considered as a control variable. The flow rate used for both meshes was $Q_{I-II} = 0.2616 \text{ m}^3\text{s}^{-1}$. The asymptotic convergence ranges obtained are ≈ 1 , indicating that they are within the asymptotic range. The maximum error calculated was up to 9.40% for a coarse mesh and 5.65% for a fine mesh, both for mesh II. The calculation of the GCI is summarized in Table 4.

Table 4. Estimation of CGI.

Grid	r	p	h_1 (m)	h_2 (m)	h_3 (m)	Richardson Extrapolate (m)	$\mathcal{E}_{21}$	$\mathcal{E}_{32}$	GCI_{21} (%)	GCI_{32} (%)	Asymptotic Range of Convergence
I	1.66	1.56	0.1247	0.1282	0.1359	0.1218	0.004	0.008	2.92	6.25	0.97
II	1.70	1.02	0.1514	0.1563	0.1647	0.1445	0.005	0.008	5.65	9.40	0.97

In the graph in Figure 3, the hydraulic head approaches an asymptotic value of zero as the mesh separation approaches zero.

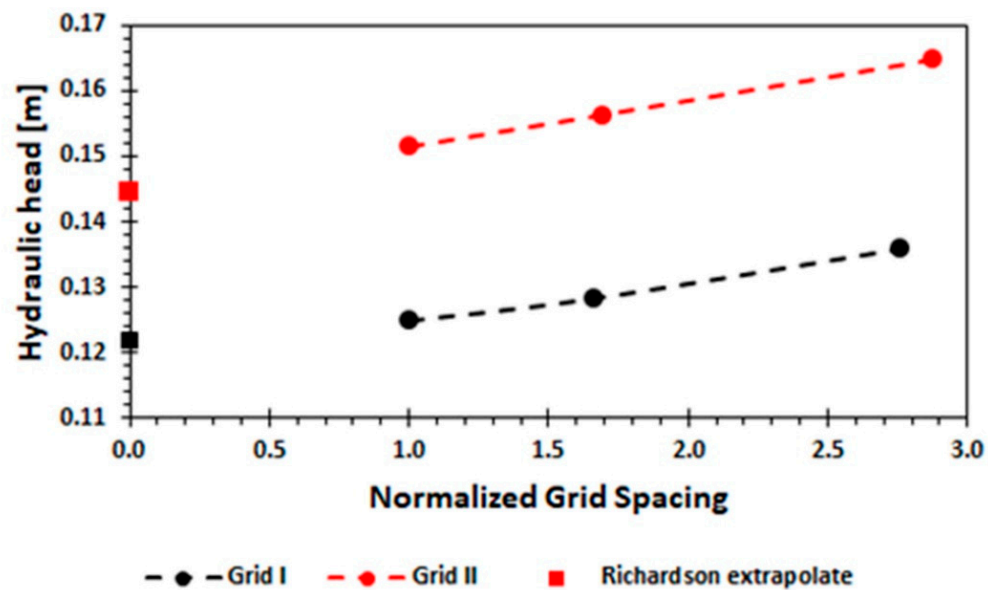


Figure 3. Hydraulic head with variable grid spacing.

2.3. Numerical Model Evaluation

Twenty-one scenarios were modeled numerically. Scenarios 1–11 correspond to the validation and evaluation by statistical methods of the numerical results of the discharge coefficient of the trapezoidal labyrinth weir with respect to the experimental values. Scenarios 12–21 are numerical simulations of the semicircular labyrinth weir; the geometry of the weir crest is of the half-rounded type. In their experimental observations, Willmore [8] and Mattos-Villarreal et al. [2] indicate that half-rounded crest profiles are more efficient than sharp and quarter-rounded crests. Table 5 details the design flow and geometric characteristics of the different simulated scenarios.

Table 5. Simulation scenarios.

Scenario	α (°)	Diameter θ (m)	P (m)	L_{cycle} (m)	w/P	N	Flow Rate ($m^3 s^{-1}$)	Crest Profile	Cycle Geometry
1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11	15	–	0.305	2.00	2.024	2	0.0126, 0.0379, 0.1065, 0.1839, 0.2616, 0.3362, 0.4071, 0.4746, 0.5394, 0.6026, 0.6651	QR	Trapezoidal
12, 13, 14, 15, 16, 17, 18, 19, 20, 21	–	0.617	0.305	1.00	4.048	1	0.0126, 0.0379, 0.1065, 0.1839, 0.2616, 0.3362, 0.4071, 0.4746, 0.5394, 0.6026	HR	Circular

Note: QR: Quarter-round; HR: Half-round; L_{cycle} : Cycle length (m).

To evaluate the reliability of the computational model, the numerical results of the discharge coefficient for scenarios 1–11 are compared with the experimental values reported by Crookston and Tullis [19]. The discharge coefficient value is calculated from the hydraulic head height and using the general equation for linear weirs (Equation (5)).

$$Q = \frac{2}{3} \sqrt{2g} C_d L H_T^{3/2} \quad (5)$$

where Q is the flow rate ($\text{m}^3 \text{s}^{-1}$), g is the acceleration due to gravity (m s^{-2}), C_d is the discharge coefficient (dimensionless), L is the characteristic length of the weir (m), and H_T is the total hydraulic head (m).

To evaluate the agreement between the results of the physical experiment and the numerical models, statistical techniques such as Pearson's coefficient of determination (R^2) (Equation (6)) were applied; to measure the dispersion between numerical and experimental results, the relative percentage error (Er) and the mean absolute error (MAE) were used to compare the approximation of experimental values with numerical values and to determine the mean of the errors (Equations (7) and (8)).

$$R^2 = \frac{\text{Cov}(num, exp)^2}{\sigma_{num}^2 \sigma_{exp}^2} \quad (6)$$

$$Er(\%) = \frac{|Y_{num} - Y_{exp}|}{Y_{exp}} \times 100 \quad (7)$$

$$MAE = \frac{\sum_i^n |Y_{num,i} - Y_{exp,i}|}{n} \quad (8)$$

where $\text{Cov}(num, exp)$ is the covariance of the numerical and experimental results, σ_{exp}^2 and σ_{num}^2 are the variance of the experimental and numerical results, respectively, Y_{num} and Y_{exp} are the numerical and experimental values, respectively.

The discharge coefficients obtained numerically show excellent agreement with the experimental values, lying very close to the diagonal line and yielding a high Pearson coefficient of determination ($R^2 = 0.999$); see Figure 4.

