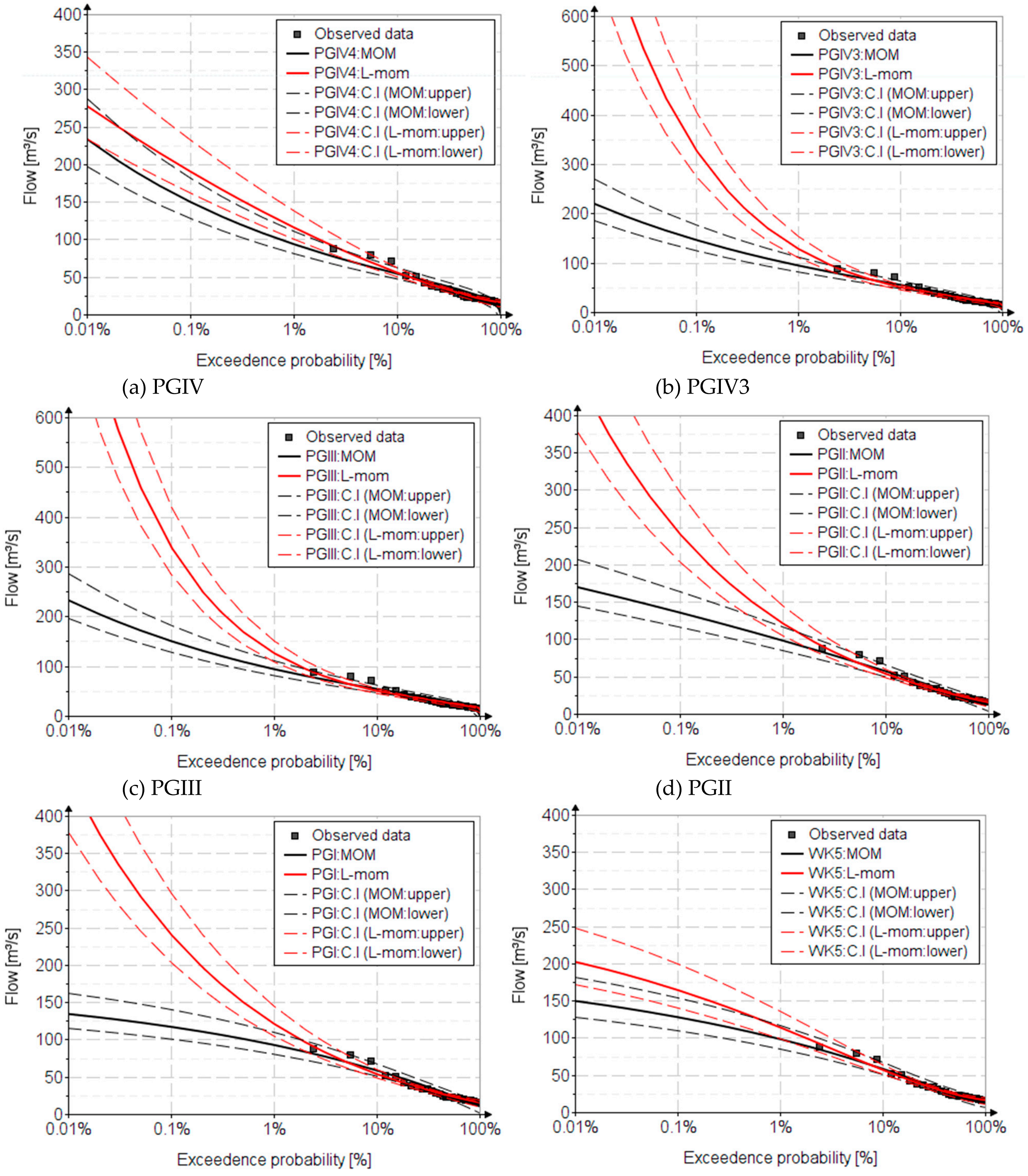
Figure S1 presents the results of the analysed distributions, highlighting the confidence interval (C.I) for each distribution, for both parameter estimation methods. Results are presented for AES analysis.



(a) PGIV (b) PGIV3

(c) PGIII (d) PGII

(e) PGI (f) WK5

Figure S1. The probability distributions curves with Confidence Intervals

The raw and central moments of the analysed distributions were determined using the following methodology presented below, by substitution using the probability density function.

For the L-moments method, variable substitution is used, using the inverse function

Generalized Pareto Type II distribution

To determine the raw moments of the Generalized Pareto Type II distribution, the following substitutions in the expression for the probability density function are required:

$$f(x)=\frac{1}{\beta}\cdot\left(1-\frac{\alpha}{\beta}\cdot(x-\gamma)\right)^{\frac{1}{\alpha}-1}$$

It is noted:

$$t=\left(1-\frac{\alpha}{\beta}\cdot(x-\gamma)\right)^{\frac{1}{\alpha}-1} \qquad x=\gamma+\frac{\beta}{\alpha}-\frac{\beta\cdot t^{\alpha}}{\alpha} \qquad dx=-\beta\cdot t^{\alpha-1}\cdot dt$$

Thus, the first six raw moments are:

$$m_1^{'}=\int_{\gamma}^{\infty}x\cdot f(x)=\int_{\gamma}^{\infty}x\cdot\frac{1}{\beta}\cdot\left(1-\frac{\alpha}{\beta}\cdot(x-\gamma)\right)^{\frac{1}{\alpha}-1}dx=\int_{1}^{\infty}\left(\frac{\beta\cdot t^{\alpha}}{\alpha}-\frac{\beta}{\alpha}-\gamma\right)\cdot dt=-\frac{\beta}{\alpha\cdot(\alpha+1)}+\frac{\beta}{\alpha}+\gamma=\frac{\beta}{\alpha+1}+\gamma$$

$$m_2^{'}=\int_{\gamma}^{\infty}x^2\cdot f(x)=\int_{\gamma}^{\infty}x^2\cdot\frac{1}{\beta}\cdot\left(1-\frac{\alpha}{\beta}\cdot(x-\gamma)\right)^{\frac{1}{\alpha}-1}dx=\int_{1}^{\infty}-\left(\gamma+\frac{\beta}{\alpha}-\frac{\beta\cdot t^{\alpha}}{\alpha}\right)^2\cdot dt=$$

$$\int_{1}^{\infty}\left(\frac{-\beta^2\cdot t^{2\cdot\alpha}}{\alpha^2}+\frac{2\cdot\beta^2\cdot t^{\alpha}}{\alpha^2}+\frac{2\cdot\gamma\cdot\beta\cdot t^{\alpha}}{\alpha}-\frac{\beta^2}{\alpha^2}-\frac{2\cdot\gamma\cdot\beta}{\alpha}-\gamma^2\right)\cdot dt=\gamma^2+\frac{2\cdot\beta\cdot(\gamma\cdot(2\cdot\alpha+1)+\beta)}{(\alpha+1)\cdot(2\cdot\alpha+1)}$$

