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A GIS-Based Probabilistic Spatial Multicriteria Roof Water Inrush Risk Evaluation Method Considering Decision Makers' Risk-Coping Attitude

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Abstract: A combination of geographic information system (GIS) and spatial multicriteria decision making (MCDA) in mine water inrush risk evaluation is widely used, but the randomness in the process of index weight determination and the risk-coping attitude of decision makers are not considered in the decision making process. Therefore, this paper proposes a probability-based roof water inrush risk evaluation method (GIS-MCDA) by combining the Monte Carlo analytic hierarchy process (MAHP) and ordered weighted averaging (OWA) operator. This method uses MAHP to determine the weight of the evaluation indicators, reducing the randomness of the analytic hierarchy process (AHP) to determine the weight of the evaluation indicators using the OWA operator to quantify the five risk-coping attitudes of decision makers and incorporate the risk attitude of decision makers into the evaluation process. Taking the Liangshuijing Coal Mine in northern Shaanxi as an example, the application of the GIS-MCDA method showed that the method makes the risk results of roof water inrush more objective and comprehensive and reduces or avoids the risk of decision making due to human subjective tendency change.

Keywords: risk assessment of roof water inrush; Monte Carlo analytic hierarchy process; risk-coping attitude; decision makers; Liangshuijing Coal Mine

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1. Introduction

With the depletion of coal resources in the east, the northwest mining area has become the main coal-producing area in China [1–4]. Roof flood is the main mine geological disaster faced by the northwest mining area, which restricts the safe and efficient mining of coal resources. There have been numerous roof water inrush accidents in recent years, resulting in numerous casualties and significant economic losses. On 5 July 2021, a roof water inrush and quicksand accident occurred at the 30,108 working face of the Haojialiang Coal Mine in Yulin, Shaanxi Province, which killed five people and had a direct economic loss of RMB 13.828 million. On 14 August 2021, a roof water collapse accident occurred at the Qinghai Caidaer Coal Mine, resulting in 20 deaths and a direct economic loss of RMB 53.9102 million. Therefore, objectively assessing the risk of roof water inrush is significant for preventing and controlling roof water damage, developing preventive and control measures in advance, and ensuring mine safety [3,4].

There are many fields of study in which multicriteria decision making (MCDA) has been applied such as engineering, economics, disaster assessment, and others [5–9]. Roof water inrush is a complex geological disaster under the action of multiple factors and has strong spatiality, so GIS-MCDA can be effectively applied to the risk evaluation of roof water inrush [10–15]. In recent years, the GIS-MCDA method has been widely used in mine water inrush risk evaluation [16–23]. To evaluate the working face water inrush risk from roofs and floors, Wu Qiang proposed three methods: three-map double prediction method; the water-rich index method; and vulnerability index method [17,24–30]. Combining the improved analytic hierarchy process and entropy weight method, Gao et al. [31] put forward a water inrush risk assessment method for the work face after water drainage measures. Li et al. [18] established a more accurate floor water inrush risk assessment model based on an improved analytic hierarchy process and logic regression method. Cheng et al. [19] suggests a method of assessing roof water inrush in multi-coal seam mining and Liu et al. [32] proposed a separate layer water inrush risk assessment model.

The traditional GIS-MCDA method has been widely applied to the field of mine water inrush risk assessment, but it has strong randomness and uncertainty in the decision making process, which leads to strong uncertainty in the evaluation results. For example, if different evaluation methods are used in the same mining area, different evaluation results can be obtained [18,33–35]. Evaluation results are uncertain because of the uncertainty in the decision-making process. Many factors contribute to uncertainty including the original data errors, processing methods, the number of evaluation criteria, and weighting. Among them, the weight of the evaluation criteria is the most influential factor, which causes the evaluation results to be controversial and uncertain [36,37]. AHP is often used to calculate the index weight in all kinds of roof water inrush risk assessment methods, but it has been criticized for not adequately handling the uncertainties and imprecisions inherent in pairwise comparisons [38–42]. Based on the analytic hierarchy process, calculation methods such as the improved analytic hierarchy process, fuzzy analytic hierarchy process, combined weighting of the analytic hierarchy process, and entropy weight method can be derived. These methods play an improved role in dealing with the uncertainty of the evaluation criteria weight, but do not completely eliminate the uncertainty of the evaluation of the criteria weight determination [43–47]. Therefore, a reasonable determination of the evaluation criteria weight is significant to eliminate the uncertainty of the evaluation results.

In addition, none of the existing roof water inrush risk assessment methods have considered the impact of the decision makers' risk-coping attitudes on roof water inrush. However, accident statistics show that the decision makers' risk-coping attitude has a significant impact on roof water inrush [48–57]. For example, a water seepage accident that occurred in Fengyuan Coal Mine in April 2021 caused 21 deaths. The cause of the accident was that the work of detecting and releasing water was not strictly implemented in the early stage. In November 2020, the Yuanjiangshan Coal Mine flooding accident caused five deaths and was caused by cross-border mining. The reason for such accidents is that the decision makers were optimistic about the roof water inrush and did not take risk-coping measures. The risk-coping attitude of decision makers plays an important role in the occurrence of roof water inrush accidents. Therefore, when making decisions on roof inrush risk assessment, the decision makers' risk-coping attitudes should be taken into consideration.

In this paper, the Monte Carlo analytic hierarchy process (MAHP) and ordered weighted averaging (OWA) operator were combined, and a probability-based roof water inrush risk evaluation method (GIS-MCDA) was proposed. This method used MAHP to calculate the criteria weights. The MAHP method integrates the Monte Carlo simulation with probability distribution and solves the index weight uncertainty very well. Based on the OWA operator, the paper explores the influence of the risk-coping attitude of decision makers on the evaluation results of roof water inrush risk by incorporating the risk-coping attitude of decision makers into the decision-making process. The GIS-MCDA method was applied to the Liangshuijing Coal Mine in northern Shaanxi as an example.

2. Study Area

Liangshuijing Coal Mine is located in Shaanxi Province, about 16 km away from Shenmu County. Figure 1 shows the geographical location of the coal mine. The mine is 10.89 km long from north to south, 10.15 km wide from east to west, and covers an area of 68.9 km^2. The mining area is located in the northwest inland, belongs to the temperate semi-arid continental climate and has an average annual precipitation of 435.7 mm and a mean annual evaporation of 1774.1 mm.

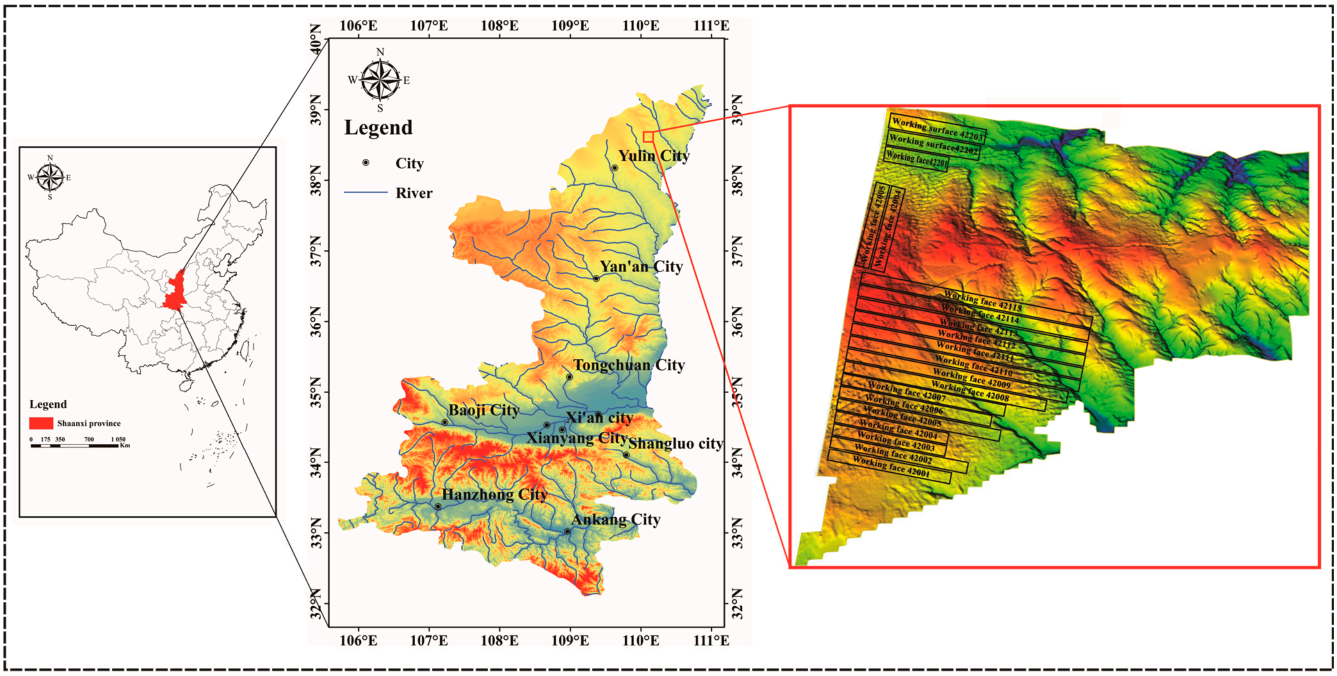


Figure 1. Geographical location of the mining area.

