

Article

Study on Optimal Allocation of Water Resources Based on Uncertain Multi-Objective Fuzzy Model: A Case of Pingliang City, China

Yun Zhao ¹, Rui Zhang ^{1,*}, Heping Shu ¹, Zhi Xu ², Shangbin Fan ³, Qiang Wang ², Yaxian Li ¹ and Yapeng An ¹

¹ College of Water Resources and Hydropower Engineering, Gansu Agricultural University, Lanzhou 730070, China; zy_010124@163.com (Y.Z.); shuhp@gsau.edu.cn (H.S.); 13884553552@163.com (Y.L.); 18793563633@163.com (Y.A.)

² Gansu Provincial Department of Water Resources Soil and Water Conservation Center, Lanzhou 730030, China; 15099166690@163.com (Z.X.)

³ Pingliang City Water Affairs Bureau of Gansu Province, Pingliang 744000, China; 13201397303@163.com

* Correspondence: zhr_1029@163.com; Tel.: +86-151-9354-0615

Abstract: Water shortages are serious in northwest China due to the level of social and economic development, engineering, resource shortages, and other factors being restricted, so the conflict between supply and demand for water resources is prominent in different regions and different water use sectors. Therefore, Pingliang City was selected as the research object in this study. The membership function was introduced, and an interactive algorithm to correct model parameters based on the fairness constraint was derived. An uncertain multi-objective fuzzy programming model was also established. The results show that the optimal allocation of water will be $38,051.9\sim 40,740 \times 10^4 \text{ m}^3$ and $39,938.4\sim 41,317.5 \times 10^4 \text{ m}^3$ under a normal year ($p = 50\%$) and a dry year ($p = 75\%$) in 2025, respectively, and the corresponding water shortage rates will be 4.2% and 6.7%. In 2035, the optimal water allocation will be $45,644.1\sim 49,245.9 \times 10^4 \text{ m}^3$ and $46,442.4\sim 50,044.2 \times 10^4 \text{ m}^3$ and the water shortage rates will be 7.0% and 7.0%, respectively. The proportion of groundwater supply will decrease by 8.8% and 13.8% in 2025 and 2035 after the optimal allocation, the proportion of surface water supply will increase by 9.6% and 12.2%, and the proportion of reclaimed water will increase by -0.78% and 2.1% , respectively. The results can provide a technical reference for the development and utilization of water resources in other cities and similar areas in semi-arid regions.

Keywords: optimal allocation of water resources; uncertainty; fuzzy programming; fairness; membership function



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1. Introduction

Water resources are not only a security guarantee for the development of the national economy but also an indispensable material resource for the ecological evolution of the Earth, human survival, and development [1,2]. China's total water resources are large; however, the per capita occupancy is low, the distribution of time and space is also uneven, and the utilization rate of water resources is not high, so the contradiction in respect to regional water resource shortages is becoming increasingly serious [3,4], especially in the arid and semi-arid regions of northwest China. Due to lesser rainfall and high evaporation in these regions, agricultural water consumption accounts for up to 80% of the total water, and water competition among various regions has intensified [5,6]. In addition, the optimal allocation of water resources is crucial for addressing the conflict between the supply and demand of water resources and the utilization rate of water resources; it also plays an important role in coordinating water resources [7]. Therefore, the optimal allocation of water resources is an important tool by which to alleviate the shortage of water resources under climate change and is also one of the most effective means by which to improve the utilization efficiency of water resources in arid and semi-arid areas [8].

The challenges of water allocation are exacerbated by the sole pursuit of economic efficiency, intersectoral competition for water, and the integration of multiple water sources [9]. Due to there being many influencing factors in the process of optimal allocation of water resources, it has great uncertainty, fuzziness, and randomness [10,11]. Over the past few decades, many models and methods of fuzzy programming of water resources have been studied, for example, the optimization model (inexact two-stage stochastic programming (ITSP)) [12], linear programming [13], interval parameter programming [14], and inexact fuzzy chance-constrained programming [15]. Some studies have shown that fuzzy optimization models are superior to non-fuzzy optimization models in optimizing multiple objective functions and they can effectively deal with these uncertainties [16–18]. Rodriguez et al. [19] and Rodriguez et al. [20] also confirmed that fuzzy logic can model and assess uncertain conditions and has been maturely utilized in engineering and environmental management decision-making. Therefore, in their water resource optimal allocation model, Zhang et al. proposed an uncertain two-stage allocation model, representing the uncertainty in water resources in the Sanjiang Plain, China, via interval values and probability density. They realized a scheme with maximum benefit and less risk; it also reduced the penalty caused by insufficient supply and reduced water resource waste [21]. Ji et al. proposed an imprecise fuzzy chance-constrained programming model; four water policies were designed for Tianjin and the advantages and disadvantages of the water distribution scheme were evaluated, realizing that strict policies help to save water and optimize the water supply structure [15]. Ahmad et al. also proposed a two-level multi-objective linear programming model via simultaneous compromise constraints [22]. It was applied to the Swat River Basin in Pakistan in order to balance the fairness among water use sectors and solve four allocation schemes under certain allocation priorities. The advantages of this method for solving bi-level programming problems were verified. However, although the above planning models verified the versatility of the fuzzy algorithm, the fairness of water resource allocation has not been well reflected, and in the optimal allocation of water resources, it is difficult to use sufficient data to describe the probability distribution of uncertainty. Therefore, in order to increase the fairness of a water distribution system, the Gini coefficient may be selected as its key constraint. The Gini coefficient is one of the most common indicators of intra-generational equity, and it has been widely used to measure the fairness of water resource allocation [23,24]. Many scholars [25–27] have used the Gini coefficient to balance the fairness of water use and economy in the sector and have applied it to the Karkheh Basin, Minjiang River, and Qujiang River Basin. They verified that the Gini coefficient is an effective method for the fair distribution of water resources. In order to provide the optimal water distribution solution, addressing issues such as the solution interval being too wide and difficult to determine, a trapezoidal fuzzy method is usually selected for transformation solution; this can clearly reflect the fuzzy concept [28]. For example, Li et al. [29] and Ren et al. [30] used the trapezoidal membership function in the Wuwei water distribution system. They divided the fuzzy set into different membership levels to better describe the fuzzy nature of parameter uncertainty and verified that the method was conducive to identifying the optimal water distribution scheme under the uncertainty and the multi-objective model.

In terms of optimal allocation methods, multi-objective programming, stochastic programming [31], interval parameter programming [32], fuzzy mathematical programming [33], or a combination of various methods have been used to solve the coordination problem in respect to water resources and complex systems, such as society and the economy. Among them, multi-objective models with interval coefficients were difficult to solve; an improved two-step method developed by Wang and Huang was proven to be an effective interval planning method for the problem [34]. For example, Khosrojerdi et al. proposed an interval fuzzy two-stage stochastic programming method to alleviate the contradiction between supply and demand. They designed four water distribution schemes in Kerman Province, Iraq [35]; the water demand was met and the utilization rates of surface water and groundwater were improved. Suo et al. proposed the fuzzy

interval dynamic programming (FIDP) method and applied it in Handan City [36]. The uncertainty was expressed by the interval number and the overall satisfaction was used to solve the water allocation conflict, with water allocation and water shortages at different stages presented in the form of intervals; however, the interval for solving the model was too large. Fatemeh et al. proposed robust fuzzy stochastic programming and other combination methods for the Mashhad Plain, and they obtained the optimal water allocation under scarce conditions [37]. However, fair allocation based on the sacrifice of agricultural and domestic water uses did not guarantee that every region had a fair water right; to this end, satisfaction was introduced as a fairness indicator. The accurate prediction of water resources is helpful for an accurate allocation. Because of the advantages of the gray prediction model, such as simple calculation and its ability to interpret uncertain data and unclear relationships, it has high applicability in water resource prediction and is widely used in water, energy, and water pollution prediction [38,39]. Despite the previous studies having effectively addressed data uncertainty through optimal allocation models and methods, they also exhibit limitations, such as the trade-off between efficiency and fairness, biased water flow, and optimization of a water supply structure. However, while there is extensive research on optimizing model selection and method or reducing the impact of uncertainty factors in the model, relatively few studies focus on balancing and dynamically allocating water resources with multi-objective comprehensive benefits in the arid and semi-arid regions of northwest China. Therefore, prior to selecting membership functions and interval multi-objective programming to address distribution system uncertainty, the Gini coefficient is utilized to regulate water distribution balance, which facilitates the development of specific and reliable strategies for the management of multiple water resources in arid and semi-arid regions.

The year 2020 was taken as the base year and Pingliang City was selected as the research area. The Gini coefficient was used to measure the fairness of water distribution, and the membership function and interval programming method were used to deal with the uncertainty of the system. An uncertain multi-objective fuzzy programming model was constructed to achieve the optimal allocation of water resources under a normal year ($p = 50\%$) and a dry year ($p = 75\%$) in 2025 and 2035. The main research objectives included the following: (1) The Gini coefficient was employed as the constraint condition, and the fairness red line value of the Gini coefficient was adopted to strictly control the fairness of the water distribution among various departments. This aims to prevent low-benefit departments from receiving disproportionately small water allocations. (2) The membership function was introduced, the upper and lower bounds corresponding to the membership function were transformed into two determined sub-models, and the interval solution was obtained through interval fuzzy linear programming. (3) We studied the water distribution plan and water shortage situation in Pingliang City and proposed policies, projects, and other water replenishment measures. The results can provide a reference for urban water resource allocation in similar arid and semi-arid areas.

2. Study Area

Pingliang City is located in the northwest part of Gansu Province in China (Figure 1). It belongs to the temperate semi-arid continental monsoon climate zone in the south of Longzhong, and the climate division of the province is the Jingwei cold temperate sub-humid zone. It has an average annual temperature of 7.9 °C and an annual average precipitation of 484.7 mm; the annual rainfall is mainly concentrated from June to September, accounting for 67.4% of the annual precipitation. The annual evaporation from the water surface is 880.7 mm and the drought index is 1.83 [40]. Kongtong (KT), Huating (HT), Chongxin (CX), Jingchuan (JC), and Lingtai (LT) in the east are located in the Jing River Basin, and Zhuanglang (ZL) and Jingning (JN) in the west are located in the Hulu River, a tributary of the Weihe River Basin [41].