$$m_3^{'}=\int_{\gamma}^{\infty}x^3\cdot f(x)=\int_{\gamma}^{\infty}x^3\cdot\frac{1}{\beta}\cdot\left(1-\frac{\alpha}{\beta}\cdot(x-\gamma)\right)^{\frac{1}{\alpha}-1}dx=\int_{1}^{\infty}-\left(\gamma+\frac{\beta}{\alpha}-\frac{\beta\cdot t^{\alpha}}{\alpha}\right)^3\cdot dt=$$

$$\int_{1}^{\infty}\left(\begin{array}{l}\frac{\beta^3\cdot t^{3\cdot\alpha}}{\alpha^3}-\frac{3\cdot\beta^3\cdot t^{2\cdot\alpha}}{\alpha^3}-\frac{3\cdot\gamma\cdot\beta^2\cdot t^{2\cdot\alpha}}{\alpha^2}+\frac{3\cdot\beta^3\cdot t^{\alpha}}{\alpha^3}+\frac{6\cdot\gamma\cdot\beta^2\cdot t^{\alpha}}{\alpha^2}+\\ \frac{3\cdot\gamma^2\cdot\beta\cdot t^{\alpha}}{\alpha}-\frac{\beta^3}{\alpha^3}-\frac{3\cdot\gamma\cdot\beta^2}{\alpha^2}-\frac{3\cdot\gamma^2\cdot\beta}{\alpha}-\gamma^3\end{array}\right)\cdot dt=$$

$$\gamma^3+\frac{3\cdot\beta\cdot\left(\gamma^2\cdot\left(6\cdot\alpha^2+5\cdot\alpha+1\right)+\beta\cdot\gamma\cdot(6\cdot\alpha+2)+2\cdot\beta^2\right)}{(\alpha+1)\cdot(2\cdot\alpha+1)\cdot(3\cdot\alpha+1)}$$

$$m_4^{'}=\int_{\gamma}^{\infty}x^4\cdot f(x)=\int_{\gamma}^{\infty}x^4\cdot\frac{1}{\beta}\cdot\left(1-\frac{\alpha}{\beta}\cdot(x-\gamma)\right)^{\frac{1}{\alpha}-1}dx=\int_{1}^{\infty}-\left(\gamma+\frac{\beta}{\alpha}-\frac{\beta\cdot t^{\alpha}}{\alpha}\right)^4\cdot dt=$$

$$\int_{1}^{\infty}\left(\begin{array}{l}\frac{-\beta^4\cdot t^{4\cdot\alpha}}{\alpha^4}+\frac{4\cdot\beta^4\cdot t^{3\cdot\alpha}}{\alpha^4}+\frac{4\cdot\gamma\cdot\beta^3\cdot t^{3\cdot\alpha}}{\alpha^3}-\frac{6\cdot\beta^4\cdot t^{2\cdot\alpha}}{\alpha^4}-\frac{12\cdot\gamma\cdot\beta^3\cdot t^{2\cdot\alpha}}{\alpha^3}-\\ \frac{6\cdot\gamma^2\cdot\beta^2\cdot t^{2\cdot\alpha}}{\alpha^2}+\frac{4\cdot\beta^4\cdot t^{\alpha}}{\alpha^4}+\frac{12\cdot\gamma\cdot\beta^3\cdot t^{\alpha}}{\alpha^3}+\frac{12\cdot\gamma^2\cdot\beta^2\cdot t^{\alpha}}{\alpha^2}+\frac{4\cdot\gamma^3\cdot\beta\cdot t^{\alpha}}{\alpha}-\\ \frac{\beta^4}{\alpha^4}-\frac{4\cdot\gamma\cdot\beta^3}{\alpha^3}-\frac{6\cdot\gamma^2\cdot\beta^2}{\alpha^2}-\frac{4\cdot\gamma^3\cdot\beta}{\alpha}-\gamma^4\end{array}\right)\cdot dt=$$

$$\gamma^4+\frac{4\cdot\beta\cdot\left(\gamma^3\cdot\left(24\cdot\alpha^3+26\cdot\alpha^2+9\cdot\alpha+1\right)+\beta\cdot\gamma^2\cdot\left(36\cdot\alpha^2+21\cdot\alpha+3\right)+\beta^2\cdot\gamma\cdot(24\cdot\alpha+6)+6\cdot\beta^3\right)}{(\alpha+1)\cdot(2\cdot\alpha+1)\cdot(3\cdot\alpha+1)\cdot(4\cdot\alpha+1)}$$

$$m_5^{'}=\int_{\gamma}^{\infty}x^5\cdot f(x)=\int_{\gamma}^{\infty}x^5\cdot\frac{1}{\beta}\cdot\left(1-\frac{\alpha}{\beta}\cdot(x-\gamma)\right)^{\frac{1}{\alpha}-1}dx=\int_{1}^{\infty}-\left(\gamma+\frac{\beta}{\alpha}-\frac{\beta\cdot t^{\alpha}}{\alpha}\right)^5\cdot dt=$$

$$\int_{1}^{\infty}\left(\begin{array}{l}\frac{\beta^5\cdot t^{5\cdot\alpha}}{\alpha^5}-\frac{5\cdot\beta^5\cdot t^{4\cdot\alpha}}{\alpha^5}-\frac{5\cdot\gamma\cdot\beta^4\cdot t^{4\cdot\alpha}}{\alpha^4}+\frac{10\cdot\beta^5\cdot t^{3\cdot\alpha}}{\alpha^5}+\frac{20\cdot\gamma\cdot\beta^4\cdot t^{3\cdot\alpha}}{\alpha^4}+\\ \frac{10\cdot\gamma^2\cdot\beta^3\cdot t^{3\cdot\alpha}}{\alpha^3}-\frac{10\cdot\beta^5\cdot t^{2\cdot\alpha}}{\alpha^5}-\frac{30\cdot\gamma\cdot\beta^4\cdot t^{2\cdot\alpha}}{\alpha^4}-\frac{30\cdot\gamma^2\cdot\beta^3\cdot t^{2\cdot\alpha}}{\alpha^3}-\frac{10\cdot\gamma^3\cdot\beta^2\cdot t^{2\cdot\alpha}}{\alpha^2}+\\ \frac{5\cdot\beta^5\cdot t^{\alpha}}{\alpha^5}+\frac{20\cdot\gamma\cdot\beta^4\cdot t^{\alpha}}{\alpha^4}+\frac{30\cdot\gamma^2\cdot\beta^3\cdot t^{\alpha}}{\alpha^3}+\frac{20\cdot\gamma^3\cdot\beta^2\cdot t^{\alpha}}{\alpha^2}+\frac{5\cdot\gamma^4\cdot\beta\cdot t^{\alpha}}{\alpha}\\ -\frac{\beta^5}{\alpha^5}-\frac{5\cdot\gamma\cdot\beta^4}{\alpha^4}-\frac{10\cdot\gamma^2\cdot\beta^3}{\alpha^3}-\frac{10\cdot\gamma^3\cdot\beta^2}{\alpha^2}-\frac{5\cdot\gamma^4\cdot\beta^2}{\alpha}-\gamma^5\end{array}\right)\cdot dt=$$

