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Scrutinization of Waste Discharge Concentrations in Eyring-Powell Nanofluid Past a Deformable Horizontal Plane Surface

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Abstract: Nanomaterials have been the focus of intense study and growth in the modern era across the globe because of their outstanding qualities, which are brought about by their nanoscale size; for instance, increased adsorption and catalysis capabilities plus significant reactivity. Multiple investigations have verified the fact that nanoparticles may successfully remove a variety of pollutants from water, and, as a result, they have been utilized in the treatment of both water and wastewater. Therefore, the current research intent is to examine the nonlinear heat source/sink influence on the 3D flow of water-based silver nanoparticles incorporated in an Eyring–Powell fluid across a deformable sheet with concentration pollutants. Silver particles have been used intensively to filter water, due to their potent antibacterial properties. The leading equations involving partial differential equations are renewed into the form of ordinary ordinary differential equations through utilizing the appropriate similarity technique. Then, these converted equations are solved by utilizing an efficient solver *bvp4c*. Visual displays and extensive exploration of the different impacts of the non-dimensional parameters on the concentration, temperature, and velocity profiles are provided. Also, the important engineering variables including skin friction, the rate of heat, and mass transfer are examined. The findings suggest that the mass transfer rate declines due to pollutant parameters. Also, the results suggest that the friction factor is uplifted by about 15% and that the heat transfer rate, as well as the mass transfer rate, declines by about 21%, due to the presence of the nanoparticle volume fraction. We believe that these results may improve the flow rate of nanofluid systems, improve heat transfer, and reduce pollutant dispersal.

Keywords: nanofluid; Eyring–Powell fluid; pollutant concentration; nonlinear heat source/sink; dual solutions



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1. Introduction

Environmental harms caused by water pollution also contribute to the dangerous consequences of pollution in the air on human health. Water contamination also has a negative effect on the social and economic advancement of the afflicted cultures and nations. In the 21st century, it is increasingly difficult to find clean, fresh water; dirty water is dangerous to human life, according to the latest UN study [1,2]. When undesired materials penetrate into water reservoirs or bodies and when water is polluted or contaminated, it cannot be

used for other purposes or for consumption by humans. With the objective of solving this rising issue, numerous chemical, mechanical, and physical techniques exist. Additionally, researchers are still exploring many new procedures to improve inexpensive processes of water filtration [3,4]. The recently created field of nanotechnology is currently offering an intriguing way of purifying water with affordable prices, significant operating efficacy in removing contaminants, and recycling ability [5]. Cintolesi et al. [6] focused on the interface of thermal and inertial forces in urban canyons and how these forces affect turbulent properties and pollution removal. Chinyoka and Makinde [7] investigated the kinetics of polymeric pollutant dispersal of Newtonian fluid flow through a rectangular channel. The numerical model offers assessments for identifying contamination levels and offers perspectives on pollution hazards arising from an inappropriate release of hydrocarbon products. Recently, Vinutha et al. [8] inspected the flow of H₂O-based ternary nanofluids from divergent/convergent channels by incorporating concentrated contaminants and a nonlinear heat source/sink.

Materials with structural elements between 1 and 100 nm in size are referred to as nanomaterials [9]. Nanomaterials are radically distinct from normal materials in terms of their magnetic, optical, electrical, and mechanical properties, because of their nanoscale dimension. Many different types of nanomaterials exhibit catalysis, high reactivity, and adsorption. Over the past few decades, nanomaterials have undergone significant research and study and have successfully been applied in a number of fields, such as medicine [10], catalysis [11], biology [12], and sensing [13]. Silver nanomaterials are employed in a variety of industries, including those of medicines, medical devices, food and beverage, cosmetics, and water purification, due to their antibacterial qualities. More specifically, the application of nanoparticles to the purification of water and wastewater has attracted a lot of interest. Due to being tiny, nanomaterials have significant surface vicinity and high-adsorption properties. Furthermore, nanoparticles are highly mobile in solutions [14]. The exclusion of organic pollutants [15], heavy metals [16], inorganic anions [17], and microorganisms [18] by dissimilar nanomaterial kinds has been reported. Lu et al. [19] presented a complete review of the significance of nanomaterial for purifying as well as treating waste water. Yaqoob et al. [20] discussed the nanomaterial role in the treatment of water. Recently, Zahmatkesh et al. [21] discussed the capability of nanomaterials to eliminate viruses, fungus, and organic matter from wastewater. Numerous studies have shown that nanoparticles hold considerable promise for use in water and wastewater treatment.

