

Article

Estimation of Hydraulic Characteristics of Unsaturated Loess with SEM Images Based on Fractal Theory

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Abstract

The accurate determination of the soil-water characteristic curve (SWCC) and unsaturated hydraulic conductivity is vital across multiple disciplines, including hydrogeology, soil science and geotechnical engineering. Nevertheless, conventional techniques for measuring these unsaturated soil parameters are often laborious and time-consuming, posing significant practical challenges. This research presents a new technique for estimating SWCC and unsaturated hydraulic conductivity by employing fractal theory and utilizing a three-dimensional fractal dimension (D_s). The results revealed that all three soils exhibited fractal characteristics in their particle surfaces, with D_s values of 2.611 for Malan loess, 2.688 for paleosol, and 2.771 for remolded loess. The complexity of the pore structure was in the order of remolded loess > paleosol > Malan loess. The test results of the soil-water characteristic curve indicate that the water storage capacity of the three soils was in the order of paleosol > remolded loess > Malan loess. Compared with the Brooks-Corey fitting curve, the fractal model is feasible in predicting the soil-water characteristic curve. Two models were used to predict the unsaturated hydraulic conductivities of three types of soil, and the results were compared with the measured values. By comparing the R^2 and $RMSE$ values of the fractal model and the Brooks-Corey model, it was found that the fractal model proposed in this paper can effectively predict the unsaturated hydraulic properties of these three types of soil. This study provides a simple and effective alternative for predicting the SWCC and unsaturated hydraulic conductivity of unsaturated soils, with potential applications in various earth science fields.

Keywords: soil-water characteristic curve; fractal dimension; microstructure; water retention characteristics; unsaturated soil



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1. Introduction

The soil-water characteristic curve (SWCC) and the unsaturated hydraulic conductivity of soil are key parameters for characterizing the hydraulic behaviors of unsaturated soil [1–3]. They are of great significance in the prevention and control of geological disasters, the simulation of hydrological processes, and the design of geotechnical engineering [4,5]. As a typical aeolian unsaturated soil in China, the unique vertical joints and macro porous structure of Chinese loess leads to significant nonlinearity and complexity in its hydraulic response [6,7]. Under the superimposed influence of climate change and human activities, disasters such as landslides and collapsibility frequently occur in the Chinese Loess

Plateau region, and their triggering mechanisms are closely related to the water migration process [8–10]. However, traditional SWCC prediction methods (such as empirical formula methods and physical-empirical model methods) are limited by problems such as the vague physical meaning of parameters and large prediction deviations in the high suction range. These shortcomings hinder the ability of such methods to accurately represent the structural properties inherent to natural loess [11–13].

Accurate estimation of unsaturated soil hydraulic conductivity constitutes a fundamental requirement across multiple geoscience disciplines, such as civil/petroleum engineering, hydrogeology, pedology, vadose zone hydrology, and engineering geology [14–16]. The SWCC serves as the cornerstone for analyzing unsaturated soil hydraulic properties, yet conventional laboratory testing method faces significant challenges such as time-intensive procedures, high costs, and susceptibility to equipment limitations and operator bias [17]. To address these constraints, three primary modeling approaches have emerged for SWCC prediction: empirical formulations, physico-empirical models, and fractal geometry techniques [18,19]. Empirical models, exemplified by van Genuchten, Brooks-Corey, and Fredlund & Xing equations, establish mathematical relationships between soil physical indices and SWCC parameters [20–22]. While widely implemented across various soil types, these models suffer from inherent limitations such as ambiguous parameter physicality and constrained applicability. For instance, the van Genuchten model demonstrates superior fitting performance for clays, whereas Brooks-Corey shows particular efficacy for sandy soils. For other soil types, they may generate big prediction errors. The Fredlund & Xing model, despite its computational precision, lacks mechanistic interpretation of parameters, impeding fundamental understanding [23,24]. Physico-empirical models attempt to bridge particle size distribution (PSD) with SWCC characteristics through semi-theoretical frameworks. However, their practical utility remains constrained by notable prediction deviations in high-suction regimes and insufficient generalization capabilities [11,25]. In contrast, fractal geometry approaches have revolutionized SWCC prediction by leveraging the intrinsic self-similarity of soil pore-grain systems [26–29]. Many scholars such as Rieu and Sposito [30], Tyler and Wheatcraft [31], Xu [32] and Tao et al. [33] have established fractal models which effectively linked the microscopic soil structure with the macroscopic hydraulic behavior. These models eliminate empirical parameter arbitrariness by directly linking fractal dimensions (quantifying pore/granular complexity) to measurable soil physical properties. The fractal paradigm offers dual advantages: (1) Enhanced prediction accuracy through physically grounded parameterization of pore topology-water retention relationships; (2) Mechanistic insights into how hierarchical soil structures govern hydraulic responses. By integrating multiscale pore network characteristics, fractal models circumvent experimental variability while elucidating structure-property interactions, emerging as an efficient alternative for concurrent SWCC and unsaturated hydraulic conductivity determination [34,35].

Since its inception by Mandelbrot [36], fractal theory has undergone profound development in soil science, evolving into a multidimensional research framework. In porous media studies, three key parameters, particle fractal dimension, pore fractal dimension, and three-dimensional fractal dimension, have emerged as critical descriptors of structural characteristics, each presenting unique advantages and limitations [37–39]. The particle fractal dimension primarily characterizes the complexity of particle size distribution. Its strength lies in directly reflecting particle size distribution patterns, facilitating understanding of spatial packing characteristics and filling modes [40]. This proves particularly valuable for investigating porous media properties such as packing density and specific surface area. For instance, in soil research, it enables analysis of particle aggregation states to evaluate soil aeration and water retention capacity. However, this approach focuses exclusively

on particulate matter while neglecting pore architecture and particle-pore interactions, thereby failing to comprehensively capture complex physical processes within porous media [41]. Conversely, the pore fractal dimension quantifies the structural complexity of void spaces, encompassing pore size distribution, morphology, and connectivity [37,39,42]. This parameter proves indispensable for analyzing fluid transport mechanisms, particularly in petrophysical studies of seepage patterns. Nevertheless, its principal limitation stems from examining pore networks in isolation, disregarding particle-induced pore constraints and their three-dimensional synergistic effects, resulting in incomplete characterization of complex porous systems [43,44]. In contrast, the three-dimensional fractal dimension demonstrates superior comprehensive capabilities. This holistic parameter integrates the structural complexity of porous media in three-dimensional space, simultaneously accounting for particle distribution patterns, pore network characteristics, and their spatial interdependencies. Such integration enables more authentic and complete representation of actual porous structures [38,45]. Natural loess, as a typical unsaturated porous medium, exhibits heterogeneous particle assemblages and intricate pore networks [6,7]. Compared to conventional fractal dimensions, the three-dimensional approach significantly enhances prediction accuracy for fluid flow patterns by overcoming previous limitations. It enables quantitative characterization of particulate and porous features while objectively reflecting loess physical properties and textural composition [46,47].

