

Article

An Investigation of Silty Sediment Erodibility Considering the Effects of Upward Seepage and Slope Gradient

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Abstract: The phenomenon of extensive erosion of silty submarine slopes in the Yellow River delta has been well documented in numerous studies. Due to poor drainage and high compressibility, silty sediments are particularly prone to pore pressure buildup and accumulated seepage under wave and current action, which can influence sediment erodibility (e.g., the critical bed shear stress and the erosion rate under various bed shear stresses). To date, there remains a lack of parametric formulation to quantitatively characterize the erodibility of silty sediments with the coupled effects of the hydraulic gradient of upward seepage and the slope gradient. In this study, a series of laboratory experiments were conducted to explore the erodibility of silt sediments from the Yellow River delta under varying hydraulic gradients of upward seepage and slope gradients. The results reveal that both upward seepage and increased slope gradients can enhance the erodibility of silty sediments. Specifically, as the seepage gradient increases from 0.1 to 0.8, the critical Shields parameter required for initiating silty particle motion decreases linearly, with a reduction rate of 0.01 per 0.1 increase in the seepage gradient, independently of changes in slope gradient. Additionally, the erosion coefficient of silty sediments grows exponentially with rising seepage gradients, with its average growth rate accelerating with increasing slope inclination. For flat sediment beds, the erosion coefficient influenced by upward seepage can be up to five times that in the absence of seepage. An empirical formula for calculating the critical Shields parameter and an erosion model incorporating upward seepage gradient and slope effects were developed through multiple regression analysis, providing an experimental basis for numerical simulations of scour in silty submarine slopes under combined waves and currents.

Keywords: sediment erodibility; upward seepage; slope gradient; silty seabed; erosion rate



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1. Introduction

Over the last 40 years, the subaqueous Yellow River delta has been experiencing extensive erosion due to the decreasing water discharge and sediment load delivered to the sea [1–3]. This phenomenon is closely linked to the unique characteristics of the silty sediments (e.g., silty sand and silt) that are prevalent across the Yellow River delta, as well as the complex hydrodynamic conditions in the region (e.g., waves and currents) [4]. Coastal and nearshore erosions in these areas can lead to the seaward retreat of the shoreline and cause localized scour around coastal infrastructures, thereby compromising their stability [5]. Therefore, understanding sediment erodibility is crucial for evaluating sediment transport and ensuring the stability of offshore structures such as submarine pipelines, cables, and breakwaters.

The critical conditions for incipient motion of sediment particles (e.g., critical bed shear stress or critical Shields parameter) and the erosion rate are fundamental parameters for describing sediment erodibility and predicting coastal erosion. Sediment erodibility is influenced by both intrinsic factors such as particle size, sediment density, water content, cohesiveness, and permeability [6], and external factors like seepage flow induced by waves and currents [7,8]. Studies have shown that for cohesionless sediments (e.g., sand), upward seepage force can reduce the effective gravity of soil particles, thereby lowering the critical bed shear stress required for incipient motion, ultimately enhancing sediment erodibility [7,9,10]. However, other research has demonstrated that upward seepage may also reduce current-induced bed shear stress, which can impede sediment particle movement and decrease the erosion rate [11–13]. Ramakrishna Rao et al. [13] suggested that the effect of seepage on bed shear stress can either increase or decrease erosion depending on factors such as bed shear stress, critical shear stress, and seepage rate, etc. Thus, the influence of upward seepage on cohesionless sediment erodibility is complex and context-dependent, with most studies reporting an increase in erosion rate, particularly for fine-grained soils under high hydraulic gradients [8,14,15], while some studies have observed the opposite trend [16,17].

Unlike cohesionless sandy sediments, silty sediments possess some degree of cohesion and relatively smaller particles. In addition, they have poor drainage conditions, making them prone to pore water accumulation and even liquefaction under cyclic wave loading [18]. This leads to more intensive seepage flow within silty sediments, complicating the erosion mechanisms associated with the seepage effects. Existing studies show that wave-induced seepage and liquefaction can significantly reduce the erodibility of silty sediments [14,15,19]. Experimental results indicate that in the Yellow River delta, the critical bed shear stress for silty sediment movement can decrease by 12% to 53% under waves with 5-year return periods, and by up to 72% under waves with 50-year return periods [19]. In addition to influencing effective particle gravity and bed shear stress, upward seepage reduces sediment strength and alters its properties, facilitating particle movement initiation [14]. Furthermore, intensive upward seepage can lead to internal erosion, where fine particles migrate from the sediment interior to the surface and are subsequently resuspended in the overlying water, further increasing the erosion rate [15,20].

To address the effect of wave-induced pore pressure accumulation and liquefaction on erosion, Lambrechts et al. [21] developed an empirical model for calculating bed erosion rates by incorporating wave height, wave frequency, and water depth. However, this model does not account for seepage effects before liquefaction occurs. Zhang et al. [15] carried out a series of annular flume tests on silty sediment from the Yellow River delta with varying degrees of consolidation. Their results showed that a linear erosion model, which allows critical shear stress to vary with depth, could accurately describe the erosion characteristics. An empirical expression for erosion rate was proposed by incorporating the excess pore water pressure into the erosion coefficient. However, this expression is unsuitable for numerical simulations of the sediment erosion process due to the vanishing excess pore pressure at the seafloor surface. Additionally, the effect of slope gradient on the sediment erodibility was not considered.

The present study aimed to investigate the combined effects of upward seepage and slope gradient on the erodibility of silty sediment through flume experiments. To ensure controllability of seepage intensity and facilitate the data analysis, the upward-directed seepage was generated directly by a waterhead difference, similar to the seepage induced by waves or interactions between waves and currents. The characteristics and mechanisms of the critical conditions of particle entrainment and erosion rates in silty sediments under varying seepage and slope conditions are analyzed and discussed. Empirical expressions for the critical Shields parameter and erosion rate, incorporating hydraulic gradient of upward seepage and the slope angle, are established to facilitate further numerical simulations and practical engineering applications.

2. Methods and Materials

2.1. Experimental Setup

The sediment erosion experiments were conducted in a self-designed circulating flume system. As illustrated in Figure 1, the flow velocity can be adjusted by controlling the flow rate of the submersible pump (ranging from 4000 to 12,000 L/h) installed in the water collection tank. The flow rate was monitored using a flowmeter. Two velocimeters with a precision of 0.01 m/s were employed to monitor the flow velocity above the scouring zone at a frequency of 1 Hz. Based on velocities of multiple measurements at different locations above the sediment bed, the averaged near-bed current velocity was calculated and used for subsequent calculation and analysis. To measure the suspended sediment concentration (SSC) in the water, a nephelometer (RBR Concerto) with a resolution of 0.01 NTU was used, recording at a frequency of 0.33 Hz. The SSC data were then used to calculate the erosion mass of the sediments. Prior to the experiments, the nephelometer was calibrated.