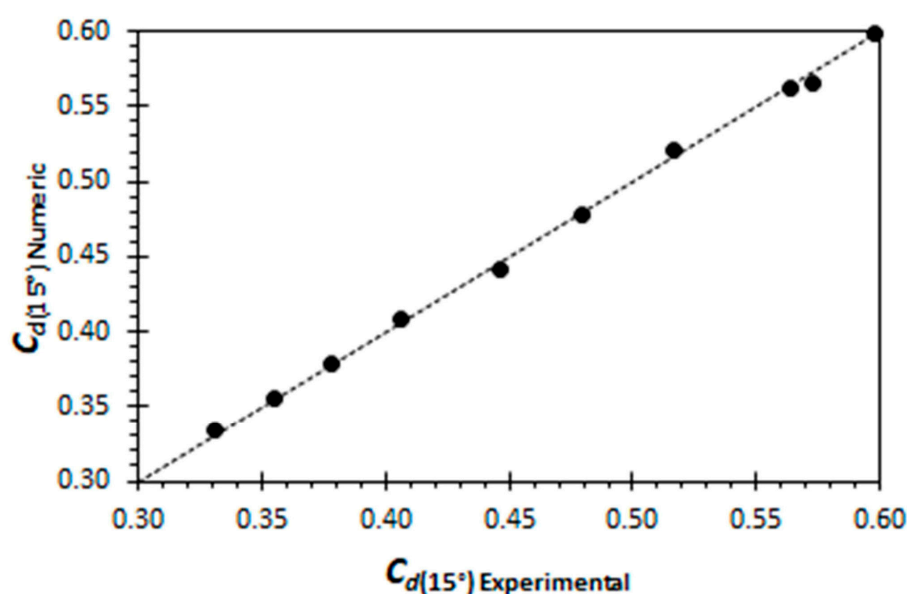


Figure 4. Numerical values vs. experimental values of the discharge coefficient of the trapezoidal labyrinth weir.

The maximum relative percentage error obtained from the discharge coefficient is 1.30% (Figure 5), and the calculated mean absolute error is 0.002.

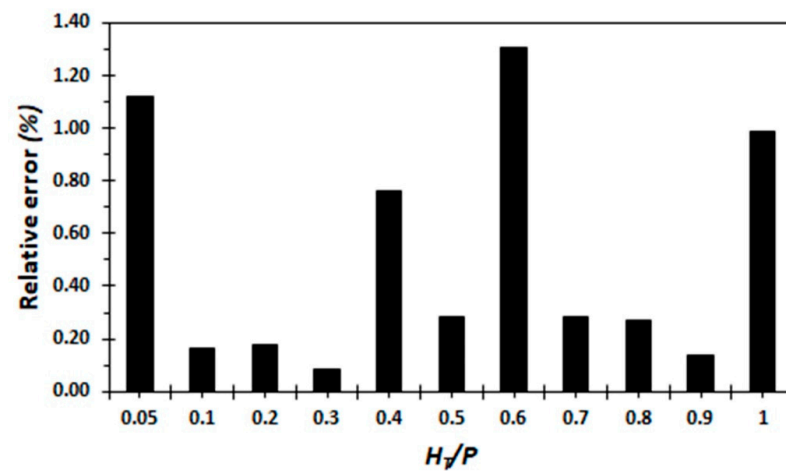


Figure 5. Relative percentage error as a function of H_T/P .

2.4. Description of the Computational Model

The conceptual model consists of a labyrinth weir composed of a semicircular cycle in the opposite direction to the flow, with a diameter equal to half the width of the channel and with a half-rounded profile crest (Figure 6). The labyrinth weir is located within a rectangular channel of the same width as the experimental channel reported by Crookston and Tullis [19]. The geometric characteristics of the labyrinth weir are shown in Table 6.

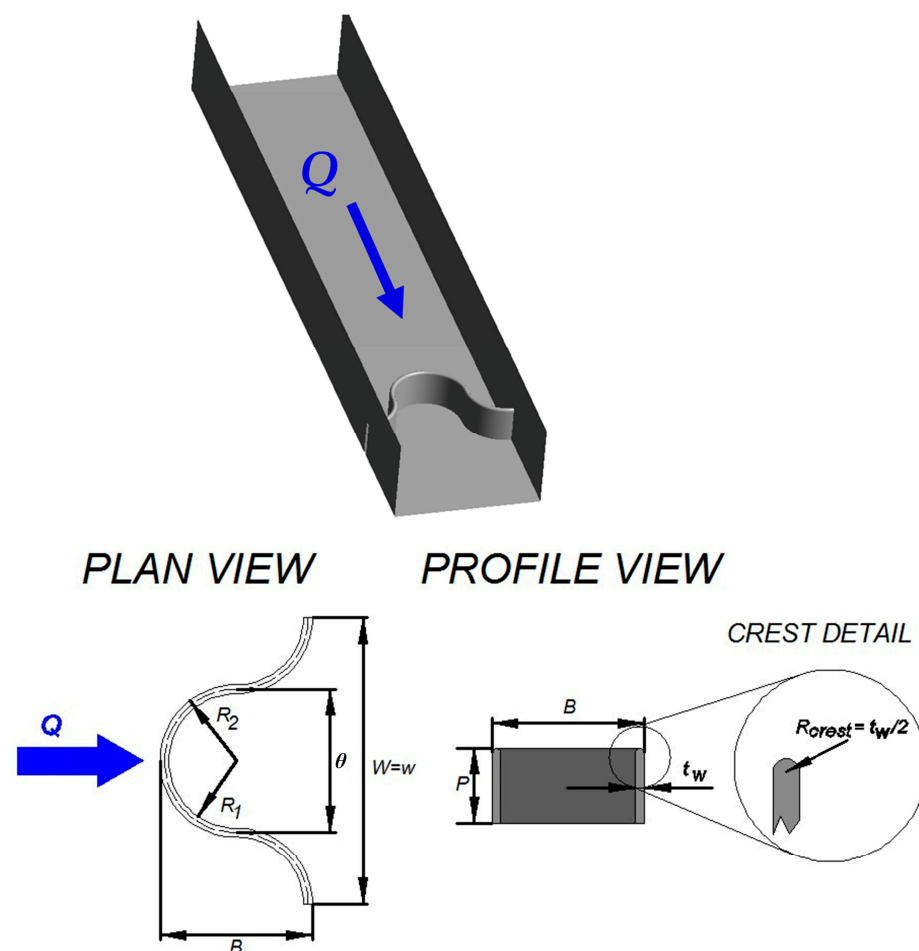


Figure 6. Geometric variables of the conceptual model. Where θ is the diameter of the cycle apex (m) referenced to the center of the weir wall, R_1 and R_2 are the internal and external radii of the weir cycle apex (m), respectively, and R_{crest} is the radius of the weir crest (m).