The 4^{-2} coal seam is the main coal seam in the mining area, with a thickness of 3.40–4.20 m, a buried depth of 13.45–160.92 m, and a mining elevation of 1120–1080 m.

The geological structure in the mining area is simple, and folds, faults, and magmatism are not developed. The stratum is gentle, the dip angle is less than 1°, and only the undulating anticline structure with very wide and gentle amplitude is developed.

From old to new, the mine's strata are: Upper Triassic Yongping Formation (T_{3y}), Middle Jurassic Yan'an Formation (J_{2y}), Zhiluo Formation (J_{2z}), Upper Neogene Xintong Baode Formation ($N_2{}^b$), Quaternary Middle Pleistocene Lishi Formation (Q_21), Upper Pleistocene Salawusu Formation ($Q_3{}^s$), and Holocene Aeolian Sand ($Q_4{}^{eol}$). The stratigraphic profile is shown in Figure 2.

The weathered bedrock aquifer at the top of the Yan'an Formation is the main water-filled aquifer during coal mining, which has strong water richness. The thickness of the weathered bedrock aquifer is 7-41 m, the average thickness is 24 m, the average unit water inflow(q) is 0.034 L/s·m, and the average permeability coefficient(k) is 0.16 m/d.

The distance between the 4^{-2} coal and weathered bedrock aquifer is 2.84–80.1 m, with an average of 47.5 m. After 4^{-2} coal mining, the average height of the water flowing fractured zone is 51.72 m. The height of the water flowing fractured zone is greater than the distance between coal seam and aquifer, which can easily connect to the aquifer and affect coal mining.

Stratigraphic unit	Lithology	Average thickness(m)	Geological column	Rmark
Q_4	Aeolian sand	26.15		
Q_3^S	Siltstone	14.35		
Q_2^1	Loess	13.05		
N_2b	Laterite	7.05		
J_2y	Fine sandstone	8.7		Aquifer
	Medium sandstone	11.2		
	Sandy mudstone	4.1		
	Siltstone	19.6		Aquiclude
	Medium sandstone	10.0		
	Fine sandstone	12.0		
	Siltstone	5.9		
	Coal	3.6		4^{-2}Coal

Figure 2. Hydrogeological profile of the study area.

3. Probabilistic Spatial Multicriteria Analysis Method

This article integrated the MAHP and OWA method to construct a probabilistic spatial multicriteria decision framework for the roof water inrush risk evaluation (Figure 3). Among them, MAHP was used to determine the criteria weights for the evaluations, which eliminates uncertainty in the process of determining the criteria weights. As a decision rule, the ordered weighted averaging method (OWA) can quantify the decision makers' risk-coping attitude and assemble the criteria attribute values and criteria weights into a comprehensive evaluation result.

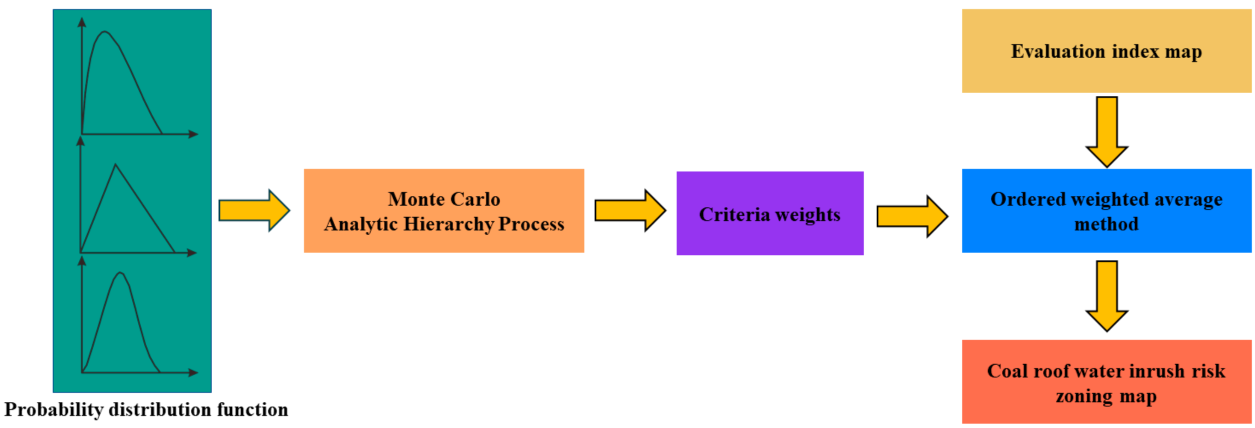


Figure 3. Probabilistic spatial multicriteria decision framework.

3.1. Monte Carlo Analytical Hierarchy Process (MAHP)

Satyr proposed the analytic hierarchy process (AHP), which is widely used in various fields. However, traditional AHP still has the following deficiencies [41,42,58–61]:

(1) Experts are required to use an accurate numerical value to describe the relative importance between criteria, but it is often difficult to give a precise numerical description.
(2) The unbalanced criterion judgment scale is used to quantify the relative importance of the criteria.
(3) When the relative importance of multiple criteria is very close, it is impossible to determine which criterion is the most important.
(4) The various possibilities of the relative importance of each element criterion in the pairwise judgment matrix are not fully examined.

Regarding the above deficiencies, the method of fuzzy mathematics can solve the above deficiencies of (1) and (2), while the Monte Carlo analytical hierarchy process used in this paper can solve all of the above problems [41,42,58–61].

MAHP integrates Monte Carlo simulation and the analytic hierarchy process. MAHP uses probability distribution to describe the distribution of elements in the pairwise judgment matrix, combining Monte Carlo to further explore the criteria weight uncertainty [62–65]. Figure 4 shows the process of calculating the criteria weight by the Monte Carlo analytical hierarchy process [66].

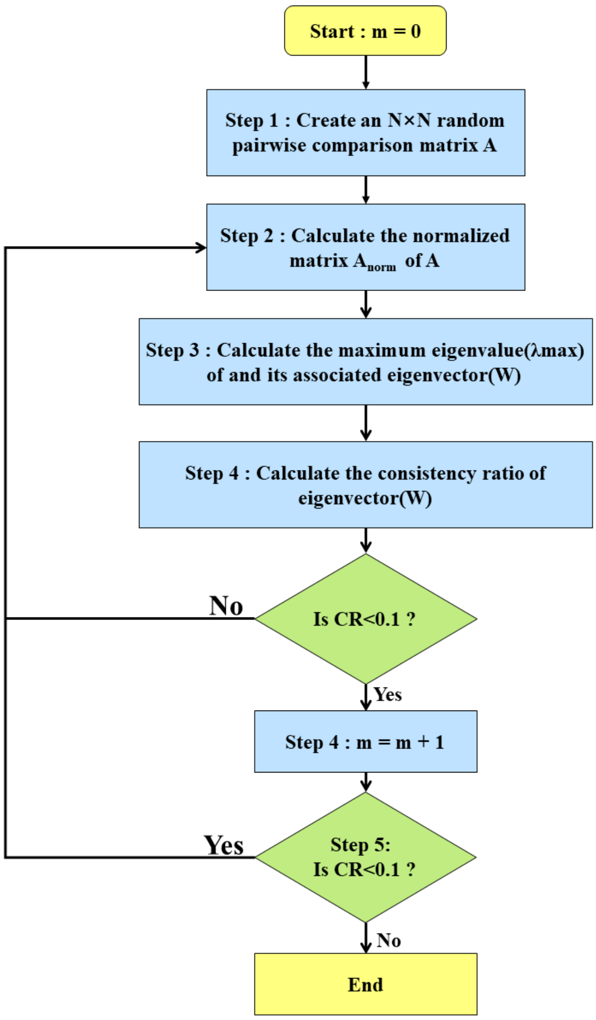


Figure 4. Flowchart for calculating the weights of the evaluation criteria by the Monte Carlo analytical hierarchy process.

Using MAHP to calculate the weight of evaluation criteria is as follows [59–61]:

Step 1: Use Monte Carlo to randomly generate $N \times N$ judgment matrix A (PCAM), and its element a_{ij} represents the pairwise comparison between the decision criteria.

$$A = [a_{ij}]\ i, j = 1, 2, \ldots, n \tag{1}$$

Consider the element a_{ij} as a continuous random variable and use the Beta-PERT probability distribution to describe the continuous random variable. The Beta-PERT probability distribution has a small amount of data and can better fit the uniform distribution and normal distribution, so it is very suitable for describing expert scoring in the decision making process [60,61,65,66].