In this study, a joint prediction mode was constructed by combining fractal theory and unsaturated soil mechanics to predict the SWCC and unsaturated hydraulic conductivity of three different soils, aiming to explore the ability of the three-dimensional fractal theory in characterizing their hydraulic properties. This research provides a theoretical basis for the risk assessment of geological disasters and the precise simulation of unsaturated seepage in the Chinese Loess Plateau and promoting the in-depth application of fractal theory in environmental geotechnical engineering.

2. Materials and Methods

2.1. Sampling and Preparation

The study area is located in Jingyang County (Xianyang City, Shaanxi Province), in the Guanzhong Plain. The site spans $108^{\circ}29'40''$ – $108^{\circ}58'23''$ E and $34^{\circ}26'37''$ – $34^{\circ}44'57''$ N, covering 780 km^2 of terrain sloping from NW to SE. Well-exposed, complete loess-paleosol sequences characterize the terrace strata (Figure 1a).

The loess layers in the southern plateau of Jingyang are well exposed. The schematic diagram of the typical loess profile in the southern plateau of Jingyang is shown in Figure 1b. The planting soil is located at the top layer, with a thickness of about 2–3 m. Next is the Malan loess, with a thickness of about 10 m. Below the Malan loess, there is a layer composed of about 4.5 m of paleosol and calcium nodules, with the paleosol being light purplish red and about 3.8 m thick. The lower part contains a layer of about 0.7 m of calcium nodules. The alternating sequence of the Middle Pleistocene Lishi loess and paleosol is the main stratum of the southern plateau of Jingyang, with a thickness of approximately 70 m. The Lishi loess is mainly composed of fine particles and has a color of grayish yellow. Some layers have a large number of snail shells distributed in them. The bottom of the paleosol layer contains calcium nodules.

In the study area, the maximum depth affected by water infiltration is approximately 10 m underground. The depth of Malan loess from the ground surface was approximately 2–3 m, and that of the first paleosol layer was approximately 13–15 m (Figure 1b). The depth of the Malan loess layer and the first layer of paleosol is precisely within this range. In engineering projects in the loess regions, Malan loess, which is widely used as a natural foundation and fill material, undergoes structural reformation due to disturbance during

the construction process, thereby significantly altering its hydraulic properties. To study the applicability of fractal theory in loess region, this research takes the undisturbed Malan loess, undisturbed paleosol soil, and remolded loess as the research objects (For convenience, the three types of soils will be collectively referred to as Malan loess, paleosol, and remolded loess in the subsequent sections of this paper). According to the undisturbed soil sample preparation method stipulated in the Chinese Standard for Geotechnical testing method [48], undisturbed Malan loess and undisturbed paleosol were collected from the distinct loess strata with obvious outcrops. The baseline characteristics of the soil samples are provided in Table 1.

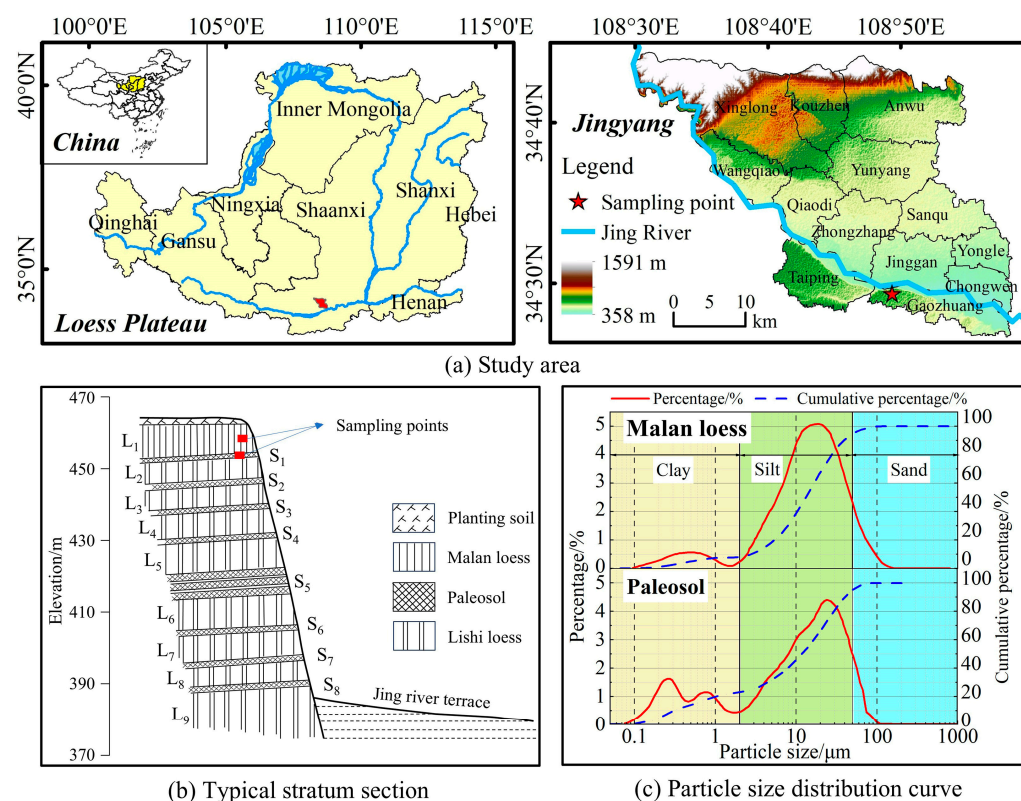


Figure 1. Sampling point location and sample particle size distribution. (a) sampling point location, (b) distribution of strata at the sampling points, (c) particle size distribution curve.

Table 1. Basic characteristics of Malan loess and Paleosol.

Parameters	Malan Loess	Paleosol	Remolded Loess
Nature density ρ (g/cm ³)	1.50	1.78	—
Nature mass moisture content ω (g/g)	0.154	0.140	0.120
Dry density ρ_d (g/cm ³)	1.30	1.56	1.4
Specific gravity G_s	2.70	2.72	2.71
Saturated mass moisture content (g/g)	0.400	0.272	0.349
Saturated volume moisture content θ_s (cm ³ /cm ³)	0.520	0.424	0.489
Void ratio e_0	1.085	0.744	0.935
USCS classification	Silt	Silt	Silt

Grain-size analysis (Figure 1c) revealed distinct differences between Malan loess and paleosol: Malan loess is silt-dominated with low clay content, while paleosol exhibits significantly higher clay content, indicative of contrasting depositional and pedogenic processes [49].