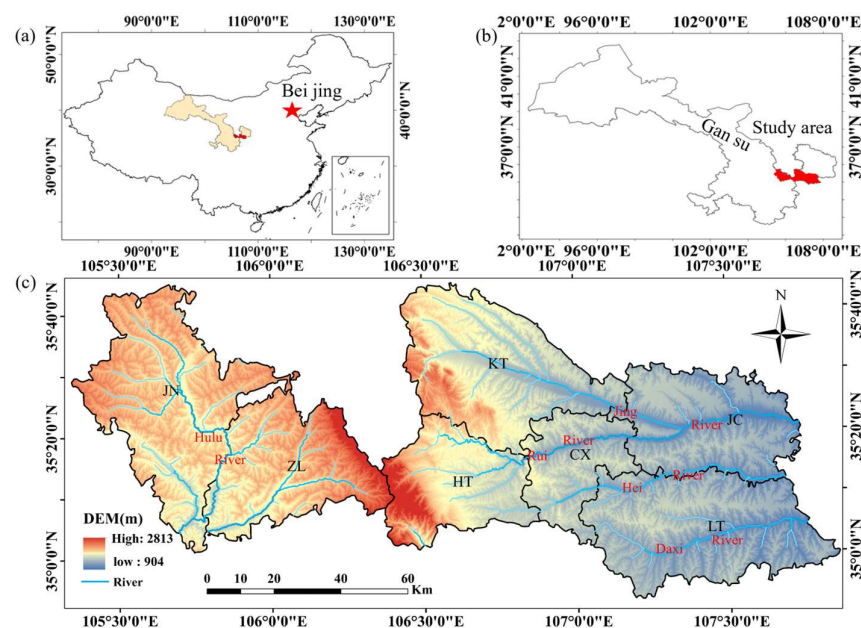


Figure 1. Geographical location of the study area: (a) map of China showing the location of Gansu Province, (b) map of Gansu highlighting Pingliang, (c) specific details of Pingliang City and its water resources' distribution.

Water resources are scarce in this region, with more people and less water. The average annual total available water resources are $6.92 \times 10^8 \text{ m}^3$; surface water resources are $5.87 \times 10^8 \text{ m}^3$ and underground water resources are $1.05 \times 10^8 \text{ m}^3$ [42]. The major rivers are experiencing increasing water scarcity with climate change and declining groundwater levels; Figure 2 shows the use of water resources by region for each water sector in the base year. The permanent resident population was 1.82×10^6 in 2020, and the annual GDP reached CNY 6.42×10^9 , with a per capita GDP of CNY 3.52×10^4 . The research area is rich in mineral resources, with relatively concentrated reserves of various resources, including coal deposits of 9.56×10^8 tons [43,44], and it is one of the three major coal-producing areas in northwest China.

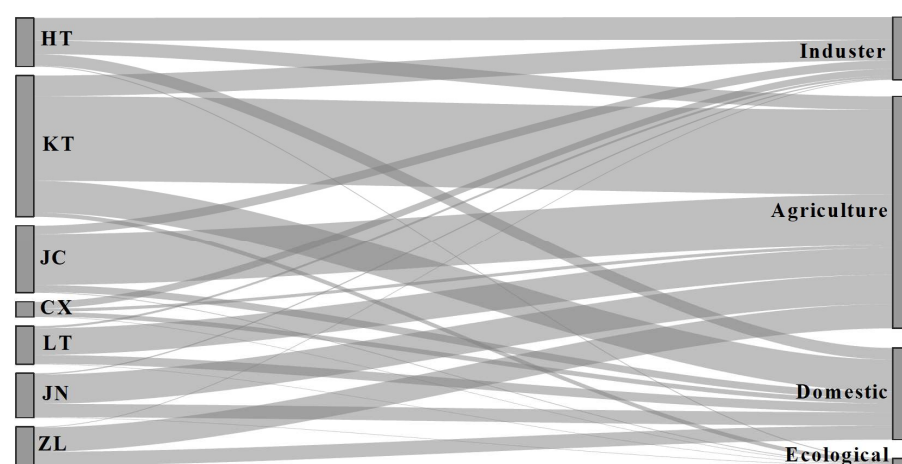


Figure 2. Current water uses in the different counties and districts of the study area. The left side shows the different regions, and the right shows the four major water use sectors. The length of the bar indicates the amount of water resources, the tributary between the different regions and the water use sector indicates the amount of water allocated to the regions by the water use sector, and the width of the tributary indicates the corresponding value of the amount of water.

3. Materials and Methods

3.1. Data

The data used mainly included socio-economic development data and long-term water resource data (Table 1). Other basic data were calculated according to survey data in the study area.

Table 1. Sources of research data.

Data	Name	Source	Time Sequence/Year
Social and economic development	Gansu provincial statistical yearbook	Gansu Provincial Water Resources Department Lanzhou, China	2007~2021
	Bulletin of National Economic and Social Development	KT, HT, JC, CX, LT, JN, ZL people's government website Pingliang, China	2010~2022
Water resources	Gansu water resources bulletin	Gansu Provincial Water Resources Department Lanzhou, China	2007~2022
	Pingliang water resources bulletin	Pingliang City People's Government Pingliang, China	2021
	Pingliang City 14th Five-Year Plan for water conservancy development		
Planning outline	The 14th Five-Year Plan for Pingliang's national economic and social development and the outline of the 2035 vision goals	Pingliang City People's Government Pingliang, China	2022
			2021
DEM (30 m)	Pingliang City range vector data	https://www.gscloud.cn/search Geospatial data cloud, China	1 March 2023

3.2. Method

3.2.1. Gray Prediction Model

The gray prediction model is a short- and medium-term prediction model and can simulate and describe the required data by using the change rule of some known information in order to solve the problem of few data points and high information uncertainty [45].

Preprocessing of data

(1) Original column

$$x^{(0)}(1) = \{x^{(0)}(1), x^{(0)}(2), \dots, x^{(0)}(m)\} \quad (1)$$

(2) Accumulation of data

$$x^{(1)}(i) = \left\{ \sum_{i=1}^m x^{(0)}(i) \mid i = 1, 2, \dots, m \right\} \quad (2)$$

(3) Accumulated data reduction

$$\Delta x^{(1)}(i) = x^{(1)}(i) - x^{(1)}(i-1) = x^{(0)}(i) \quad (3)$$

Here, $i = 1, 2, \dots, m$, $x^{(0)}(0) = 0$.

A new discrete data column with more regular randomness reduction was generated via one-time accumulation; then, the basic form of the model was obtained by establishing a differential equation model:

$$x^{(0)}(k) + \alpha z^{(1)}(k) = b \quad (4)$$

The prediction equation is as follows:

$$x^{(1)}(t) = \left(x^{(0)}(1) - \frac{b}{a} \right) e^{-a(t-1)} + \frac{b}{a} \quad (5)$$

where the main parameter a represents the development coefficient and b represents the gray action.

The prediction equation can be used to obtain the cumulative value of the prediction, and the prediction value needs to be inversely calculated by subtraction. The value of decreasing reduction was the required prediction value. This method was used to forecast water demand and development indicators [46]. In addition, the local water conservancy development plan, population growth, the implementation of industrial water-saving measures, and the construction of high-standard farmland water-saving irrigation were strictly considered in the process of forecasting. This method was used to forecast water demand and development indicators. In addition, the local water conservancy development plan, population growth, the implementation of industrial water-saving measures, and the construction of high-standard farmland water-saving irrigation were strictly considered in the process of forecasting.

3.2.2. Fairness Constraint

The Gini coefficient is often used to measure income inequality, as well as land and water inequality [47]. In order to realize the fair distribution of regional water resources and maximize the benefits of the regional objectives, the Gini coefficient [48] was introduced as the fairness index of the water supply capacity [27,49].

$$\sum_{k=1, k'=2}^K \sum_{k'=2}^K \left\{ \sum_{j=1, j'=2}^J \sum_{j'=2}^J \frac{\left| \frac{W(k,j)}{D(k,j)} - \frac{W(k',j')}{D(k',j')} \right|}{KJN} \right\} \leq \alpha \quad (6)$$

Here, $W(k, j)$ is the water allocation of the water department of j in sub-region k ; $D(k, j)$ is the water demand of water department j in sub-region k ; N is the sum of the ratio of water allocation and water demand of all sub-regions and water consumption departments; K is the number of sub-regions; J is the number of water consumption departments; k, k' are sub-regions; j, j' are sub-region water consumption departments; and α is the Gini coefficient. The Gini coefficient, commonly used to gauge economic equality, is also applicable in the context of water resources. A smaller Gini coefficient indicates a more evenly distributed allocation of water resources, while a larger coefficient suggests greater inequality in distribution. This can be understood as the ratio of water allocation to total supply across different regions falling within an acceptable range. In essence, if a region yields higher net benefits, it will receive a larger share of water allocation, potentially disadvantaging regions with lower benefits. Therefore, the introduction of the Gini coefficient serves to assess the fairness of water resource distribution and prevent excessive concentration in areas with higher benefits. According to the regulations of international organizations, less than 0.2 means absolutely fair, 0.2~0.3 means relatively fair, 0.3~0.4 means basically fair, 0.4~0.5 means a large gap, 0.5 or more means unfair, and 0.4 represents the red line standard; α is 0.4, which can be used as the upper limit value of constraint condition in the model [50]. The solutions solved are within the range of basic fairness in this way, and many times iterations that solutions satisfying the constraints are solved.

3.2.3. Membership Functions

Membership functions are the core idea of fuzzy set theory. The theory involves a representation in which binary relations are transformed into continuous uncertainties and evaluated using membership functions [51]. Because of the uncertainty of water use factors in a water resource system, fuzzy sets and membership functions were used to represent

the uncertainty in our model [52]. When constructing the nonlinear membership function of multi-objective fuzzy programming, the objective function was divided into maximum optimal and minimum optimal membership functions according to its characteristics, and the nonlinear function was introduced to transform the objective functions in the model construction.

Fuzzy membership function is introduced to represent the three objective functions of a constructed model. If we set f to represent any objective function, then the corresponding nonlinear membership function can be expressed as follows:

Maximum optimal type:

$$\mu_f(x) = \begin{cases} 0 & f \leq f_a \\ \left(\frac{f_b - f}{f_b - f_a}\right)^{\beta_1} & f_a < f < f_b \\ 1 & f \geq f_b \end{cases} \quad (7)$$

Minimum optimal type:

$$\mu_f(x) = \begin{cases} 1 & f \leq f_a \\ \left(\frac{f_b - f}{f_b - f_a}\right)^{\beta_2} & f_a < f < f_b \\ 0 & f \geq f_b \end{cases} \quad (8)$$

where $\mu_f(x)$ is the membership function of f ; f_b, f_a are the maximum and minimum values of the objective function. $\beta(\beta_1, \beta_2)$ is the shape index of the nonlinear membership function; $\beta=1$ indicates linearity and $\beta > 1$ and $0 < \beta < 1$ indicate nonlinearity. We introduce variable γ , the original model of the objective function in the form of membership functions, by $[\mu_f(x)]^{\beta_n} \geq \gamma (0 \leq \gamma \leq 1)$ converted into constraints. A single decision problem with the objective function $\max f(x) = \gamma^\pm$ was constructed together with the original constraints.