Beta-PERT distribution:

$$f(x) = \{ \begin{matrix} \frac{x^{v-1}(1-x)^{w-1}}{B(v,w)} & 0 \le x \le 1;\ v, w > 0 \\ 0 & other \end{matrix} \tag{2}$$

In Equation (2), $B(v,w)$ is a standard Beta function, and its expression is:

$$B(v, w) = \int_0^1 t^{v-1}(1-t)w^{-1}dt \tag{3}$$

$$v = [\frac{x_{\text{mean}} - x_{\text{min}}}{x_{\text{max}} - x_{\text{min}}}][\frac{2x_{\text{mode}} - x_{\text{min}} - x_{\text{max}}}{x_{\text{mode}} - x_{\text{mean}}}] \tag{4}$$

$$w = v[\frac{x_{\text{max}} - x_{\text{mean}}}{x_{\text{mean}} - x_{\text{min}}}] \tag{5}$$

$$x_{\text{mean}} = (1/(\lambda + 2))(x_{\text{min}} + \lambda x_{\text{mode}} + x_{\text{max}}) \tag{6}$$

In Equations (3)–(6), x_{min}, x_{max}, x_{mode}, x_{mean} are the minimum value, maximum value, most probable value, and average value, respectively. The default value λ is 4.

Step 2: Determine the normalized matrix A_{norm} of the judgment matrix A. The element $\bar{a}_{ij}$ of A_{norm} is calculated as:

$$\bar{a}_{ij} = a_{ij} / \sum_{i=1}^{n} a_{ij}\ \ i, j = 1, 2, \ldots, n \tag{7}$$

Step 3: Calculate the maximum eigenvalue λ_{max} and the corresponding largest eigenvector W of the normalized matrix A_{norm}:

$$A_{\text{norm}} W = \lambda_{\text{max}} W \tag{8}$$

$$w_i = \frac{\sum_{j=1}^{n} \bar{a}_{ij}}{\sum_{i=1}^{n} \sum_{j=1}^{n} \bar{a}_{ij}} i, j = 1, 2, \ldots, n \tag{9}$$

Step 4: Consistency check.

$$\text{CI} = \frac{\lambda_{\text{max}} - n}{n - 1} \tag{10}$$

The consistency ratio CR is calculated as follows:

$$\text{CR} = \frac{\text{CI}}{\text{RI}} \tag{11}$$

In Equation (11), RI is the average random consistency index [38].

When CR is less than 0.1, the judgment matrix passes the consistency check and meets the requirements; when CR is greater than 0.1, return to step 1.

Step 5: Repeat steps 1–4 until the M group of criteria weights are obtained.

3.2. Ordered Weighted Averaging (OWA)

Ordered weighted averaging is a kind of aggregation method of multicriteria decision information proposed by Yager in 1988. The calculation equation of the ordered weighting method is shown in Equation (12) [67–69].

$$\mathrm{OWA} = \sum_{j=1}^{n} \frac{u_j v_j z_{ij}}{\sum\limits_{j=1}^{n} u_j v_j} \tag{12}$$

Among them, OWA is the roof water inrush risk evaluation score; $z_{i1} > z_{i2} > \ldots > z_{in}$ is the queue reordered by size after normalizing the evaluation criteria attribute value a_{ij}; u_j is the weight of the criteria; v_j is the order weight, which has nothing to do with a_{ij} but is only related to the sorting position of the criteria, v_1 is assigned to z_{i1}, v_2 is assigned to z_{i2}, and so on, v_n is assigned to z_{in}.

There are two types of weights in ordered weighted averaging: the criteria weight, $w_1, w_2, \ldots, w_n$ $(0 < w_j < 1, \sum w_j)$, which represents the relative importance between the evaluation criteria; and the order weight, $v_1, v_2, \ldots, v_n$ $(0 < v_j < 1, \sum v_j)$, which represents the decision maker's attitude to the roof water inrush risk [70,71].

Figure 5 presents the flowchart of the ordered weighted averaging for spatial multicriteria decisions, and the calculation process is as follows:

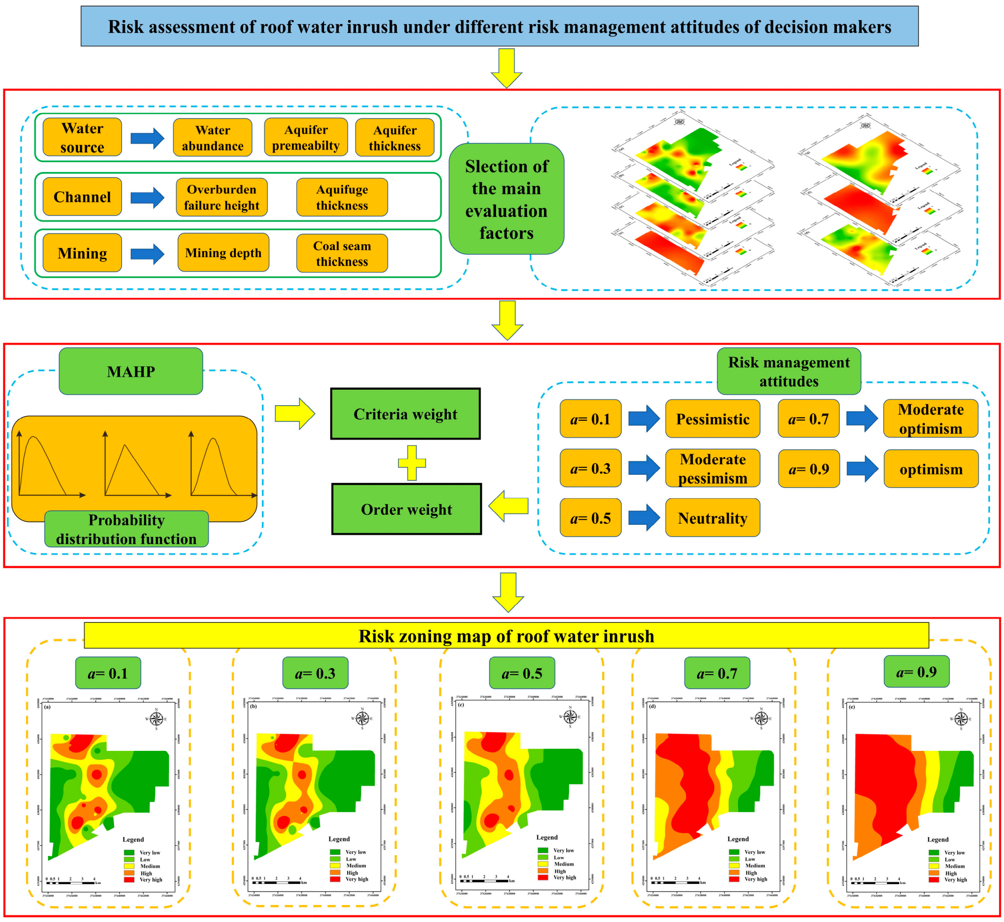


Figure 5. Ordered weighted averaging flowchart.

Step 1: Standardization of the evaluation criteria

To eliminate the conflict between different dimensions of evaluation criteria, it is necessary to standardize the attribute values of the evaluation criteria. In this paper, the range transformation method was used to standardize the spatial criteria, and the calculation equations are as follows:

$$\bar{a}_{ij} = \frac{a_{ij} - \min\{a_{ij}\}}{r_j} \tag{13}$$

$$\bar{a}_{ij} = \frac{\max\{a_{ij}\} - a_{ij}}{r_j} \tag{14}$$

where $\min\{a_{ij}\}$ and $\max\{a_{ij}\}$ are the minimum and maximum values of the criteria attribute value, respectively; $r_j = \max\{a_{ij}\} - \min\{a_{ij}\}$; $\bar{a}_{ij}$ is the attribute value after the criteria normalization.

Step 2: Calculate the weight of the evaluation criteria

In this paper, in Section 3.1, the steps for calculating the criteria weight are shown using MAHP.

Step 3: Arrange the normalized criteria attribute value at each spatial position in descending order.

Step 4: Sort based on the weights corresponding to the standardized criteria attribute values after sorting.

Step 5: Calculate the order weight of the evaluation criteria.

A key issue in the OWA approach is to quantify the decision makers' risk-coping attitude corresponding to the criteria order weight. This paper used the ORness measure and tradeoff measure to calculate the order weight [69–72].