The mineral composition of Malan loess and paleosol is shown in Table 2. Malan loess is usually formed in arid or semi-arid regions, where wind-sediment deposits undergo relatively weak weathering and leaching processes. The primary carbonates and calcium-containing minerals in the sediments, when subjected to leaching, form secondary carbonates that are preserved. Therefore, the carbonate content in loess is relatively high. Paleosol is usually formed under warmer and more humid climatic conditions, which are conducive to the decomposition and transformation of minerals in the soil. Clay minerals are usually gradually formed during the soil formation process through the weathering, leaching, and deposition of parent rocks. In warm and humid climates, these processes are more active, so the content of clay minerals is also relatively high. Additionally, the content of clay minerals is also affected by soil leaching processes. During soil formation, elements that are easily mobile, such as Na, Ca, and Mg, undergo significant leaching loss, while elements with relatively weak mobility, such as Al and Si, are relatively enriched. These processes further influence the formation and distribution of clay minerals, resulting in a higher content of clay minerals in paleosol than in loess.

Table 2. The mineral composition of Malan loess and paleosol (%).

Samples	Quartz	Plagioclase	Feldspar	Illite	Muscovite	Kalbite	Calcite	Hematite
Malan loess	42.2	13.8	12.2	10.8	10.1	7.7	2.5	0.7
Paleosol	43.6	15.4	4.3	16.8	13.2	6.7	0.0	0.0

2.2. Theoretical Background

2.2.1. Fractal Model of SWCC and Unsaturated Hydraulic Conductivity

Fractal theory is an effective method to predict soil hydraulic properties based on soil structure and has made great contributions to the development of unsaturated soil theory and application. According to Xu [32] and Tao et al. [33], the fractal model describing the three-dimensional fractal dimension and SWCC can be described as follows:

$$S_e = \begin{cases} \frac{\theta - \theta_r}{\theta_s - \theta_r} = \left(\frac{\psi_{aev}}{\psi} \right)^{3-D_s}, & \psi \geq \psi_{aev} \\ 1, & \psi < \psi_{aev} \end{cases} \quad (1)$$

where S_e is effective saturation (%), ψ is matrix suction (kPa), θ is soil volumetric water content (cm^3/cm^3), θ_s is soil saturated volumetric water content (cm^3/cm^3), ψ_{aev} is air-entry value (kPa), θ_r is the residual moisture content (cm^3/cm^3) and D_s is the three-dimensional fractal dimension.

Regarding the unsaturated hydraulic conductivity, Mualem [50] proposed the model (Equation (2)) for determining the relative hydraulic conductivity of unsaturated soil as:

$$k_r(\theta) = S_e^{0.5} \left(\int_{\theta_r}^{\theta} \frac{dx}{\psi(x)} / \int_{\theta_r}^{\theta_s} \frac{dx}{\psi(x)} \right)^2 \quad (2)$$

where $k_r(\theta)$ is the relative hydraulic conductivity, and $k_r(\theta) = k(\theta)/k_s$, $k(\theta)$ is the unsaturated hydraulic conductivity (cm/s), k_s is the saturated hydraulic conductivity (cm/s).

To derive the fractal model for determining the relative hydraulic conductivity for unsaturated soils, first, the derivative of Equation (1) yields:

$$d(\theta) = (\theta_s - \theta_r)(D_s - 3)\psi_{aev}^{3-D_s}\psi^{D_s-4}d\psi \quad (3)$$

If $\theta = x$, then $d\theta = dx$. Then, substituting Equation (3) into Equation (2) gives:

$$k_r(\psi) = S_e^{0.5} \left[\frac{\int_{\psi_d}^{\psi} (\theta_s - \theta_r)(D_s - 3)\psi_{aev}^{3-D_s}\psi^{D_s-5}d\psi}{\int_{\psi_d}^{\psi_{aev}} (\theta_s - \theta_r)(D_s - 3)\psi_{aev}^{3-D_s}\psi^{D_s-5}d\psi} \right]^2 \quad (4)$$

where ψ_d is the maximum suction value (kPa). Since ψ_{aev} much less than ψ_d , and $D_s < 3$, $(\psi_{aev}/\psi_d)^{4-D_s}$ is close to 0 and can be neglected. Therefore, Equation (4) can be further simplified as:

$$k_r(\psi) = S_e^{0.5} \left[\left(\frac{\psi_{aev}}{\psi} \right)^{4-D_s} \right]^2 \quad (5)$$

Substitute the expression of S_e (Equation (1)) into Equation (5), and according to the relationship among the relative hydraulic conductivity, unsaturated hydraulic conductivity and saturated hydraulic conductivity ($k_r(\psi) = k(\psi)/k_s$), the mathematical expression of unsaturated hydraulic conductivity based on fractal theory is:

$$\begin{cases} k(\psi) = k_s, & \psi \leq \psi_{aev} \\ k(\psi) = k_s \left(\frac{\psi_{aev}}{\psi} \right)^{9.5-2.5D_s}, & \psi > \psi_{aev} \end{cases} \quad (6)$$

2.2.2. Theory of Three-Dimensional Fractal Dimension Calculation

One of the defining characteristics of the fractal dimension (D_s) of porous media is that it is determined by the measurement scale r . As shown in Figure 2a, as the calculation scale r is gradually increased, the information of the surface undulation of the image will gradually decrease, and the image becomes blurrier. The three-dimensional fractal dimension (D_s) of the soil microstructure was calculated based on the scale-dependent surface area method. Surface areas ($A(r)$) were computed at multiple measurement scales (r), where r was defined as an integer multiple of the SEM image pixel size. Surface area $A(r)$ at each scale was derived from three-dimensionally reconstructed surfaces of SEM images (Figure 2b). This reconstruction process generated triangular meshes representing the imaged surface topography, and the total surface area of the mesh at scale r provided $A(r)$.

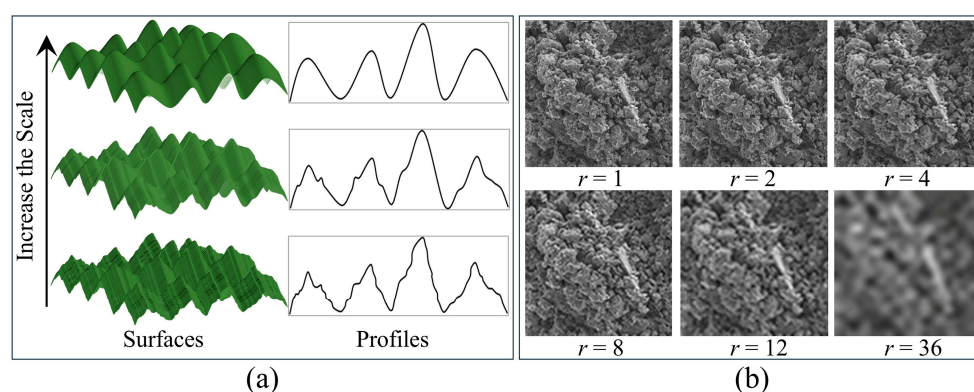


Figure 2. Fractal dimension scale effect (a) illustration of increased surface fractals, (b) SEM image of partially calculated scale.