The study of flow by incorporating non-Newtonian liquids has received a lot of interest recently. These liquids are used often in the production of coated sheets, plastic polymers, food, drilling muds, optical fibers, etc. These fluids have very complex connections between the flow field and shear stress, which presents researchers with fascinating difficulties. Despite these obstacles, the field's experts are even contributing significantly to the study of non-Newtonian fluids [22–25]. The research into Powell–Eyring liquid across a constantly movable porous plane was scrutinized by Jalil et al. [26]. They observed that the examination of the Powell–Eyring fluid model has a benefit, as it is developed from the kinetic theory of gases rather than from empirical relations. Roşca and Pop [27] looked into flow features and heat transfer through a shrinkable surface. To identify a stable solution, they carried out a stability analysis and observed that the first solution is stable and acceptable whilst the second solution is unstable and physically not acceptable. Rahimi et al. [28] utilized the collocation approach to achieve the outcome of the fluid flow of Eyring–Powell liquid across a stretchy sheet. The velocity is seen to decrease when the fluid substance parameter is increased, and to increase when the Eyring–Powell fluid substance parameter is expanded. The magneto influence on the radiative flow of Eyring–Powell fluid from a stretchable defaming surface with slip effects and chemical reaction was inspected by Reddy et al. [29]. Larger radiative heat transfer values have been found to affect temperature more significantly. Larger radiative heat transport values have been found to affect temperature more significantly. Khan et al. [30] scrutinized the Joule heating influence on the MHD flow and heat transfer induced by Eyring–Powell fluid with irregular

viscosity. Recently, Hassan et al. [31] examined the characteristics of fluid flow past an irregular thicker sheet with Eyring–Powell fluid, with an entropy effect. It was discovered that both temperature and velocity are affected differently by the time-scale factor.

The phenomena of an irregular heat sink or source have applications in both medical and engineering fields, such as the metallic sheet cooling, the design of thrust bearings, the revitalization of unrefined oil, etc. Tawade et al. [32] discussed the motion of unsteady thin film and heat transfer from a stretchable sheet with magnetic and irregular heat sink/source effects. The performance of heat transport is impacted by irregular heat parameters. Das et al. [33] inspected the impact of heat source/sink on the fluid flow and heat transfer, incorporating a water-based nanofluid moving past a shrinkable sheet. Thumma et al. [34] inspected the magnetic impact on the convective motion of nanofluid with an unpredictable heat sink/source caused by a stretching sheet. The outcome is obtained by utilizing the well-known Keller box numerical technique. Acharya et al. [35] examined the impact of a non-uniform heat source on the magneto flow across a time-dependent stretchable cylinder. Ramadevi et al. [36] inspected the consequences of an inconsistent thermal sink/source on the flow of an electrically conductive fluid through a varied thickness sheet. Lately, Khan et al. [37] have examined the characteristics of a jet flow incorporated H_2O/Al_2O_3 nanofluid moving past a slippery stretchable surface in a porous medium with an erratic heat source/sink. It has been concluded that the temperature is enhanced due to the heat source and declined owing to the heat sink factor.

According to the literature, no study has looked at the effects of pollutant concentration and irregular heat source/sink on the movement of water-based silver nanoparticles induced by Eyring–Powell fluid across a stretching/shrinking sheet. The aforementioned consequences have been explored in the present research work. The distinguished numerical bvp4c solver is employed to construct and work out the leading equations for current flow scenarios. There is a visual impact on many flow characteristics with pertinent physical significance. Notably, the present research is the initial attempt to examine the flow of heat and mass transfer in a non-Newtonian Eyring–Powell fluid incorporating the discharge concentration. The modeling of the nanofluid flow led to the development of the following research questions:

- What effect does the heat sink/source have on the temperature disparity within the fluid layers near the boundary and the temperature distribution on the behavior of nanofluid?
- What are the effects of skin friction, Nusselt number, and Sherwood number on the behavior of Eyring–Powell fluid in the presence of silver nanoparticles?
- How do the silver nanoparticles affect the reduction in the pollutant dispersal?