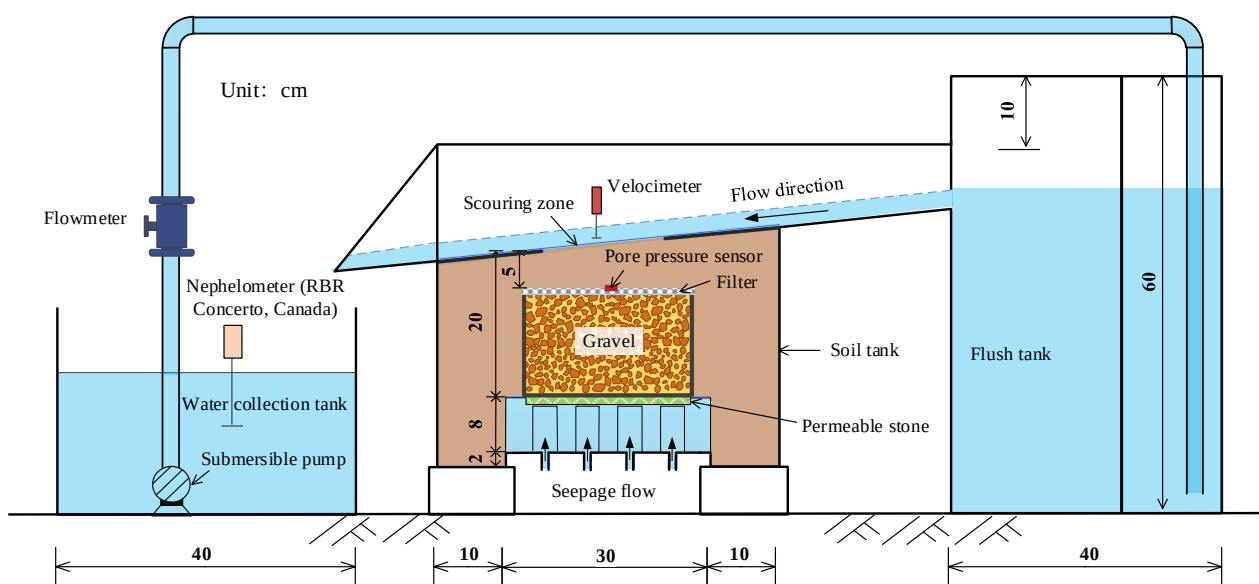


Figure 1. Schematic of the scouring flume system.

For silty sediments, waves or combined wave and current action readily induces the accumulation of excess pore water pressure. The non-uniform distribution of residual pore water pressure with depth can lead to cumulative seepage, with the seepage direction being upward from the peak region of the residual pore water pressure. This phenomenon is similar to waterhead difference seepage [20]. Consequently, in this study, the upward-directed seepage in silty sediments was simulated by creating a waterhead difference. This method is convenient to control the seepage intensity during scouring experiments. To mitigate preferential flow along the sidewalls of the soil tank, which could have distorted experimental results, a highly permeable gravel layer was placed in the central region of the sediments (Figure 2a). This arrangement shortened the upward-directed seepage path to the sediment bed surface and lengthened the seepage path along the sidewalls, ensuring that the pore water would drain out from the sediment surface (i.e., the scouring area, as shown in Figure 2b,c) rather than along the sidewalls. A micropore pressure sensor was placed at the top of the gravel layer and used to record excess pore water pressure, which was then used to calculate the hydraulic gradient of the seepage within the sediments.

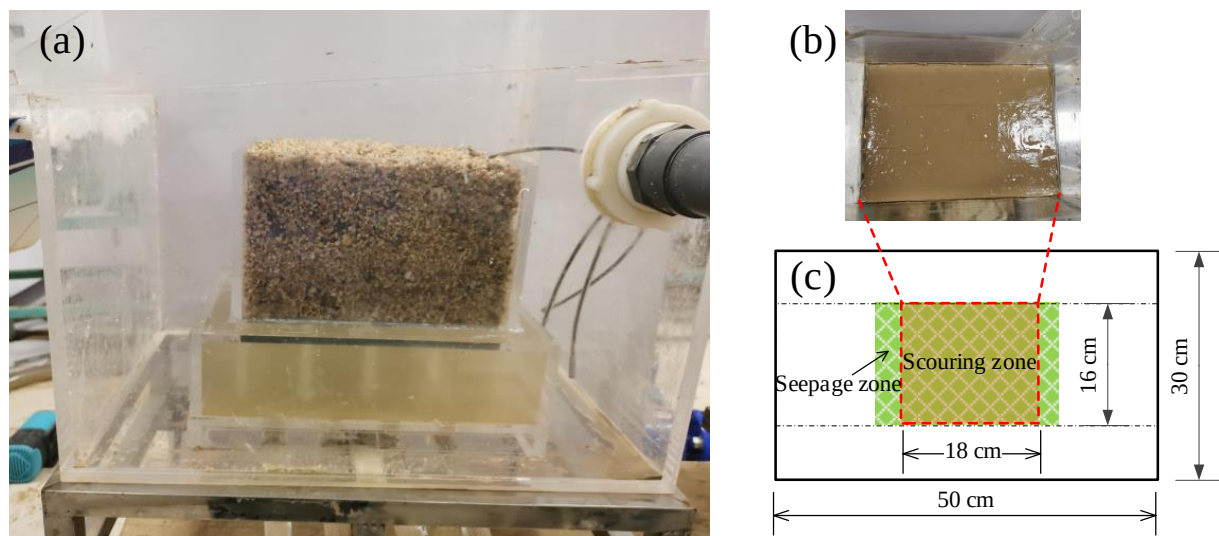


Figure 2. (a) Seepage setup; (b) scouring zone; (c) plan of the scouring flume.

2.2. Test Sediments

The sediments used in this study were collected from the tidal flats of the Yellow River estuary, comprising fine sand (approximately 35.5%), silt (about 58.6%), and clay (around 5.7%) with a median grain diameter of 0.0413 mm, as depicted in Figure 3. To achieve more homogeneous samples, a slurry with a soil-to-water mass ratio of 1:3.5 was prepared and placed into the soil tank to consolidate under gravity. The properties of the consolidated sediment were as follows: water content was 30%, specific gravity was 2.71, soil density was 1.89 g/cm³, and porosity was 0.45. Shear tests indicated that the cohesion of the sediment was 4.3 kPa, while the internal friction angle measured 28°.