The fractal dimension D_s is given by the negative slope of the linear regression line in the log-log plot of normalized surface area $A(r)/r^2$ versus scale r (Equation (7)):

$$\lg\left(\frac{A(r)}{r^2}\right) = -D_s \lg r + \lg C_0 \quad (7)$$

The geometric parameters such as the surface area and volume of soil pores and particles are the basis for calculating the fractal dimension. Assuming that the surfaces of microscopic particles (pores) in the soil have fractal properties, the SEM images are processed in three dimensions using Arcgis 10.3, and then the surface area $A(r)$ is calculated by applying the principle of irregular triangular grids. The scale r is taken as an integer multiple of the minimum pixel size of SEM (Figure 2b). As shown in Figure 3a, a plane parallel to the x-y plane is drawn at the lowest point of the three-dimensional image. The space between this lowest plane and the interface is regarded as soil particles, and similarly, the space between the highest plane and the interface is regarded as pores. As shown in Figure 3b, four points A, B, C, and D are selected on the three-dimensional undulating surface. Connecting the coordinate points can obtain two spatial triangles, and the sum of their areas can be regarded as the approximate value of the surface area of the region enclosed by these four points. The volume of the space between this region and any plane can be approximated by the volume between the spatial triangle and the plane. By reducing the spacing between points A, B, C, and D, the approximate value can be gradually approached to the exact value.

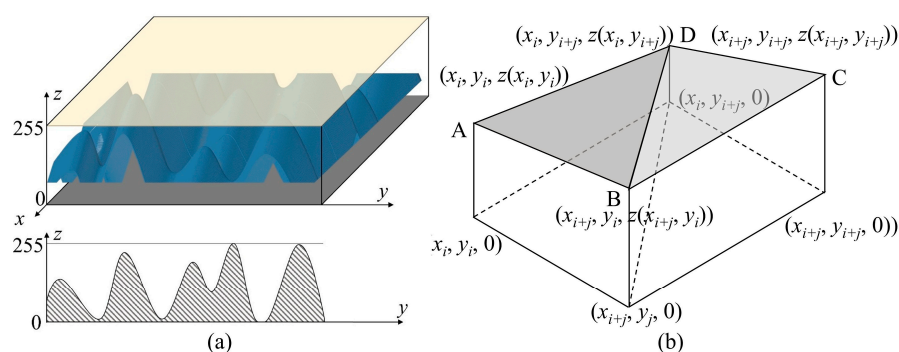


Figure 3. The principle of three-dimensional space surface area calculation. (a) three-dimensional schematic diagram, (b) diagram of spatial triangle coordinates.

In Figure 3b, the sum of the areas of the spatial triangles can be calculated by using the Heron's formula (Equation (8)):

$$A_i = \sqrt{p_i(p_i - |AB|)(p_i - |BD|)(p_i - |AD|)} + \sqrt{q_i(q_i - |BC|)(q_i - |CD|)(q_i - |BD|)} \quad (8)$$

$$p_i = \frac{1}{2}(|AB| + |BD| + |AD|), \quad q_i = \frac{1}{2}(|BC| + |CD| + |BD|) \quad (9)$$

where A_i represents the sum of the areas of the irregular grid spaces of the i th one; $|AB|$ indicates the distance between points A and B in the figure, and so on. The distances between each point are calculated using spatial coordinates, taking $|AB|$ as an example.

$$|AB| = \sqrt{(x_{i+j} - x_i)^2 + (y_j - y_i)^2 + [z(x_{i+j}, y_j) - z(x_i, y_i)]^2} \quad (10)$$

Introduce $\Delta x = x_{i+j} - x_i$, $\Delta y = y_{i+j} - y_i$, and assume that the SEM image consists of $m \times n$ pixels. Then, the three-dimensional surface area $A(r)$ of the SEM image can be calculated by the following equation:

$$A(r) = \lim_{\substack{\Delta x \rightarrow 0 \\ \Delta y \rightarrow 0}} \sum_{i=0}^m \sum_{j=0}^n A_i \quad (11)$$

When calculating the fractal dimension, it is necessary to perform scaling processing on the pixels of the image. Moreover, according to the calculation method of fractal dimension, it can be known that when the soil has fractal characteristics, in the logarithmic coordinate axis, $A(r)/r^2$ and r under different scaling multiples are in a linear relationship. Therefore, the magnification factor of the original image has little influence on this study. In this study, we choose to use an image with a magnification factor of 800 times to calculate the three-dimensional fractal dimension.

2.2.3. Calculation of Key Parameters

The fractal model for characterizing soil-water properties, as expressed in Equation (1), incorporates three principal unknown parameters in addition to the fractal dimension. These include the saturated water content (θ_s), the residual water content (θ_r), and the air entry value (ψ_{aev}). The saturated water content (θ_s , cm^3/cm^3) refers to the water content of soil or rock-soil medium when it is in a fully saturated state (all pores are filled with water), and it is determined by the inundation saturation method [51,52]. The residual moisture content (θ_r , cm^3/cm^3) is an important parameter for the hydraulic characteristics of unsaturated soil and plays a significant role in the study of the hydraulic and mechanical properties of unsaturated soil. This paper adopts the natural air-drying method to determine the residual moisture content of soil samples. ψ_{aev} is the critical point on the SWCC, indicating that the maximum pores in the soil are unable to resist the applied suction and thus lose water [53]. Based on the Young-Laplace equation [54], the expression of ψ_{aev} is:

$$\psi_{\text{aev}} = \frac{2T_s \cos \alpha}{R} \quad (12)$$

where T_s signifies the surface tension of the liquid (typically 0.072 N/m), and α represents the contact angle (ψ_{aev} indicates that the largest pores in the soil cannot withstand the applied suction force. At this point, the soil is nearly saturated. it is usually assumed that $\alpha = 0^\circ$ and $\cos \alpha = 1$), and R is the maximum pore radius of the soil and can be obtained from the quantitative analysis of SEM images.

The IPP 6.0 software was used for the quantitative analysis of SEM image. Initially, the original images underwent brightness and contrast enhancement to improve the discernibility of pore and particle boundaries. A median filter was then applied to correct isolated pixel errors. This was followed by the use of a low-value filter to remove random noise, thereby simplifying and increasing the accuracy of image segmentation. Finally, pore boundaries were automatically detected based on the HSI color model, and the image was binarized. In the resulting binary image, each pixel is represented as either white (soil particles) or black (pores). The detailed steps can be found in Xu et al. [55] and Wen et al. [56].

2.2.4. Error Analysis

The root mean square error ($RMSE$) and the coefficient of determination (R^2) are commonly used evaluation metrics in regression models, which are employed to assess the predictive performance and fitting effect of the model. R^2 represents the proportion of variance in the dependent variable that is predictable from the independent variable [57]. The closer the coefficient of determination is to 1, the better the model fits. The definition of the coefficient of determination is:

$$R^2 = 1 - \frac{SSR}{SST} \quad (13)$$

where SSR is the sum of squared residuals, representing the sum of the squared deviations between the model's predicted values and the actual values, reflecting the unexplained

variation. *SST*, the total sum of squares, quantifies the total variation in the observed data. It is computed as the cumulative total of squared differences between each data point and the global mean of the response variable, reflecting the total variation in the data. *SSR* and *SST* can be calculated using the following formulas:

$$SSR = \sum_{i=1}^n (y_i - Y_i)^2 \quad (14)$$

$$SST = \sum_{i=1}^n (y_i - \bar{Y})^2 \quad (15)$$

where y_i is the observed value of the i th data point, Y_i is the predicted value of the i th data point by the model, and $\bar{Y}$ is the mean of all the predicted value.