Table 6. Geometric characteristics of the semicircular labyrinth weir.

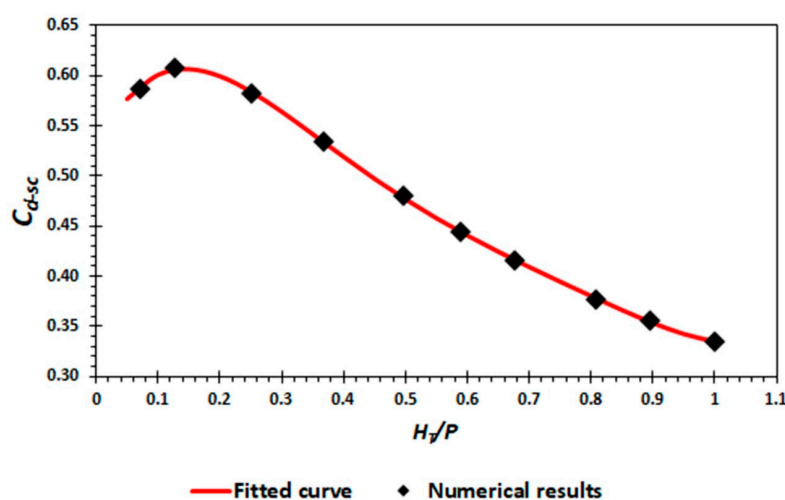
θ (m)	N	L (m)	w (m)	P (m)	W (m)	R_{crest} (m)	R_1 (m)	R_2 (m)	Crest Profile
0.617	1	1.938	1.235	0.305	1.235	0.038	0.289	0.328	Half-round

3. Results

3.1. Discharge Coefficient, Weir and Cycle Efficiency

The numerical results of the hydraulic head allow us to present the results of the discharge coefficients of the semicircular labyrinth weir (C_{d-sc}) for free flow for ratios of $0.05 \leq H_T/P \leq 1$. The results of the discharge coefficient of the semicircular labyrinth weir are shown graphically in Figure 7 as a function of the H_T/P ratio. Additionally, a regressive mathematical model is proposed to estimate the discharge coefficient for $H_T/P \leq 1$ (Equation (9)). It should be noted that this type of model does not stem from a dimensional analysis but rather from a statistically derived relationship based on observed data of the phenomenon. Accordingly, a fifth-degree polynomial was obtained using the least-squares method. The polynomial coefficients were determined so that the sum of the squared errors between the experimental and predicted values was minimized. By applying statistical techniques, such as Pearson's coefficient of determination, the accuracy of the proposed model was evaluated, yielding a value of 0.99. Therefore, it can be inferred that the results are reliable for the estimation of the discharge coefficient.

$$C_{d-sc} = a\left(\frac{H_T}{P}\right)^5 + b\left(\frac{H_T}{P}\right)^4 + c\left(\frac{H_T}{P}\right)^3 + d\left(\frac{H_T}{P}\right)^2 + e\left(\frac{H_T}{P}\right) + f \quad (9)$$

**Figure 7.** Fitted curve of C_{d-sc} as a function of H_T/P for semicircular labyrinth weir.

The coefficients in Equation (9) take the following values: $a = 3.0844$, $b = -9.7568$, $c = 11.8325$, $d = -6.6396$, $e = 1.2869$, $f = 0.5276$.

To better understand the behavior of the discharge coefficient relative to other geometries, comparisons are made between the discharge coefficient of the semicircular labyrinth weir with a half-rounded crest and trapezoidal and linear labyrinth weirs ($\alpha = 90^\circ$) in with a half-rounded and quarter-rounded crest reported by Crookston and Tullis [19] for angles $6^\circ \leq \alpha \leq 35^\circ$, as a function of the H_T/P ratio (Figure 8A,B). Likewise, it is compared with the discharge coefficient of the trapezoidal labyrinth weir with a circular apex and half-rounded crest profile (Figure 8C) reported by Mattos-Villarreal et al. [20] for angles $6^\circ \leq \alpha \leq 20^\circ$.

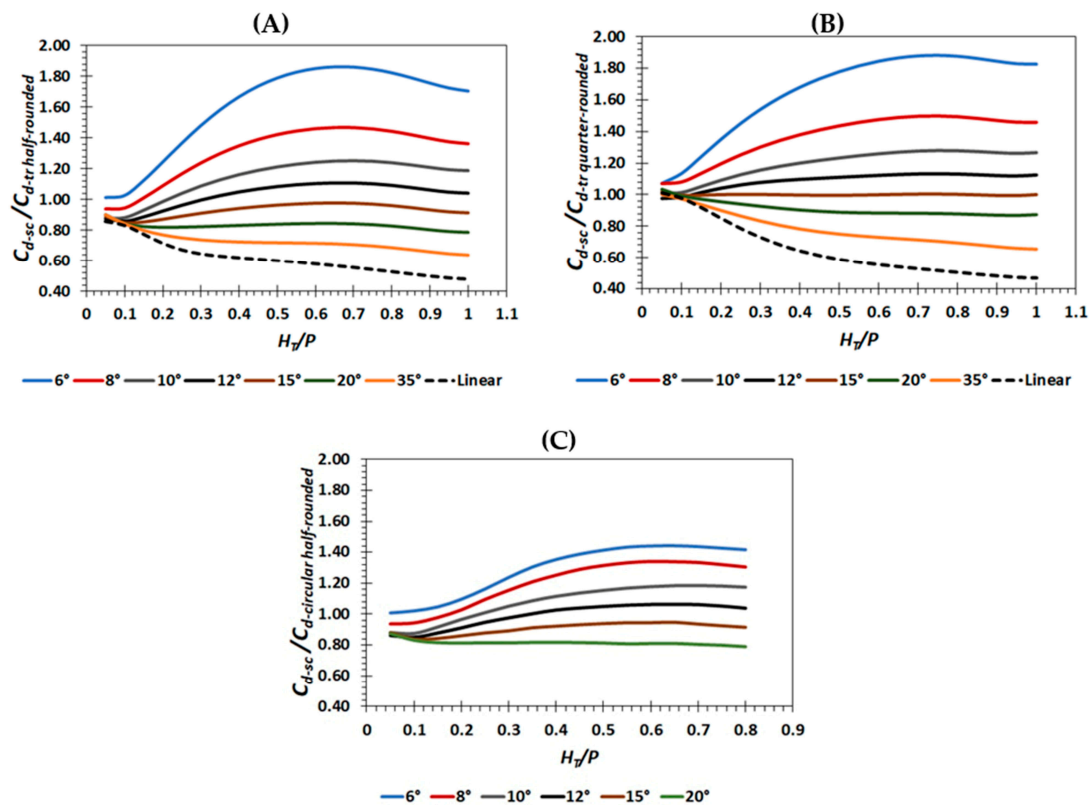


Figure 8. Relation of the discharge coefficient of the semicircular labyrinth weir with a half-rounded crest profile to other geometries. (A) Trapezoidal labyrinth weir with half-rounded crest profile, (B) trapezoidal labyrinth weir with quarter-rounded crest profile, (C) trapezoidal labyrinth weir with circular apex.