Based on the ORness measure, a is defined as [70]:

$$a = \sum_{j=1}^{n} \frac{n-j}{n-1} v_j, 0 \leq a \leq 1 \tag{15}$$

In Equation (15), a represents the decision maker's attitude to roof water inrush risk, and v_j is the order weight of the evaluation criteria. The higher the value of a, the more optimistic the decision maker is about the roof water inrush risk. When $0.5 \geq a \geq 0$, it means that decision makers are unwilling to accept high-risk solutions and tend to accept less risky solutions; when $a = 0.5$, it indicates that decision makers are neither willing to accept high-risk schemes nor willing to accept low-risk schemes; when $1 \geq a \geq 0.5$, it indicates that decision makers prefer to risk it and tend to accept more risky options [69–72].

The normalized form of the dispersion measure is [73]:

$$disp(W) = -\sum_{j=1}^{n} \frac{v_j \ln v_j}{\ln n} \tag{16}$$

The dispersion measure is used to measure the extent to which the criteria information is fully utilized in the aggregation process. The larger the value, the more information is used. When $\varphi = 0$, $v_j = 1$, other weights are 0, when $\varphi = 1$, the order weight $v = [n^{-1}, n^{-1}, \ldots, n^{-1}]$.

Based on the ORness measure and dispersion measure, O'Hagan proposed a method of determining the order weight of OWA operators using the maximum entropy principle. The method determines the order weight of the evaluation criteria by solving a constrained nonlinear optimization problem when the optimism of the decision maker is known. Equation (17) was used for the calculation [74,75].

$$\max imize\ \phi = -\sum_{j=1}^{n} \frac{v_j \ln v_j}{\ln n} \tag{17}$$

where the constraints are:

$$\sum_{j=1}^{n} \frac{n-j}{n-1} v_j = \alpha, \ \sum_{j=1}^{n} v_j = 1, \\ 0 \leq \alpha \leq 1, \ 0 \leq v_j \leq 1, \ j = 1, 2, \ldots, n \tag{18}$$

Step 6: Multiply the standardized criteria attribute value with the sorting criteria weight to obtain the weighted criteria attribute normalized value, multiplied by the order weight.

Step 7: Superimpose and sum the results of each criteria attribute map to obtain each spatial position's comprehensive evaluation value of the ordered weighted averaging.

4. Results and Discussion

4.1. Determining Evaluation Criteria of Roof Water Inrush

In this paper, water abundance of the aquifer, aquifer permeability, aquifer thickness, mining depth, coal seam thickness, overburden failure height, and aquifuge thickness were selected as the seven evaluation criteria shown in Figure 6. The specific analysis of the selected factors is as follows:

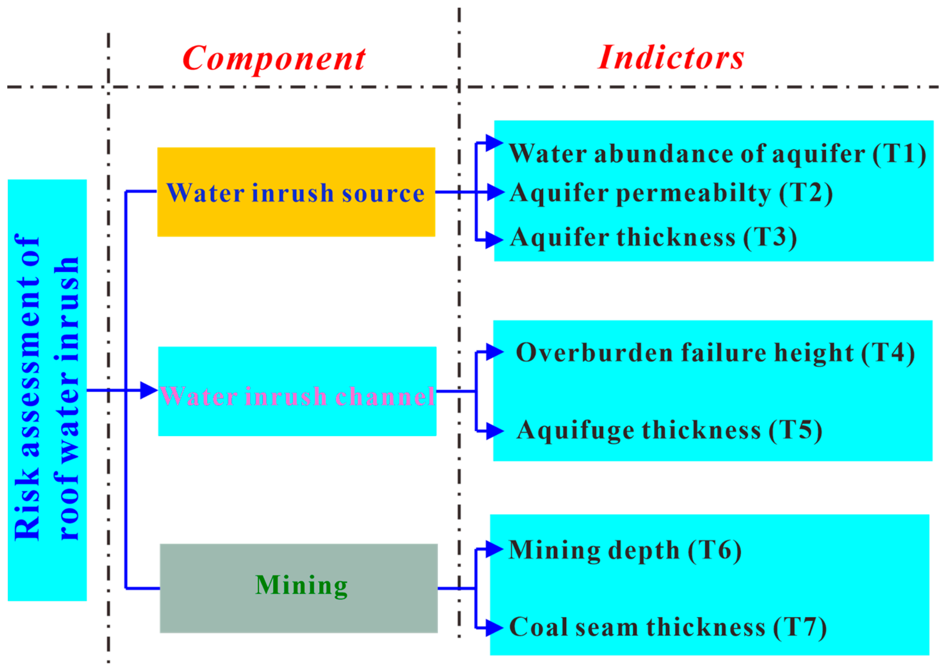


Figure 6. Roof water inrush risk evaluation index system.

Water abundance of the aquifer (T1): The water abundance of the aquifer is used to measure the amount of water yield of the aquifer during mining. The stronger the water abundance of the aquifer, the more water will be released from the aquifer during coal seam mining, and the higher the risk of roof water inrush. The water abundance of the aquifer is quantified by the unit water inflow (q) of the borehole.

Aquifer permeability (T2): The size of the permeability coefficient reflects the permeability of the aquifer. The larger the permeability coefficient of the aquifer, the stronger the permeability of the aquifer, and the higher the risk of roof water inrush.

Aquifer thickness (T3): The thickness of the aquifer directly determines the amount of water stored in the aquifer. The greater the thickness of the aquifer, the more water is stored in the aquifer, and the higher the risk of roof water inrush when the coal seam is mined.

Mining depth (T4): The deeper the coal seam is buried, the higher the risk of roof water inrush. The depth of coal seam directly determines the stress in overburden and the failure height of roof overburden after mining. The deeper the coal seam is buried, the greater the original rock stress in the roof overburden, the more obvious the roof pressure during coal seam mining, and the more likely this is to lead to the occurrence of a water inrush accident.

Coal seam thickness (T5): The greater the thickness of coal seam mining, the greater the disturbance of roof overburden, the more serious the deformation and failure of roof overburden, and the greater the risk of roof water inrush during coal seam mining.

Overburden failure height (T6): The mining of the coal seam causes the overburden to move, deform, and destroy, forming a fracture zone and a collapse zone within the overburden damage height. When the fracture zone connects to the aquifer, it becomes a water inrush channel. The deeper the water conducting fracture zone enters the aquifer, the greater the damage degree of the aquifer, and the greater the risk of roof water inrush during coal mining.

Aquifuge thickness (T7): The aquifuge can block the hydraulic connection between the aquifer and the coal seam, and reduce the development height of mining fractures. The greater the thickness of the roof aquifuge, the less the possibility of the water permeating through the aquifer, and the lower the risk of roof water inrush during coal seam mining.

4.2. Standardization of Evaluation Criteria

The contribution of the six factors (T1, T2, T3, T4, T6, T7) to roof water inrush was positively correlated by using Equation (13) to standardize them. The contribution of aquifuge thickness to roof water inrush was negatively correlated, and Equation (14) was used for standardization. The dimensionless thematic map (Figure 7) for each evaluation criterion was obtained in ArgcGIS based on the exploration borehole data of the Liangshuijing Mine.

