

## Supplementary Materials

Text S1 Set Pair Analysis evaluation process

Let A represent the spatial set of evaluation indicators of functional connectivity. B represents the attribute space denoting the evaluation levels of functional connectivity. The study considers primary subsystems ( $m$ ,  $1 \leq m \leq 2$ ) and secondary subsystems ( $mq$ ,  $1 \leq q \leq 2$ ) within them, as well as attribute indicators ( $m q k$ ,  $1 \leq k \leq 3$ ) at different levels. The indicators ( $I_m$ ,  $I_{mq}$ ,  $I_{mqk}$ ) correspond to target, criterion, and indicator layers, respectively.  $t_{mqk}$  as the measured values of  $I_{mqk}$  in study area. The graded of functional connectivity as n-level scale. Using an n-dimensional correlation coefficient ( $\mu$ ), the functional connectivity at different levels and the overall connectivity can be evaluated. Evaluation levels are determined based on equal division principles.

1) It is necessary to determine the type of each indicator value, which is either cost-type or benefit-type. Indicators with smaller measured values are considered cost-type and have higher evaluation levels, while those with larger measured values are considered benefit-type and have higher evaluation levels. Based on this, the comprehensive evaluation n-tuple connection coefficient of the water system functional connectivity indicator layer  $I_{mqk}$  in Table S1 is calculated:

$$\mu_{mqk} = r_{mqk1} + r_{mqk2}i_1 + \dots + r_{mqk(n-1)}i_{n-2} + r_{mqkn}j \quad (5-3)$$

The equation includes  $r_{mqk1}$ ,  $r_{mqk2}$ , ...,  $r_{mqkn}$  as correlation coefficients for evaluation grades at each level,  $i_1$ ,  $i_2$ , ...,  $i_{n-2}$  as coefficients for uncertainty difference components, and  $j = -1$  as the opposition coefficient.

2) Calculate the comprehensive evaluation n-tuple correlation coefficient of the water

system functional connectivity objective layer  $I_{mq}$ .

$$\mu_{mq} = r_{mq1} + r_{mq2}i_1 + \dots + r_{mq(n-1)}i_{n-2} + r_{mqn}j \quad (5-4)$$

$$r_{mq1} = \sum w_{mqk}r_{mqkl} (1 \leq l \leq n) \quad (5-5)$$

Where  $w_{mqk}$  is the weight of the third-level indicator  $I_{mqk}$ , and  $r_{mqkl}$  is the connectivity component of the third-level indicator relative to the first-level subsystem.

3) Calculate the comprehensive evaluation n-tuple connection coefficient of the total indicator for the functional connectivity of the water system.

$$\mu = r_1 + r_2i_1 + r_3i_2 + \dots + r_nj \quad (5-6)$$

4) Calculate the main value of the n-tuple connection: According to the principle of equal distribution, the value of the n-tuple connection obtained when  $j = -1$ .

5) To evaluate the level of water system connectivity, divide the range  $[-1,1]$  into  $n$  equal parts, where each part corresponds to a level of evaluation. Compare the main value of the connection with each level, with higher values indicating better connectivity as Table S2 shows.

Applying the principle of equal weighting, with  $i_1 = 1/3$ ,  $i_2 = -1/3$ , and  $j = -1$ , the main linkage values between the target layer and the overall index, as well as the comprehensive evaluation grades, are obtained in Table 3.

Table S1 Comparative analysis of two different n-dimensional correlation coefficient calculation methods for distinct types of indicators using the Set Pair analysis approach

| N-element coefficient   | Cost-based indicators           | Benefit-based indicators        |
|---|---------------------------------|---------------------------------|
| $1 + 0i + \dots + 0i_{n-2} + j$   | $I_{mqk} \leq a_1$              | $I_{mqk} \geq a_1$              |
| $\left  \frac{t - a_2}{a_1 - a_2} \right  + \left  \frac{t - a_1}{a_2 - a_1} \right  i_1 + \dots + 0j$                  | $a_1 \leq I_{mqk} \leq a_2$     | $a_1 \geq I_{mqk} \geq a_2$     |
| $0 + \left  \frac{t - a_3}{a_2 - a_3} \right  i_1 + \left  \frac{t - a_2}{a_2 - a_3} \right  i_2 + \dots + 0j$          | $a_2 \leq I_{mqk} \leq a_3$     | $a_2 \geq I_{mqk} \geq a_3$     |
| $0 + \dots + \left  \frac{t - a_n}{a_{n-1} - a_n} \right  i_{n-2} + \left  \frac{t - a_{n-1}}{a_{n-1} - a_n} \right  j$ | $a_{n-1} \leq I_{mqk} \leq a_n$ | $a_{n-1} \geq I_{mqk} \geq a_n$ |
| $0 + 0i + \dots lj$   | $I_{mqk} \geq a_n$              | $I_{mqk} \leq a_n$              |

$a_1, a_2, a_3, \dots, a_n$  are the evaluation standards for the indicators. For cost-type indicators,  $a_1 \geq a_2 \geq \dots \geq a_n$ , and for benefit-type indicators,  $a_1 \leq a_2 \leq \dots \leq a_n$ .

Table S2 Relation number of the criterion layer comprehensive evaluation using Set Pair analysis

| Criterion layer                         | Comprehensive evaluation relation number |                 |                 |                   |                 |
|---|--|-----------------|-----------------|-------------------|-----------------|
|   | 2000                                     | 2005            | 2010            | 2015              | 2020            |
| River breakage rate                     | $0.75+0.25i_l$                           | $0.95+0.05i_l$  | 1               | 1                 | 1               |
| Water quality compliance rate           | $0.25i_l+0.75i_2$                        | $0.2+0.8i_l$    | $0.29+0.71i_l$  | $0.14+0.86i_l$    | 1               |
| Percentage of urban water supply        | $0.46i_l+0.54i_2$                        | $0.3i_l+0.7i_2$ | $0.8i_l+0.2i_2$ | $0.77i_l+0.23i_2$ | $0.8i_l+0.2i_2$ |
| Percentage of industrial water supply   | $0.4i_l+0.6j$                            | $i_2$           | $0.85+0.15i_l$  | $0.25+0.75i_l$    | $0.35+0.65i_l$  |
| Percentage of agricultural water supply | $0.9i_l+0.1i_2$                          | $0.8i_2+0.2j$   | $0.7i_l+0.3i_2$ | $0.8i_l+0.2i_2$   | $0.6+0.4i_l$    |

$i_l, i_2$ , as coefficients for uncertainty difference components;  $j$  as the opposition coefficient.