3.2. Weir and Cycle Efficiency

Equation (10), known as the efficiency of the labyrinth weir ($\mathcal{E}'$), shows the advantage of increasing the length of the labyrinth weir in relation to a linear weir [24].

$$\mathcal{E}' = \frac{C_d(\alpha)}{C_d(90^\circ)} \frac{L}{W} \quad (10)$$

Equation (11) allows the efficiency of the labyrinth spillway cycle ($\mathcal{E}''$) to be calculated, which is useful for operation with small hydraulic heads [8].

$$\mathcal{E}'' = C_d(\alpha) \frac{L_{\text{cycle}}}{w} \quad (11)$$

The graphs in Figure 9A,B represent the efficiency of the weir and the cycle, respectively.

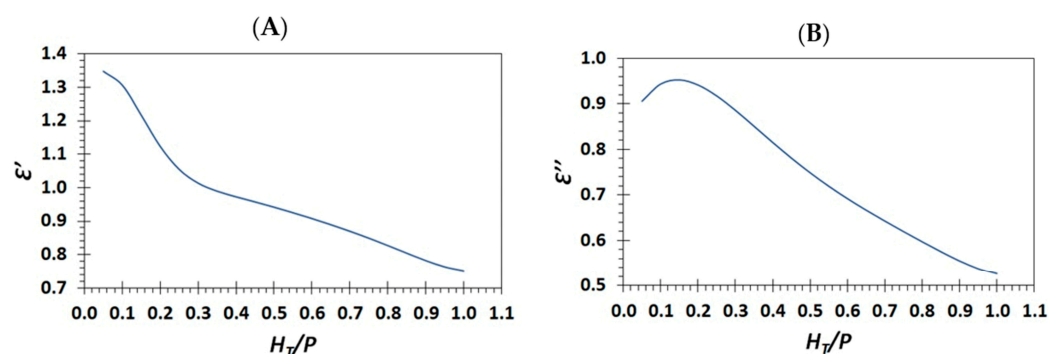


Figure 9. (A) Semicircular labyrinth weir efficiency. (B) Semicircular cycle efficiency.

3.3. Design Sequence for the Semicircular Labyrinth Weir

The design procedure is illustrated through an example based on data from a physical model of the Standley Lake labyrinth weir, located near Denver, and tested at the Utah Water Research Laboratory (UWRL) [48]. Table 7 shows the design of the Standley Lake labyrinth weir presented by Tullis et al. [16]. The crest profile is of the quarter-round type, the weir has a height of 3.05 m and operates with a head of 1.98 m for a flow rate of 1538 m³/s.

Table 7. Standley Lake labyrinth weir design (adapted from Tullis et al. [16]).

Concept	Symbol	Value-Unit	Observations
(i) Initial data			
Flow rate	Q	1538 m ³ /s	Input
Reservoir elevation	–	1680.91 m.a.s.l.	Input
Approach channel elevation	–	1675.75 m.a.s.l.	Input
Crest elevation	–	1678.80 m.a.s.l.	Input
Total head	H_T	1.975 m.a.s.l.	–
(ii) Assumed data			
Estimated inlet loss at Q	–	0.13 m	Estimated
Number of cycles	N	13	$3 \leq w/P \leq 4$
Crest height	P	3.05	$P \approx 1.4 H_T$
Angle of side legs	α	8°	$8^\circ \leq \alpha \leq 16^\circ$
(iii) Calculated data			
Thickness of wall	t_w	0.51 m	$t_w \approx P/6$
Inside width at apex	A	0.95 m	$t_w \leq A \leq 2t_w$
Outside width of apex	D	1.83 m	–
Head water ratio	H_T/P	0.65	–
Discharge coefficient	C_d	0.3255	–
Effective crest length	L	576.6 m	–
Length of apron	B	22.72 m	–
Actual length of side leg	l_c	22.45 m	–
Total length of walls	L_1	620 m	–
Cycle width	w	9.42 m	–
Weir width	W	123 m	–
Length of linear weir for same flow	–	249 m	–
Vertical aspect ratio	w/P	3.09	$3 \leq w/P \leq 4$
(iv) Concrete volume			
Wall concrete volume	–	964.41 m ³	–
Apron concrete volume	–	1425.22 m ³	–
Total	–	2389.63 m ³	–

The flowchart in Figure 10 describes the design sequence for the semicircular labyrinth weir, and Table 8 presents a spreadsheet of the design for the semicircular labyrinth weir at Standley Lake, following the proposed design sequence.

Table 8. Spreadsheet for the design of the semicircular labyrinth weir.

Concept	Symbol	Value-Unit	Equations and Limits
(i) Input data			
Desing flow	Q	1538.00 m ³ /s	
Weir width	W	123.00 m	
Weir height	P	3.05 m	
Upstream total head	H_T	1.98 m	
(ii) Definition of cycle width (w) and number of cycles			
Cycle width	w	9.15 m	$w = 3 P$
Number of cycles	N	13.00	$N = W/w$
New cycle width	w	9.46 m	$w = W/N$
Vertical aspect ratio	w/P	3.10	$2 \leq w/P \leq 4$
(iii) Calculation of geometric variables, weir and cycle efficiencies			
Geometric variables			
Head water ratio	H_T/P	0.65	$0.05 \leq H_T/P \leq 1.0$
Labyrinth weir discharge coefficient	C_{d-sc}	0.425	$C_{d-sc} = f(H_T/P)$. Equation (9)
Total centerline length of weir	L	441.14 m	$L = 1.5 Q/[Cd H_T^{1.5} (2 g)^{0.5}]$
Wall width	t_w	0.38 m	$t_w = P/8$
Crest radius	R_{crest}	0.19 m	$R_{crest} = t_w/2$
Diameter of the cycle apex	θ	4.73 m	$\theta = w/2$
Internal radius of the apex	R_1	2.17 m	$R_1 = (w - P/4)/4$
External radius of the apex	R_2	2.56 m	$R_2 = (w + P/4)/4$
Length of apron	B	5.11 m	$B = w + P/4$
Weir and cycle efficiencies			
Magnification ratio	M	3.59	$M = L/W$
Linear weir coefficient discharge	$C_{d(90^\circ)}$	0.75	$C_{d(90^\circ)} = 1/[-8.609 + 22.65 \times (H_T/P) + 1.812/(H_T/P)] + 0.6375$ [19]
Weir efficiency	$\mathcal{E}'$	2.03	$\mathcal{E}' = C_{d-sc} M / C_{d(90^\circ)}$
Cycle efficiency	$\mathcal{E}''$	1.53	$\mathcal{E}'' = C_{d-sc} M$
(iv) Submergence [18]			
Downstream total head	H_d	0.98 m	
Head ratio	H_d/H_T	0.50 m	
Submergence upstream total head	H^*	2.11 m	
Submergence level	S	0.46	$S = H_d/H^*; 0 \leq S \leq 1$
Submerged weir discharge coefficient	C_{d-sum}	0.385	$C_{d-sum} = C_{d-sc} (H_T/H)^{1.5}$
(v) Volume concrete			
Weir concrete volume	–	512.96 m ³	Volume = $L P t_w$
Apron concrete volume *	–	239.72 m ³	Volume = $W B t_w$
Total	–	752.68 m ³	