$$\gamma^5+\frac{5\cdot\beta\cdot\left(\begin{array}{l}\gamma^4\cdot\left(120\cdot\alpha^4+154\cdot\alpha^3+71\cdot\alpha^2+14\cdot\alpha+1\right)+\beta\cdot\gamma^3\cdot\left(240\cdot\alpha^3+188\cdot\alpha^2+48\cdot\alpha+4\right)+\\ +\beta^2\cdot\gamma^2\cdot\left(240\cdot\alpha^2+108\cdot\alpha+12\right)+\beta^3\cdot\gamma\cdot(120\cdot\alpha+24)+24\cdot\beta^4\end{array}\right)}{(\alpha+1)\cdot(2\cdot\alpha+1)\cdot(3\cdot\alpha+1)\cdot(4\cdot\alpha+1)\cdot(5\cdot\alpha+1)}$$

$$m_6^{'}=\int_{\gamma}^{\infty}x^6\cdot f(x)=\int_{\gamma}^{\infty}x^6\cdot\frac{1}{\beta}\cdot\left(1-\frac{\alpha}{\beta}\cdot(x-\gamma)\right)^{\frac{1}{\alpha}-1}dx=\int_{1}^{\infty}-\left(\gamma+\frac{\beta}{\alpha}-\frac{\beta\cdot t^{\alpha}}{\alpha}\right)^6\cdot dt=$$

$$\int_{1}^{\infty}\left(\begin{array}{l}\frac{-\beta^6\cdot t^{6\cdot\alpha}}{\alpha^6}+\frac{6\cdot\beta^6\cdot t^{5\cdot\alpha}}{\alpha^6}+\frac{6\cdot\gamma\cdot\beta^5\cdot t^{5\cdot\alpha}}{\alpha^5}-\frac{15\cdot\beta^6\cdot t^{4\cdot\alpha}}{\alpha^6}-\frac{30\cdot\gamma\cdot\beta^5\cdot t^{4\cdot\alpha}}{\alpha^5}-\\ \frac{15\cdot\gamma^2\cdot\beta^4\cdot t^{4\cdot\alpha}}{\alpha^4}+\frac{20\cdot\beta^6\cdot t^{3\cdot\alpha}}{\alpha^6}+\frac{60\cdot\gamma\cdot\beta^5\cdot t^{3\cdot\alpha}}{\alpha^5}+\frac{60\cdot\gamma^2\cdot\beta^4\cdot t^{3\cdot\alpha}}{\alpha^4}+\frac{20\cdot\gamma^3\cdot\beta^3\cdot t^{3\cdot\alpha}}{\alpha^3}-\\ \frac{15\cdot\beta^6\cdot t^{2\cdot\alpha}}{\alpha^6}-\frac{60\cdot\gamma\cdot\beta^5\cdot t^{2\cdot\alpha}}{\alpha^5}-\frac{90\cdot\gamma^2\cdot\beta^4\cdot t^{2\cdot\alpha}}{\alpha^4}-\frac{60\cdot\gamma^3\cdot\beta^3\cdot t^{2\cdot\alpha}}{\alpha^3}-\frac{15\cdot\gamma^4\cdot\beta^2\cdot t^{2\cdot\alpha}}{\alpha^2}+\\ \frac{6\cdot\beta^6\cdot t^{\alpha}}{\alpha^6}+\frac{30\cdot\gamma\cdot\beta^5\cdot t^{\alpha}}{\alpha^5}+\frac{60\cdot\gamma^2\cdot\beta^4\cdot t^{\alpha}}{\alpha^4}+\frac{60\cdot\gamma^3\cdot\beta^3\cdot t^{\alpha}}{\alpha^3}+\frac{30\cdot\gamma^4\cdot\beta^2\cdot t^{\alpha}}{\alpha^2}+\frac{6\cdot\gamma^5\cdot\beta\cdot t^{\alpha}}{\alpha}-\\ \frac{\beta^6}{\alpha^6}-\frac{6\cdot\gamma\cdot\beta^5}{\alpha^5}-\frac{15\cdot\gamma^2\cdot\beta^4}{\alpha^4}-\frac{20\cdot\gamma^3\cdot\beta^3}{\alpha^3}-\frac{15\cdot\gamma^4\cdot\beta^2}{\alpha^2}-\frac{6\cdot\gamma^5\cdot\beta}{\alpha}-\gamma^6\end{array}\right)\cdot dt=$$

$$\gamma^6+\frac{6\cdot\beta\cdot\left(\begin{array}{l}\gamma^5\cdot\left(720\cdot\alpha^5+1044\cdot\alpha^4+580\cdot\alpha^3+155\cdot\alpha^2+20\cdot\alpha+1\right)+\\ +\beta\cdot\gamma^4\cdot\left(1800\cdot\alpha^4+1710\cdot\alpha^3+595\cdot\alpha^2+90\cdot\alpha+5\right)+\\ +\beta^2\cdot\gamma^3\cdot\left(2400\cdot\alpha^3+1480\cdot\alpha^2+300\cdot\alpha+20\right)+\\ +\beta^3\cdot\gamma^2\cdot\left(1800\cdot\alpha^2+660\cdot\alpha+60\right)+\beta^4\cdot\gamma\cdot(720\cdot\alpha+120)+120\cdot\beta^5\end{array}\right)}{(\alpha+1)\cdot(2\cdot\alpha+1)\cdot(3\cdot\alpha+1)\cdot(4\cdot\alpha+1)\cdot(5\cdot\alpha+1)\cdot(6\cdot\alpha+1)}$$