3.2.4. Interval Fuzzy Linear Programming

The interval fuzzy linear programming improves the problems that do not correspond to the constraints of different objective functions by means of fuzzy operator decomposition and makes the accuracy of the interval results more reliable [53]. X is the closed set of bounded real numbers, and $X^\pm$ is the number of intervals with known upper and lower limits. The model is as follows:

$$\text{Max} \gamma^\pm \quad (9)$$

Subject to:

$$C_1^\pm X^\pm \geq f_1^- + \gamma^\pm (f_1^+ - f_1^-) \quad (10)$$

$$X_2^\pm X^\pm \geq f_2^- + \gamma^\pm (f_2^+ - f_2^-) \quad (11)$$

$$A^\pm X^\pm \leq B^\pm \quad (12)$$

$$X^\pm \geq 0 \quad (13)$$

$$0 \leq \gamma^\pm \leq 1 \quad (14)$$

Also, need to note:

$$C_1^\pm X^\pm = \text{Max} f_1^\pm \quad (15)$$

$$C_2^\pm X^\pm = \text{Max} f_2^\pm \quad (16)$$

Here, $C_1^\pm \in \{R_1^\pm\}^{1 \times n}$, $C_2^\pm \in \{R_2^\pm\}^{1 \times n}$, $A_i^\pm \in \{R_3^\pm\}^{1 \times n}$, $B_i^\pm \in \{R_4^\pm\}^{n \times 1}$, $X^\pm \in \{R_4^\pm\}^{n \times 1}$, ($R^\pm$ means a set of interval numbers), 1 and 2 are maximizing and minimizing of objective functions, f^+, f^- are the upper and lower limit of $f^\pm$; the higher the value of $\gamma^\pm$, the more reliable the result.

3.3. Uncertain Multi-Objective Fuzzy Programming Model

3.3.1. Objective Function

- (1) Social objective (10^4 m^3): measured indirectly in terms of minimizing water shortages in the region as a whole.

$$F_1(x) = \min f_1^\pm(x) = \sum_{k=1}^K \sum_{j=1}^J \sum_{i=1}^I (Q_{kj}^\pm - B_{ij}x_{kij}^\pm) \quad (17)$$

Here, k represents 7 regions (KT, HT, JC, CX, LT, JN, ZL) that are numbered as sub-regions ($k = 1, 2, \dots, K$); K is the number of sub-regions; j represents the four water use departments (domestic, industrial, agriculture, ecology); i represents the three main sources of water; $Q_{kj}^\pm$ represents the water demand of water use sector j in region K ; $x_{kij}^\pm$ represents the water supply from source i to water department j in region k ; and B_{ij} denotes the water supply from source i to water department j ; 1 indicates water supply, 0 indicates no water supply.

- (2) Economic goal (million CNY): after the optimal allocation, the goal is to maximize the economic benefits of the region directly reflected.

$$F_2(x) = \max f_2^\pm(x) = \sum_{k=1}^K \sum_{j=1}^J \left[\sum_{i=1}^I (b_{kj}^\pm - c_{kj}^\pm) B_{ij}x_{kij}^\pm \alpha_{ik} \right] \beta_{kj} \omega_k \quad (18)$$

Here, $b_{jk}^\pm$ is the water benefit coefficient generated by the water supply from water source i to water department j in region k ; $c_{jk}^\pm$ is the water supply cost coefficient generated by the water source supplying water to water department j in region k ; α_{ik} is the water supply sequence coefficient of unit water source i in region k ; β_{kj} is the water equity coefficient of user j in region k ; ω_k is the weight coefficient of region k .

- (3) Ecological objective (tons): indirectly expressed as a minimum value (COD) that represents the pollution component of the area.

$$F_3(x) = \min f_3^\pm = \left[\sum_{k=1}^K \sum_{j=1}^J 0.01 d_{kj}^\pm p_{kj}^\pm \sum_{i=1}^I B_{ij}x_{kij}^\pm \right] \quad (19)$$

Here, $d_{kj}^\pm$ is the COD content of the user's unit wastewater discharge in region k ; $p_{kj}^\pm$ is the sewage discharge coefficient of user j of region k .

3.3.2. Constraint Condition

- (1) Constraints on the available water supply

The total amount of water in each water department should not be more than the available water supply.

$$\sum_{k=1}^K \sum_{j=1}^J x_{kij}^\pm B_{ij} \leq W_i^\pm \quad (20)$$

Here, $W_i^\pm$ is the water supply capacity.

- (2) User water requirement constraint

The amount of water provided by each water source must not be lower than the minimum water consumption of the user or exceed the maximum water consumption of the user.

$$Q_{kj\min} \leq \sum_{i=1}^I B_{ij}x_{kij}^\pm \leq Q_{kj\max} \quad (21)$$

Here, $Q_{kj\min}^\pm$ is user j 's minimum water requirement in region k ; $Q_{kj\max}^\pm$ is user j 's maximum water requirement in region k .

(3) Pollutant discharge restraint

The pollutant emission content should not be higher than the pollutant emission red line stipulated by the government.

$$\sum_{j=1}^J 0.01 d_{kj}^{\pm} p_{kj}^{\pm} \sum_{i=1}^I B_{ij} x_{kij}^{\pm} \leq C_k^{\pm} \quad (22)$$

Here, $C_K^{\pm}$ denotes the maximum pollutant (COD) emissions of region k.

(4) Fairness constraint

The water use equity constraint (3) was transformed.

$$\sum_{k=1, k'=2}^K \sum_{k' > k}^K \left\{ \sum_{j=1, j'=2}^J \sum_{j' > j}^J \left| \frac{\sum_{i=1}^I B_{ij} x_{kij}^{\pm}}{Q_{kj}^{\pm}} - \frac{\sum_{i=1}^I B_{ij'} x_{k'ij'}^{\pm}}{Q_{k'j'}^{\pm}} \right| \right\} \leq \alpha \quad (23)$$

(5) Variables are not negatively constrained

The water supply of any water sector in each sub-region must not be negative.

$$x_{kij}^{\pm} \geq 0 \quad (24)$$

3.3.3. Determination of Model Parameters

(1) Water use efficiency coefficient

Equations (22)–(24) were used for calculation [54]. The water uses' efficiency coefficients for life, industry, agriculture, and ecology in 2025 will be 51.58 CNY/m³, 29.73 CNY/m³, 48.61 CNY/m³, and 31.37 CNY/m³, respectively. In 2035, they will be 68.17 CNY/m³, 39.29 CNY/m³, 64.24 CNY/m³, and 41.45 CNY/m³, respectively.

$$R_g^{\pm} = \frac{\mu}{W} \quad (25)$$

$$Y_{kj}^{\pm} = \varepsilon_{kj}^{\pm} R_g^{\pm} \quad (26)$$

$$\varepsilon_{kj}^{\pm} = \begin{cases} \omega_{kj}^{\pm} & 0 < Q_{kj}^{\pm} \leq Q_{kjmin}^{\pm} \\ \frac{[\partial_{kj}^{\pm} Q_{kjmin}^{\pm} + \mu_{kj}^{\pm} (Q_{kj}^{\pm} - Q_{kjmin}^{\pm})]}{Q_j} & Q_{kjmin}^{\pm} < Q_{kj}^{\pm} \leq Q_{kjmax}^{\pm} \\ \frac{[\partial_{kj}^{\pm} Q_{kjmin}^{\pm} + \mu_{kj}^{\pm} (Q_{kjmax}^{\pm} - Q_{kjmin}^{\pm})]}{Q_j} & Q_{kj}^{\pm} > Q_{kjmax}^{\pm} \end{cases} \quad (27)$$

Here, $\partial_{kj}^{\pm}$, $\mu_{kj}^{\pm}$, $\varepsilon_{kj}^{\pm}$ are reduction coefficients; the coefficients were determined using the Delphi method as $\partial_{kj}^{\pm} = [1.8 \ 1.1 \ 1.7]$, $\mu_{kj}^{\pm} = [0.5 \ 0.2 \ 0.4]$. $R_g^{\pm}$ denotes the benefits of unilateral industrial water use. W is the water consumption in respect to CNY 10,000 industrial added value. $Y_{kj}^{\pm}$ is the water use efficiency coefficient in region k.

(2) Water supply cost coefficient

The water supply cost coefficient was based on the local water fee collection standard of Pingliang City. Finally, the cost coefficients of domestic, industry, agriculture, and ecology in 2025 were determined to be 2.5 CNY/m³, 3.5 CNY/m³, 0.6 CNY/m³, and 1.5 CNY/m³, respectively. In 2035, these will be rounded on the basis of 2025, giving 3.0 CNY/m³, 4.0 CNY/m³, 1.0 CNY/m³, and 2.0 CNY/m³, respectively.

(3) Water use equity coefficient and water supply order coefficient

We optimized the regional water structure to ensure the overall goals of regional economic and social development. The results calculated using Equation (25) were 0.5 for local surface water [55], 0.17 for groundwater, and 0.33 for reclaimed water. The water distribution priority coefficients of the four water use departments (domestic, industry, agriculture, and ecological) of the water source were determined; the results were as follows: domestic water 0.4, agricultural water 0.1, industrial water 0.3, and ecological water 0.2.

$$\rho_{kj}^{\pm} = \frac{1 + n_{kj,\max}^{\pm} - n_{kj}^{\pm}}{\sum_{n=1}^N (1 + n_{kj,\max}^{\pm} - n_{kj}^{\pm})} \quad (28)$$

Here, $\rho_{kj}^{\pm}$ is the water supply order coefficient; $n_{kj}^{\pm}$ is the water use serial number of water department j in region k ; $n_{kj,\max}^{\pm}$ is the maximum water use serial number of water department j in region k .

(4) Water weight coefficient

The weight coefficient of the study area represents the degree of importance among the areas. The weight coefficient is calculated using the analytic hierarchy process (AHP) [56]. When AHP was used to calculate the importance weight of water consumption sector, six aspects were considered: tourism, population, industrial added value, area, GDP, and water consumption. The consistency ratio (CR) did not exceed 0.1, and they proved that the judgment matrix was consistent. Therefore, the calculated weights are reasonable. The calculated weight coefficients of KT, HT, CX, JC, LT, JN, and ZL were 0.3, 0.2, 0.12, 0.05, 0.07, 0.20, and 0.06, respectively.