RMSE is the average value of the squared errors between the predicted values of the regression model and the actual observed values. The calculation formula is as shown in Equation (9). The smaller the value of *RMSE*, the more accurate the model's prediction.

$$RMSE = \sqrt{\frac{\sum_{i=1}^n (y_i - Y_i)^2}{n}} \quad (16)$$

2.3. Laboratory Test

2.3.1. SEM Test

Soil microstructure (pores and particles) was examined using environmental scanning electron microscopy (SEM) on sample fresh surfaces [58]. Undisturbed soil samples were trimmed to cylinders (approximately 2 cm height $\times$ 1 cm diameter) and freeze-dried. To stabilize particles and obtain a fresh fracture surface, samples were prepared following Xu et al. [7] and Luo et al. [59]. Samples were then gold-coated and imaged using an FEI Nova NanoSEM 230 (Thermo Fisher, Waltham, MA, USA) scanning electron microscope.

2.3.2. Measurement of SWCC and Unsaturated Hydraulic Conductivity

To validate the fractal model of the SWCC detailed above, measuring and determining the SWCC is needed. In this study, the filter paper method was used to measure the SWCC of soil samples during the humidification process. The operation of filter paper method of measuring SWCC includes sample drying and weighing, sample humidification, sample water balance, measurement and weighing, etc. [60,61]. Two soil samples with a quality difference in no more than 3 g were selected as one group for water addition. The soil samples were humidified using the water film transfer method. The soil samples after adding water were wrapped in protective films and sealed. Two pre-prepared soil samples with the same moisture content were placed in the same sealed bag. They were left to stand in a constant temperature and humidity environment for 3 days to allow the moisture in the soil samples to diffuse evenly. After 3 days, the weights of the ring knife and the soil samples were re-measured to calculate the actual moisture content of the soil samples. If the error between the actual moisture content and the pre-determined moisture content does not exceed 3%, then it is considered that the soil sample has met the test standard. When the water in the filter paper and the soil reaches the equilibrium state, the matrix suction can be obtained through the weighing of the filter paper and the conversion of the determination curve, and the water content can be obtained through the mass calculation of the upper and lower soil samples. In this paper, Whatman's No. 42 filter paper, which is widely used, was selected. And the rate curve equation of this filter paper is shown in Equation (17) [61].

$$\begin{cases} \log \psi = 5.327 - 0.0779w_f & w_f < 45.3\% \\ \log \psi = 2.412 - 0.0135w_f & w_f \geq 45.3\% \end{cases} \quad (17)$$

where ψ is matric suction (kPa), w_f (g/g) is the mass moisture content of the filter paper. The specific operation procedures can be referred to in ASTM [61] and Li et al. [62].

Transient profile method is an important technical means for modern research on unsaturated soil mechanics [62,63]. In this study, the filter paper method and the transient profile method were combined to construct a set of high-precision transient hydraulic parameter testing system. The test device is shown in Figure 4a, consisting of the upper water supply system and the lower soil column. The entire soil column was placed on a semi-circular acrylic stand to ensure its stability. The entire device is enclosed in an acrylic cylinder to prevent evaporation and the environmental temperature is 20 °C.

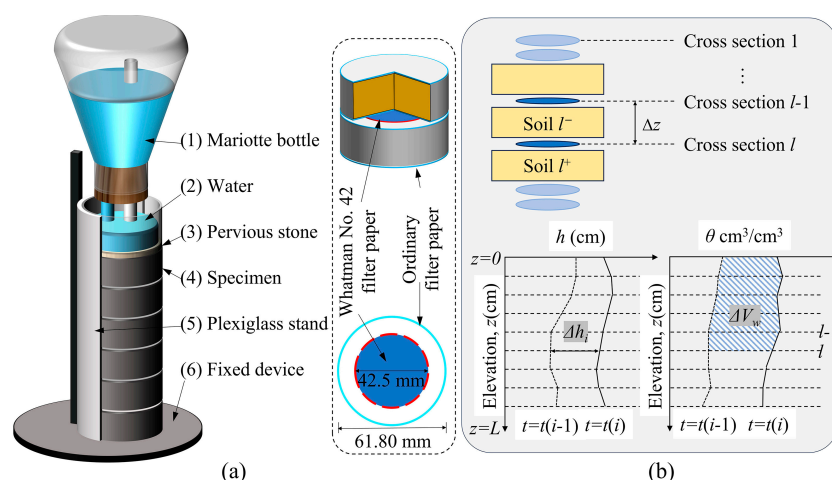


Figure 4. Experimental setup and experimental principle of transient profile method. (a) test device of the transient profile test device, (b) experimental principle of transient profile method.

The volume water content profiles and water head profiles along the soil column at different times were measured during the test. Using these profiles, the unsaturated hydraulic conductivity curve can be calculated (Figure 4b). The detailed process of calculating the unsaturated hydraulic conductivity using transient profile method can be found in Li et al. [62] and Ngyuen et al. [63].

3. Results and Discussion

3.1. Three-Dimensional Fractal Dimension

Based on the scale effect of the three-dimensional surface area of the pores, set the calculation scale $r = 1, 2, 3, 4, 6, 8, 9, 12, 24, 36$. Utilizing the principle of irregular triangular grids, the particle (pore) surface area $A(r)$ was calculated. The results were shown in Figure 5. Figure 5 shows the double logarithmic relationship curve between the calculated values of particle (pore) surface area and the measurement scale ($A(r)/r^2 - r$). The calculated values of surface area decrease significantly with the continuous increase in the calculation scale. And the three soil samples can be linearly fitted, confirming that the microscopic particle (pore) surface areas of the three soil bodies have fractal properties. The fractal dimension D_s is the negative slope of the fitted straight line and the fitting parameters were shown in Table 3. The fractal dimensions D_s of Malan loess, paleosol and remolded loess are 2.611, 2.688 and 2.771, respectively. As shown in Figure 5 and Table 3, for the same type of soil, the obtained D_s values from different shooting areas are relatively close, with differences in less than 1% among the three. The influence of sampling area variability on D_s can be disregarded.