Note: * Assumed that apron thickness is the same as wall.

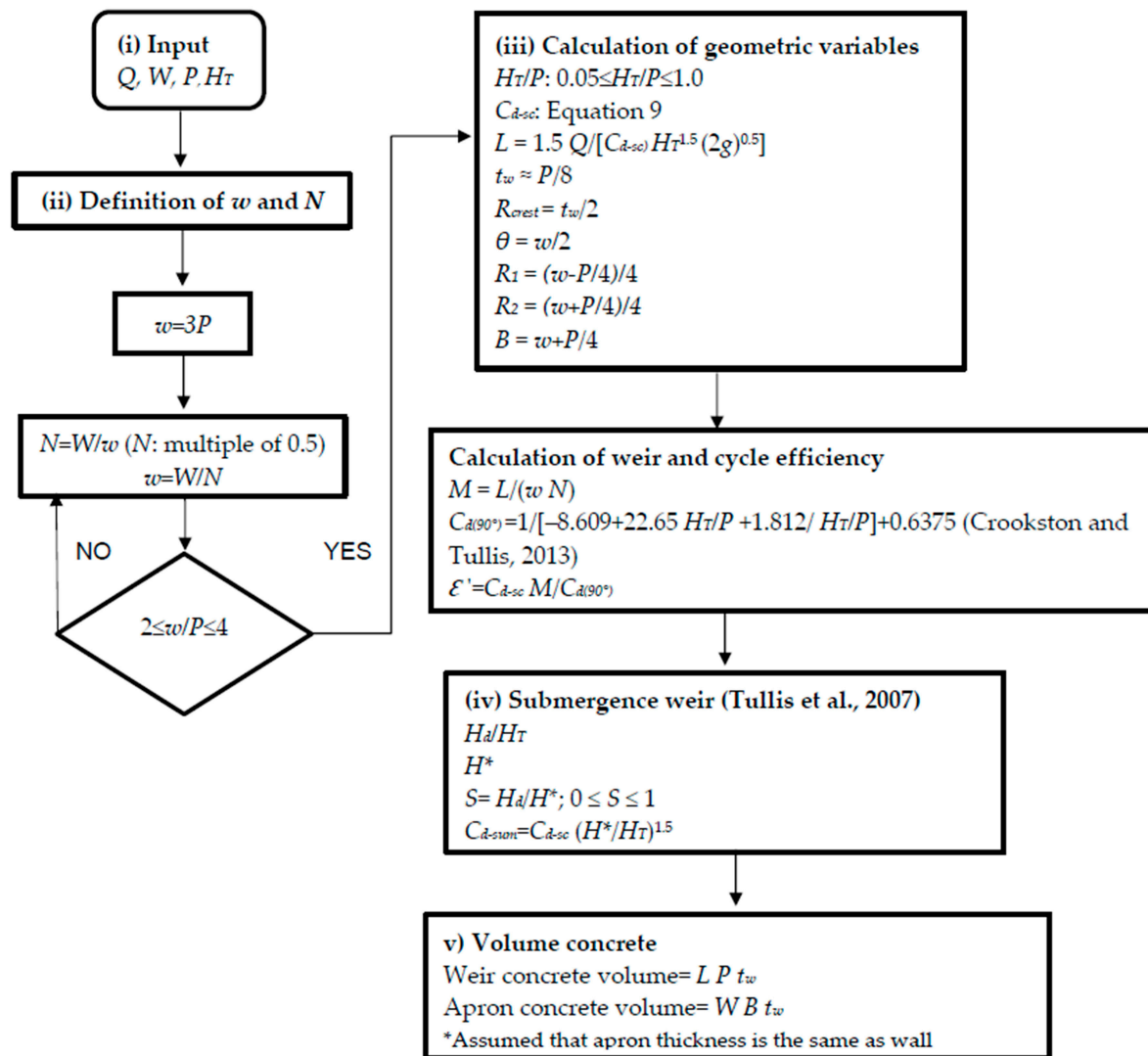


Figure 10. Design sequence for the semicircular labyrinth weir [18,19].

4. Discussion

4.1. Discussion of Discharge Coefficient

The hydraulic behavior of a labyrinth weir is a function of its discharge coefficient and depends on the complex geometry of the hydraulic structure and the discharge flow conditions. The numerical results obtained for the discharge coefficient represent an approximation of those that would be obtained from a physical model; therefore, the curve in Figure 7 can be considered reliable for determining the discharge coefficient. The discharge coefficient C_{d-sc} reaches its maximum values when $0.05 \leq H_T/P \leq 0.25$, and the nappe adhered to the weir wall. Its maximum value $C_{d-sc} = 0.61$ is obtained when $H_T/P = 0.13$ (Figure 11A). However, it gradually decreases when the $H_T/P > 0.25$, and aeration occurs beneath the nappe.

When $H_T/P > 0.8$, the slope of the curve in Figure 7 becomes gentler, local submergence occurs at the upstream apex, the nappe thickens, and no bubbles or air pockets are observed beneath the nappe. Near the weir wall, the flow is turbulent, and helical streamlines are generated. Under these conditions, the nappe is in a state of drowning (Figure 11B).