3.3.4. Solving Procedure

The objective function of this study represents the maximum economic benefit, the minimum social water shortage, and the minimum environmental pollution discharge. According to the characteristics of the objective function, the maximum economic benefit belongs to the maximum optimal type; the membership function was calculated using Equation (4). The minimum water shortage and minimum COD emission belong to the minimum optimal type, and the membership function was calculated using Equation (5). The multi-objective problem of the original model was transformed into a single objective problem; the transformation model of the objective function and constraints was as follows.

Objective function

$$\max f(x) = \gamma^{\pm} \quad (29)$$

Constraint conditions

$$\sum_{k=1}^K \sum_{j=1}^J \omega_k B_{ij} \sum_{i=1}^I B_{ij} \beta_{kj} x_{kij}^{\pm} \geq f_1(x)_{\min} + \gamma^{\pm} (f_1(x)_{\max} - f_1(x)_{\min}) \quad (30)$$

$$\sum_{k=1}^K \sum_{j=1}^J \sum_{i=1}^I (Q_{kj}^{\pm} - B_{ij} x_{kij}^{\pm}) \geq \gamma^{\pm} (f_2(x)_{\max} - f_2(x)_{\min}) - f_2(x)_{\max} \quad (31)$$

$$\sum_{k=1}^K \sum_{j=1}^J 0.01 d_{kj}^{\pm} p_{kj}^{\pm} \sum_{i=1}^I B_{ij} x_{kij}^{\pm} \geq \gamma^{\pm} (f_3(x)_{\max} - f_3(x)_{\min}) - f_3(x)_{\max} \quad (32)$$

Here, $f_1(x)_{\max}$, $f_1(x)_{\min}$, $f_2(x)_{\max}$, $f_2(x)_{\min}$, and $f_3(x)_{\max}$, $f_3(x)_{\min}$ correspond to objective functions 1, 2, and 3 in the deterministic model. Equations (17)–(21) were the maximum and minimum values to be solved under the constraints of the single objective model. The results in respect to water demand and available water supply prediction were input into the model, and the results of the membership function were solved according to linear and

nonlinear functions with little difference in the model, so we considered the membership function a linear function [57].

According to the parameter and water supply relationship setting, LINGO 11.0 was used for programming. The model was transformed into a single objective interval fuzzy linear programming problem (IFLP) [53], and the single objective model was decomposed into two deterministic sub-models; corresponding to the upper and lower limits of the interval, the interval solution was obtained by interval linear programming (ITSM) [34]. The solution steps can be summarized as follows: 1. Each target with Equations (17)–(21) was solved by using interval linear programming. The corresponding f^- and f^+ were obtained and transformed into a multi-objective fuzzy programming model with membership function and fuzzy number characteristics. 2. The multi-objective fuzzy programming model was integrated into a single objective model and decomposed into two sub-models (upper and lower limits). Firstly, we solved the γ^+ sub-model to obtain the corresponding x_{kij}^+ and γ^+ ; then, we solved the γ^- sub-model to obtain the corresponding x_{kij}^- and γ^- . 3. We integrated the solutions of the two sub-models with all the target values by calculating all the values x_{kij}^+ and x_{kij}^- of the three targets. The values with the highest satisfaction and the most consistency with $\gamma^\pm$ were solved. Then, the optimal result interval $[x_{kij}^-, x_{kij}^+]$ of the model was obtained.

4. Results

4.1. Supply and Demand Balance Analysis

4.1.1. Water Demand Forecasting

This study used the quota method to predict water demand. In the planning year, the lower limit of the domestic water demand and the upper limit of the industrial and ecological water demand were forecast. The upper limit of the domestic water demand and the lower limit of the industrial and ecological water demand were 120%, 80%, and 95% of the forecast water demand, respectively. The lower limit of the agricultural water demand was 105% and 95% of the forecast water demand [58].

For the years 2025 and 2035, the regional domestic, industrial, agricultural, and ecological water demand were predicted under a normal year and a dry year (Table 2). In 2025, compared with 2020, the water demand of the four major water consumption sectors will have increased, of which agriculture will still be the main water demand sector. In 2035, compared with 2025, JN and ZL industrial water demand decreases and the agricultural growth rate shows a downward trend. The differences are that the ecological water demand will increase in JC and JN in 2025, while the water demand in other regions has a downward trend. In 2035, the ecological water demand of KT, JC, and ZL will increase compared with that of 2025, while the ecological water demand of HT, CX, and LT will decrease.

Table 2. Forecast results of water demand in 2025 and 2035 (10^4 m^3).

Year	Area	Domestic	Industrial	Agricultural 50%	Agricultural 75%	Ecology
2025	KT	[1896.92, 2276.31]	[1163.87, 1454.84]	[5882.39, 6501.59]	[6316.73, 6981.65]	[121.41, 127.8]
	HT	[690, 828]	[1814.78, 2268.47]	[2040.11, 2254.85]	[2154.44, 2381.22]	[38.57, 40.6]
	JC	[818.88, 982.65]	[198.32, 247.9]	[5020.75, 5549.25]	[5388.09, 5955.25]	[61.37, 64.6]
	CX	[356.75, 428.1]	[959.34, 1199.17]	[1838.25, 2031.75]	[1944.92, 2149.64]	[23.09, 24.3]
	LT	[746.24, 895.49]	[315.54, 394.42]	[2346.94, 2593.98]	[2486.27, 2747.99]	[31.54, 33.2]
	JN	[1333.93, 1600.71]	[161.91, 202.39]	[4160.34, 4598.28]	[4435.67, 4902.59]	[68.97, 72.6]
	ZL	[1219.06, 1462.88]	[305.18, 381.47]	[3548.96, 3922.54]	[3734.96, 4128.12]	[68.12, 71.7]
	KT	[2277.64, 2733.16]	[1552.1, 1940.12]	[6076.07, 6715.65]	[6289.66, 6951.72]	[211.68, 222.82]
2035	HT	[876.91, 1052.3]	[2680.38, 3350.48]	[3863.67, 4270.37]	[3919.88, 4332.5]	[34.52, 36.34]
	JC	[854.39, 1025.27]	[810.43, 1013.04]	[6724.75, 7432.61]	[6905.41, 7632.29]	[95.25, 100.26]
	CX	[469.77, 563.73]	[1169.95, 1462.44]	[2240.49, 2476.33]	[2292.9, 2534.26]	[19.98, 21.03]
	LT	[953.62, 1144.34]	[551.04, 688.8]	[2508.82, 2772.9]	[2577.38, 2848.68]	[29.17, 30.7]
	JN	[1660.77, 1992.92]	[156.35, 195.44]	[4236.74, 4682.72]	[4372.12, 4832.34]	[64.7, 68.1]
	ZL	[1538.8, 1846.56]	[256.48, 320.6]	[4269.14, 4718.52]	[4360.65, 4819.67]	[71.18, 74.93]

There is a downward trend with industrial water in JN and ZL due to the implementation of water-saving policies for industrial water; the reuse potential of enterprises will increase and the water demand will decrease. With respect to agricultural water, an overall growth trend was presented; the reason for the increase in agricultural water demand is the development of animal husbandry rather than the amount of farmland irrigation. HT, CX, and LT are located in the transition zone of Liupan Mountain. Because rainfall is decreasing year by year and vegetation is wilting, this leads to vegetation coverage decreasing, so ecological water demand is reduced.

4.1.2. Water Supply Forecasting

The data used mainly included long-term water resource data. Other basic data were calculated according to survey data for the study area. The lower limit of the water supply was the water supply actually predicted by trend analysis, and 110% of the lower limit was taken as the upper limit [58] (Table 3).

Table 3. Forecast results of water supply from different sources (10^4 m^3).

Planning Year	Source of Water	Water Supply	Water Supply Relationship B_{ij}			
			Domestic	Industrial	Agricultural	Ecological
2025	Surface water	[30,000, 33,000]	1	1	1	1
	Underground water	[6067.00, 6673.7]	1	1	1	0
	Reclaimed water	[4556.06, 5011.67]	0	1	1	1
2035	Surface water	[35,000, 385.00]	1	1	1	1
	Underground water	[5120.67, 5632.74]	1	1	1	0
	Reclaimed water	[6965.01, 7661.51]	0	1	1	1

4.1.3. Supply and Demand Balance Analysis

The total water demand of the study area is $37,231.53 \sim 42,509.84 \times 10^4 \text{ m}^3$ and $38,854.87 \sim 44,304.06 \times 10^4 \text{ m}^3$ for 2025, respectively, under a normal year and a dry year (Table 2). The total water demand is $46,254.78 \sim 52,952.48 \times 10^4 \text{ m}^3$ and $47,053.11 \sim 53,834.84 \times 10^4 \text{ m}^3$ for 2035, respectively, representing an overall increase in water demand in each planned year. The maximum water demand is even lower than the total water consumption index of $45,250 \times 10^4 \text{ m}^3$ for 2025; due to the industrial and agricultural water-saving modes, the planned annual total water demand is strictly controlled within the red line of water quantity under the water consumption constraint of the government.

Figure 3 shows that agriculture has always been the sector with the largest water demand. In 2025, the water demand in a dry year will have increased by $1623.34 \sim 1794.22 \times 10^4 \text{ m}^3$ compared with that in a normal year, and by $798.32 \sim 882.36 \times 10^4 \text{ m}^3$ in 2035, indicating that the growth rate of the agricultural water demand slows down. The maximum departments of domestic water consumption have always been KT, JN, and ZL. The largest departments of industrial water demand are HT, CX, and KT, and these will change to HT, KT, CX, and JC in 2035. The water demand of JC increases rapidly, while the water demand of JN and ZL decreases. The ecological water demand of KT is the largest over all time, and the ecological water demand of HT, CX, LT, and JN decreases slightly.

The water supply was $25,000 \times 10^4 \text{ m}^3$ in the base year, and the total water supply is $40,623.06 \sim 47,496.04 \times 10^4 \text{ m}^3$ and $47,085.68 \sim 59,425.17 \times 10^4 \text{ m}^3$ in 2025 and 2035 (Table 3). Compared with the base year, the water supply increased in the planning year, and the water supply and demand should be balanced, according to the lower limit of the total water demand and the lower limit of the total water supply. The total water supply can basically meet the water demand requirements in 2025 and 2035. However, the balance of the total water supply and demand does not mean that the water demand of each department is satisfied. Therefore, in order to avoid the concentration of surplus water and water shortages in each water department, it is necessary to optimize the allocation of

water resources and to achieve a fair water distribution and efficient use of water resources under the goal of optimal comprehensive benefits.