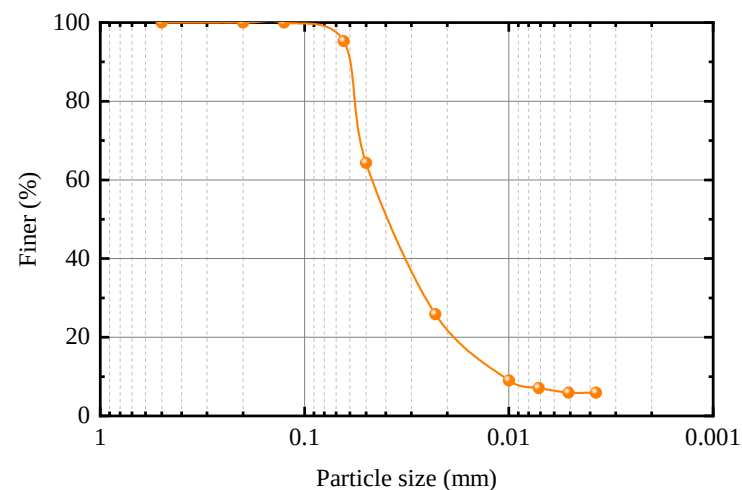


Figure 3. Particle size distribution of the test sediments.

2.3. Experimental Procedure

A submarine slope typically exhibits a mild slope angle; therefore, in this study, the sediment bed angles (α) were set at 0°, 2.7°, and 5.4° to investigate the erosion characteristics of silty sediments under varying hydraulic gradients of seepage. The specific experimental steps were as follows.

- (1) The silty sediments were mixed with water at a mass ratio of 1:3.5 to create a homogeneous slurry, which was slowly poured into the soil tank until it slightly exceeded

the top. A micropore water pressure transducer was positioned at the top of the gravel layer.

- (2) The sediment sample consolidated under its own weight, monitored by a pore pressure sensor. A thin layer of water was maintained on the sediment surface to prevent evaporation during consolidation. After consolidation, the experiment proceeded to the next phase.
- (3) Upward seepage was generated within the sediments by applying a differential waterhead, which remained constant throughout each test. Once the seepage flow stabilized, the scouring stage commenced.
- (4) The flow rate of the submersible pump was adjusted to create different flow velocities, gradually increasing the rate to scour the sediment bed. Once the suspended sediment concentration stabilized at each flow velocity level, the flow velocity was increased to the next stage. Throughout the experiments, parameters such as flow velocity, water depth, and suspended sediment concentration were recorded, and the experimental phenomena were documented with photographs.

3. Results and Discussion

3.1. Incipient Movement of Silt Particles with the Effect of Upward Seepage and Slope Gradient

Incipient motion of sediment particles on a bed surface normally occurs when the flow velocity surpasses a specific threshold value, triggering particle mobilization [22]. In coastal engineering, determining the threshold of incipient movement of sediment particles is essential for predicting erosion and scour in coastal regions. The critical conditions for sediment entrainment are often expressed using critical shear velocity (U_{cr}), critical bed shear stress (τ_{cr}), or the critical Shields parameter (θ_{cr}). Current-induced bed shear stress at the sediment bed surface, τ_b , can be calculated as [14]:

$$\tau_b = \frac{1}{8} f \rho_w U^2 \quad (1)$$

where ρ_w is the density of water, U is the near-bed averaged current velocity, and f is the drag coefficient, which is influenced by roughness and Reynolds number and can be expressed as:

$$\frac{1}{\sqrt{f}} = 2 \lg \left(Re \sqrt{f} \right) - 0.8 \quad (2)$$

where Re is the Reynolds number, defined as $Re = Ud/\nu$. Here, d represents the width of the scouring zone (16 cm) and ν is the kinematic viscosity of water ($0.01 \text{ cm}^2 \text{ s}^{-1}$).

The Shields parameter, θ_b , a dimensionless measure of bed shear stress, is expressed by the following formula:

$$\theta_b = \frac{\tau_b}{\rho_w (s-1) g d_{50}} \quad (3)$$

where s is the specific gravity of soil particles (2.71), g is the gravitational acceleration (9.8 N/kg), and d_{50} is the median grain size of the sediment particles (0.0413 mm).

To avoid potential bias from visual observations in determining the threshold for sediment particle movement, this study employed the extrapolation method by extrapolating the linear regression equation that relates eroded sediment mass to bed shear stress. The critical bed shear stress can be determined at zero transported sediment flux. This method has been widely applied in studies of sediment initiation [23,24].

As an example, the determination of the critical bed shear stress is demonstrated using experimental data from a flat sediment bed without seepage flow. Since all eroded silty sediments, including both bedload and suspended load particles, are transported into and suspended in the water collection tank, the eroded sediment mass can be calculated based on changes in the suspended sediment concentration (SSC) in the water. Figure 4 shows the variation in bed shear stress and the corresponding eroded sediment mass over time at different flow velocities ranging from 38.9 cm/s to 64.8 cm/s. It is evident that at each level

of bed shear stress, the eroded sediment mass increases substantially at the initial stage, then gradually stabilizes. As bed shear stress increases, the number of eroded particles increases until a new steady state is reached over time. It is worth noting that as the flow velocity increases, the morphology of the sediment bed (such as ripples, small gullies and pits, as described in Section 3.3) may influence the velocity. As a first approximation, the averaged near-bed flow velocity, calculated based on velocities measured at different locations, has been used for analysis.

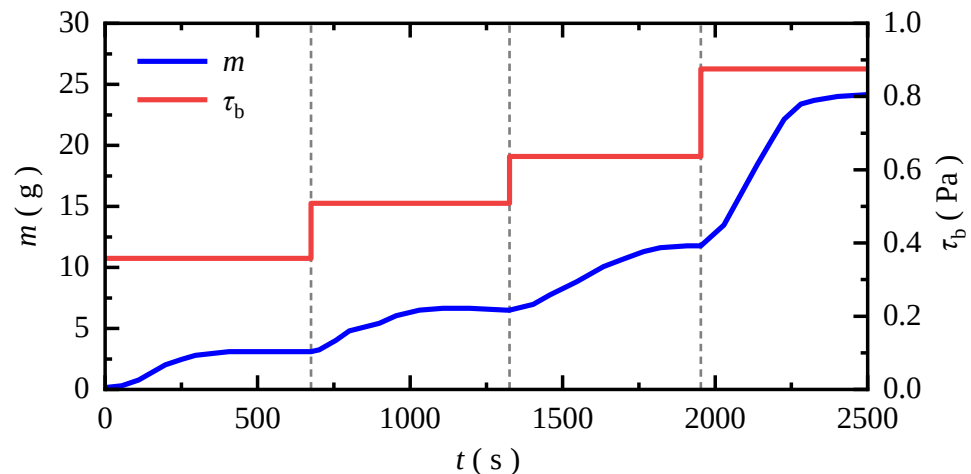


Figure 4. Time histories of bed shear stress and eroded sediment mass without the effect of upward seepage or slope gradient.