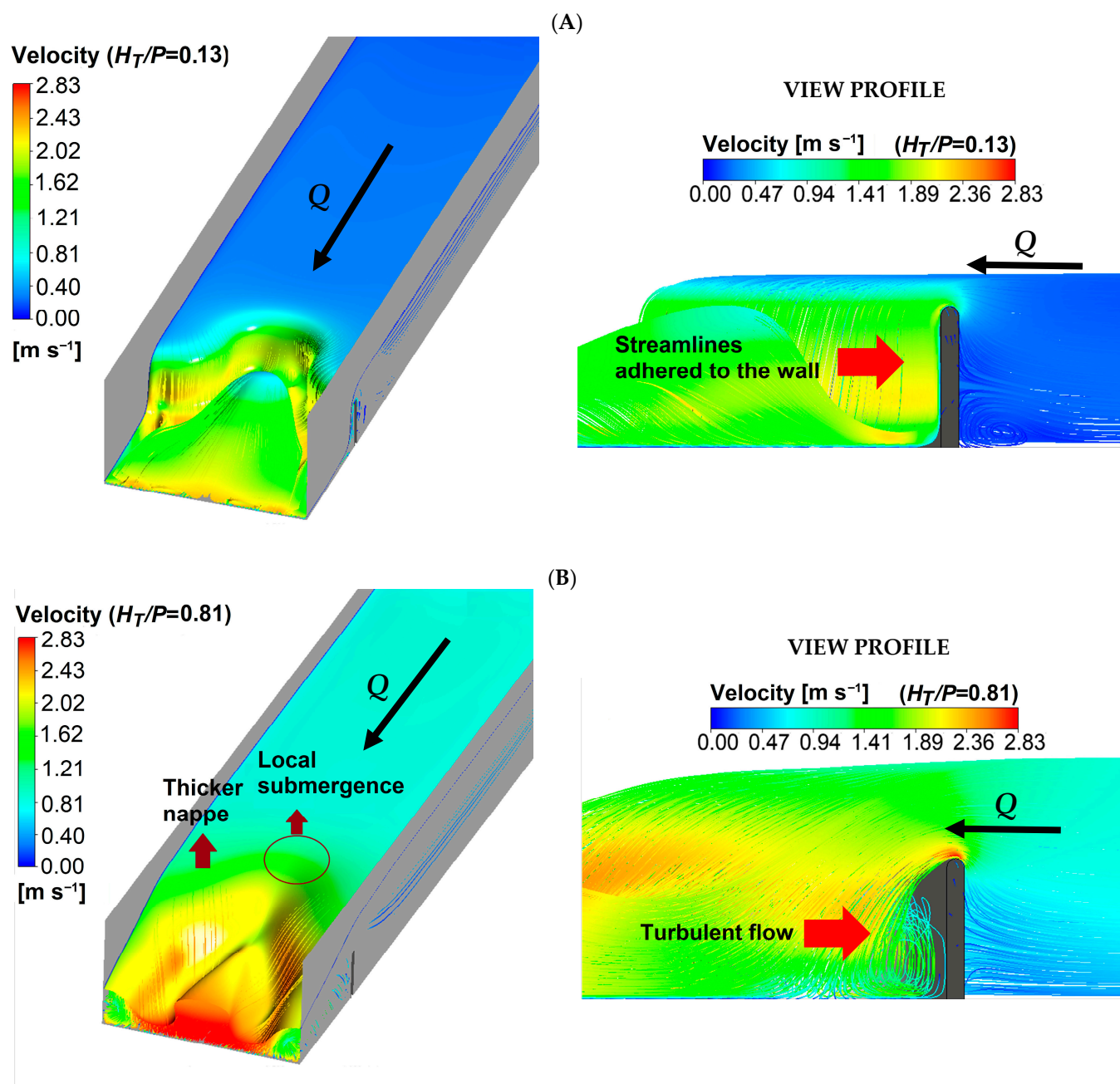


Figure 11. Nappe aeration. (A) Nappe adhered to the wall. (B) Nappe drowned.

The C_{d-sc}/C_{d-tr} ratio for quarter-round and half-round crest profiles shows that the discharge coefficient of the weir with semicircular cycles is 1% to 86% higher than that of the trapezoidal weir with a half-round crest profile and straight apex when $\alpha \leq 12^\circ$. The semicircular labyrinth weir has a higher discharge coefficient than the trapezoidal labyrinth weir when $H_T/P \geq 0.05, 0.15, 0.25$, and 0.35 when $\alpha = 6^\circ, 8^\circ, 10^\circ$, and 12° , respectively. Its maximum values for the C_{d-sc}/C_{d-tr} ratio are reached when $H_T/P \approx 0.65$, which is when the flow of the semicircular labyrinth weir is partially aerated. In the case of a trapezoidal weir with a quarter-round crest profile and straight apex, the discharge coefficient of the weir with semicircular cycles is 1% to 88% higher when $\alpha \leq 12^\circ$. When $\alpha = 15^\circ$, both weirs have similar discharge coefficient values; their maximum values are reached when $H_T/P \geq 0.75$, and the flow in the semicircular cycle weir goes from being partially aerated to drowned. However, unlike the half-round crest profile, the C_{d-sc}/C_{d-tr} ratio values of the quarter-round crest profile decrease slightly after reaching their maximum value, remaining

almost constant when $10^\circ \leq \alpha \leq 20^\circ$. The discharge coefficient of the semicircular labyrinth weir with respect to the trapezoidal weir with a circular apex is up to 44% higher when $\alpha = 6^\circ$, and when $\alpha = 12^\circ$ and $H_T/P \geq 0.35$, it is up to 6% higher. Conversely, when $\alpha = 20^\circ$ it is up to 20% lower. In all cases when $\alpha \leq 12^\circ$, $C_{d-sc} > C_{d-tr}$, but the cycles of the trapezoidal weir with a straight and circular apex have the advantage of having a greater weir length. However, the narrow space between the apexes upstream of the trapezoidal labyrinth weir causes interference from the nappe and local drowning at the apex upstream of the weir, which reduces its discharge coefficient. In the case of linear weirs, both have similar discharge coefficient behavior from $H_T/P \geq 0.45$, and are more efficient than the semicircular labyrinth weir, 39% more efficient when $H_T/P = 0.45$ and up to 53% more efficient when $H_T/P = 1$. However, the discharge coefficient of the linear weir with a quarter-round crest behaves similarly to the semicircular labyrinth weir for $0.05 \leq H_T/P \leq 0.1$.