2. Problem Formulation

Consider a 3D flow of water-based silver nanoparticles with heat and mass transport phenomenon in a non-Newtonian (Eyring–Powell) nanofluid past a deformable (stretching/shrinking) horizontal plane surface. The impacts of pollutant concentration and irregular heat source/sink are incorporated. Figure 1 shows the Eyring–Powell fluid flow structure when silver nanoparticles are included, where x_q , y_q and z_q are Cartesian coordinates with velocities u_q , v_q , and w_q , respectively. Here, the z_q –axis is normal to the deformable sheet, and the x_q – and y_q –axes are placed along the streamwise and spanwise directions, respectively. Assume that the streamwise and spanwise velocities at the surface of the horizontal plane are described by $u_{wq}(x_q) = Ax_q^n$ and $v_{wq}(x_q) = Bx_q^n$, where A , and B refer to the constant stretching rates in the x_q – and y_q –directions, respectively (see Weidman [38]). In addition, $w_q(x_q) = w_0$ communicates the mass transpiration velocity with $w_0 > 0$, and $w_0 < 0$ refer to the suction and injection, respectively, while the impermeable surface of the plane is symbolized by $w_0 = 0$. Additionally, T_{wq} and T_∞ are thought to stand for the far-field temperature and the constant surface temperature, respectively. Furthermore, C_{wq} and C_∞ illustrate, respectively, the consistent concentration at the wall

surface and the outside (far-field) flow condition. The Eyring–Powell constitutive model has the following form and is in accordance with Narsimhan [39] and Ara [40].

$$\tau_{ij} = \mu_{nf} \frac{\partial u_i}{\partial x_j} + \frac{1}{\beta_q} \sinh^{-1} \left(\frac{1}{\delta_q} \frac{\partial u_i}{\partial x_j} \right), \quad (1)$$

The first expression describes the viscosity effect, and the second expression describes the elastic component. Taking into account the Maclaurin series from the second term, this has the form as follows:

$$\sinh^{-1} \left(\frac{1}{\delta_q} \frac{\partial u_i}{\partial x_j} \right) \cong \frac{1}{\delta_q} \frac{\partial u_i}{\partial x_j} - \frac{1}{6} \left(\frac{1}{\delta_q} \frac{\partial u_i}{\partial x_j} \right)^3 + \frac{3}{4} \left(\frac{1}{\delta_q} \frac{\partial u_i}{\partial x_j} \right)^5 - \frac{5}{12} \left(\frac{1}{\delta_q} \frac{\partial u_i}{\partial x_j} \right)^7 + \dots, \quad (2)$$

or

$$\sinh^{-1} \left(\frac{1}{\delta_q} \frac{\partial u_i}{\partial x_j} \right) \cong \sum_{m=0}^{\infty} \frac{(-1)^m (2m-1)}{(2m+1)(2m)!} \left(\frac{1}{\delta_q} \frac{\partial u_i}{\partial x_j} \right)^{2m+1}, \quad \left| \frac{1}{\delta_q} \frac{\partial u_i}{\partial x_j} \right| \ll 1. \quad (3)$$

By ignoring extraneous components and only making use of the first term and second term from the approximations, one can derive

$$\sinh^{-1} \left(\frac{1}{\delta_q} \frac{\partial u_i}{\partial x_j} \right) \cong \frac{1}{\delta_q} \frac{\partial u_i}{\partial x_j} - \frac{1}{6} \left(\frac{1}{\delta_q} \frac{\partial u_i}{\partial x_j} \right)^3. \quad (4)$$

As a result, the fundamental model for Eyring–Powell in Equation (1) changes to the form:

$$\tau_{ij} = \mu_{nf} \frac{\partial u_i}{\partial x_j} + \frac{1}{\delta_q \beta_q} \frac{\partial u_i}{\partial x_j} - \frac{1}{6 \beta_q} \left(\frac{1}{\delta_q} \frac{\partial u_i}{\partial x_j} \right)^3, \quad (5)$$

where β_q and δ_q indicate the Eyring–Powell fluid or material parameters and μ_{nf} refers to the nanofluid viscosity. Considering these presuppositions, the leading governing equations of the nanofluid in terms of PDEs can be constructed as follows, with the assistance of the aforementioned premises and the boundary layer approximations [8,26–29]:

$$\frac{\partial u_q}{\partial x_q} + \frac{\partial v_q}{\partial y_q} + \frac{\partial w_q}{\partial z_q} = 0, \quad (6)$$