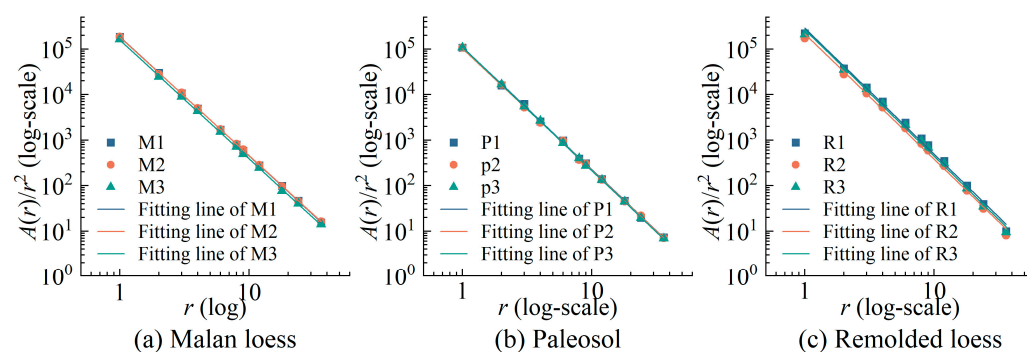


Figure 5. Linear fitting of three-dimensional fractal dimensions.

Table 3. The fitting results of the fractal dimensions of the three types of soil.

Samples		Expression of Fitting Line	r^2	D_s	$\bar{D}_s$
Malan loess	M1	$y = -2.615x + 5.268$	0.997	2.615	2.611
	M2	$y = -2.606x + 5.262$	0.998	2.606	
	M3	$y = -2.612x + 5.195$	0.993	2.612	
Paleosol	P1	$y = -2.698x + 5.033$	0.996	2.698	2.688
	P2	$y = -2.686x + 5.012$	0.997	2.686	
	P3	$y = -2.680x + 5.041$	0.996	2.680	
Remolded loess	R1	$y = -2.761x + 5.342$	0.996	2.761	2.771
	R2	$y = -2.770x + 5.315$	0.996	2.770	
	R3	$y = -2.781x + 5.246$	0.997	2.781	

Fractal dimension is an important parameter for quantifying the complexity and heterogeneity of soil pore structure. Its value reflects the self-similarity characteristics of the pore space. The complexity of a pore system is directly indicated by its fractal dimension; values closer to 3 denote greater intricacy, manifested through a broader pore size distribution and more irregular pore morphologies [64]. Therefore, in terms of the complexity of pore networks, remolded loess > paleosol > Malan loess. During the process of soil formation, Malan loess mainly undergoes physical weathering and retains the sorting characteristics of the original aeolian sediments. The particle size is relatively uniform, mainly composed of silt, lacking fine clay and organic matter. The pore structure is simple and highly interconnected, presenting a relatively regular pore network with a low fractal dimension [65,66]. Paleosol has undergone long-term chemical weathering and biological modification, resulting in the decomposition of primary minerals into secondary clay minerals, an increase in clay content, and the presence of organic matter. The pore system becomes more complex, featuring the coexistence of micropores, mesopores and macropores at multiple levels, and the fractal dimension increases [42,67]. For remolded loess, during the remolding process, the soil structure of loess changes. The primary large pores are destroyed, the contact between particles becomes tighter, and the number of small pores increases [68]. In addition, compaction occurred during the remolding process. The compaction process causes changes in the soil pore structure, thereby increasing the irregularity of the pore structure and maximizing the fractal dimension [68].

3.2. Prediction Curve of Soil-Water Characteristics Based on Fractal Theory

3.2.1. The Test Data of SWCC

The traditional Brooks-Corey equation was used to fit the SWCC test data measured by the filter paper method. The parameter ‘n’ characterizes the rate of inflection observed within the transient zone and serves as an indicator of how uniformly distributed the pore sizes are. Values closer to unity suggest a more homogeneous pore structure, whereas lower

values indicate greater variability. A summary of the fitted parameters for the Brooks-Corey model is provided in Table 4, and the corresponding model curves are displayed in Figure 6.

Table 4. The fitting parameters of the Brooks-Corey model for the SWCC.

Samples	θ_r (cm ³ /cm ³)	ψ_{aev} (kPa)	n	r^2
Malan loess	0.0708	7.203	3.203	0.889
Paleosol	0.0827	6.541	3.026	0.854
Remolded loess	0.0623	2.74	2.726	0.962

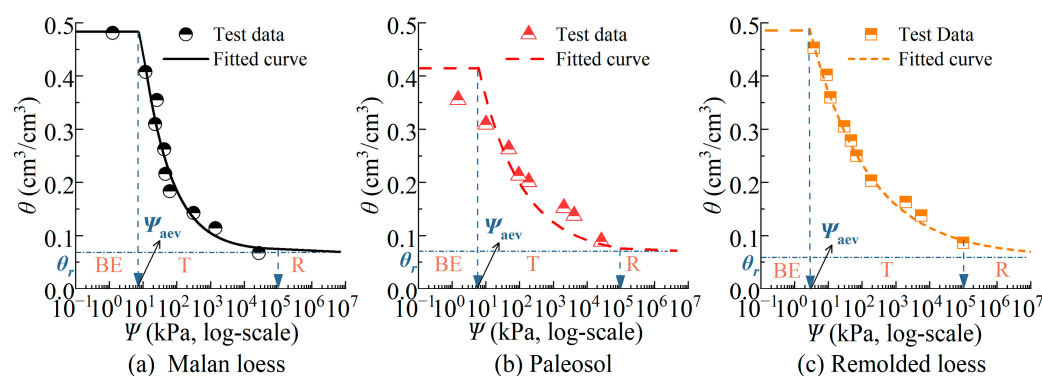


Figure 6. The test data of SWCC obtained by the filter paper method and the fitting curve of the Brooks-Corey model (BE, Boundary effect zone; T, Transition zone; R, Residual zone).

Figure 6 displays the SWCCs for the three investigated soil types, which were obtained via the filter paper technique and subsequently modeled using the Brooks-Corey equation. Marked discrepancies can be seen in both the water retention at equivalent suction levels and the slopes of the corresponding curves, reflecting distinct hydraulic properties. Malan loess exhibits a steeper decline in the transitional zone and covers a broader saturated-transitional region compared to paleosol (Figure 6a,b), indicating faster moisture release and easier water movement. In contrast, paleosol transitions earlier into the residual stage, confirming its stronger water retention capacity [68,69]. The remolded loess shows a transitional zone wider than that of Malan loess but with a gentler slope than paleosol (Figure 6c). Based on the Brooks-Corey model fitting, the water storage capacity ranks as follows: paleosol > remolded loess > Malan loess.

The variation in SWCC across soil samples is primarily governed by particle size distribution and the compaction degree of the soil skeleton. Soils with higher proportions of fine-grained materials exhibit enhanced particle adsorption capacity. Furthermore, increased clay content correlates with a reduced inflection rate in the transitional section of the curve [60]. Particle composition also directly affects pore structure: coarse particles form larger pores, whereas fine particles promote smaller pore channels [70].