Figure 5 shows the relationship between bed shear stress and eroded sediment mass, along with a linearly fitted equation. This reveals a significant positive correlation between bed shear stress and the corresponding eroded mass of silty sediments. The linear equation indicates that the critical bed shear stress of silty sediments is 0.293 Pa, corresponding to zero eroded mass. From Equation (3), the critical Shields parameter is calculated to be 0.423. It is worth noting that the regression equation proposed in this study is based on a limited range of bed shear stress (0.36–0.88 Pa) that is greater than the critical bed shear stress (0.293 Pa). From a physical standpoint, sediment particles will not be entrained into the overlying water if the bed shear stress is below the critical value. Therefore, the regression equation in Figure 5 is only applicable for bed shear stress values that are equal to or greater than the critical bed shear stress.

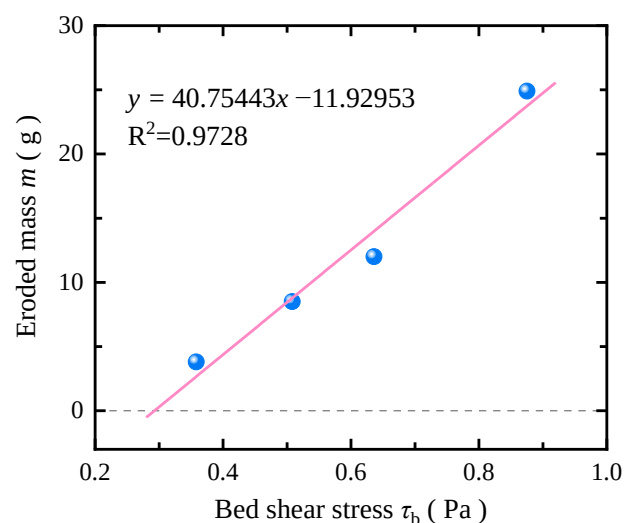


Figure 5. Eroded sediment mass vs. bed shear stress.

Using the same method, the critical Shields parameter for silty sediment was calculated under the combined influence of upward-directed seepage and slope gradient. Figure 6 shows the variation in the critical Shields parameter with hydraulic seepage gradients for different slope angles, along with the results of the linear regression analysis. Overall, the critical Shields parameter of silty sediments decreases linearly as the hydraulic gradient of upward seepage increases from 0.1 to 0.8, with a reduction rate of 0.01 per 0.1 increase in seepage gradient, regardless of changes in slope angle. For a horizontal sediment bed, as the hydraulic gradient increases from 0 to 0.75, the critical Shields parameter decreases from 0.423 to 0.342, representing a reduction of approximately 20%.

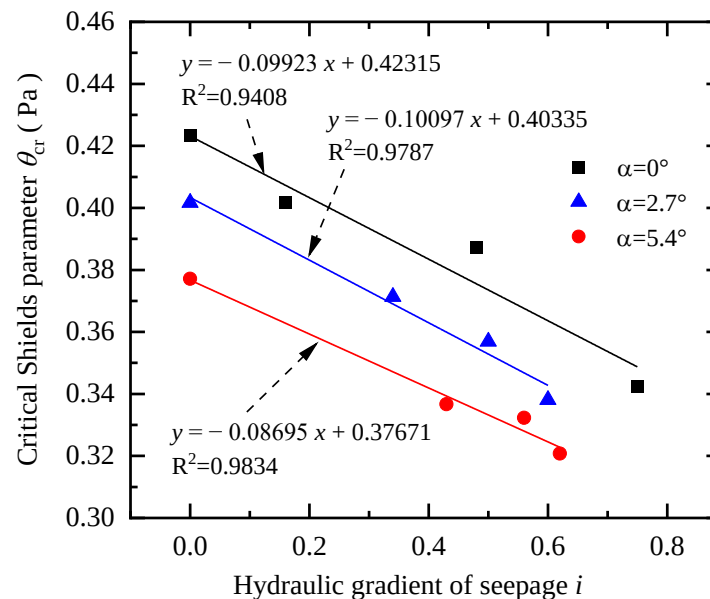


Figure 6. Linear regression between critical Shields parameters and hydraulic gradients with different slope angles.

Figure 6 also implies that slope angles can affect the critical conditions for sediment transport initiation, even if the slope angle is small. As the slope angle increases, the critical Shields parameter for silty particles decreases. This is due to the fact that as the slope gradient increases, the gravitational component of soil particles along the slope bed increases, increasing the downward force of the particles. At the same time, the component of gravity in the direction perpendicular to the slope bed decreases, reducing the frictional force acting on the particles. As a result, the sediment particles are more susceptible to movement under currents. In addition, where there is a slope, there may also be some downslope seepage, which can make the sediment particles more susceptible to scour.

A multiple linear regression analysis was performed on the critical Shields parameter for the incipient motion of silty sediments, with the hydraulic gradient of upward seepage and the slope gradient (tangent of the slope angle) as variables. The resulting regression formula for the critical Shields parameter is as follows:

$$\theta_{cr}/\theta_{cr,0} = 0.997 - 1.041 \tan \alpha - 0.227 i \quad (R^2 = 0.968) \quad (4)$$

where α is the slope angle of sediment bed, i is the hydraulic gradient of seepage, θ_{cr} is the critical Shields parameter considering the effect of upward seepage and slope gradient, and $\theta_{cr,0}$ is the critical Shields parameter without these effects. The results obtained from the regression formula (Equation (4)) were compared with experimental data and results of the theoretical formula (Equation (5)) derived by Cheng and Chiew [7], as shown in Figure 7.

$$\theta_{cr}/\theta_{cr,0} = 1 - i \cdot \tan \varphi / [(s - 1)(1 - n)(\cos \alpha \tan \varphi - \sin \alpha)] \quad (5)$$

Here, φ is the friction angle of the sediment (28°), n is the porosity (0.45), and other variables can be found in the previous context.