$$\rho_{nf} \left(u_q \frac{\partial u_q}{\partial x_q} + v_q \frac{\partial u_q}{\partial y_q} + w_q \frac{\partial u_q}{\partial z_q} \right) = \left(\mu_{nf} + \frac{1}{\beta_q \delta_q} \right) \frac{\partial^2 u_q}{\partial z_q^2} - \frac{1}{2 \beta_q \delta_q^3} \left(\frac{\partial u_q}{\partial z_q} \right)^2 \frac{\partial^2 u_q}{\partial z_q^2}, \quad (7)$$

$$\rho_{nf} \left(u_q \frac{\partial v_q}{\partial x_q} + v_q \frac{\partial v_q}{\partial y_q} + w_q \frac{\partial v_q}{\partial z_q} \right) = \left(\mu_{nf} + \frac{1}{\beta_q \delta_q} \right) \frac{\partial^2 v_q}{\partial z_q^2} - \frac{1}{2 \beta_q \delta_q^3} \left(\frac{\partial v_q}{\partial z_q} \right)^2 \frac{\partial^2 v_q}{\partial z_q^2}, \quad (8)$$

$$\begin{aligned} (\rho c_p)_{nf} \left(u_q \frac{\partial T_q}{\partial x_q} + v_q \frac{\partial T_q}{\partial y_q} + w_q \frac{\partial T_q}{\partial z_q} \right) &= k_{nf} \frac{\partial^2 T_q}{\partial z_q^2} \\ &+ \frac{k_{nf} u_{wq}(x_q)}{x_q v_{nf}} [A_b^* (T_{wq} - T_{\infty}) e^{-\xi} + B_b^* (T_q - T_{\infty})], \end{aligned} \quad (9)$$

$$u_q \frac{\partial C_q}{\partial x_q} + v_q \frac{\partial C_q}{\partial y_q} + w_q \frac{\partial C_q}{\partial z_q} = D_f \frac{\partial^2 C_q}{\partial z_q^2} + Q_q \exp(b^* (C_q - C_{\infty})), \quad (10)$$

The physical boundary conditions (BCs) listed below are

$$\begin{aligned} u_q &= \gamma_b u_{wq}(x_q), \quad v_q = \gamma_b v_{wq}(x_q), \quad w_q = w_0, \quad T_q = T_{wq}, \quad C_q = C_{wq} \quad \text{at } z_q = 0, \\ u_q &\rightarrow 0, \quad v_q \rightarrow 0, \quad T_q \rightarrow T_{\infty}, \quad C_q \rightarrow C_{\infty} \quad \text{as } z_q \rightarrow \infty. \end{aligned} \quad (11)$$

Here, β_q and δ_q indicate the Eyring–Powell fluid or material parameters, T_q the nanofluid temperature, C_q the nanofluid concentration, and D_f the concentration molecu-

lar diffusivity; Q_q and b^* correspond to the external pollutant strengths, A_b^* refers to the exponentially decaying space coefficient, and B_b^* signifies the time-dependent heat absorption/generation. In addition, the phenomenon of heat source and heat sink is created by the positive and negative consistent values of A_b^* and B_b^* . Also, γ_b states the deformable (stretching and shrinking) horizontal plane surface with $\gamma_b < 0$ for shrinking case and $\gamma_b > 0$ for stretching case, whereas, $\gamma_b = 0$ is the case of a static plane surface.