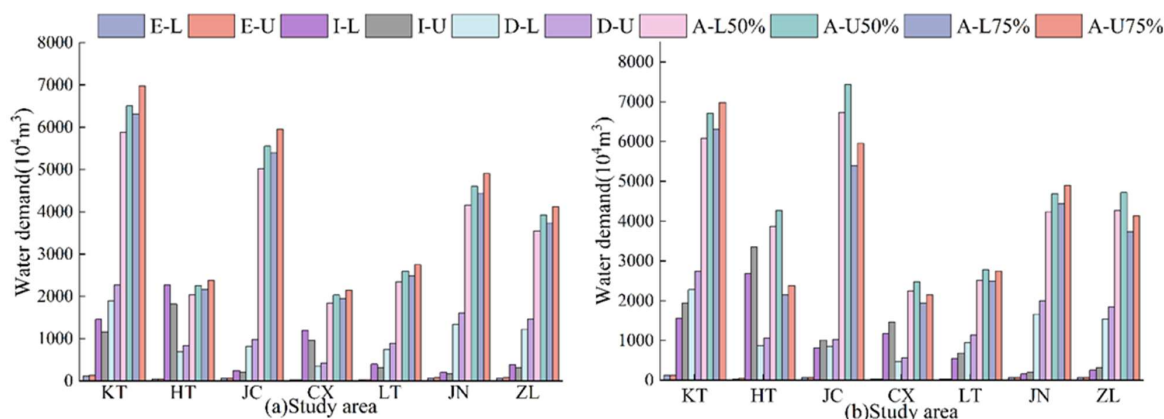


Figure 3. Results of the planned annual water demand forecast. (a) Forecast interval value of water demand in 2025. (b) Forecast interval value of water demand in 2035. D, domestic; I, industry; A, agriculture; E, ecology; U, upper limit; L, lower limit. (For example, E-L represents the lower limit of ecology).

4.2. Optimal Allocation of Water Resources

4.2.1. Water Resource Optimal Distribution Program

In 2025, KT is projected to have the highest agricultural water allocation at $6501.59 \times 10^4 \text{ m}^3$ and $6434.08 \times 10^4 \text{ m}^3$, respectively. Conversely, CX is expected to have the lowest agricultural water distribution at $1838.25 \times 10^4 \text{ m}^3$ and $1944.91 \times 10^4 \text{ m}^3$, respectively. HT will lead in industrial water distribution with a total of $2268.47 \times 10^4 \text{ m}^3$, while JN will have the smallest range at $155.71\sim 168.11 \times 10^4 \text{ m}^3$. The maximum domestic water distribution for KT is estimated to be between 2226.65 and approximately $2276.31 \times 10^4 \text{ m}^3$, whereas CX is anticipated to have the minimum of $356.75\sim 428.95 \times 10^4 \text{ m}^3$.

In 2035, it is projected that KT will maintain its position with the largest agricultural water allocation of approximately $6076.07 \times 10^4 \text{ m}^3$ and $6289.66 \times 10^4 \text{ m}^3$, respectively; meanwhile, CX will continue to hold the smallest share at $2240.49 \times 10^4 \text{ m}^3$ and $2292.9 \times 10^4 \text{ m}^3$. The industrial water distribution of HT is expected to range from $2599.17\sim 3368.62 \times 10^4 \text{ m}^3$, and that of JN will decrease to $153.25\sim 159.46 \times 10^4 \text{ m}^3$. The domestic water distribution for KT is $2288.93\sim 2747.14 \times 10^4 \text{ m}^3$. CX's domestic water allocation increased to $457.95\sim 575.55 \times 10^4 \text{ m}^3$. (Table 4 shows detailed results of different regions' planned years' sectoral allocations.)

Table 4. Results of optimal allocation of water resources in 2025 and 2035 (10^4 m^3).

Year	Area	Domestic	Industrial	Agricultural 50%	Agricultural 75%	Ecological
2025	KT	[2226.65, 2276.31]	[1454.84, 1454.84]	[6501.59, 6501.59]	[6434.08, 6434.08]	[127.8, 127.8]
	HT	[690, 831.39]	[2268.47, 2268.47]	[2040.11, 2040.11]	[2154.44, 2154.44]	[38.57, 38.57]
	JC	[818.88, 1043.07]	[192.18, 204.52]	[5126.86, 5126.86]	[5388.09, 5388.09]	[61.37, 61.37]
	CX	[356.75, 428.95]	[865.623, 1053.05]	[1838.25, 1838.25]	[1944.92, 1944.92]	[23.09, 23.09]
	LT	[746.24, 898.89]	[310.84, 319.29]	[2346.94, 2346.94]	[2486.27, 2486.27]	[31.54, 31.54]
	JN	[1333.93, 1600.72]	[155.71, 168.11]	[4598.28, 4598.28]	[4435.67, 4435.67]	[68.97, 68.97]
	ZL	[1219.07, 1462.88]	[301.28, 309.07]	[3548.96, 3548.96]	[3734.96, 3734.96]	[68.12, 68.12]
2035	KT	[2288.93, 2747.14]	[1519.1, 1585.05]	[6076.07, 6076.07]	[6289.66, 6289.66]	[210.1, 234.91]
	HT	[862.25, 1066.97]	[2599.17, 3368.62]	[3863.67, 3863.67]	[3919.88, 3919.88]	[33.64, 35.42]
	JC	[839.91, 1039.75]	[783.08, 837.79]	[6724.75, 6724.75]	[6905.41, 6905.41]	[94.43, 96.07]
	CX	[457.95, 575.55]	[1141.44, 1198.47]	[2240.49, 2240.49]	[2292.9, 2292.9]	[17.54, 22.42]
	LT	[938.47, 1159.49]	[519.39, 582.69]	[2508.82, 2508.82]	[2577.38, 2577.38]	[27.71, 30.62]
	JN	[1449.36, 2204.33]	[153.25, 159.46]	[4236.74, 4236.74]	[4372.12, 4372.12]	[62.16, 67.24]
	ZL	[1404.39, 1980.98]	[252.77, 260.18]	[4269.14, 4269.14]	[4360.65, 4360.65]	[69.3, 73.07]

Figure 4 shows the relationship between water allocation and water demand in 2025 and 2035. The water allocation for residential use exceeds the minimum water demand in 2025 (Figure 4a), ensuring sufficient supply. However, there may be potential water shortages in JN and ZL due to lower domestic water distribution in 2035 (Figure 4b). Industrial water consumption in KT and HT is guaranteed in 2025, while other regions may experience slightly reduced industrial water allocation below the minimum demand. In 2035, industrial water allocation in HT, CX, JC, and LT falls below the minimum demand. Regarding the relationship between agricultural allocation in normal and dry years, the water allocation of KT agriculture is close to the upper limit of the water demand in normal years in 2025, and the other allocation results are close to the lower limit of water demand; 2035 experiences the lower limit water demand. Regarding the allocation relationship of the ecology sector, in 2025 and 2035, the amount of water allocated by KT is close to the upper limit, and the rest is in the lower limit.

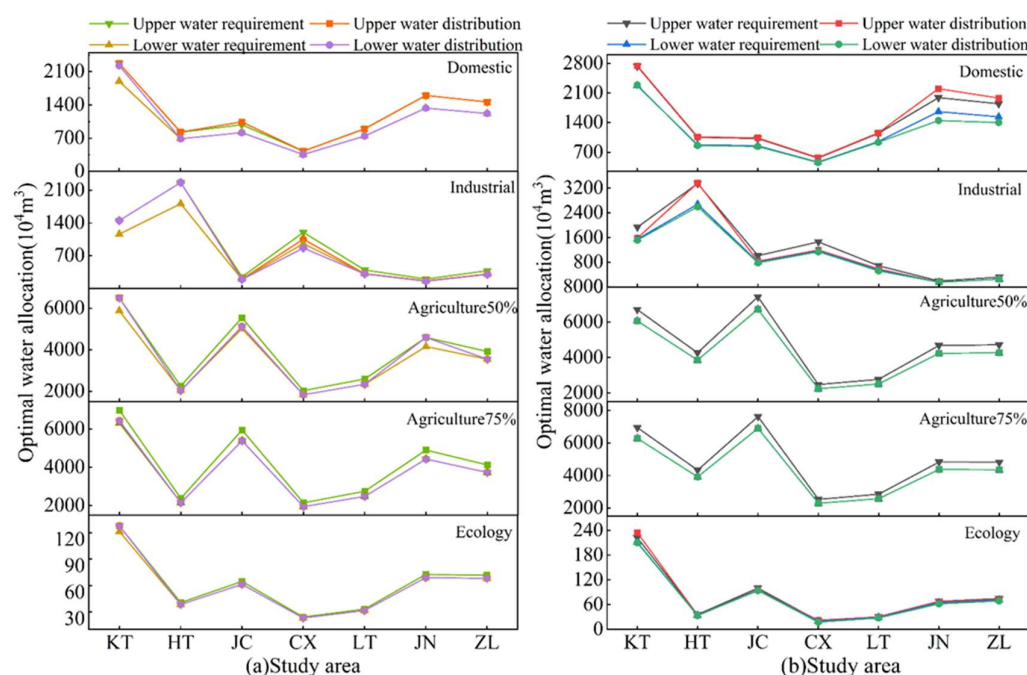


Figure 4. Results of water supply and demand allocation in different regions and departments (10^4 m^3): (a) comparison of water allocation and water demand in different regions in 2025; (b) comparison of water allocation and water demand in different regions in 2035.

4.2.2. Water Distribution Benefit Analysis

The benefit of water distribution is similar for 2025 and 2035 (Figure 5). Take KT as an example in 2025. When KT's domestic, industrial, agricultural, and ecological water use efficiency coefficient is 17.5, 7.0, 12.7, and 9.7, the corresponding allocation of water is $2276.31 \times 10^4 \text{ m}^3$, $1454.84 \times 10^4 \text{ m}^3$, $6501.59 \times 10^4 \text{ m}^3$, and $127.8 \times 10^4 \text{ m}^3$ (Figure 5b). The results indicate that, due to the calculation of the benefit coefficient, the benefits of domestic, ecological, and agricultural water use are relatively high, and the model-solving priority is given to the supply of water for domestic, ecology, industry, and agriculture use based on the actual situation. Therefore, a large water shortage occurs in industry and agriculture due to the smallest benefit coefficient, and water resources are not concentrated on water supply for domestic and ecology uses due to the Gini coefficient of trade-off fairness being introduced into the model. A large amount of surplus water for domestic and ecological uses is presented, and serious water shortages take place in other sectors. Instead, water flow to the four water consumption sectors is balanced on the basis of limited, fair, and efficient water supply under the constraint of fairness. Therefore, this proves the fairness

of the Gini coefficient constraint on the efficient utilization of water resources and the matching of the optimal water resource allocation model.