The first six central moments (except the arithmetic mean) have the following expressions:

$$m_1 = m_1^{'} = \mu = \gamma + \frac{\beta}{\alpha+1}$$

$$m_2 = \sigma^2 = m_2^{'} - m_1^{'2} = \frac{\beta^2}{(\alpha+1)^2 \cdot (2\cdot\alpha+1)}$$

$$m_3 = m_3^{'} - 3\cdot m_2^{'}\cdot m_1^{'} + 2\cdot m_1^{'3} = \frac{-2\cdot\beta^3\cdot(\alpha+1)}{(\alpha+1)^3\cdot(2\cdot\alpha+1)\cdot(3\cdot\alpha+1)}$$

$$m_4 = m_4^{'} - 4\cdot m_3^{'}\cdot m_1^{'} + 6\cdot m_2^{'}\cdot m_1^{'2} - 3\cdot m_1^{'4} = \frac{3\cdot\beta^4\cdot\left(2\cdot\alpha^2-\alpha+3\right)}{(\alpha+1)^4\cdot(2\cdot\alpha+1)\cdot(3\cdot\alpha+1)\cdot(4\cdot\alpha+1)}$$

$$m_5 = m_5^{'} - 5\cdot m_4^{'}\cdot m_1^{'} + 10\cdot m_3^{'}\cdot m_1^{'2} - 10\cdot m_2^{'}\cdot m_1^{'3} + 4\cdot m_1^{'5} =$$
$$= \frac{-4\cdot\beta^5\cdot\left(6\cdot\alpha^2+5\cdot\alpha+11\right)\cdot(\alpha-1)}{(\alpha+1)^5\cdot(2\cdot\alpha+1)\cdot(3\cdot\alpha+1)\cdot(4\cdot\alpha+1)\cdot(5\cdot\alpha+1)}$$

$$m_6 = m_6^{'} - 6\cdot m_5^{'}\cdot m_1^{'} + 15\cdot m_4^{'}\cdot m_1^{'2} - 20\cdot m_3^{'}\cdot m_1^{'3} + 15\cdot m_2^{'}\cdot m_1^{4} - 5\cdot m_1^{'6} =$$
$$= \frac{5\cdot\beta^6\cdot\left(24\cdot\alpha^4+2\cdot\alpha^3+19\cdot\alpha^2-26\cdot\alpha+53\right)}{(\alpha+1)^6\cdot(2\cdot\alpha+1)\cdot(3\cdot\alpha+1)\cdot(4\cdot\alpha+1)\cdot(5\cdot\alpha+1)\cdot(6\cdot\alpha+1)}$$

Skewness (γ_1), kurtosis (γ_2), γ_3 and γ_4 are determined with the following expressions:

$$\gamma_1 = C_s = \frac{m_3}{m_2^{3/2}} = \frac{-2\cdot(\alpha-1)\cdot(2\cdot\alpha+1)^{0.5}}{3\cdot\alpha+1}$$

$$\gamma_2 = C_k = \frac{m_4}{m_2^{2}} = \frac{(2\cdot\alpha+1)\cdot\left(6\cdot\alpha^2-3\cdot\alpha+9\right)}{(3\cdot\alpha+1)\cdot(4\cdot\alpha+1)}$$

$$\gamma_3 = \frac{m_5}{m_2^{\frac{5}{2}}} = \frac{-(4\cdot\alpha-4)\cdot\left(6\cdot\alpha^2+5\cdot\alpha+11\right)\cdot(2\cdot\alpha+1)^{\frac{3}{2}}}{(3\cdot\alpha+1)\cdot(4\cdot\alpha+1)\cdot(5\cdot\alpha+1)}$$

$$\gamma_4 = \frac{m_6}{m_2^{3}} = \frac{(2\cdot\alpha+1)^2\cdot\left(120\cdot\alpha^4+10\cdot\alpha^3+95\cdot\alpha^2-130\cdot\alpha+265\right)}{(3\cdot\alpha+1)\cdot(4\cdot\alpha+1)\cdot(5\cdot\alpha+1)\cdot(6\cdot\alpha+1)}$$

The first three L-moments of the Generalized Pareto Type II distribution are obtained as follows:

$$L_1 = \int_0^1 x(p)\cdot dp = \int_0^1\left(\gamma+\frac{\beta}{\alpha}\cdot\left(1-p^{\alpha}\right)\right)\cdot dp = \int_0^1 \gamma\cdot dp + \int_0^1 \frac{\beta}{\alpha}\cdot dp - \int_0^1 \frac{\beta}{\alpha}\cdot p^{\alpha}\cdot dp =$$
$$\gamma + \frac{\beta}{\alpha} - \frac{\beta}{\alpha}\cdot\frac{1}{\alpha+1} = \gamma + \frac{\beta}{\alpha+1}$$

$$L_2 = \int_0^1 x(p)\cdot(1-2\cdot p)\cdot dp = \int_0^1\left(\gamma+\frac{\beta}{\alpha}\cdot\left(1-p^{\alpha}\right)\right)\cdot(1-2\cdot p)\cdot dp = \int_0^1 \gamma\cdot dp - 2\cdot\int_0^1 \gamma\cdot p\cdot dp +$$
$$\int_0^1 \frac{\beta}{\alpha}\cdot\left(1-p^{\alpha}\right)\cdot dp - \int_0^1 \frac{\beta}{\alpha}\cdot\left(1-p^{\alpha}\right)\cdot 2\cdot p\cdot dp = \frac{\beta}{\alpha}\cdot\left(\frac{2}{\alpha+2}-\frac{1}{\alpha+1}\right) = \frac{\beta}{(\alpha+1)\cdot(\alpha+2)}$$

$$L_3 = \int_0^1 x(p)\cdot\left(6\cdot p^2-6\cdot p+1\right)\cdot dp = \int_0^1\left(\gamma+\frac{\beta}{\alpha}\cdot\left(1-p^{\alpha}\right)\right)\cdot\left(6\cdot p^2-6\cdot p+1\right)\cdot dp =$$
$$\int_0^1 \gamma\cdot dp - 6\cdot\int_0^1 \gamma\cdot p\cdot dp + 6\cdot\int_0^1 \gamma\cdot p^2\cdot dp + \int_0^1 \frac{\beta}{\alpha}\cdot\left(1-p^{\alpha}\right)\cdot dp - \int_0^1 \frac{\beta}{\alpha}\cdot\left(1-p^{\alpha}\right)\cdot 6\cdot p\cdot dp +$$
$$\int_0^1 \frac{\beta}{\alpha}\cdot\left(1-p^{\alpha}\right)\cdot 6\cdot p^2\cdot dp = \frac{\beta}{\alpha}\cdot\left(\frac{6}{\alpha+2}-\frac{6}{\alpha+3}-\frac{1}{\alpha+1}\right) = \frac{\beta\cdot(1-\alpha)}{(\alpha+1)\cdot(\alpha+2)\cdot(\alpha+3)}$$