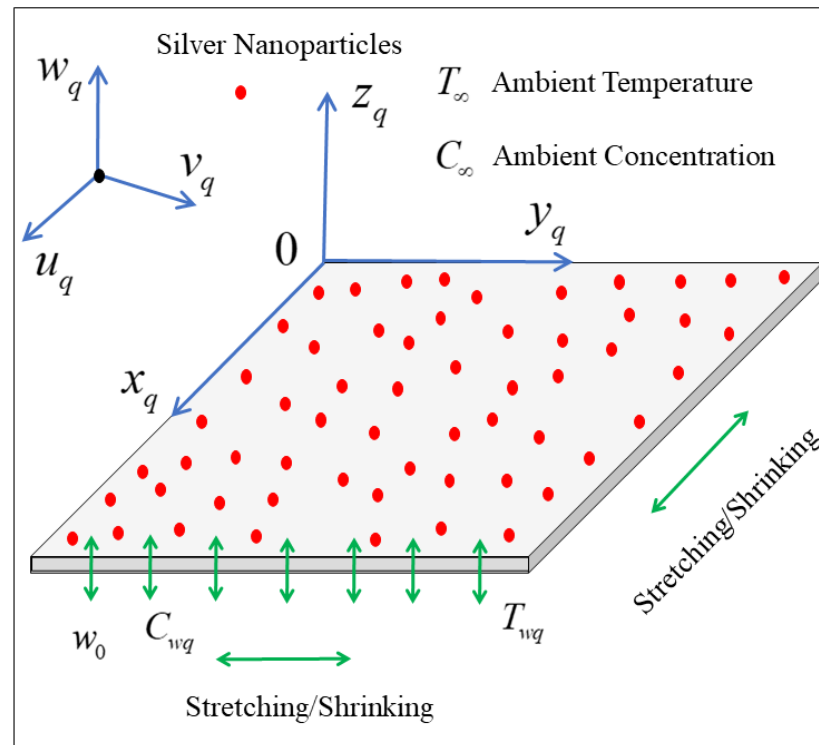


Figure 1. Physical model of the problem.

In addition, the rest of the mathematical symbols utilized in the aforesaid equations for the physical correlations of the silver nanofluid are ρ_{nf} the density, k_{nf} the thermal conductivity, and $(\rho c_p)_{nf}$ the specific heat capacity. The correlations are placed in Table 1.

Table 1. The correlations of water-based silver nanoparticles [25,37].

Properties	Correlations
Dynamic viscosity	$\mu_{nf} = \mu_f (1 - \varphi_{sc})^{-2.5}$
Density	$\rho_{nf} = \rho_f \left[\varphi_{sc} \left(\frac{\rho_{sc}}{\rho_f} \right) + (1 - \varphi_{sc}) \right]$
Heat capacity	$(\rho c_p)_{nf} = (\rho c_p)_f \left[\varphi_{sc} \left(\frac{(\rho c_p)_{sc}}{(\rho c_p)_f} \right) + (1 - \varphi_{sc}) \right]$
Thermal conductivity	$k_{nf} = k_f \left[\frac{(k_{sc} + 2k_f) - 2\varphi_{sc}(k_f - k_{sc})}{(k_{sc} + 2k_f) + \varphi_{sc}(k_f - k_{sc})} \right]$

The notation φ_{sc} suggests the solid-fraction volume of nanoparticles, whereas the particular case $\varphi_{sc} = 0$ shrinks the above correlations into a regular water-based fluid. Also, the subscripts nf , sc , and f indicate the corresponding nanofluid, nanoparticles, and regular fluid. However, c_p signifies the heat capacitance at constant or uniform pressure. Table 2 is constructed to show the data of the water liquid and the silver nanoparticles obtained from the experiment.

Table 2. The thermophysical properties of the base fluid and the silver nanoparticles [41].

Properties	ρ (kg/m ³)	c_p (J kgK)	k (W/mK)	Pr
H ₂ O	997.1	4179	0.613	6.2
Silver	10,500	235	429	-

The similarity variables are presented which obey the equation of the continuity Equation (6) as:

$$\begin{aligned}\xi &= \left(A/v_f\right)^{1/2} C(x_q) z_q, \quad u_q = Ax_q^n F'(\xi), \quad v_q = Bx_q^n G'(\xi), \\ w_q &= -\frac{\sqrt{Av_f}}{C} \left[nx_q^{n-1} F(\xi) + x_q^n \frac{C_{x_q}}{C} \{ \xi F'(\xi) - F(\xi) \} \right],\end{aligned}\quad (12)$$

where ξ is the pseudo-similarity variable and prime signifies the derivative with respect to ξ , n the power-law index. Also, $C = C(x_q)$ and $C_{x_q} = \frac{dC(x_q)}{dx_q}$. Using Equation (12), the momentum Equations (7) and (8) in the streamwise and spanwise directions are altered into the following form:

$$\begin{aligned}\frac{\rho_f}{\rho_{nf}} \left(\frac{\mu_{nf}}{\mu_f} + \frac{1}{\delta_q \beta_q \mu_f} \right) C^2 F''' - \frac{\rho_f}{\rho_{nf}} \frac{A^3 x_q^{2n} C^4}{2\beta_q \delta_q^3 \mu_f v_f} F''^2 F''' + \\ \left(nx_q^{n-1} - \frac{C_{x_q}}{C} x_q^n \right) FF'' - nx_q^{n-1} F'^2 = 0,\end{aligned}\quad (13)$$