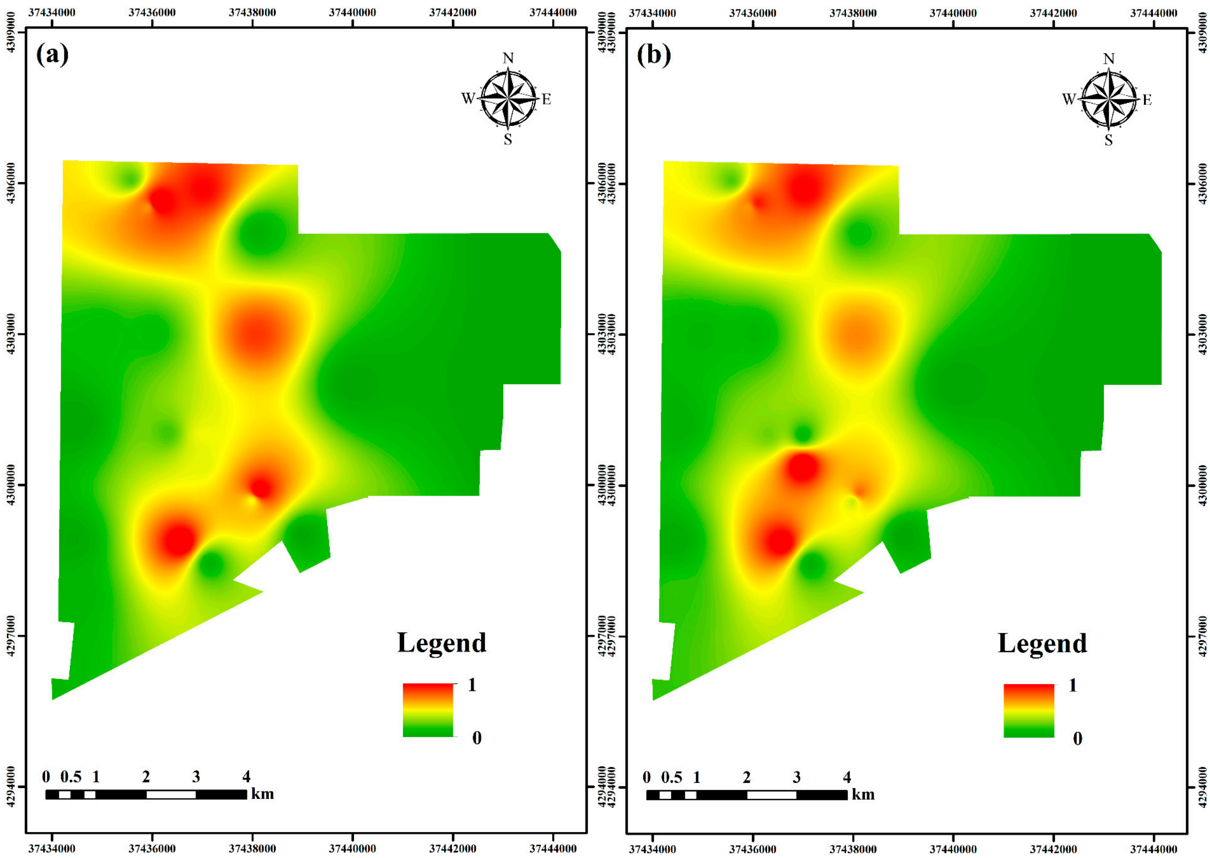
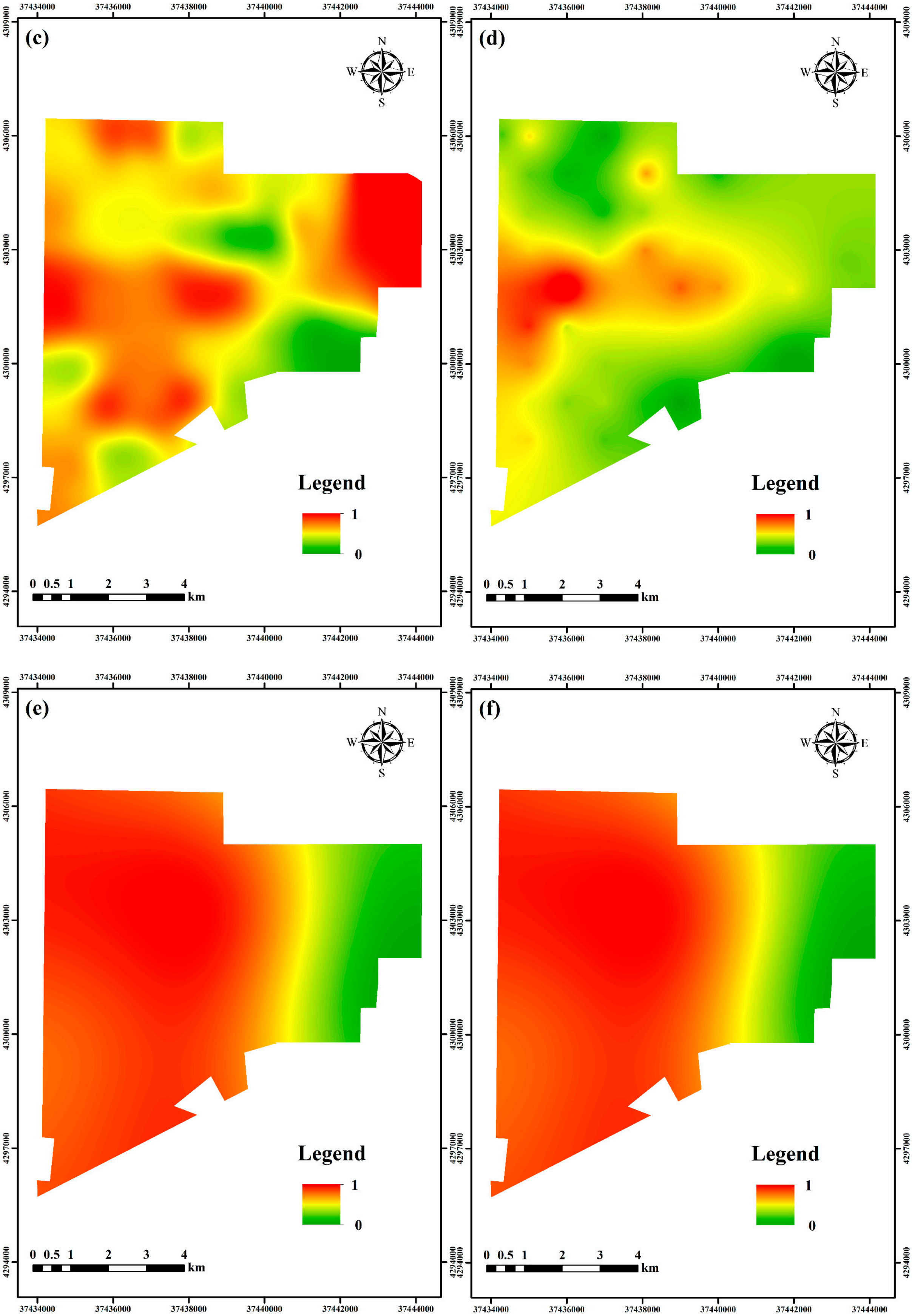


Figure 7. *Cont.*



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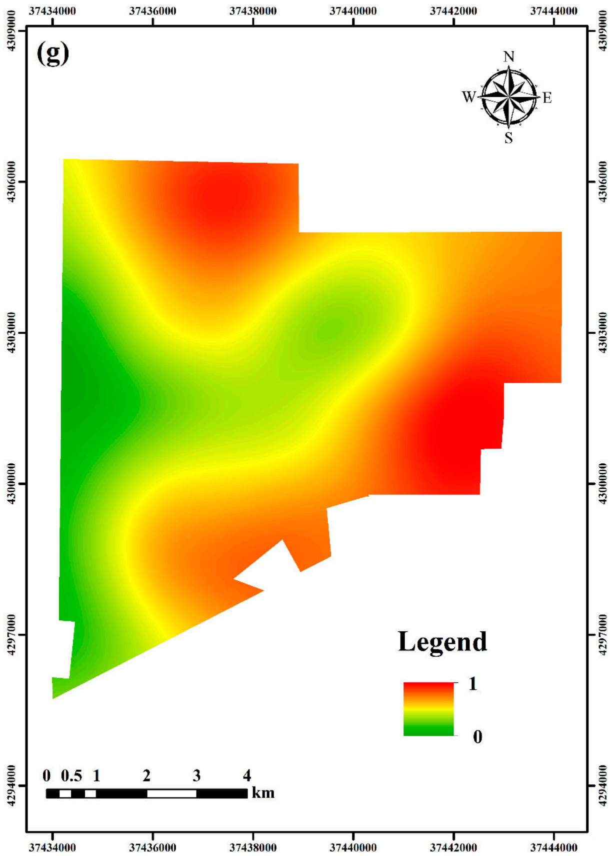

Figure 7. The dimensionless thematic map for the evaluation criteria: (**a**) T1; (**b**) T2; (**c**) T3; (**d**) T4; (**e**) T5; (**f**) T6; (**g**) T7.

4.3. Evaluation Criteria Weight

4.3.1. Criteria Weight

Regarding the criteria weight, based on the probability distribution function of each element in the pairwise comparison judgment matrix, Monte Carlo simulation uses random number sampling to generate 10,000 judgment matrices. After a consistency check of the analytic hierarchy process, there were 259 groups of qualified weight samples. Figure 8 shows the probability density estimation of the criteria weight samples obtained by the MAHP method, and Figure 9 is the weight distribution function of each evaluation criterion.

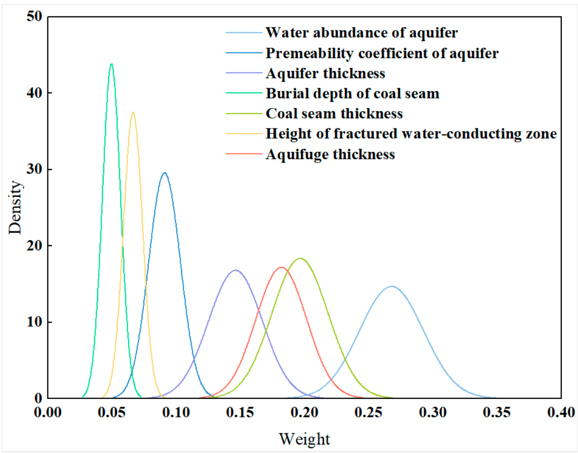

Figure 8. Criteria weight distribution function.