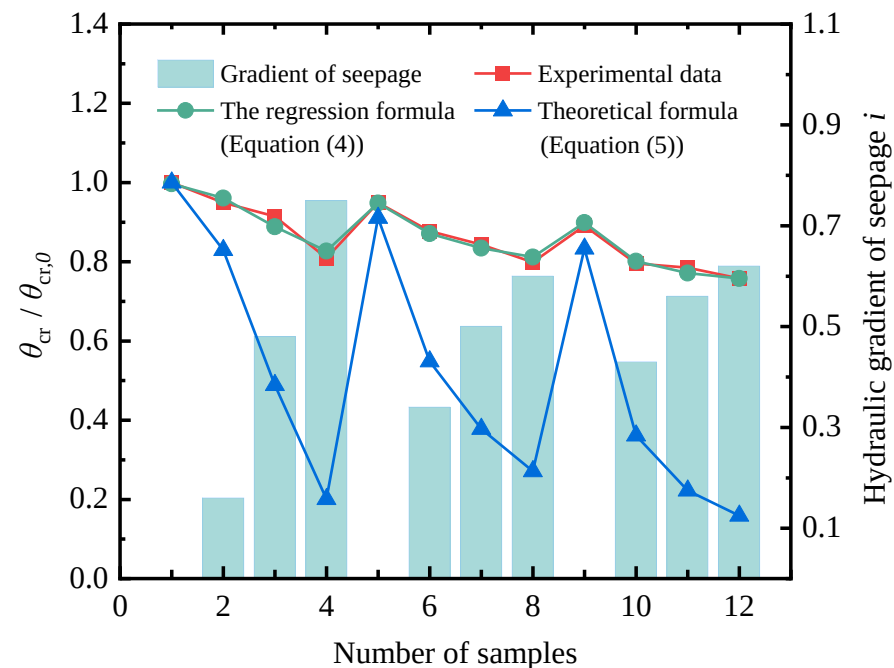


Figure 7. Comparison of critical Shields parameter between experiments and predictions.

It can be observed that the predicted values from the regression formula closely match the experimental data, while there is a significant deviation between the theoretical values and the experimental data when upward seepage is considered. Moreover, the deviation increases as the hydraulic gradient of upward seepage increases. This can be mainly attributed to the fact that the particle size of silty soil considered in this experimental study (0.0413 mm) was much smaller than that in the experiments of Cheng and Chiew [7] (ranging from 0.63 mm to 5.83 mm). Moreover, there was some cohesion within the silty sediment considered in this study. The complicated interaction between soil particles results in a certain discrepancy between the theoretical analysis of the force equilibrium of soil particles and the actual conditions of fine grains. Therefore, further research is needed in the future to investigate the erodibility of silty sediments under the influence of seepage flow.

3.2. Erosion Rate of Silty Sediments Considering the Effect of Upward Seepage and Slope Gradient

In addition to the critical bed shear stress for the incipient movement of sediments, the erosion rate characteristics are also crucial for describing sediment erodibility. The erosion rate of silty sediments can be modeled using a linear erosion relationship, even when the critical shear stress varies with depth [6,15], i.e.,

$$E_r = M_e \tau_{exc} = M_e (\tau_b - \tau_{cr}) \quad (6)$$

where E_r is the erosion rate, representing the mass of eroded sediment per unit area of the bed surface and unit time, with units of $\text{kg}/(\text{m}^2 \cdot \text{s})$, M_e is the erosion coefficient (s/m), and τ_{exc} is the excess bed shear stress, defined as the amount by which the bed shear stress exceeds the critical bed shear stress.

It is noteworthy that in the present study, the critical bed shear stress τ_{cr} increased with depth into the sediments. This was validated by the observation that at the end of each flow velocity level (i.e., each increment of bed shear stress), no additional sediment particles were detected in suspension. Consequently, the critical bed shear stress corresponding to

the bed shear stress at level n , $\tau_{b,n}$, can be considered as the previous bed shear stress $\tau_{b,n-1}$ at level $n - 1$. Therefore, for $n \geq 2$, $\tau_{cr,n} = \tau_{b,n-1}$, while for $n = 1$, the critical shear stress is the initial value corresponding to zero eroded sediment mass, as analyzed in Section 3.1.

The erosion rate of sediments on the bed surface, E_r , can be calculated from experimental data as follows:

$$E_r = \frac{cV}{At} = \frac{\phi}{t} \quad (7)$$

where c is the suspended sediment concentration, V is the volume of water in the experiment, A is the scouring area, and t is the scouring time. The variable ϕ represents the mass of sediment eroded per unit area during a given time t , and its variation with time can be determined by fitting the experimental data. The initial erosion rate, E_i , which corresponds to a new level of bed shear stress, can be calculated by the following equation [15]:

$$E_i = \lim_{t \rightarrow 0} \frac{\partial \phi}{\partial t} \quad (8)$$

Figure 8 shows time series of the scatters of eroded mass per area under different bed shear stresses with an upward seepage gradient of 0.48, marked with optimal fitted curves (in blue) and initial erosion rates (indicated by red arrow). Initial erosion rates for other cases were also obtained. Linear regressions were then performed between these initial erosion rates and their corresponding excess bed shear stresses to derive erosion coefficients (M_e) for various slope angles under different hydraulic gradients, as shown in Figure 9.