$$\begin{aligned}\frac{\rho_f}{\rho_{nf}} \left(\frac{\mu_{nf}}{\mu_f} + \frac{1}{\beta_q \delta_q \mu_f} \right) C^2 G''' - \frac{\rho_f}{\rho_{nf}} \frac{AB^2 x_q^{2n} C^4}{2\beta_q \delta_q^3 \mu_f v_f} G''^2 G''' + \\ \left(nx_q^{n-1} - \frac{C_{x_q}}{C} x_q^n \right) FG'' - nx_q^{n-1} F'G' = 0.\end{aligned}\quad (14)$$

The following concept was chosen as follows, in order to achieve the similarity equations (see Weidman [38]):

$$\frac{C_{x_q}}{C} = \frac{K_b n}{x_q}.\quad (15)$$

Applying integration by parts, one now arrives at

$$C = D_b x_q^{K_b n}\quad (16)$$

where K_b and D_b are the requisite arbitrary constants. Additionally, $\delta_q = \delta_b x_q^{\frac{3n-1}{2}}$ and $\beta_q = \beta_b x_q^{-\left(\frac{3n-1}{2}\right)}$ are the Eyring–Powell fluid or material parameters with constants δ_b and β_b . Using Equations (15) and (16) and the above working variable expressions in the streamwise and spanwise momentum Equations (13) and (14) yield:

$$\begin{aligned}\frac{\rho_f}{\rho_{nf}} \left(\frac{\mu_{nf}}{\mu_f} + \frac{1}{\beta_b \delta_b \mu_f} \right) F''' - \frac{\rho_f}{\rho_{nf}} \frac{1}{2\beta_b \delta_b \mu_f} \frac{A^3 x_q^{2n} D_b^2 x_q^{2K_b n}}{\delta_b^2 x_q^{3n-1} v_f} F''^2 F''' + \\ \left(\frac{nx_q^{n-1} - K_b nx_q^{n-1}}{D_b^2 x_q^{2K_b n}} \right) FF'' - \frac{nx_q^{n-1}}{D_b^2 x_q^{2K_b n}} F'^2 = 0,\end{aligned}\quad (17)$$

$$\begin{aligned}\frac{\rho_f}{\rho_{nf}} \left(\frac{\mu_{nf}}{\mu_f} + \frac{1}{\beta_b \delta_b \mu_f} \right) G''' - \frac{\rho_f}{\rho_{nf}} \frac{1}{2\beta_b \delta_b \mu_f} \frac{AB^2 x_q^{2n} D_b^2 x_q^{2K_b n}}{\delta_b^2 x_q^{3n-1} v_f} G''^2 G''' + \\ \left(\frac{nx_q^{n-1} - K_b nx_q^{n-1}}{D_b^2 x_q^{2K_b n}} \right) FG'' - \frac{nx_q^{n-1}}{D_b^2 x_q^{2K_b n}} F'G' = 0.\end{aligned}\quad (18)$$

Hence, we set the coefficients of FF'' and FG'' as equal to unity, so that

$$D_b = \sqrt{\frac{n+1}{2}}, \quad \text{and} \quad K_b = \frac{n-1}{2n},\quad (19)$$

and Equations (17) and (18) are simplified into the following form:

$$\frac{1}{\rho_{nf}/\rho_f} \left(\frac{\mu_{nf}}{\mu_f} + \Sigma_b \right) F''' - \left(\frac{n+1}{4} \right) \frac{\Sigma_b \Sigma_c}{\rho_{nf}/\rho_f} F''^2 F''' + FF'' - \frac{2n}{n+1} F'^2 = 0, \quad (20)$$

$$\frac{1}{\rho_{nf}/\rho_f} \left(\frac{\mu_{nf}}{\mu_f} + \Sigma_b \right) G''' - \left(\frac{n+1}{4} \right) \frac{\Sigma_b \Sigma_c}{\rho_{nf}/\rho_f} E_b^2 G''^2 G''' + FG'' - \frac{2n}{n+1} F'G' = 0. \quad (21)$$

Here, the dimensionless influential parameters are $\Sigma_b = \frac{1}{\beta_b \delta_b \mu_f}$, $\Sigma_c = \frac{A^3}{\delta_b^2 v_f}$ the Eyring–Powell fluid parameters, and $E_b = \frac{B}{A}$ the stretching-rate ratio parameter.