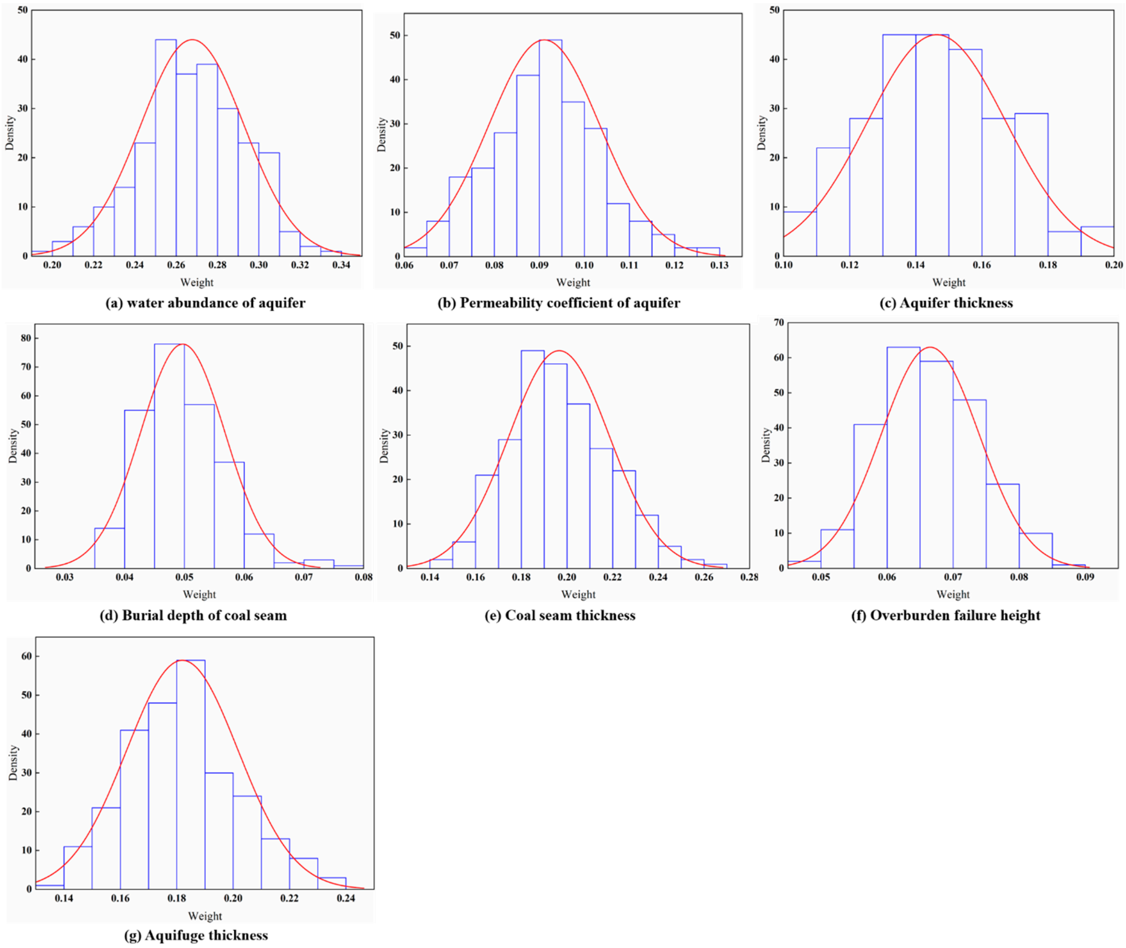

Figure 9. Evaluation criteria weight distribution function.

In Table 1, the criteria weight data are shown, where the value in parentheses is the confidence interval for the mean of the criteria weight with a confidence level of 95%. Figures 1 and 9 are the weight distribution function of the evaluation criteria. From Table 1 and Figure 4, most of the criteria weights calculated by MAHP were distributed within a narrow confidence interval range around the mean value. The narrower the confidence interval width, the more concentrated the distribution of the criteria weight and the lower the uncertainty. This method greatly reduces the uncertainty in the weight calculation of the water inrush evaluation index.

In the traditional analytic hierarchy process, the relative importance of the valuation criteria is not clear due to different experts. Different experts obtain different criteria weight ranking results that have strong randomness and uncertainty. This paper used MAHP, which introduces the probability density function of evaluation criteria, solves the randomness and uncertainty in the process of determining the evaluation index weight, and gives a clear ranking result of the relative importance of the water inrush evaluation criteria from the most to the least as follows: water abundance of the aquifer, coal seam thickness, aquifuge thickness, aquifer thickness, aquifer permeability, overburden failure height, and mining depth.

It is worth noting that in the criteria weight results obtained by the Monte Carlo analytic hierarchy process, the weight value of the coal seam thickness was large while the weight values of aquifer permeability and overburden failure height were small. This is not in line with previous perceptions. Traditionally, it is believed that the water abundance of the aquifer, aquifer thickness, aquifer permeability, and overburden failure height significantly influence the roof water inrush, therefore, their weight values of the evaluation criteria should be large. However, this was not the case for the evaluation criteria weight calculations obtained in the Monte Carlo analytic hierarchy process. This is because two evaluation criteria had a strong correlation: coal seam thickness and the overburden failure height, and the water abundance of the aquifer and aquifer permeability all had a strong correlation. The coal seam thickness determines the overburden failure height. The size of the aquifer permeability determines the water abundance of the aquifer. To ensure the rationality of the criteria weight distribution, when the weight value of one of them was relatively large, the weight value of the other with correlation was small, which demonstrates the rationality of determining the criteria weights using MAHP.

Table 1. The criteria weight statistics.

Criteria	Mean	Min	Max	Standardization	Confidence Interval
Water abundance of the aquifer	0.2678	0.1974	0.3301	0.02498	(0.2647–0.2708)
Aquifer permeability	0.0911	0.0633	0.1288	0.01233	(0.0895–0.0925)
Aquifer thickness	0.1463	0.1044	0.1985	0.0209	(0.1437–0.1488)
Mining depth	0.0497	0.0370	0.0761	0.0070	(0.0488–0.0505)
Coal seam thickness	0.1965	0.1401	0.2613	0.0220	(0.1938–0.1992)
Overburden failure height	0.0665	0.0484	0.0875	0.0074	(0.0656–0.0674)
Aquifuge thickness	0.1819	0.1395	0.2398	0.01979	(0.1795–0.1843)

Furthermore, in the traditional AHP, it is difficult for experts to give the relative importance of the three factors: water abundance of the aquifer, aquifer thickness, and aquifuge thickness. Different experts obtain different results. However, in the Monte Carlo analytic hierarchy process, the relative importance of the three can be obtained based on the interpretation of probability statistics.

4.3.2. Order Weight

Regarding the order weight, this paper assumed that decision makers have five risk attitudes in response to roof water inrush: pessimistic, moderately pessimistic, neutral, moderately optimistic, and optimistic. The corresponding a values were 0.1, 0.3, 0.5, 0.7, and 0.9. After calculation by Equations (15)–(18), the order weights of each criterion are shown in Table 2 [72].

Table 2. The order weights of the evaluation index under different a values.

a	0.1	0.3	0.5	0.7	0.9
Order weight v_1	0.002	0.0438	0.1428	0.3096	0.6226
Order weight v_2	0.0052	0.0607	0.1428	0.2236	0.2367
Order weight v_3	0.0136	0.0841	0.1428	0.1614	0.09
Order weight v_4	0.0353	0.1165	0.1429	0.1166	0.0342
Order weight v_5	0.0918	0.1614	0.1429	0.0842	0.0130
Order weight v_6	0.2391	0.2236	0.1429	0.0608	0.0049
Order weight v_7	0.6224	0.3097	0.1429	0.0439	0.0019

4.4. Influence of Risk Attitudes on Roof Water Inrush Risk Evaluation Results

By using the dimensionless thematic map, evaluation criteria weights, and order weights obtained in Section 4.1, Section 4.2 and Section 4.3, the zoning map of the water inrush risk evaluation under different risk-coping attitudes of decision makers (a) was obtained using ArcGIS. As shown in Figure 10, the evaluation result map was classified into five water inrush risk levels (very high, high, medium, low, and very low) using the natural breaks classification method.