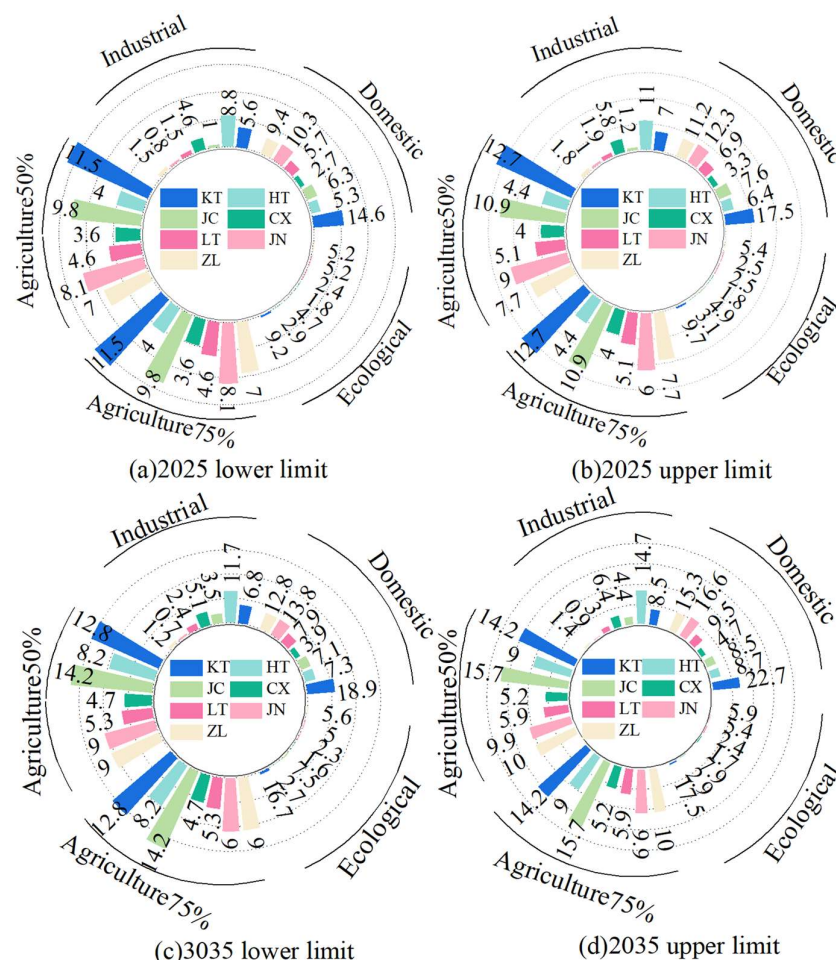


Figure 5. The water distribution corresponding to the benefit value. (a,b) Allocation of water resources corresponding to the lower limit and upper limit of water efficiency in 2025. (c,d) Allocation of water resources corresponding to the lower limit and upper limit of water efficiency in 2035.

4.2.3. Water Shortage Rate

In 2025, the water shortage and water shortage rate under the two scenarios of normal and dry years are $1769.83 \times 10^4 \text{ m}^3$ and $2986.61 \times 10^4 \text{ m}^3$, and 4.2% and 6.7%, respectively. For 2035, they are $3706.58 \times 10^4 \text{ m}^3$ and $3790.62 \times 10^4 \text{ m}^3$, and 7.0% and 7.0%, respectively. In the normal year of 2025 (Figure 6a), there are different degrees of water shortage in other regions, except the water shortage rate is 0 in KT. The water shortage rates of CX, JN, ZL, JC, and HT are 9.2%, 0.6%, 7.7%, 6.0%, and 4.0%, respectively. In the case of dry years, water shortages occur in all seven regions, except for the water shortage rates of 5.1% in KT and 7.4% in JN; the water shortage in other regions is basically the same as that in normal years. Compared with 2025, the water shortage rate is essentially identical in normal and dry years in 2035. In addition, the aggravation of the water shortage in KT is 8.3%. The water shortage rates of CX, JC, and HT increase by 0.2%~1.6% compared with 2025, and those for LT, JN, and ZL decrease by 0.5%~3.0%.

In 2025, the domestic water sector will meet the relevant water requirements; however, the domestic and ecology sector requirements will be met in 2035. In 2025, the industrial water shortage rate is 6.0%, and the agricultural water shortage rate is 5.3% and 9.1% in normal and dry years, respectively; the corresponding water scarcity rate will increase by 3.9%, 4.3%, and 0.4% in 2035. In 2025, the water shortage rate in the ecology water

sector is the lowest, at 3.5%. In view of the high water shortage rates in the industrial and agricultural water sectors, it is necessary to analyze these in detail.

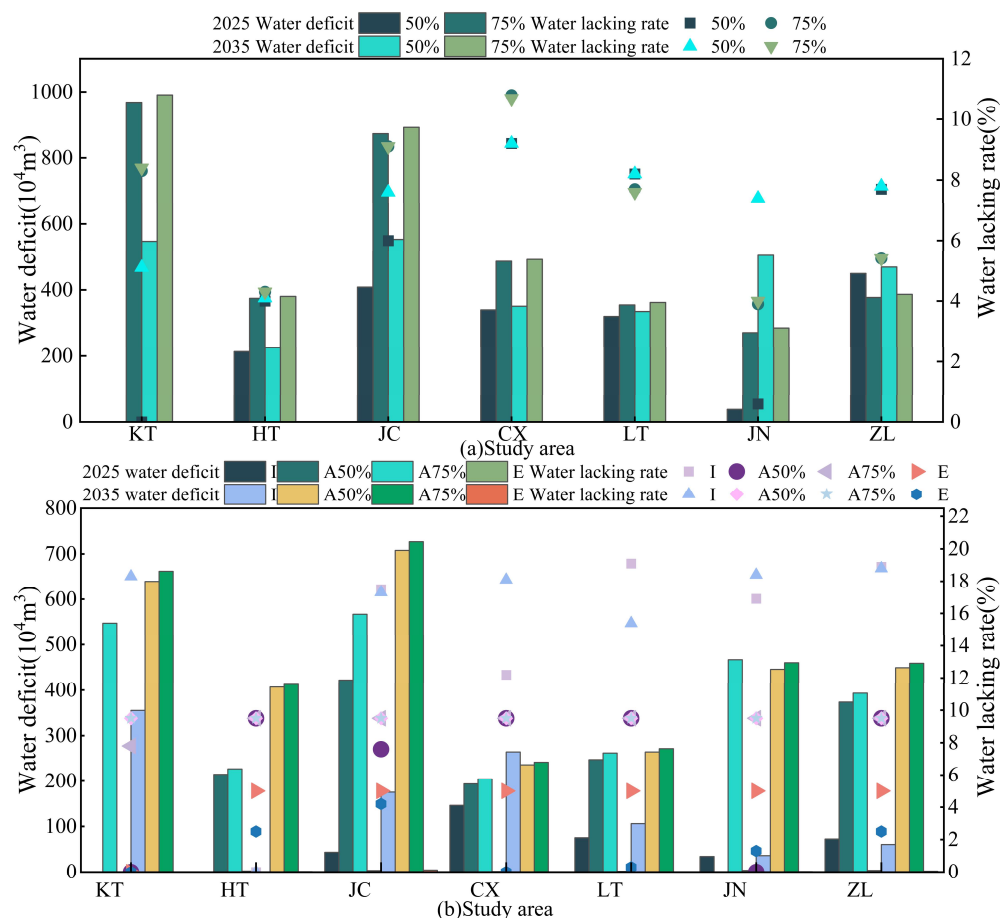


Figure 6. The rate of water scarcity for different regions. (a) Water shortages and water shortage rates under a normal year ($p = 50\%$) and a dry year ($p = 75\%$) in the planning. (b) The water shortage rate of different water uses' sectors in the planning year.

In 2025, the industrial water shortage rates of JC, LT, JN, and ZL are higher. KT and CX show an upward trend in 2035, and other areas show a downward trend (Figure 6b). Under the dry year scenario in 2025, the agricultural water shortage rate in KT and JN is 0. In the dry year scenario, the water shortage rate remains at 10.0%, and by 2035, the agricultural water shortage rate decreases to 9.5%. In 2025, the KT ecological water shortage rate is 0, and other water shortage rates remain at 5.0%. In 2035, the highest water shortage rate is in JC, and the water shortage rate in other areas is below 3.0%. In general, domestic and ecological water rates are basically safe, and the industrial water shortage is the largest among all water consumption departments because the industrial unilateral water benefit is lower than in the other three water consumption departments. The water shortage rate of the study area is high in each planning year, and the water shortage is serious, so deep and extreme water savings should be carried out continuously to improve the utilization efficiency of water resources.

4.2.4. Water Allocation from Different Water Sources

In 2025, the proportion of surface water supply will increase from 63.7% in the base year to 73.3%. Due to the limited exploitation of groundwater, the reconstruction and expansion of reservoirs, and the implementation of rainwater harvesting projects, groundwater will decrease from 24.2% to 15.4% and reclaimed water will not change significantly, i.e., to 11.3% (Figure 7). In 2035, the scale of inter-basin water transfer increases, the proportion

of surface water supply increases from 63.7% in the base year to 75.9%, groundwater decreases to 10.4%, the intensity of unconventional water reuse is strengthened, water-saving enterprises are established, and reclaimed water increases from 12.1% to 14.2%. Figure 7 shows the proportion of the water supply sources in the planning year. The water resources optimization framework is conducive to improving the utilization rate of surface water and reclaimed water and further protecting groundwater resources and conducive to the efficient use of water resources by adjusting the water supply structure.