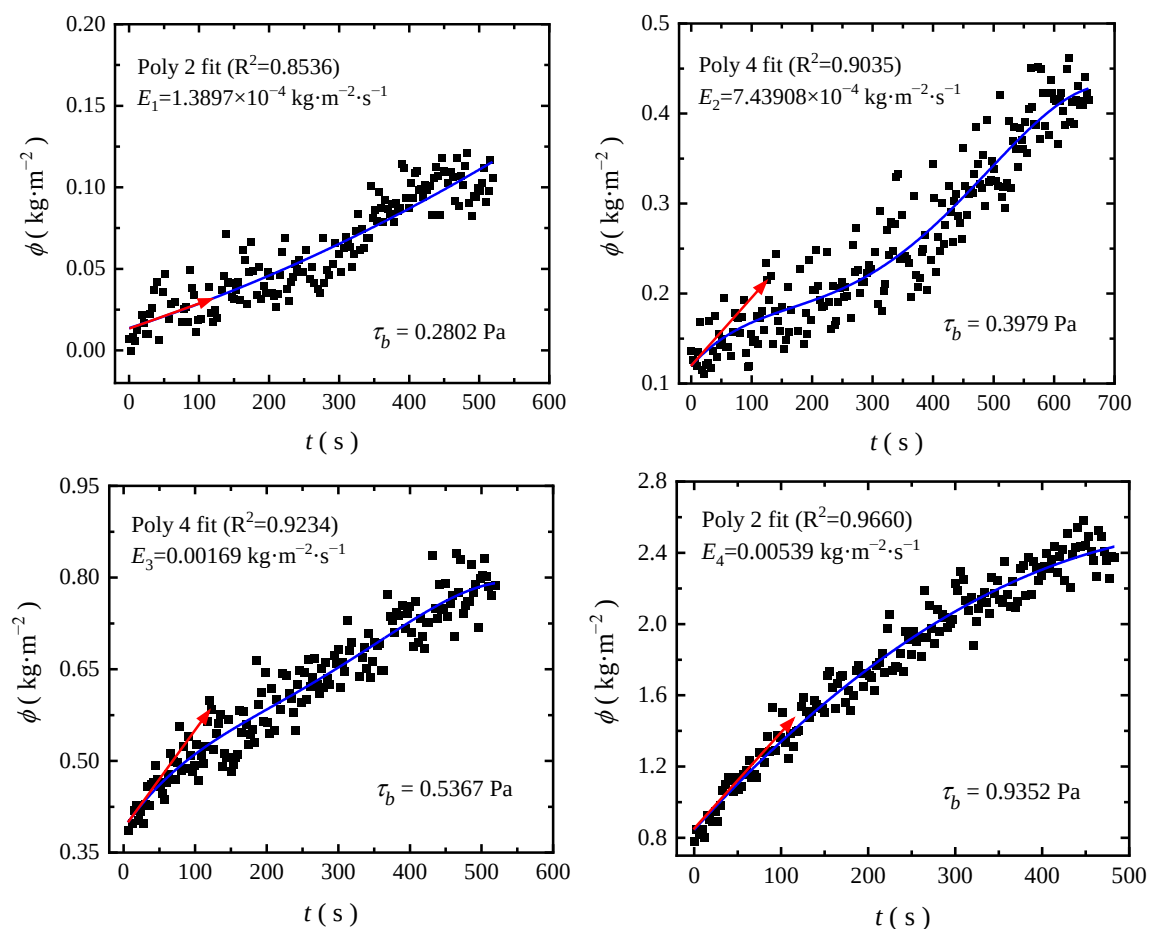


Figure 8. Time series of the scatters of eroded mass per area corresponding to various bed shear stresses with an upward seepage gradient of 0.48, marked with the optimal fitted curves (blue) and initial erosion rates (red arrows).

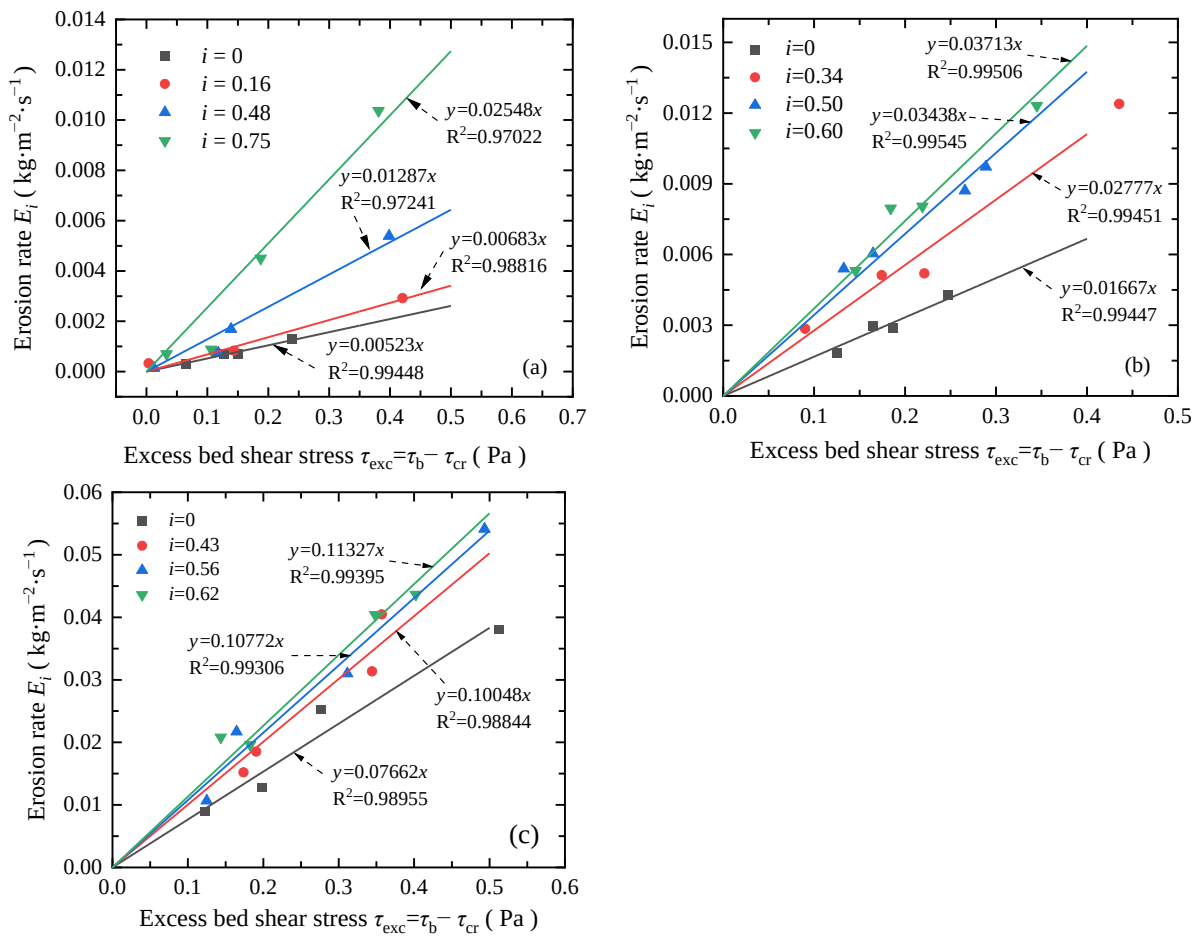


Figure 9. Linear regressions between erosion rates and excess bed shear stresses for erosion coefficients for slope angles of (a) 0° , (b) 2.7° , and (c) 5.4° .