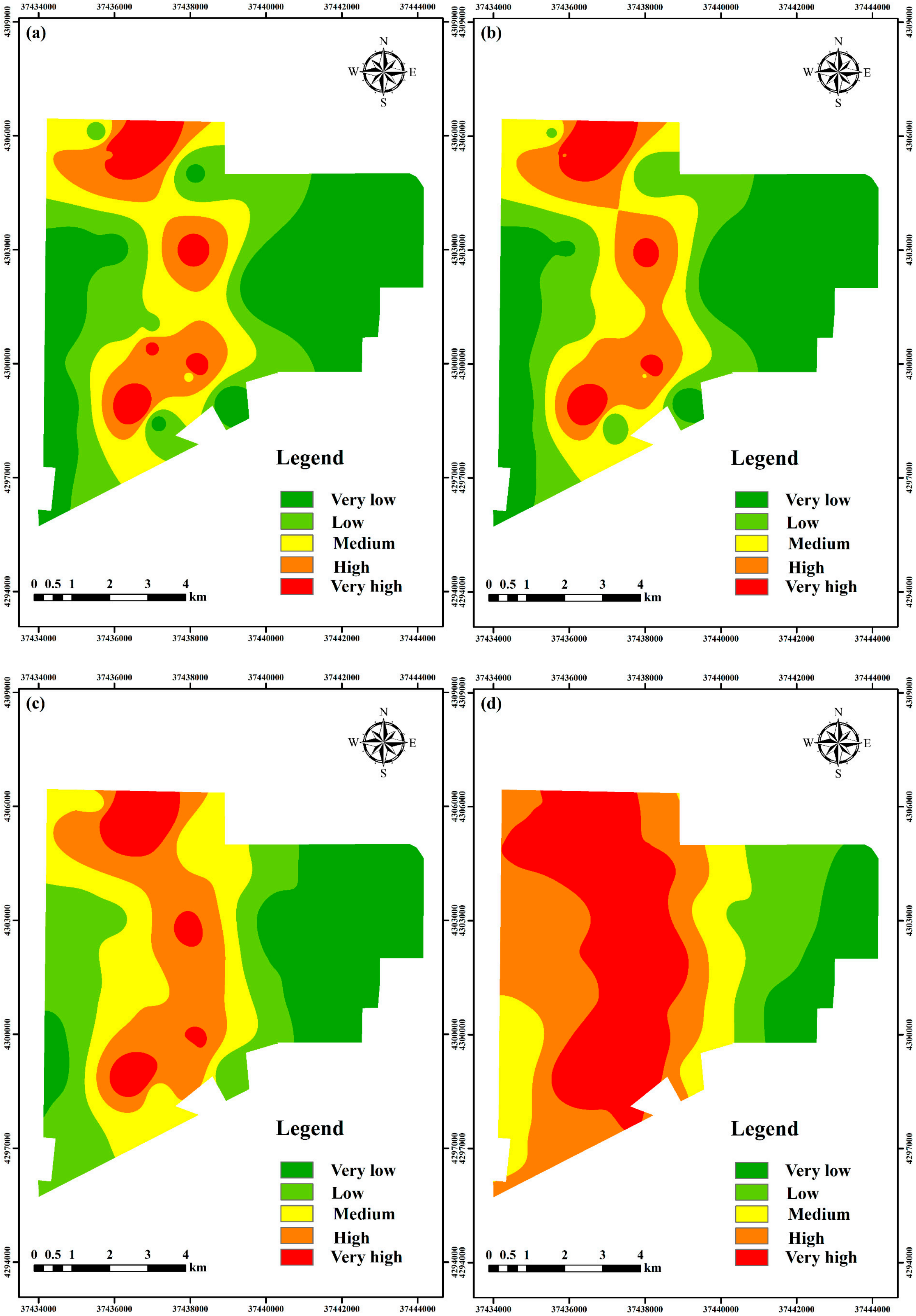

Figure 10. *Cont.*

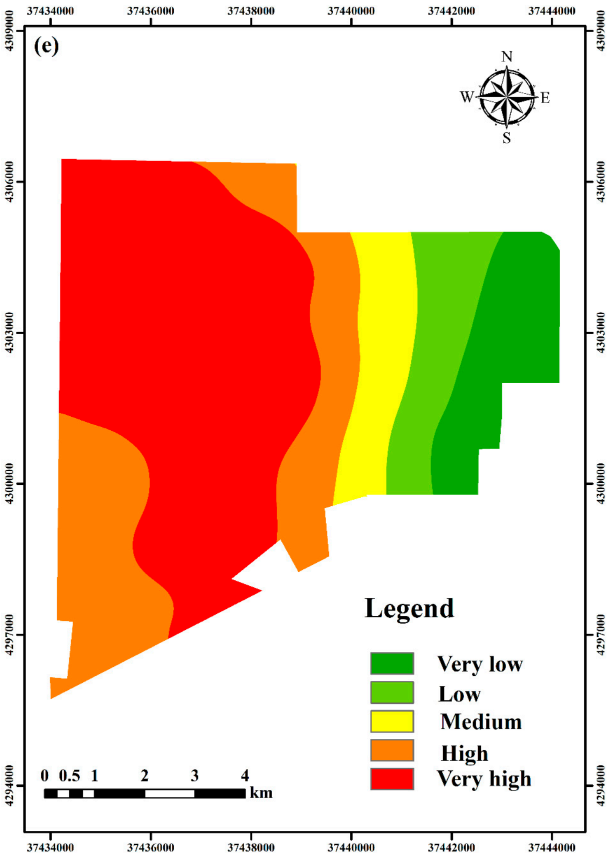

Figure 10. The evaluation result map of the roof water inrush risk under different risk attitudes: (**a**) a = 0.1; (**b**) a = 0.5; (**c**) a = 0.7; (**d**) a = 0.9; (**e**) a = 0.9.

To further explore the impact of different risk-coping attitudes of decision makers on the risk evaluation results of roof water inrush, the number of pixels of very high and high-risk areas for roof water inrush under different risk-coping attitudes were counted (Table 3). Combining Figure 10 and Table 3, as the value of a gradually increased, that is, the risk-coping attitude of decision makers gradually changed from pessimistic to neutral, and finally to optimistic, and the area of very-high and high-risk areas for water inrush significantly increased. This indicates that the roof water inrush evaluation results strongly depend on the risk-coping attitude of the decision makers. Even the slightest change in the risk-coping attitude can have a significant impact on the final risk evaluation results.

Table 3. The number of pixels of the roof water inrush risk areas in Figure 10.

Risk Attitude	a	Number of Pixels
Pessimistic	0.1	13,774,955
Moderately pessimistic	0.3	15,245,159
Neutral	0.5	19,685,325
Moderately optimistic	0.7	43,416,472
Optimistic	0.9	50,369,478

4.5. Verification and Comparison of Evaluation Results under Different Risk Attitudes

In order to obtain the accuracy of the risk evaluation results of roof water inrush under different risk-coping attitudes of decision makers, the 25 water inrush points known in the mining area were compared with the roof water inrush risk zoning map obtained under different risk-coping attitudes (Figure 11). Figure 12 is the prediction accuracy of the roof water inrush risk zoning evaluation results under different risk attitudes. Among them, when the risk attitude was 0.1, 0.3, 0.5, 0.7, and 0.9, the prediction accuracy (including very high and high-risk) was 48%, 60%, 68%, 96%, and 100%, respectively. This shows that as decision makers become more optimistic about risk-coping, the roof water inrush prediction accuracy will be higher and higher. This is because the size of the water inrush danger area in a mining area is larger, so there are more geographical units that can reach high and very high-risk levels, and therefore a higher prediction accuracy. However, this accuracy improvement is at the cost of increasing the overall area of very high and high-risk areas for water inrush. The increase in this area is not necessarily consistent with the actual situation, but also reduces the pertinence and effectiveness of roof water treatment work. Therefore, water inrush risk evaluation under different risk-coping attitudes can provide decision making references in various situations for the prevention and control of roof water inrush, but a blindly optimistic attitude should not be taken to deliberately improve the prediction accuracy.