4.2. Discussion of Weir and Cycle Efficiency

The curves in Figure 9 represent a numerical approximation of the physical behavior of the weir and cycle efficiencies. From a physical perspective, the maximum efficiency of the weir and the cycle occurs at small values of H_T/P . The weir efficiency represented by the curve in Figure 9A shows an immediate decrease in slope when $0.1 \leq H_T/P \leq 0.3$. When $H_T/P = 0.3$, there is an inflection point in the curve, the slope is gentle, and the weir efficiency decreases slowly. The weir cycle efficiency has higher values when $H_T/P \leq 0.25$. Its maximum cycle efficiency value coincides with the maximum discharge coefficient value of the weir when $H_T/P = 0.13$ and the nappe adhered to the weir wall.

4.3. Discussion of the Design Sequence

The type of weir crest profile affects the weir discharge efficiency. A rounded crest profile downstream allows the nappe to remain adhered to the weir wall, favoring discharge capacity. However, research indicates that the half-round profile proposed in this study is more efficient than the quarter-round profile [2,3]. The $w/P = 3$ ratio proposed here takes a middle ground between the limits proposed by Hay and Taylor [12] and Tullis et al. [16] to ensure weir efficiency. On the other hand, the proposed design method considers that the minimum weir wall thickness is equal to $P/8$. A significant decrease in thickness has a significant effect on weir efficiency, separates the nappe from the weir wall, and reduces the discharge coefficient value, contrary to what would happen with a thicker wall, which prevents the separation of the nappe.

The number of cycles of the semicircular and trapezoidal labyrinth weirs is identical; however, the length of the semicircular weir decreases 29% in relation to the length of the trapezoidal labyrinth weir. Similarly, the length of platform B decreases from 22.72 m to 5.11 m, resulting in less concrete material and construction cost. In the proposed design sequence, a preliminary economic analysis can be performed by comparing the volume of concrete required for different numbers of cycles. The volume of concrete decreases with increasing cycles, while the cost of the channel increases with increasing W . Topographical conditions limit the width of the weir. Finally, the hydraulic design of the weir must be evaluated in relation to the final cost of its construction.

4.4. Pressure Between the Walls of the Weir

Figure 12 shows the reference plane where the pressure distribution maps are obtained. The pressure distribution maps between the walls of the downstream weir cycle (Figure 13) suggest the presence of subatmospheric pressure zones. Indeed, along with the gradual increase in the H_T/P ratio, the subatmospheric pressure and its area increase, approaching the weir wall simultaneously with the development of turbulent flow.

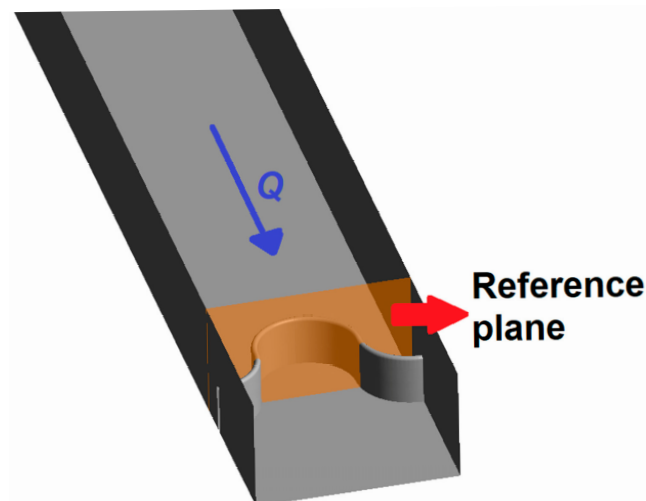


Figure 12. Reference plane for pressure distribution maps.

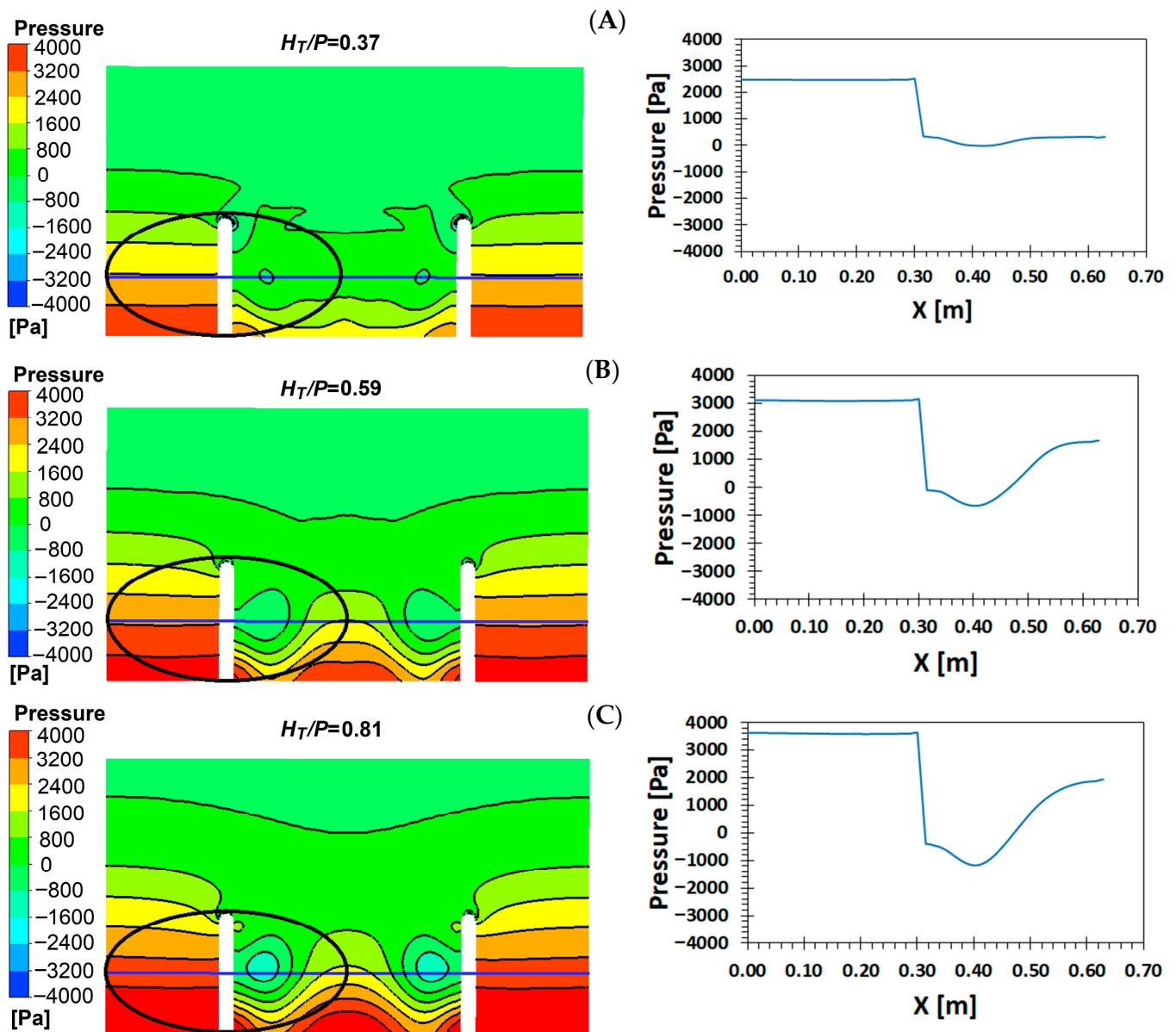


Figure 13. Pressure distribution. (A) $H_T/P = 0.37$, (B) $H_T/P = 0.59$, (C) $H_T/P = 0.81$.