The results indicate a remarkable positive correlation between erosion rates and the excess bed shear stresses of silty sediments, consistent with previous findings [25,26]. The erosion coefficients for different hydraulic gradients and slope angles are plotted in Figure 10, with the results of a nonlinear (exponential) fitting. It is observed that as the hydraulic gradients of upward seepage increase, the erosion coefficients increase exponentially, particularly at high hydraulic gradients. For example, as the upward hydraulic gradient in a flat sediment bed increases from 0 to 0.75, the erosion coefficient increases from 0.00523 s/m to 0.02548 s/m, approximately a fourfold increase. This is primarily because upward seepage reduces the critical bed shear stress needed for particle entrainment, making sediments more prone to erosion. Moreover, internal erosion may further enhance the erodibility of silty sediments, which will be elaborated on in the following section.

The slope angle also significantly influenced the erosion coefficient of silty sediments. Without upward seepage, the erosion coefficients of silty beds with slope angles of 2.7° and 5.4° were 3.2 and 14.6 times that of a flat bed, respectively. The erosion coefficient of a silty bed with a slope angle of 5.4° was 4.6 times that of a 2.7° slope.

To quantitatively analyze the combined effects of upward seepage and slope gradient on the erosion rate of silty sediments, a multiple regression analysis was conducted on the erosion coefficient using the hydraulic gradient of upward seepage and the slope gradient as variables. The regression formula for the erosion coefficient of silty sediments is given as:

$$M_e = e^{29.46 \tan \alpha - 5.38 + (2.25 - 17.67 \tan \alpha)i} \quad (9)$$

Substituting this equation into the linear erosion model (Equation (6)), the empirical formula for calculating the erosion rate is expressed as:

$$Er = e^{29.46 \tan \alpha - 5.38 + (2.25 - 17.67 \tan \alpha) i} (\tau_b - \tau_{cr}) \quad (10)$$

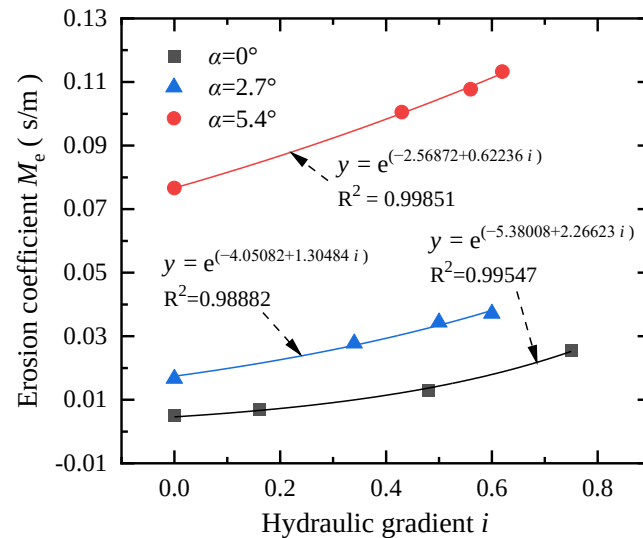


Figure 10. Erosion coefficients vs. hydraulic gradients of seepage for different slope angles.

This empirical relationship suggests that the erosion rate is proportional to the excess bed shear stress and depends on the exponential terms of the hydraulic gradient of upward seepage and the slope gradient. The erosion rate (E_c) calculated using this empirical formula is compared to the erosion rate (E_e) directly obtained from the experimental data, as shown in Figure 11. The blue data points indicate errors less than 30%, while red data points indicate relative errors greater than 30%. It can be seen that over 90% of the data points fall within the confidence intervals of $\pm 30\%$ relative error, indicating that the regression formula has potentially strong applicability. Using the regression Formula (10), the erosion rate of silty sediment under various hydrodynamic environments can be predicted, provided that the bed shear stress and the upward seepage are known.

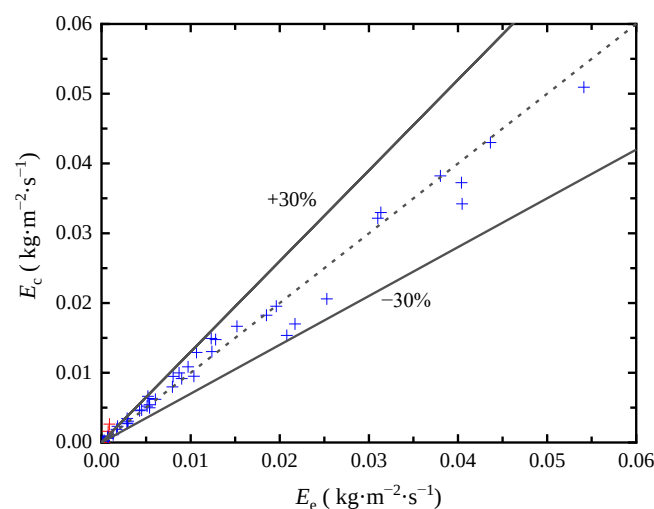


Figure 11. Comparison between predicted erosion rates and experimental data.

3.3. Morphology of the Eroded Silty Sediment Bed and the Effect of Internal Erosion

Figure 12 shows snapshots of the sediment surface during scour and erosion under the influence of flowing currents alone. The results indicated that sediment particles were

lifted and suspended in the overlying water column, creating a blurred interface between the sediment bed and water (Figure 12a,b). The morphology of the eroded silty bed varied significantly depending on the intensity of current-induced scour. At the initial stage of the experiment, with a relatively low flow velocity and short scouring duration, the amount of eroded and resuspended sediment mass was minimal, resulting in the formation of ripples on the silty bed (Figure 12a). As the flow velocity increased, the amount of eroded sediment increased, causing the ripples to disappear and leading to a more irregular erosion pattern. By the end of the experiments, the sediment bed exhibited substantial erosion, with localized areas experiencing deeper erosion, creating a complex morphology marked by small gullies and pits of varying shapes and sizes (Figure 12c).