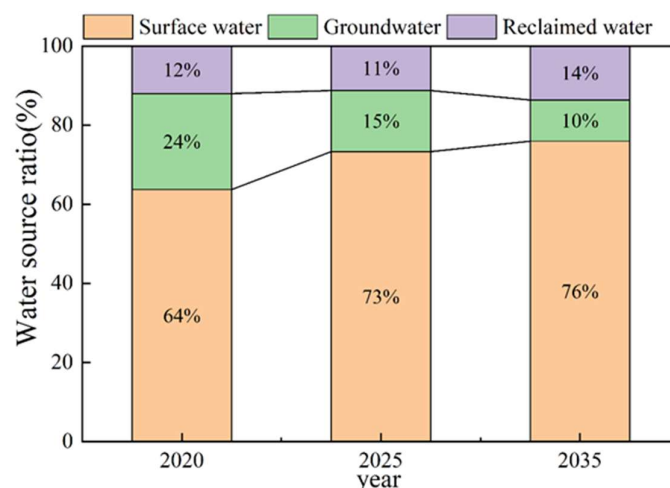


Figure 7. The proportion of water supplied by different sources in the base year and the planning year.

In 2025, the amount of surface water allocated is $28,852.46 \sim 29,047.13 \times 10^4 \text{ m}^3$ and $29,265.79 \sim 29,460.46 \times 10^4 \text{ m}^3$ in normal and dry years (Table 5, Figure 8a), and the water allocation rate is 96.2% and 97.6%, respectively. The groundwater distribution is $6063.61 \sim 7146.75 \times 10^4 \text{ m}^3$ and $6108.36 \sim 7191.5 \times 10^4 \text{ m}^3$, and the water distribution rate is 100.0%. The allocated amount of reclaimed water is $4436.68 \sim 4556.04 \times 10^4 \text{ m}^3$ and $4556.06 \sim 4675.42 \times 10^4 \text{ m}^3$, and the water distribution rate is 97.4% and 100.0%, respectively. In 2035, the surface water allocation will be $34,659.64 \sim 37,017.44 \times 10^4 \text{ m}^3$ and $34,500.05 \sim 36,857.85 \times 10^4 \text{ m}^3$ in normal and dry years (Figure 8b), respectively, with water allocation rates of 99.0% and 98.6%. The water distribution of groundwater is $4748.78 \sim 5255.48 \times 10^4 \text{ m}^3$ and $4985.91 \sim 5495.71 \times 10^4 \text{ m}^3$, and the water distribution rate is 92.7% and 97.4%, respectively. The amount of reclaimed water allocated is $6235.75 \sim 6972.99 \times 10^4 \text{ m}^3$ and $6956.43 \sim 7693.67 \times 10^4 \text{ m}^3$, and the water distribution rate is 89.5% and 100.0%, respectively.

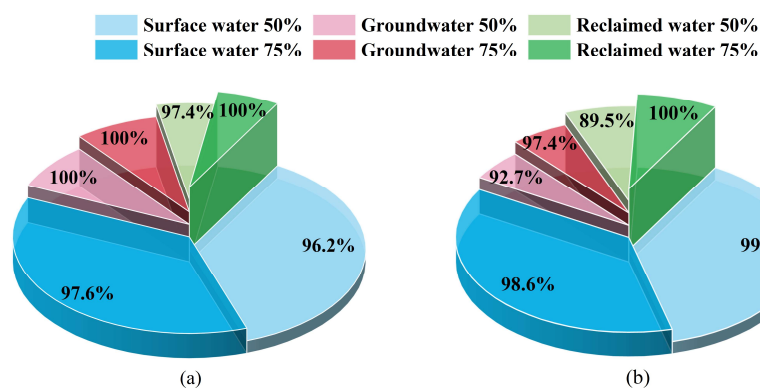


Figure 8. Water distribution rate of the source. (a) Water distribution rates of different water sources under a normal year and a dry year in 2025. (b) Water distribution rates of different water sources under a normal year and a dry year in 2035. The size of the fan indicates the amount of water allocated to the source.

Table 5. Water distribution results of each water source in 2025 and 2035 (10⁴ m³).

Year	Area	Domestic			Industrial Underground	Reclaimed	Surface	Agricultural 50% Underground	Reclaimed	Surface	Agricultural 75%			Ecological Reclaimed
		Surface	Underground	Surface							Underground	Reclaimed	Surface	
2025	KT	[2207.68, 2227.77]	[18.97, 48.53]	[1431.56, 1431.56]	[11.64, 11.64]	[11.64, 11.64]	[6383.94, 6383.94]	[58.82, 58.82]	[58.82, 58.82]	[6307.75, 6307.75]	[63.16, 63.16]	[63.16, 63.16]	[126.59, 126.59]	[1.21, 1.21]
	HT	[351.00, 354.40]	[338.99, 476.99]	[2232.17, 2232.17]	[18.15, 18.15]	[18.15, 18.15]	[1026.18, 1026.18]	[487.92, 487.92]	[526.01, 526.01]	[1160.94, 1160.94]	[567.545, 567.55]	[425.95, 425.95]	[38.18, 38.18]	[0.39, 0.39]
	JC	[424.21, 484.62]	[394.67, 558.45]	[71.52, 77.72]	[61.41, 63.10]	[59.20, 63.70]	[3918.52, 3918.52]	[427.34, 427.34]	[781, 781]	[2830.15, 2830.15]	[625.44, 625.44]	[1932.5, 1932.5]	[60.76, 60.76]	[0.6, 0.6]
	CX	[179.88, 180.73]	[176.87, 248.22]	[427.95, 505.55]	[228.48, 244.59]	[209.20, 302.91]	[958.89, 958.89]	[420.64, 420.64]	[458.72, 458.72]	[1391.38, 1391.38]	[347.57, 347.57]	[205.97, 205.97]	[22.85, 22.85]	[0.23, 0.23]
	LT	[379.12, 382.52]	[367.12, 516.37]	[106.98, 111.68]	[103.98, 103.98]	[100.82, 104.57]	[897.69, 897.69]	[705.58, 705.58]	[743.67, 743.67]	[1163.92, 1163.92]	[731.98, 731.98]	[590.38, 590.38]	[31.22, 31.22]	[0.32, 0.32]
	JN	[1320.59, 1320.59]	[13.34, 280.13]	[59.38, 65.58]	[49.27, 50.97]	[47.06, 51.56]	[2286.23, 2286.23]	[1079.84, 1079.84]	[1232.20, 1232.20]	[2469.32, 2469.32]	[1266.37, 1266.37]	[699.98, 699.98]	[36.89, 41.40]	[27.57, 32.08]
	ZL	[633.55, 633.55]	[585.52, 829.33]	[103.53, 106.33]	[100.53, 101.63]	[97.23, 101.12]	[3098.94, 3098.94]	[414.53, 414.53]	[35.49, 35.49]	[3660.26, 3660.26]	[37.35, 37.35]	[37.35, 37.35]	[36.46, 40.97]	[27.15, 31.65]
	KT	[2063.96, 2508.2]	[224.97, 238.94]	[1189.28, 1222.24]	[155.96, 173.51]	[173.9, 189.30]	[4909.69, 4909.69]	[548.45, 548.45]	[617.93, 617.93]	[4490.51, 4490.51]	[547.90, 547.90]	[1251.25, 1251.25]	[125.10, 137.19]	[85, 97.72]
2035	HT	[728.75, 918.8]	[133.50, 148.17]	[2000.47, 2091.83]	[219.38, 241.11]	[379.32, 1035.68]	[2689.81, 2689.81]	[409.58, 409.58]	[764.28, 764.28]	[2648.8, 2648.8]	[452.33, 452.33]	[818.76, 818.76]	[13.65, 14.54]	[19.99, 20.88]
	JC	[707.22, 892.58]	[132.69, 147.17]	[517.98, 545.34]	[129.82, 144.86]	[135.28, 147.59]	[4902.01, 4902.01]	[457.46, 457.46]	[1365.28, 1365.28]	[5065.63, 5065.63]	[413.19, 413.19]	[1426.59, 1426.59]	[43.11, 43.93]	[51.32, 52.14]
	CX	[328.62, 434.40]	[129.33, 141.15]	[850.70, 879.21]	[141.28, 156.76]	[149.46, 162.50]	[1778.41, 1778.41]	[206.27, 206.27]	[255.81, 255.81]	[1874.29, 1874.29]	[186.64, 186.64]	[231.97, 231.97]	[7.19, 9.63]	[10.35, 12.79]
	LT	[802.19, 1008.06]	[136.28, 151.43]	[282.57, 314.22]	[116.34, 133.11]	[120.48, 135.36]	[1970.73, 1970.73]	[224.19, 224.19]	[313.90, 313.90]	[2096.57, 2096.57]	[205.14, 205.14]	[275.67, 275.67]	[11.08, 12.53]	[16.63, 18.09]
	JN	[1281.86, 1825.42]	[167.50, 378.91]	[48.74, 51.85]	[55.25, 55.63]	[49.26, 51.98]	[2922.03, 2922.03]	[527.18, 527.08]	[787.64, 787.64]	[2894.17, 2894.17]	[673.06, 673.06]	[804.89, 804.89]	[27.51, 30.05]	[34.65, 37.19]
	ZL	[1389, 1831.18]	[15.39, 149.8]	[95.97, 99.67]	[79.09, 83.03]	[77.71, 77.48]	[2941.46, 2941.46]	[538.87, 538.87]	[788.81, 788.81]	[2884.58, 2884.58]	[670.87, 670.87]	[805.20, 805.20]	[30.55, 32.43]	[38.75, 40.64]

5. Discussion

5.1. Optimal Allocation Model

The proposed uncertain multi-objective fuzzy programming model in this paper was validated for its validity, reliability, and scientific accuracy. Suo et al. [36] introduced the fuzzy interval dynamic programming (FIDP) and applied it to provide a water distribution scheme and assess the water shortage rate of Handan City during different periods. The results indicate that the target interval values obtained by the FIDP model were more accurate than those obtained by the FILP model [59], leading to a lower water shortage rate. According to the FIDP model, the total water distribution was lower than the water supply and there were different degrees of water shortage in different regions. Figure 4 illustrates the interval solution for allocating water resources across different regions and sectors in the planning year, demonstrating that the optimal solution complies with constraints. Additionally, when comparing with the lower limit of the planning annual water demand and water supply, it was observed that the lower limit of water demand was less than the allocated water. For instance, domestic water demand for KT ranges from $1896.92\sim 2276.3 \times 10^4 \text{ m}^3$ in 2025, while the actual distributed quantity falls within $2226.65\sim 2276.31 \times 10^4 \text{ m}^3$, confirming result reliability. For the multi-objective fuzzy interval programming model, the solved configuration interval is constrained within a small range, leading to more accurate and specific results [36]. The FIDP model's calculated water allocation interval value exhibits a significant error compared to the boundary value of water demand, resulting in a wider water allocation solution interval. This contradicts the purpose of introducing fuzzy membership function to solve narrow intervals. In this study, the error between the water allocation boundary and the water demand boundary was 3.1%, much smaller than FILP's 19.9%. Suo et al. [58] proposed an interval multi-objective programming model that combines fuzzy programming with an improved two-step method. The solution not only provides optimized benefits and water distribution solutions for each user but also allows for external water transfer in combination with local water conservancy to alleviate water shortages. Finally, the stability of the model is verified using integral and weighting methods.