Generalized Pareto Type III distribution

To determine the raw moments of the Generalized Pareto Type III distribution, the following substitutions in the expression for the probability density function are required:

$$f(x)=\frac{\alpha\cdot\left(\frac{x-\gamma}{\beta}\right)^{\alpha-1}\cdot\left(\left(\frac{x-\gamma}{\beta}\right)^{\alpha}+1\right)^{-2}}{\beta}$$

It is noted:

$$t=\left(\frac{x-\gamma}{\beta}\right)^{\alpha} \qquad x=\beta\cdot t^{\frac{1}{\alpha}}+\gamma \qquad dx=\frac{\beta\cdot t^{\frac{1}{\alpha}-1}}{\alpha}\cdot dt$$

Thus, the first six raw moments are:

$$m_1^{'}=\int_{\gamma+\frac{\alpha}{\beta}}^{\infty}x\cdot f(x)=\int_{\gamma+\frac{\alpha}{\beta}}^{\infty}x\cdot\frac{\alpha\cdot\left(\frac{x-\gamma}{\beta}\right)^{\alpha-1}\cdot\left(\left(\frac{x-\gamma}{\beta}\right)^{\alpha}+1\right)^{-2}}{\beta}dx=\int_0^{\infty}\left(\beta\cdot t^{\frac{1}{\alpha}}+\gamma\right)\cdot(t+1)^{-2}\,dt=$$
$$\int_0^{\infty}\beta\cdot t^{\frac{1}{\alpha}}\cdot\frac{1}{(t+1)^2}dt+\int_0^{\infty}\gamma\cdot\frac{1}{(t+1)^2}dt=\beta\cdot\Gamma\left(\frac{1}{\alpha}+1\right)\cdot\Gamma\left(1-\frac{1}{\alpha}\right)+\gamma$$

$$m_2^{'}=\int_{\gamma+\frac{\alpha}{\beta}}^{\infty}x^2\cdot f(x)=\int_{\gamma+\frac{\alpha}{\beta}}^{\infty}x^2\cdot\frac{\alpha\cdot\left(\frac{x-\gamma}{\beta}\right)^{\alpha-1}\cdot\left(\left(\frac{x-\gamma}{\beta}\right)^{\alpha}+1\right)^{-2}}{\beta}dx=\int_0^{\infty}\left(\beta\cdot t^{\frac{1}{\alpha}}+\gamma\right)^2\cdot(t+1)^{-2}\,dt=$$
$$\int_0^{\infty}\beta^2\cdot t^{\frac{2}{\alpha}}\cdot\frac{1}{(t+1)^2}dt+\int_0^{\infty}2\cdot\gamma\cdot\beta\cdot t^{\frac{1}{\alpha}}\cdot\frac{1}{(t+1)^2}dt+\int_0^{\infty}\gamma^2\cdot\frac{1}{(t+1)^2}dt=$$
$$\beta^2\cdot\Gamma\left(\frac{2}{\alpha}+1\right)\cdot\Gamma\left(1-\frac{2}{\alpha}\right)+2\cdot\gamma\cdot\beta\cdot\Gamma\left(\frac{1}{\alpha}+1\right)\cdot\Gamma\left(1-\frac{1}{\alpha}\right)+\gamma^2$$

$$m_3^{'}=\int_{\gamma+\frac{\alpha}{\beta}}^{\infty}x^3\cdot f(x)=\int_{\gamma+\frac{\alpha}{\beta}}^{\infty}x^3\cdot\frac{\alpha\cdot\left(\frac{x-\gamma}{\beta}\right)^{\alpha-1}\cdot\left(\left(\frac{x-\gamma}{\beta}\right)^{\alpha}+1\right)^{-2}}{\beta}dx=\int_0^{\infty}\left(\beta\cdot t^{\frac{1}{\alpha}}+\gamma\right)^3\cdot(t+1)^{-2}\,dt=$$
$$\int_0^{\infty}\beta^3\cdot t^{\frac{3}{\alpha}}\cdot\frac{1}{(t+1)^2}dt+\int_0^{\infty}3\cdot\gamma\cdot\beta^2\cdot t^{\frac{2}{\alpha}}\cdot\frac{1}{(t+1)^2}dt+\int_0^{\infty}3\cdot\gamma^2\cdot\beta\cdot t^{\frac{1}{\alpha}}\cdot\frac{1}{(t+1)^2}dt+\int_0^{\infty}\gamma^3\cdot\frac{1}{(t+1)^2}dt=$$
$$\beta^3\cdot\Gamma\left(\frac{3}{\alpha}+1\right)\cdot\Gamma\left(1-\frac{3}{\alpha}\right)+3\cdot\gamma\cdot\beta^2\cdot\Gamma\left(\frac{2}{\alpha}+1\right)\cdot\Gamma\left(1-\frac{2}{\alpha}\right)+3\cdot\gamma^2\cdot\beta\cdot\Gamma\left(\frac{1}{\alpha}+1\right)\cdot\Gamma\left(1-\frac{1}{\alpha}\right)+\gamma^3$$

$$m_4' = \int_{\gamma+\frac{\alpha}{\beta}}^{\infty} x^4 \cdot f(x) = \int_{\gamma+\frac{\alpha}{\beta}}^{\infty} x^4 \cdot \frac{\alpha \cdot \left(\frac{x-\gamma}{\beta}\right)^{\alpha-1} \cdot \left(\left(\frac{x-\gamma}{\beta}\right)^{\alpha} + 1\right)^{-2}}{\beta} dx = \int_0^{\infty} \left(\beta \cdot t^{\frac{1}{\alpha}} + \gamma\right)^4 \cdot (t+1)^{-2} dt =$$

$$\int_0^{\infty} \beta^4 \cdot t^{\frac{4}{\alpha}} \cdot \frac{1}{(t+1)^2} dt + \int_0^{\infty} 4 \cdot \gamma \cdot \beta^3 \cdot t^{\frac{3}{\alpha}} \cdot \frac{1}{(t+1)^2} dt + \int_0^{\infty} 6 \cdot \gamma^2 \cdot \beta^2 \cdot t^{\frac{2}{\alpha}} \cdot \frac{1}{(t+1)^2} dt + \int_0^{\infty} 4 \cdot \gamma^3 \cdot \beta \cdot t^{\frac{1}{\alpha}} \cdot \frac{1}{(t+1)^2} dt +$$

$$\int_0^{\infty} \gamma^4 \cdot \frac{1}{(t+1)^2} dt = \beta^4 \cdot \Gamma\left(\frac{4}{\alpha}+1\right) \cdot \Gamma\left(1-\frac{4}{\alpha}\right) + 4 \cdot \gamma \cdot \beta^3 \cdot \Gamma\left(\frac{3}{\alpha}+1\right) \cdot \Gamma\left(1-\frac{3}{\alpha}\right) + 6 \cdot \gamma^2 \cdot \beta^2 \cdot \Gamma\left(\frac{2}{\alpha}+1\right) \cdot \Gamma\left(1-\frac{2}{\alpha}\right) +$$