The steep transitional zone in the curve of Malan loess reflects a rapid and concentrated water release process. Due to its formation through physical weathering, Malan loess retains the sorting characteristics of aeolian deposits, resulting in relatively uniform pore sizes [71]. This uniformity favors capillary-dominated water transport, causing significant changes in water content over a narrow suction range. In contrast, paleosol undergoes long-term chemical weathering and biological activity, transforming primary minerals into secondary clay minerals. Coarse particles are reduced through leaching or fragmentation, while fine particles accumulate, leading to poor sorting. Fine particles fill larger pores, forming a multi-level pore network [72], which increases water retention complexity and heterogeneity. Consequently, its SWCC is flatter, and water release occurs over a wider suction range. For remolded loess, the original structure is altered during remolding:

macropores are destroyed, particle contact tightens, and smaller pores increase while total porosity decreases [73]. Compaction further enhances pore irregularity and reduces sorting, thereby improving water retention and slowing down water release.

The fitting parameters are shown in Table 4. Values of θ_r , ψ_{aev} , and r^2 are $0.0708 \text{ cm}^3/\text{cm}^3$, 7.203 kPa , and 0.889 for Malan loess; $0.0827 \text{ cm}^3/\text{cm}^3$, 6.541 kPa , and 0.854 for Paleosol; and $0.0623 \text{ cm}^3/\text{cm}^3$, 2.74 kPa , and 0.962 for remolded loess. Generally, soils with high water storage capacity exhibit greater ψ_{aev} , indicating higher suction resistance to air entry and better water retention by small pores. Soils with low water retention show lower ψ_{aev} and rapid water release under low suction [60,74]. Similarly, a high θ_r reflects strong adsorption by fine particles and micropores under high suction ($>10^4 \text{ kPa}$), while a low θ_r indicates limited residual moisture [75].

However, the ψ_{aev} and θ_r values derived from the Brooks-Corey model do not align consistently with the actual water storage capacities of these soils. This discrepancy suggests that while the Brooks-Corey model accurately captures suction changes in the transitional zone, it performs less reliably in characterizing the boundary (low-suction) and residual (high-suction) zones. One reason is that filter paper measurements at very high and low water contents, specifically beyond ψ_{aev} and below θ_r , provide less reliable suction data for model fitting [60,61]. Since ψ_{aev} and θ_r are a priori unknown, all measured data points were included in the fitting process.

3.2.2. The Predicted SWCC Based on Fractal Theory

Fractal theory is an effective method for predicting soil hydraulic properties based on soil structure. In this section, the three-dimensional fractal dimension is measured based on SEM microscopic images, and combined with the θ_s , ψ_{aev} and θ_r measured in Section 2.2.3 to predict the SWCCs for the three soil types (Table 5). The predicted curves were compared against experimental data, and an error assessment was conducted to evaluate the applicability of the fractal model.

Table 5. The key parameters of the fractal model for SWCC.

Samples	Malan Loess			Paleosol			Remolded Loess		
	M1	M2	M3	P1	P2	P3	R1	R2	R3
$R \text{ (}\mu\text{m)}$	110.7	45.9	69.7	15.3	18.8	27.4	45.5	52.2	44.9
$\psi_{aev} \text{ (kPa)}$	1.335	3.221	2.119	9.689	7.849	5.397	3.245	2.832	3.292
$\theta_r \text{ (cm}^3/\text{cm}^3)$	0.0542	0.0592	0.0633	0.0821	0.0794	0.0824	0.0742	0.0734	0.0732
$\bar{\psi}_{aev} \text{ (kPa)}$		2.225			7.645			4.123	
$\bar{\theta}_r \text{ (cm}^3/\text{cm}^3)$		0.0589			0.0813			0.0716	
R^2		0.891			0.879			0.973	
RMSE		0.0037			0.0121			0.0011	

Since the ψ_{aev} and θ_r of the fractal model were actually measured, the test data points below air-entry suction were excluded during model comparison. As shown in Figures 6 and 7, the fractal-based model captures the water retention behavior across soil types. For Malan loess (Figure 7a), the predicted curve agrees well with measured data above ψ_{aev} (Boundary zone), indicating effective structural characterization. The prediction for Paleosol (Figure 7b) is generally flat and shows slight deviation in the medium-suction range (10^2 – 10^4 kPa), which can be attributed to its complex aggregated and fissured structure resulting from strong pedogenesis, leading to greater sample variability despite controlled testing conditions [76]. Remolded loess (Figure 7c) exhibits excellent agreement between prediction and measurement. The fractal model consistently ranks water storage capacity as paleosol > remolded loess > Malan loess, aligning with the Brooks-Corey model

results. The values of R^2 between predicted and measured curves are 0.891, 0.879, and 0.973, respectively. The values of RMSE between predicted and measured curves are 0.0037, 0.0121, 0.0011, respectively.

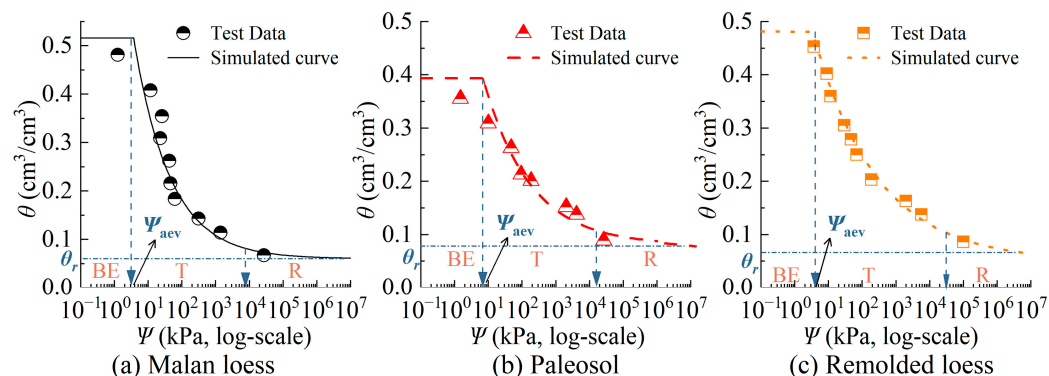


Figure 7. SWCCs predicted based on fractal theory (BE, Boundary effect zone; T, Transition zone; R, Residual zone).

Compared with the Brooks-Corey model, the fractal model of the SWCC can also well describe the water-holding capacity of the three types of soil. The three parameters of the fractal model all have actual physical meanings and can be obtained through simple tests or calculations [77]. In contrast, the parameter n of the Brooks-Corey model depends on model fitting in this study. It requires conducting SWCC tests first (such as using a pressure plate apparatus, filter paper method, etc.) to obtain data and then fitting the data to achieve the SWCC. The fractal model is much simpler than the Brooks-Corey model. The three-dimensional fractal dimension and the air-entry value can be determined directly through the analysis of SEM images, streamlining the required analytical process. Then, by using the water-saturated method and air-drying method to obtain the residual water content, the SWCC of the corresponding soil can be obtained [28,40].

3.3. Prediction of Unsaturated Hydraulic Conductivity Based on Fractal Theory

By incorporating the Young-Laplace equation, the fractal model of unsaturated hydraulic conductivity establishes a relationship between matric suction and the structural properties of the soil. This approach explicitly accounts for pore interconnectivity, thereby deriving a functional relationship between matric suction and hydraulic conductivity in unsaturated media [32,33]. To evaluate its predictive capability, the fractal model for unsaturated hydraulic conductivity was applied to generate hydraulic conductivity curves for the three investigated soil types. For comparative purposes, predictions from the Burdine-Brooks-Corey model were also generated and benchmarked against the fractal model's results [20,78,79].