The geometry of the weir has a direct effect on the behavior of the nappe and the occurrence of subatmospheric pressures. For low hydraulic heads, the weir is more efficient, and the nappe remains adhered to its walls. As the head over the weir increases, the nappe separates from the wall, and subatmospheric pressures develop. In Figure 13A, the nappe begins to detach from the weir wall, generating small areas of subatmospheric pressure; at this stage, the nappe is in an aerated regime, and the weir discharge coefficient decreases. With a higher hydraulic head, the nappe transitions from an aerated to a partially aerated state (Figure 13B); the area and magnitude of the subatmospheric pressure increase near the weir wall, while air bubbles are unstable, oscillating between the apexes of the weir wall, and can be removed and reappear. Finally, for even higher hydraulic heads, the nappe aeration changes from partially aerated to a drowned regime (Figure 13C), in which the air bubbles are removed and subatmospheric pressure near the weir wall increases, although its area decreases.

5. Conclusions

This study examined the discharge coefficient behavior of a labyrinth weir with a semicircular cycle and conducted a comparative analysis against previously reported weir discharge coefficients in the literature. For this purpose, numerical modeling of flow over the experimental weir was first performed using computational fluid dynamics (CFD) to validate the model and confirm its consistency with published experimental data.

The discharge coefficient results, based on the H_T/P ratio, were used to develop a regression model for $0.05 \leq H_T/P \leq 1$. A fifth-degree polynomial equation was proposed to estimate the discharge coefficient, yielding excellent agreement with $R^2 = 0.99$. The results indicate that the discharge coefficient attains higher values when $H_T/P \leq 0.25$, highlighting that under this condition, the nappe adhered to the weir wall.

A comparative analysis of the discharge coefficients indicates that the semicircular labyrinth weir exhibits significantly higher discharge capacity relative to its trapezoidal counterparts. Specifically, the semicircular design achieves up to 86% greater discharge than the straight-apex trapezoidal labyrinth weir with a half-rounded crest profile, up to 88% more than the straight-apex trapezoidal labyrinth weir with a quarter-rounded crest profile, and up to 44% above the circular-apex trapezoidal labyrinth weir. For all configurations evaluated, the semicircular labyrinth weir demonstrates superior discharge performance when $\alpha \leq 12^\circ$, while its behavior is comparable to that of the straight-apex trapezoidal labyrinth weir with a quarter-rounded crest profile at $\alpha = 15^\circ$.

A flowchart and accompanying spreadsheet have been developed to facilitate the design process of a semicircular labyrinth weir. The evaluation of optimal weir and cycle efficiency is incorporated as a decision-making aid within the design framework, indicating higher performance at $H_T/P \leq 0.1$ for maximum weir efficiency ($\mathcal{E}'_{\max} = 1.35$) and at $H_T/P \leq 0.25$ for maximum cycle efficiency ($\mathcal{E}''_{\max} = 0.95$). This sequential design approach also considers factors such as weir submergence and concrete volume required for construction, providing preliminary insights into economic feasibility.

It is essential to account for and mitigate subatmospheric pressures during the design phase to prevent nappe instability. The emergence of subatmospheric pressures near the walls of hydraulic structures may pose risks to the safety and structural integrity of weirs.

While constructing this type of semicircular structure presents certain challenges, its geometric configuration proves more economical than a trapezoidal labyrinth weir, attributed to the reduced lengths of both the weir and platform B , resulting in decreased construction expenditure. Additionally, when evaluating hydraulic semicircular performance, the discharge capacity surpasses that of the trapezoidal labyrinth weir for $\alpha \leq 12^\circ$, as the spacing at the upstream apex mitigates the early onset of local submergence, thereby

enhancing overall discharge capability. It is also recommended to supplement numerical modeling with physical modeling to thoroughly validate the hydraulic behavior of the semicircular labyrinth weir.

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Abbreviations

The following abbreviations are used in this manuscript:

A	inside width at apex
a	adjustment factor to obtain the discharge coefficient
B	length of apron
b	adjustment factor to obtain the discharge coefficient
c	adjustment factor to obtain the discharge coefficient
C_d	discharge coefficient
$C_{d-circular}$	discharge coefficient of circular apex weir
C_{d-sc}	semicircular labyrinth weir discharge coefficient
C_{d-sum}	submerged weir discharge coefficient
C_{d-tr}	trapezoidal labyrinth weir discharge coefficient
$C_{d(\alpha)}$	trapezoidal labyrinth weir discharge coefficient
$C_{d(90^\circ)}$	linear weir discharge coefficient
Cov	covariance
D	outside width of apex
d	adjustment factor to obtain the discharge coefficient
e	adjustment factor to obtain the discharge coefficient
F_s	security factor
f	adjustment factor to obtain the discharge coefficient
f_i	control variable
g	acceleration gravity
H_d	downstream total head
H_T	upstream total head
H^*	downstream total head of drowned weir
h	piezometric head
k	turbulent kinetic energy
L	characteristic length of the weir
L_{cycle}	cycle length
l_C	centerline length of sidewall
N	number of cycles
N_C	number of cells
P	weir height
p	order of convergence
Q	design flow
R^2	Pearson's coefficient of determination

R_1	internal radius of the apex
R_2	external radius of the apex
R_{crest}	crest radius
r	mesh refinement ratio
S	submergence level
t_w	weir wall width
W	channel width
w	cycle width
x	coordinate axis in flow direction
Y_{exp}	experimental results
Y_{num}	numerical results
y	coordinate axis in the direction of the channel width
z	coordinate axis in the direction of the channel height
α	angle of sidewall
ΔV_i	volume of the i-th cell
λ	representative cell size
ε	dissipation rate
$\mathcal{E}$	relative error
$\mathcal{E}'$	weir efficiency
$\mathcal{E}''$	cycle efficiency
θ	diameter of the cycle apex
σ_{exp}^2	variance of the experimental results
σ_{num}^2	variance of the numerical results

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