In the study, when the water demand equals the lower limit, the water supply can fully meet the requirements. However, when the water demand reaches the upper limit, both industry and agriculture face severe water shortages. Liu et al. [60] also used the uncertain two-stage multi-water source allocation model in Sanjiang Plain to investigate the satisfaction of the departmental water supply in different years and at different water supply levels. Under the multi-water supply level, the optimization goal of the agricultural sector was the lower limit and the domestic sector was basically the upper limit. After optimizing the allocation, the water distribution interval and the water demand interval of the living department are compared. The water distribution interval was within the water demand interval or close to the boundary value of water demand, indicating that the allocation of living departments has reached the optimal allocation state. The agricultural water demand of HT was $2040.11\sim 2254.85 \times 10^4 \text{ m}^3$ with a water distribution of $20,400.11\sim 2040.11 \times 10^4 \text{ m}^3$. The domestic water demand of CX was $356.75\sim 428.1 \times 10^4 \text{ m}^3$, and the water distribution of CX was $356.75\sim 428.95 \times 10^4 \text{ m}^3$. This adds credibility to the results.

Across different planning periods and guarantee rates, it was observed that the lower limit value of agricultural water allocation aligns with the corresponding lower limit value of actual demand, a finding consistent with Li's [61] research on resource allocation in Hengshui City. This suggests the potential occurrence of boundary value phenomena in meeting specific demands. Banihabid et al.'s [62] discussion of three cases concerning the outcomes of resource allocation indicates that uncertain models yield smaller results compared to deterministic ones, suggesting an inaccurate estimation by deterministic models—a conclusion supported by Zeng et al.'s findings as well [63]. Fuzzy multi-objective programming can be used to develop effective water distribution schemes. In this study, the upper and lower limits of agricultural water allocation were equal in 2025 and 2035. This shows that after fuzzy optimization, the minimum water shortage can be ensured

and the phenomenon of high water resources' allocation can be reduced. Agriculture was the last water supply sector to ensure that the combined benefits and needs of other water sectors are optimally met, so that water consumption was greater than or equal to the lower limit of water demand. In addition, when input uncertainty parameters are used for the optimal allocation of water resources, the studied system has high complexity in satisfying the three objective functions [62]. On the whole, the allocation effect is good and the water distribution rate of each water source basically reaches more than 95.0%, avoiding the waste of water resources caused by over-allocation. Therefore, the uncertain multi-objective fuzzy programming model has better optimal solution ability and strong adaptability. By introducing the membership function, the result is expressed in the form of interval, which can help the decision maker to obtain the decision space with upper and lower limits. Based on the aforementioned research, the model is well suited for managing small and irregular water quantities. It aims to optimize the objective function by maximizing social benefits, minimizing water shortages, and enhancing environmental benefits. Meanwhile, the optimal allocation scheme can not only maximize the research target benefit within the region but also represent the specific water resource allocation and water shortage situation of different water consumption departments at different stages in the form of intervals. It is helpful for the government to adjust the strategy in time for different regions and different water users and optimize the water supply structure to cope with the water shortage crisis.

5.2. Water Shortage Solutions

The amount of water transfer is very small in northwest China; in addition, river water storage is not promising. Therefore, in order to alleviate the problem of a resource-based water shortage, a series of engineering measures and policy interventions have been put forward. From the perspective of water resource management, China proposed the policy of "the strictest water resource management system" to improve the water resource management system on the basis of "three red lines" (total water use, water use efficiency, and pollutant discharge) in 2011 [64,65]. "Four systems" were added (total water use control system, water efficiency control system, water function zone pollution restriction system, water resource management assessment system) [66,67], and strict control objectives and water use constraints were formulated, effectively alleviating the shortage, waste, and pollution of water resources in China [66]. The "sponge city" development plan proposed in 2013 effectively controls urban rainfall runoff and pollutants, solves water supply, and reduces flood risk [68]. The north-to-south water diversion project in California, USA, has realized the allocation of water resources through water pricing and the water market, and China's South-to-North Water Diversion project has learned from California's successful experience and implemented the same system [69]. In terms of water supply, the use of reclaimed water as an alternative water source has become a necessity in arid and semi-arid regions of the world [70]. The over-exploitation of groundwater has broken the original law of runoff generation and confluence in these regions, and the surface runoff has been reduced [71]. It is necessary to realize the "double control" management of groundwater level and reduced groundwater overdraw [72]. Under the implementation of the policy, the water shortage problem has been effectively alleviated, but there is still a risk of water shortage under climate change and future extreme events. Therefore, in order to meet domestic and production needs, inter-basin water transfer can realize the scientific unified distribution of water resources, form a coordinated regional spatial layout, and achieve harmony between people and water.

The Fifth Plenary Session of the 19th CPC Central Committee clearly suggested that the "14th Five-Year Plan" period will promote the construction of major projects, such as the national water network and key water diversion projects. The Bailong River water diversion project and the Yintao water supply phase II project in Gansu Province are two of the 172 key water conservancy projects approved by the state council. The Bailong River project diverts water from the Bailong River (a tributary of the Jialing River) in the Yangtze River Basin to the Longdong area of the Yellow River Basin. The project benefit area

involves 20 counties and 36 industrial parks under the jurisdiction of Qingyang, Tianshui, and Pingliang in the Yellow River Basin, which basically covers the major cities with rapid development in the Weihe Basin and Jing River Basin in the future. The Bailong River's multi-year average water transfer was $7.9 \times 10^8 \text{ m}^3$, of which Pingliang City's water supply was $1.5 \times 10^8 \text{ m}^3$ (Figure 9) [73]. The Yintao water supply phase II project is an extension of the first phase of the project, fundamentally solving the extreme shortage of water resources in the semi-arid areas of central Gansu, such as Dingxi City, Baiyin City, Tianshui City, and Jingning county. It has addressed the domestic, industrial, ecological, and other water consumption requirements of 463,000 people in 21 townships in JN county, Pingliang City, with a planned allocation of $4.3 \times 10^7 \text{ m}^3$ [74–77]. The problem of the JN water shortage was basically solved by the Tao River Diversion Project. The problem of water shortage will fundamentally be solved in the Longzhong and Longdong areas after the implementation of the Bailong River Project. Therefore, by speeding up the Bailong River water supply project and improving the Yintao area supporting water supply project, the problem of water shortage can be solved in the Longdong area, consolidating the important locations of economic development strategy and proving that domestic and ecological conditions have an important role.

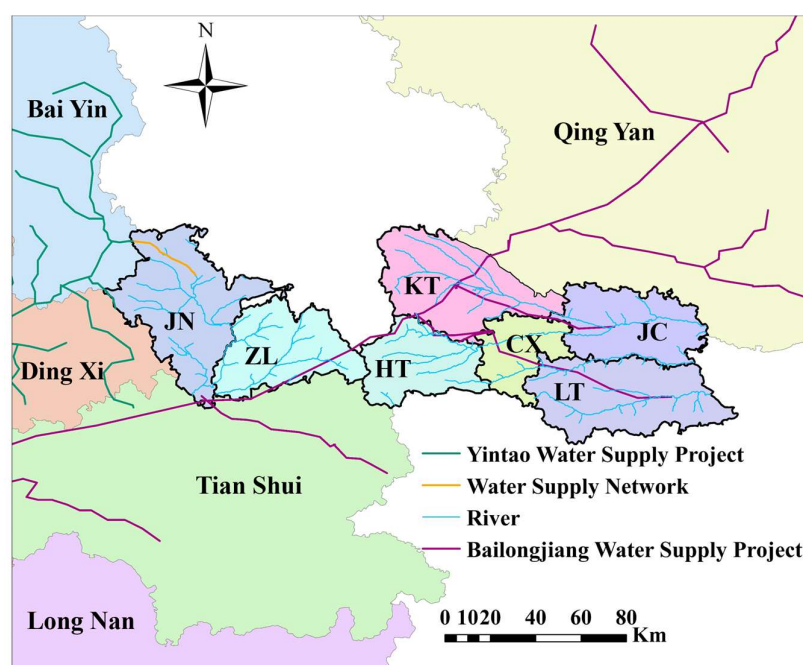


Figure 9. Planning route of key water supply projects in Pingliang City. (Currently, Pingliang City has surface water, groundwater, and reclaimed water and is waiting for improvements to the Yintao water transfer project and the planned Bailong River water transfer project.)

6. Conclusions

The optimal allocation of water resources can solve the problem of the gradual expansion of regional water scarcity. We took Pingliang City, a resource-based, extremely water-deficient, northwest, arid and semi-arid area, as an example. Considering the benefit maximization of the regional water supply, demand, sewage discharge and objective function, the membership function was adopted to deal with the uncertainty of the system and the Gini coefficient was adopted as the constraint condition to improve the fairness of water resources' allocation. An uncertain multi-objective programming model of Pingliang City was established to optimize the water resources in normal and dry years in 2025 and 2035.

The upper and lower bounds of the membership function in the interval multi-objective fuzzy model can be transformed into two sub-models, which can then be solved separately to obtain the interval solution. The findings indicate a decreasing trend in

water demand during the planning year, with a gradual reduction in water shortage rates during dry years following optimal allocation. This ensures a domestic water supply for the planning year, with shortages primarily concentrated in the industrial and agricultural sectors. Additionally, there will be an increase in the proportion of surface water and unconventional water supply, while groundwater supply will decrease. Groundwater supply was expected to remain stable due to an increase in unconventional water sources.

Taking Pingliang City as an example, the applicability of an uncertain multi-objective fuzzy programming model in the northwest semi-arid area was verified. Compared with the constraints, the obtained solution not only provides the optimal water supply scheme for the water department but also realizes the maximum benefit balance of water resources' allocation and optimizes the water supply structure. However, the external water transfer project was still in the process of improvement in this area, and the organic combination of external water transfer and existing water cannot be realized. Therefore, due to the limitation of the water resource background and the difference in water supply frequency, the joint configuration of multi-water source, multi-target, and multi-scene should be realized in the future to contribute more effectively to water management policies and regional water sustainability.

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