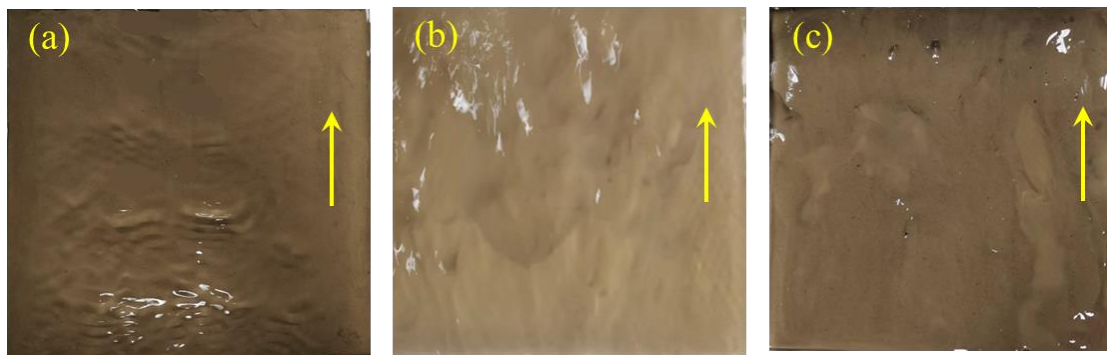


Figure 12. The eroded sediment bed under currents with no upward seepage corresponding to early, mid-, and post-scouring from (a–c), with the arrows indicating the flow direction.

When upward-directed seepage was introduced, as depicted in Figure 13a for a seepage hydraulic gradient of 0.75, small cracks known as seepage channels developed on the sediment surface due to upward seepage force. At the same time, under relatively high seepage gradients, circular seepage features, often referred to as micro-mud volcanoes, also appeared on the bed surface. Similarly to the case without seepage, ripples initially formed on the silty bed during the early stages of scouring (Figure 13b), but gradually disappeared as the eroded sediment mass increased with rising flow velocity (Figure 13c). By the end of the experiment, localized areas of the sediment bed experienced more intense erosion and scour. Notably, several visible holes corresponding to the seepage channels remained scattered across the bed surface throughout the scouring process.

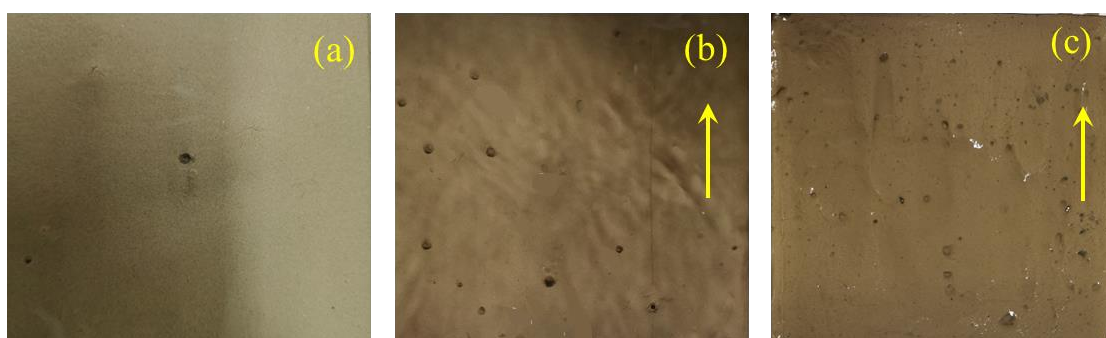


Figure 13. Sediment bed with the effect of upward seepage ($i = 0.75$) before scouring (a), during scouring (b), and after scouring (c), with the arrows indicating the flow direction.

As previously discussed, the physical mechanism by which upward seepage influences sediment erosion is complex, especially for fine-grained sediments. Figure 13 clearly illustrates the formation of seepage channels, indicating the occurrence of internal erosion. When the seepage velocity reached a sufficient magnitude, fine sediment particles were

transported to the sediment surface through these channels following the paths created by the upward-directed flow [27], which significantly enhanced sediment erosion [20]. Therefore, internal erosion emerges as a critical mechanism through which upward seepage affects the erosion characteristics of silty sediments, as the wave-induced excess pore water pressure tends to accumulate in a silty seabed, generating continuous upward seepage flows. These findings highlight the impact of seepage forces on sediment mobilization and erosion, demonstrating how seepage can modify the sediment structure and bed morphology.

4. Conclusions

A series of experiments were carried out to investigate the erodibility of silt under the combined effects of upward seepage and slope gradient. The study focused on analyzing the critical incipient motion of silt particles at the sediment bed surface and the corresponding erosion rate. Regression formulas were developed to estimate the critical Shields parameter and the erosion rate using the hydraulic gradient of seepage and the slope gradient as primary variables. The main conclusions are summarized as follows.

- (1) An increase in the hydraulic gradient of upward seepage and the slope gradient reduces the critical Shields parameter. A linear relation exists between the critical shields parameter and the hydraulic gradient, where the critical Shields parameter approximately decreases at a rate of 0.01 for every 0.1 increase in seepage gradient (for gradients below 0.8), independent of the slope angle.
- (2) The erosion coefficient of silty sediments shows an exponential growth trend with increasing hydraulic gradient, and the average growth rate is amplified as the slope angle increases. For a flat bed, the erosion coefficient in the presence of upward seepage with a hydraulic gradient of 0.75 can be up to five times that without seepage.
- (3) Based on regression analysis of the experimental data, an empirical expression for the critical Shields parameter incorporating the effects of upward seepage and slope gradient was formulated. Comparative analysis showed that this empirical equation accurately captured the effects of seepage and slope gradient on the initiation of silt particle movement, although it deviated from previous theoretical results for cohesionless sediments, particularly at higher hydraulic gradients.
- (4) An empirical formula for estimating the erosion rate of silty sediments was proposed through multiple regression analysis, accounting for the effects of upward seepage and slope gradient. Comparison between the calculated results and experimental data indicated that more than 90% of the experimental results fell within a confidence interval of $\pm 30\%$ relative error.
- (5) At the initial stage of the experiment, ripples formed on the silt bed surface as the amount of eroded sediment mass was minimal. However, these ripples gradually disappeared as the experiment progressed. By the end of the experiment, the sediment surface exhibited a complex bed morphology, characterized by small gullies and pits of varying shapes and sizes after the completion of the experiment. In the presence of upward-directed seepage at relatively high hydraulic gradients, a more uniform bed surface and deeper scours were observed. This was accompanied by the formation of small micro-mud volcanoes on the bed surface prior to the onset of erosion, while visible holes, corresponding to seepage channels, remained scattered across the surface throughout the scouring process. These features suggest that internal erosion plays a significant role in influencing the erosion process of silty sediments.

It is worth noting that there may be a discrepancy between the experimental results and real world in situ conditions due to the limited scouring area of the sediment bed in the experiments. However, the scaling effects in such scenarios are not yet fully understood. Therefore, future studies could investigate these scale effects by conducting experiments with varying scouring areas of the sediment bed. Additionally, future research can further investigate the effects of seepage and slope gradient on the erodibility of silty sediments with different densities and different particle size distributions, as well as the characteristics of silty sediment transport in complex hydrodynamic conditions.

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