$$4 \cdot \gamma^3 \cdot \beta \cdot \Gamma\left(\frac{1}{\alpha}+1\right) \cdot \Gamma\left(1-\frac{1}{\alpha}\right) + \gamma^4$$

$$m_5' = \int_{\gamma+\frac{\alpha}{\beta}}^{\infty} x^5 \cdot f(x) = \int_{\gamma+\frac{\alpha}{\beta}}^{\infty} x^5 \cdot \frac{\alpha \cdot \left(\frac{x-\gamma}{\beta}\right)^{\alpha-1} \cdot \left(\left(\frac{x-\gamma}{\beta}\right)^{\alpha} + 1\right)^{-2}}{\beta} dx = \int_0^{\infty} \left(\beta \cdot t^{\frac{1}{\alpha}} + \gamma\right)^5 \cdot (t+1)^{-2} dt =$$

$$\int_0^{\infty} \beta^5 \cdot t^{\frac{5}{\alpha}} \cdot \frac{1}{(t+1)^2} dt + \int_0^{\infty} 5 \cdot \gamma \cdot \beta^4 \cdot t^{\frac{4}{\alpha}} \cdot \frac{1}{(t+1)^2} dt + \int_0^{\infty} 10 \cdot \gamma^2 \cdot \beta^3 \cdot t^{\frac{3}{\alpha}} \cdot \frac{1}{(t+1)^2} dt +$$

$$\int_0^{\infty} 10 \cdot \gamma^3 \cdot \beta^2 \cdot t^{\frac{2}{\alpha}} \cdot \frac{1}{(t+1)^2} dt + \int_0^{\infty} 5 \cdot \gamma^4 \cdot \beta \cdot t^{\frac{1}{\alpha}} \cdot \frac{1}{(t+1)^2} dt + \int_0^{\infty} \gamma^5 \cdot \frac{1}{(t+1)^2} dt =$$

$$\beta^5 \cdot \Gamma\left(\frac{5}{\alpha}+1\right) \cdot \Gamma\left(1-\frac{5}{\alpha}\right) + 5 \cdot \gamma \cdot \beta^4 \cdot \Gamma\left(\frac{4}{\alpha}+1\right) \cdot \Gamma\left(1-\frac{4}{\alpha}\right) + 10 \cdot \gamma^2 \cdot \beta^3 \cdot \Gamma\left(\frac{3}{\alpha}+1\right) \cdot \Gamma\left(1-\frac{3}{\alpha}\right) +$$

$$+10 \cdot \gamma^3 \cdot \beta^2 \cdot \Gamma\left(\frac{2}{\alpha}+1\right) \cdot \Gamma\left(1-\frac{2}{\alpha}\right) + 5 \cdot \gamma^4 \cdot \beta \cdot \Gamma\left(\frac{1}{\alpha}+1\right) \cdot \Gamma\left(1-\frac{1}{\alpha}\right) + \gamma^5$$

$$m_6' = \int_{\gamma+\frac{\alpha}{\beta}}^{\infty} x^6 \cdot f(x) = \int_{\gamma+\frac{\alpha}{\beta}}^{\infty} x^6 \cdot \frac{\alpha \cdot \left(\frac{x-\gamma}{\beta}\right)^{\alpha-1} \cdot \left(\left(\frac{x-\gamma}{\beta}\right)^{\alpha} + 1\right)^{-2}}{\beta} dx = \int_0^{\infty} \left(\beta \cdot t^{\frac{1}{\alpha}} + \gamma\right)^6 \cdot (t+1)^{-2} dt =$$

$$\int_0^{\infty} \beta^6 \cdot t^{\frac{6}{\alpha}} \cdot \frac{1}{(t+1)^2} dt + \int_0^{\infty} 6 \cdot \gamma \cdot \beta^5 \cdot t^{\frac{5}{\alpha}} \cdot \frac{1}{(t+1)^2} dt + \int_0^{\infty} 15 \cdot \gamma^2 \cdot \beta^4 \cdot t^{\frac{4}{\alpha}} \cdot \frac{1}{(t+1)^2} dt +$$

$$\int_0^{\infty} 20 \cdot \gamma^3 \cdot \beta^3 \cdot t^{\frac{3}{\alpha}} \cdot \frac{1}{(t+1)^2} dt + \int_0^{\infty} 15 \cdot \gamma^4 \cdot \beta^2 \cdot t^{\frac{2}{\alpha}} \cdot \frac{1}{(t+1)^2} dt + \int_0^{\infty} 6 \cdot \gamma^5 \cdot \beta \cdot t^{\frac{1}{\alpha}} \cdot \frac{1}{(t+1)^2} dt + \int_0^{\infty} \gamma^6 \cdot \frac{1}{(t+1)^2} dt =$$

$$\beta^6 \cdot \Gamma\left(\frac{6}{\alpha}+1\right) \cdot \Gamma\left(1-\frac{6}{\alpha}\right) + 6 \cdot \gamma \cdot \beta^5 \cdot \Gamma\left(\frac{5}{\alpha}+1\right) \cdot \Gamma\left(1-\frac{5}{\alpha}\right) + 15 \cdot \gamma^2 \cdot \beta^4 \cdot \Gamma\left(\frac{4}{\alpha}+1\right) \cdot \Gamma\left(1-\frac{4}{\alpha}\right) +$$

$$+20 \cdot \gamma^3 \cdot \beta^3 \cdot \Gamma\left(\frac{3}{\alpha}+1\right) \cdot \Gamma\left(1-\frac{3}{\alpha}\right) + 15 \cdot \gamma^4 \cdot \beta^2 \cdot \Gamma\left(\frac{2}{\alpha}+1\right) \cdot \Gamma\left(1-\frac{2}{\alpha}\right) + 6 \cdot \gamma^5 \cdot \beta \cdot \Gamma\left(\frac{1}{\alpha}+1\right) \cdot \Gamma\left(1-\frac{1}{\alpha}\right) + \gamma^6$$

The characteristic moment of order r of the Generalized Pareto Type III distribution, has the following expression:

$$g_r = \beta^r \cdot \Gamma\left(\frac{r}{\alpha}+1\right) \cdot \Gamma\left(1-\frac{r}{\alpha}\right)$$

The first six central moments (except the arithmetic mean) have the following expressions:

$m_1 = m_1' = \mu = \gamma + g_1$

$m_2 = \sigma^2 = m_2' - m_1'^2 = g_2 - g_1^2$

$$m_3 = m_3^{'} - 3 \cdot m_2^{'} \cdot m_1^{'} + 2 \cdot m_1^{'3} = g_3 - 3 \cdot g_2 \cdot g_1 + 2 \cdot g_1^3$$
$$m_4 = m_4^{'} - 4 \cdot m_3^{'} \cdot m_1^{'} + 6 \cdot m_2^{'} \cdot m_1^{'2} - 3 \cdot m_1^{'4} = g_4 - 4 \cdot g_3 \cdot g_1 + 6 \cdot g_2 \cdot g_1^2 - 3 \cdot g_1^4$$
$$m_5 = m_5^{'} - 5 \cdot m_4^{'} \cdot m_1^{'} + 10 \cdot m_3^{'} \cdot m_1^{'2} - 10 \cdot m_2^{'} \cdot m_1^{'3} + 4 \cdot m_1^{'5} =$$
$$g_5 - 5 \cdot g_4 \cdot g_1 + 10 \cdot g_3 \cdot g_1^2 - 10 \cdot g_2 \cdot g_1^3 + 4 \cdot g_1^5$$
$$m_6 = m_6^{'} - 6 \cdot m_5^{'} \cdot m_1^{'} + 15 \cdot m_4^{'} \cdot m_1^{'2} - 20 \cdot m_3^{'} \cdot m_1^{'3} + 15 \cdot m_2^{'} \cdot m_1^{4} - 5 \cdot m_1^{'6} =$$
$$g_6 - 6 \cdot g_5 \cdot g_1 + 15 \cdot g_4 \cdot g_1^2 - 20 \cdot g_3 \cdot g_1^3 + 15 \cdot g_2 \cdot g_1^4 - 5 \cdot g_1^6$$

Skewness (γ_1), kurtosis (γ_2), γ_3 and γ_4 are determined with the following expressions:

$$\gamma_1 = C_s = \frac{m_3}{m_2^{3/2}} \qquad \gamma_2 = C_k = \frac{m_4}{m_2^2} \qquad \gamma_3 = \frac{m_5}{m_2^{\frac{5}{2}}} \qquad \gamma_4 = \frac{m_6}{m_2^3}$$

The first three L-moments of the Generalized Pareto Type III distribution are obtained as follows:

$$L_1 = \gamma + B_1 \qquad L_2 = B_2 \qquad L_3 = B_3$$

where $B_r = \frac{\beta}{\alpha^r} \cdot \Gamma\left(\frac{1}{\alpha}\right) \cdot \Gamma\left(1 - \frac{1}{\alpha}\right)$

Generalized Pareto Type IV distribution

To determine the raw moments of the Generalized Pareto Type IV distribution, the following substitutions in the expression for the probability density function are required:

$$f(x) = \frac{\lambda \cdot \left(\frac{x-\gamma}{\beta}\right)^{\frac{1}{\alpha}-1} \cdot \left(\left(\frac{x-\gamma}{\beta}\right)^{\frac{1}{\alpha}} + 1\right)^{-\lambda-1}}{\alpha \cdot \beta}$$

It is noted:

$$t = \left(\frac{x-\gamma}{\beta}\right)^{\frac{1}{\alpha}} \qquad x = \beta \cdot t^{\alpha} + \gamma \qquad dx = \alpha \cdot \beta \cdot t^{\alpha-1} dt$$

Thus, the first four raw moments are:

$$m_1^{'} = \int_0^{\infty} x \cdot f(x) = \int_0^{\infty} x \cdot \frac{\lambda \cdot \left(\frac{x-\gamma}{\beta}\right)^{\frac{1}{\alpha}-1} \cdot \left(\left(\frac{x-\gamma}{\beta}\right)^{\frac{1}{\alpha}} + 1\right)^{-\lambda-1}}{\alpha \cdot \beta} dx = \int_0^{\infty} \left(\beta \cdot t^{\alpha} + \gamma\right) \cdot \lambda \cdot (t+1)^{-\lambda-1} \cdot dt =$$
$$\beta \cdot \lambda \cdot \int_0^{\infty} t^{\alpha} \cdot \frac{1}{(t+1)^{\lambda+1}} \cdot dt + \beta \cdot \lambda \cdot \int_0^{\infty} \frac{1}{(t+1)^{\lambda+1}} \cdot dt = \beta \cdot \lambda \cdot \frac{\Gamma(\alpha+1) \cdot \Gamma(\lambda-\alpha)}{\Gamma(\lambda+1)} +$$

$$m_2^{'}=\int_0^\infty x^2\cdot f(x)=\int_0^\infty x^2\cdot\frac{\lambda\cdot\left(\frac{x-\gamma}{\beta}\right)^{\frac{1}{\alpha}-1}\cdot\left(\left(\frac{x-\gamma}{\beta}\right)^{\frac{1}{\alpha}}+1\right)^{-\lambda-1}}{\alpha\cdot\beta}dx=\int_0^\infty\left(\beta\cdot t^\alpha+\gamma\right)^2\cdot\lambda\cdot(t+1)^{-\lambda-1}\cdot dt=$$
$$\beta^2\cdot\lambda\cdot\int_0^\infty t^{2\cdot\alpha}\cdot\frac{1}{(t+1)^{-\lambda-1}}dt+2\cdot\gamma\cdot\beta\cdot\lambda\cdot\int_0^\infty t^\alpha\cdot\frac{1}{(t+1)^{-\lambda-1}}dt+\gamma^2\cdot\int_0^\infty\frac{1}{(t+1)^{-\lambda-1}}dt=$$
$$\beta^2\cdot\lambda\cdot\frac{\Gamma(2\cdot\alpha+1)\cdot\Gamma(\lambda-2\cdot\alpha)}{\Gamma(\lambda+1)}+2\cdot\gamma\cdot\beta\cdot\lambda\cdot\frac{\Gamma(\alpha+1)\cdot\Gamma(\lambda-\alpha)}{\Gamma(\lambda+1)}+\gamma^2$$