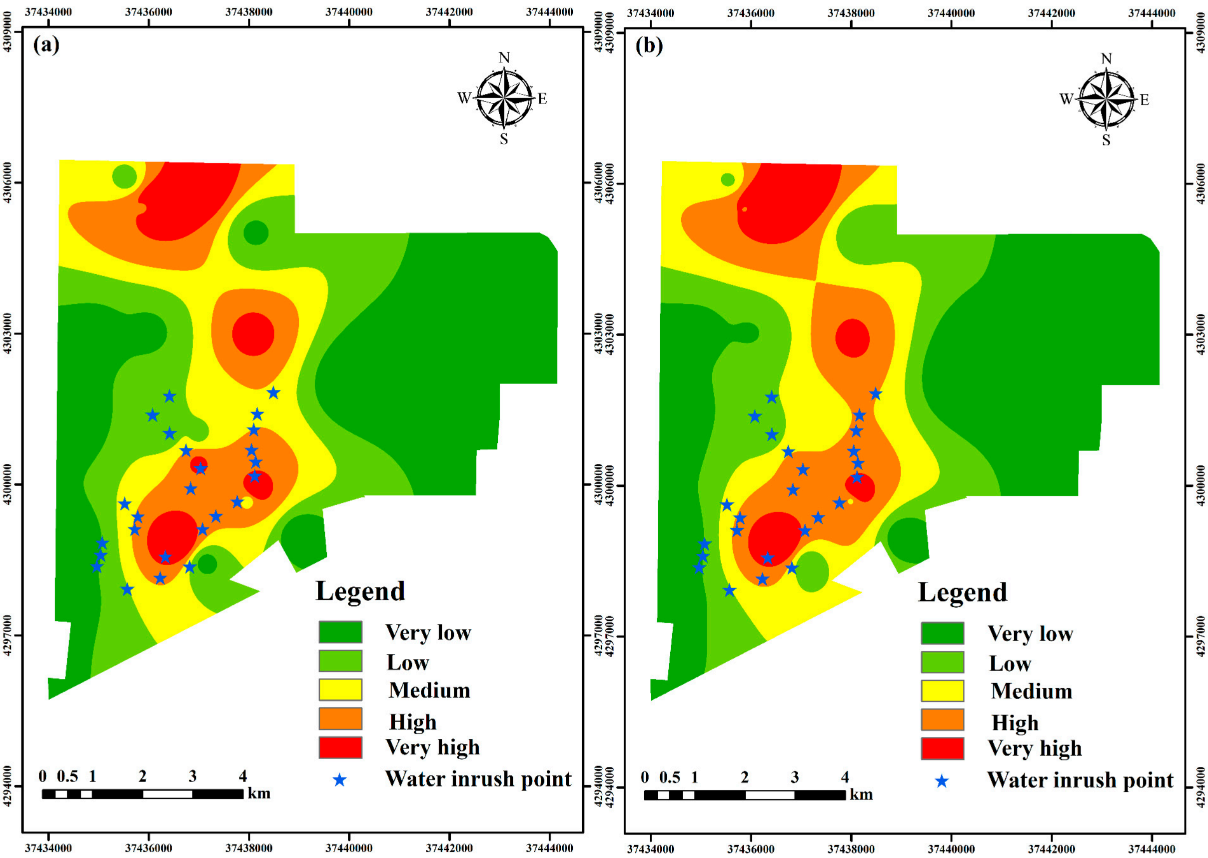

Figure 11. *Cont.*

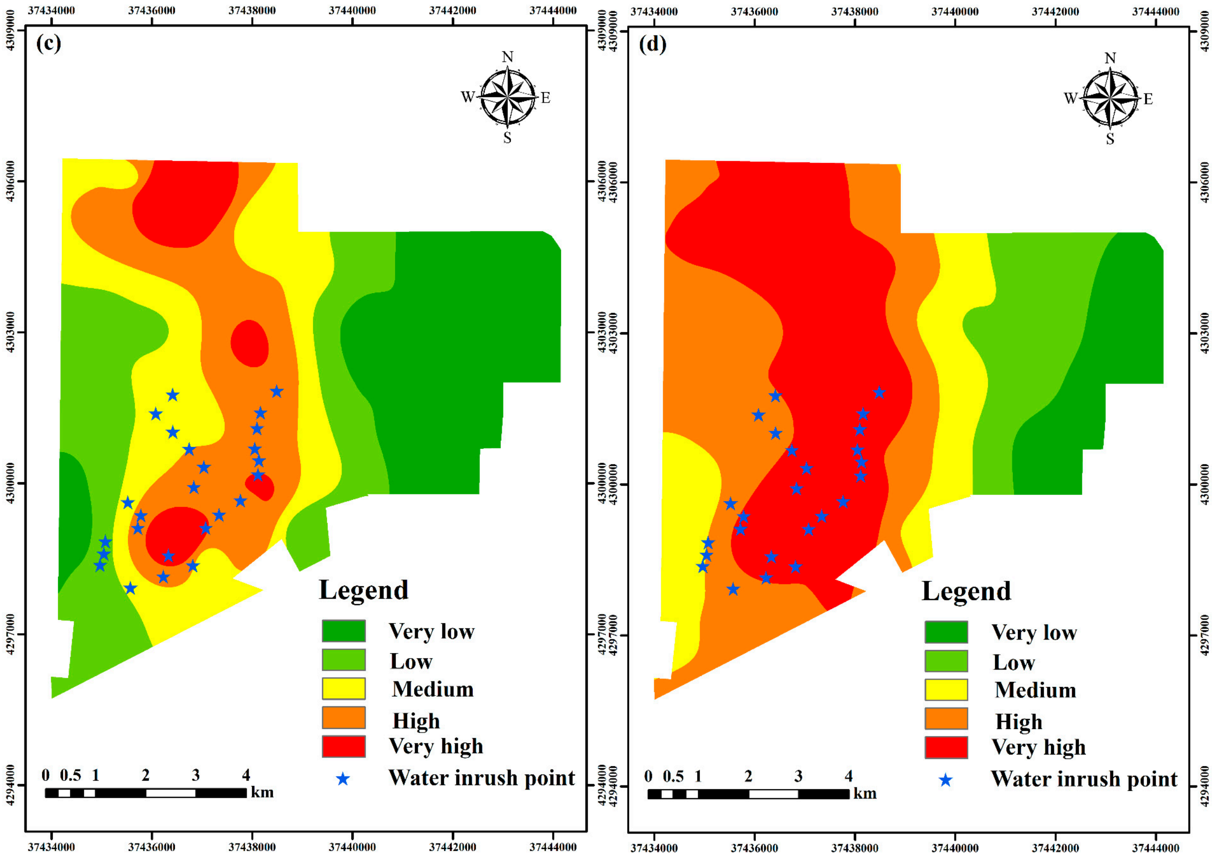

Figure 11. Verification diagram of the roof water inrush risk evaluation results under different risk-coping attitudes: (**a**) a = 0.1; (**b**) a = 0.5; (**c**) a = 0.7; (**d**) a = 0.9.

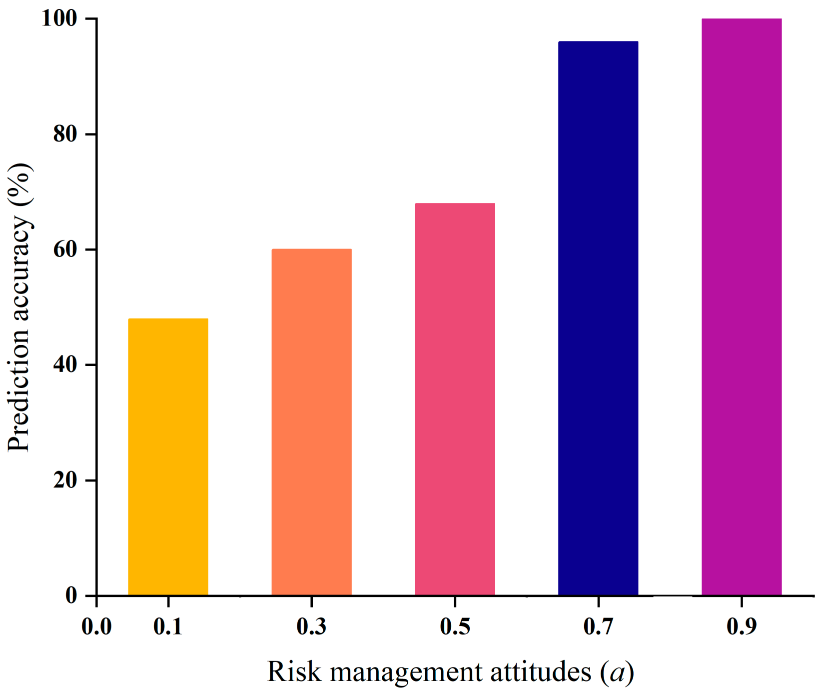

Figure 12. Prediction accuracy of different risk-coping attitudes.

5. Conclusions

(1) In this paper, the Monte Carlo analytical hierarchy process (MAHP) was used to calculate the evaluation criteria weight, which eliminates randomness and uncertainty in the process of determining the evaluation indicators in the traditional analytical hierarchy process. This gives the relative importance of the evaluation criteria in descending order: water abundance of the aquifer, coal seam thickness, aquifuge thickness, aquifer thickness, aquifer permeability, overburden failure height, and mining depth.

(2) In this paper, the risk-coping attitude of decision makers was considered during the risk evaluation of roof water inrush. The OWA operator quantifies the impact of the risk attitude of decision makers on the water inrush risk evaluation. This paper assumed that the risk-coping attitude of decision makers to deal with roof water inrush has five situations: pessimistic, moderately pessimistic, neutral, moderately optimistic, and optimistic. The corresponding a values were 0.1, 0.3, 0.5, 0.7, and 0.9, respectively.

(3) As decision makers become more optimistic about their risk-coping attitudes, the area of high-risk areas for roof water inrush within mining becomes significantly larger. The roof water inrush risk assessment results strongly depend on the risk-coping attitude of decision makers. A slight change in the decision makers' risk-coping attitude can have a significant impact on the final risk assessment results..

(4) Using the method proposed in this paper, the roof water inrush risk assessment results can be made more objective and accurate, thereby reducing or eliminating the risks associated with subjective decision making.

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