The unsaturated hydraulic conductivity values measured for the three soils are presented in Figure 8, alongside the corresponding predictive curves derived from the fractal and Burdine-Brooks-Corey models. Test data for both undisturbed soils (Malan loess and paleosol) show notable scatter, especially in Paleosol, due to inherent structural heterogeneity despite sampling efforts to minimize weight variations [80]. In contrast, the remolded loess, which original large pores were destroyed during the remolding process, has better homogeneity, and thus the test data of the unsaturated hydraulic conductivity are more concentrated [67].

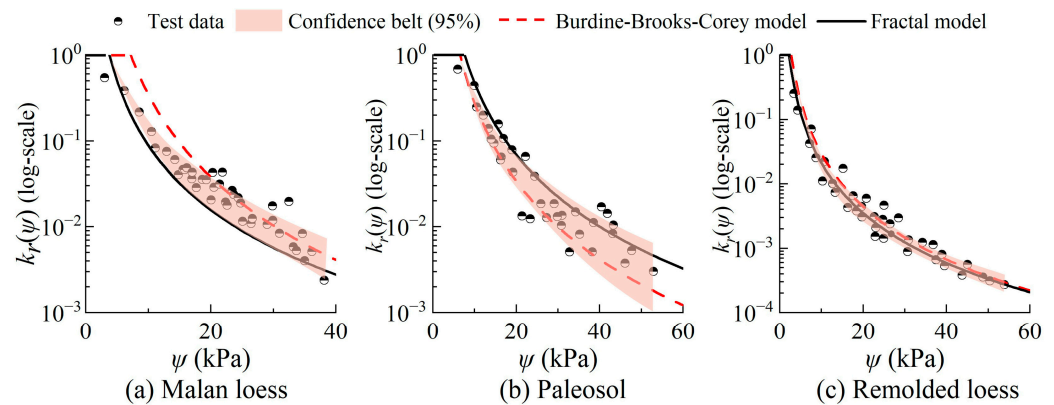


Figure 8. Comparison of the predicted curve of the unsaturated relative hydraulic conductivity with the test data.

Predictive performance differed between models. For Malan loess, the Burdine-Brooks-Corey model overestimates conductivity, while the fractal model underestimates it. However, both capture the decreasing trend with increasing suction. The fractal model aligns well with data at low suction (Figure 8a). For paleosol, the Burdine-Brooks-Corey model consistently underestimates conductivity, with growing deviation as suction rises. The fractal model slightly overestimates but shows good consistency across the entire suction range (Figure 8b). Both models perform excellently for remolded loess, matching test data closely (Figure 8c).

The calculation results of determination coefficients (R^2) for the prediction curves of the fractal model and the Burdine-Brooks-Corey model were summarized in Table 6. The R^2 values of the fractal model prediction curves for Malan loess, paleosol, and remolded loess are 0.915, 0.918, and 0.929, respectively, while those of the Burdine-Brooks-Corey model are 0.864, 0.825, and 0.925, respectively. The values of RMSE the fractal model prediction curves for Malan loess, paleosol, and remolded loess are 1.23, 1.07, and 0.043, respectively, while those of the Burdine-Brooks-Corey model are 6.37, 2.81, and 1.01, respectively (Note, when calculating the RMSE here, both the predicted value and the measured value are in units of 10^{-6} cm/s). The fractal model not only shows excellent predictive performance for homogeneous remolded loess but also outperforms the Burdine-Brooks-Corey model in predicting unsaturated hydraulic conductivity of non-homogeneous soils such as natural Malan loess, and paleosol. In this paper, the Burdine-Brooks-Corey model only shows good fitting effects for homogeneous remolded loess.

Table 6. Error analysis of the unsaturated hydraulic conductivity prediction model.

Samples	k_{sat} (cm/s)	Burdine-Brooks-Corey Model		Fractal Dimension Model	
		R^2	RMSE	R^2	RMSE
Malan loess	7.203×10^{-6}	0.864	6.37	0.915	1.23
Paleosol	4.87×10^{-6}	0.825	2.81	0.918	1.07
Remolded loess	1.68×10^{-5}	0.925	1.01	0.929	0.043

The use of 3D fractal theory to predict the unsaturated hydraulic characteristics of loess creates a high-accuracy, low-cost parametric tool, which is critical for construction, ecological recovery, and hazard prevention in these areas. This methodological advancement bridges the gap between theoretical modeling and practical applications in geotechnical engineering and environmental sciences.

4. Conclusions

This paper presents a simple and easily determinable estimation method for the SWCC and the unsaturated hydraulic conductivity of unsaturated soils based on fractal theory using the three-dimensional fractal dimension. The main conclusions drawn from this study are as follows:

- (1) Malan loess, paleosol, and remolded loess all exhibit fractal pore-surface characteristics, with three-dimensional fractal dimensions (D_s) being 2.611, 2.688, and 2.771, respectively. Pore structure complexity follows the order: remolded loess > paleosol > Malan loess.
- (2) For SWCC, the fractal model demonstrated relatively good predictive capabilities. The R^2 and RMSE values of the fractal model for the three types of soil are: 0.891, 0.879, 0.973, and 0.0037, 0.0121, 0.0011. The water storage capacity of the three soils was in the order of paleosol > remolded loess > Malan loess.
- (3) Based on the analysis of three soil types, the fractal model demonstrates superior overall performance in predicting unsaturated hydraulic conductivity compared to the Burdine-Brooks-Corey model. While both models perform excellently for homogeneous remolded loess, the fractal model significantly outperforms the Burdine-Brooks-Corey model for the more heterogeneous natural soils (Malan loess and paleosol). This is quantitatively supported by higher R^2 and substantially lower RMSE values for the fractal model's predictions across these undisturbed soils.

In terms of predicting the unsaturated hydraulic conductivity, the fractal model can predict unsaturated hydraulic conductivity more accurately for heterogeneous soils (Malan loess and paleosol). Both models perform well for homogeneous remolded loess.

The fractal model quantifies the microscopic characteristics such as the pore structure and particle distribution of loess, providing a powerful tool for understanding and predicting its macroscopic engineering behavior in real environments. For instance, the fractal model enables us to predict the soil-water characteristic curves under different stress conditions or dry densities based on a small amount of experimental data. This is crucial for simulating the changes in water content and matrix suction within slopes under rainfall infiltration conditions and can more accurately assess the stability of slopes. However, it should be noted that this model still has certain limitations. For example, it only measures the wetting curve and does not obtain the drying curve. Moreover, when using the Young-Laplace equation, α is set to 0° , while ignoring the actual contact angle lag phenomenon observed in cohesive soils, thereby underestimating the air entry value. Future research will focus on this aspect.

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