$$m_3^{'}=\int_0^\infty x^3\cdot f(x)=\int_0^\infty x^3\cdot\frac{\lambda\cdot\left(\frac{x-\gamma}{\beta}\right)^{\frac{1}{\alpha}-1}\cdot\left(\left(\frac{x-\gamma}{\beta}\right)^{\frac{1}{\alpha}}+1\right)^{-\lambda-1}}{\alpha\cdot\beta}dx=\int_0^\infty\left(\beta\cdot t^\alpha+\gamma\right)^3\cdot\lambda\cdot(t+1)^{-\lambda-1}\cdot dt=$$
$$\beta^3\cdot\lambda\cdot\int_0^\infty t^{3\cdot\alpha}\cdot\frac{1}{(t+1)^{-\lambda-1}}dt+3\cdot\gamma\cdot\beta^2\cdot\lambda\cdot\int_0^\infty t^{2\cdot\alpha}\cdot\frac{1}{(t+1)^{-\lambda-1}}dt+3\cdot\gamma^2\cdot\beta\cdot\lambda\cdot\int_0^\infty t^\alpha\cdot\frac{1}{(t+1)^{-\lambda-1}}dt+$$
$$\gamma^3\cdot\int_0^\infty\frac{1}{(t+1)^{-\lambda-1}}dt=\beta^3\cdot\lambda\cdot\frac{\Gamma(3\cdot\alpha+1)\cdot\Gamma(\lambda-3\cdot\alpha)}{\Gamma(\lambda+1)}+3\cdot\gamma\cdot\beta^2\cdot\lambda\cdot\frac{\Gamma(2\cdot\alpha+1)\cdot\Gamma(\lambda-2\cdot\alpha)}{\Gamma(\lambda+1)}+$$
$$3\cdot\gamma^2\cdot\beta\cdot\lambda\cdot\frac{\Gamma(\alpha+1)\cdot\Gamma(\lambda-\alpha)}{\Gamma(\lambda+1)}+\gamma^3$$

$$m_4^{'}=\int_0^\infty x^4\cdot f(x)=\int_0^\infty x^4\cdot\frac{\lambda\cdot\left(\frac{x-\gamma}{\beta}\right)^{\frac{1}{\alpha}-1}\cdot\left(\left(\frac{x-\gamma}{\beta}\right)^{\frac{1}{\alpha}}+1\right)^{-\lambda-1}}{\alpha\cdot\beta}dx=\int_0^\infty\left(\beta\cdot t^\alpha+\gamma\right)^4\cdot\lambda\cdot(t+1)^{-\lambda-1}\cdot dt=$$
$$\beta^4\cdot\lambda\cdot\int_0^\infty t^{4\cdot\alpha}\cdot\frac{1}{(t+1)^{-\lambda-1}}dt+4\cdot\gamma\cdot\beta^3\cdot\lambda\cdot\int_0^\infty t^{3\cdot\alpha}\cdot\frac{1}{(t+1)^{-\lambda-1}}dt+$$
$$6\cdot\gamma^2\cdot\beta^2\cdot\lambda\cdot\int_0^\infty t^{2\cdot\alpha}\cdot\frac{1}{(t+1)^{-\lambda-1}}dt+4\cdot\gamma^3\cdot\beta\cdot\lambda\cdot\int_0^\infty t^\alpha\cdot\frac{1}{(t+1)^{-\lambda-1}}dt+\gamma^4\cdot\int_0^\infty\frac{1}{(t+1)^{-\lambda-1}}dt=$$
$$\beta^4\cdot\lambda\cdot\frac{\Gamma(4\cdot\alpha+1)\cdot\Gamma(\lambda-4\cdot\alpha)}{\Gamma(\lambda+1)}+4\cdot\gamma\cdot\beta^3\cdot\lambda\cdot\frac{\Gamma(3\cdot\alpha+1)\cdot\Gamma(\lambda-3\cdot\alpha)}{\Gamma(\lambda+1)}+$$
$$6\cdot\gamma^2\cdot\beta^2\cdot\lambda\cdot\frac{\Gamma(2\cdot\alpha+1)\cdot\Gamma(\lambda-2\cdot\alpha)}{\Gamma(\lambda+1)}+4\cdot\gamma^3\cdot\beta\cdot\lambda\cdot\frac{\Gamma(\alpha+1)\cdot\Gamma(\lambda-\alpha)}{\Gamma(\lambda+1)}+\gamma^4$$

The characteristic moment of order r of the Generalized Pareto Type IV distribution, has the following expression:

$$g_r=\beta^r\cdot\frac{\Gamma(1+r\cdot\alpha)\cdot\Gamma(\lambda-r\cdot\alpha)}{\Gamma(\lambda+1)}$$

The first six central moments (except the arithmetic mean) have the following expressions:

$$m_1=m_1^{'}=\mu=\gamma+g_1$$
$$m_2=\sigma^2=m_2^{'}-m_1^{'2}=g_2-g_1^2$$
$$m_3=m_3^{'}-3\cdot m_2^{'}\cdot m_1^{'}+2\cdot m_1^{'3}=g_3-3\cdot g_2\cdot g_1+2\cdot g_1^3$$
$$m_4=m_4^{'}-4\cdot m_3^{'}\cdot m_1^{'}+6\cdot m_2^{'}\cdot m_1^{'2}-3\cdot m_1^{'4}=g_4-4\cdot g_3\cdot g_1+6\cdot g_2\cdot g_1^2-3\cdot g_1^4$$
$$m_5=m_5^{'}-5\cdot m_4^{'}\cdot m_1^{'}+10\cdot m_3^{'}\cdot m_1^{'2}-10\cdot m_2^{'}\cdot m_1^{'3}+4\cdot m_1^{'5}=$$
$$g_5-5\cdot g_4\cdot g_1+10\cdot g_3\cdot g_1^2-10\cdot g_2\cdot g_1^3+4\cdot g_1^5$$

$$m_6 = m_6' - 6 \cdot m_5' \cdot m_1' + 15 \cdot m_4' \cdot m_1'^2 - 20 \cdot m_3' \cdot m_1'^3 + 15 \cdot m_2' \cdot m_1^4 - 5 \cdot m_1'^6 =$$
$$g_6 - 6 \cdot g_5 \cdot g_1 + 15 \cdot g_4 \cdot g_1^2 - 20 \cdot g_3 \cdot g_1^3 + 15 \cdot g_2 \cdot g_1^4 - 5 \cdot g_1^6$$

Skewness (γ_1), kurtosis (γ_2), γ_3 and γ_4 are determined with the following expressions:

$$\gamma_1 = C_s = \frac{m_3}{m_2^{3/2}} \qquad \gamma_2 = C_k = \frac{m_4}{m_2^{2}} \qquad \gamma_3 = \frac{m_5}{m_2^{\frac{5}{2}}} \qquad \gamma_4 = \frac{m_6}{m_2^{3}}$$

The four L-moments of the Generalized Pareto Type IV distribution are obtained as follows:

$$L_1 = \gamma + B_1$$

$$L_2 = B_1 - B_2$$

$$L_3 = B_1 - 3 \cdot B_2 + 2 \cdot B_3$$

$$L_4 = B_1 - 6 \cdot B_2 + 10 \cdot B_3 - 5 \cdot B_4$$

where $B_r = \beta \cdot \frac{\Gamma(r \cdot \lambda - \alpha) \cdot \Gamma(1 + \alpha)}{\Gamma(r \cdot \lambda)}$