Additionally, employing Equation (12), along with $S(\xi) = \frac{T_q - T_\infty}{T_{wq} - T_\infty}$, and $H(\xi) = \frac{C_q - C_\infty}{C_{wq} - C_\infty}$, the energy Equation (9) and concentration Equation (10) are transmuted into the following posited form:

$$\frac{1}{\text{Pr}} \frac{k_{nf}/k_f}{(\rho c_p)_{nf}/(\rho c_p)_f} C^2 S'' + \left(n x_q^{n-1} - \frac{C_{xq}}{C} x_q^n \right) FS' + \frac{1}{\text{Pr}} \frac{(k_{nf}/k_f)(\rho_{nf}/\rho_f)x_q^{n-1}}{((\rho c_p)_{nf}/(\rho c_p)_f)(\mu_{nf}/\mu_f)} [A_b^* e^{-\xi} + B_b^* S], \quad (22)$$

$$\frac{1}{Sc_b} C^2 H'' + \left(n x_q^{n-1} - \frac{C_{xq}}{C} x_q^n \right) FH' + \frac{Q_q}{A(C_{wq} - C_\infty)} \exp(b^*(C_{wq} - C_\infty)H). \quad (23)$$

Using the strength of the variable external pollutant and Equations (15), (16) and (19), $Q_q = Q_b x_q^{n-1}$, the same procedure was used to update Equations (22) and (23), resulting in the following form

$$\frac{k_{nf}}{k_f} S'' + \text{Pr} \frac{(\rho c_p)_{nf}}{(\rho c_p)_f} FS' + \left(\frac{2}{n+1} \right) \frac{\left(\frac{k_{nf}}{k_f} \right) \left(\frac{\rho_{nf}}{\rho_f} \right)}{\left(\frac{\mu_{nf}}{\mu_f} \right)} [A_b^* e^{-\xi} + B_b^* S], \quad (24)$$

$$\frac{1}{Sc_b} H'' + FH' + \left(\frac{2}{n+1} \right) \delta_b \exp(\delta_a H). \quad (25)$$

Additionally, the mass transpiration parameter for the surface of a horizontal plane is provided by

$$w_0 = -\frac{\sqrt{A v_f}}{C} \left(\frac{n+1}{2} \right) x_q^{n-1} f_{wb} \quad (26)$$

and these are the new boundary conditions:

$$F(0) = f_{wb}, G(0) = 0, F'(0) = G'(0) = \gamma_b, S(0) = 1, H(0) = 1, \\ F'(\xi) \rightarrow 0, G'(\xi) \rightarrow 0, S(\xi) \rightarrow 0, H(\xi) \rightarrow 0 \text{ as } \xi \rightarrow \infty, \quad (27)$$

where $\text{Pr} = v_f/\alpha_f$ represents the Prandtl number and f_{wb} the constant mass flux parameter, with $f_{wb} > 0$ and $f_{wb} < 0$ indicating the suction and injection, respectively, while $f_{wb} = 0$, subject to the impermeable surface of the plane, $Sc_b = v_f/D_f$ is the Schmidt number, $\delta_a = b^*(C_{wq} - C_\infty)$ corresponds to the local pollutant external source parameter, and $\delta_b = \frac{Q_b}{A(C_{wq} - C_\infty)}$ denotes the local external pollution source variation parameter.

In this investigation, it should be noted that if $\Sigma_b = \Sigma_c = 0$ (Newtonian fluid), $f_{wb} = 0$ (impermeable surface of the plane), $\gamma_b = 1$ (stretched surface of the plane), and $\varphi_{sc} = 0$ (viscous fluid) are investigated, the above model reduces to Weidman [38] (see the third extension of the Bank's problem). Weidman [38] did not incorporate the heat transfer features because of the absence of the energy equation. Furthermore, Equations (20) and

(27) can be simplified to align with work by Miklavčič and Wang [42] for linear shrinkable and Fang [43] for nonlinear shrinkable when viscous liquid, Newtonian fluid, and a porous flat plate are taken into account. Similar to previous papers, none of them examined the necessary energy equation or the concentration equation.

The local Nusselt number Nu_{x_q} , the Sherwood number Sh_{x_q} , and the coefficients of the skin friction in the streamwise (C_f) and spanwise (C_g) directions are the gradients of engineering physical significance.

$$\begin{aligned} C_f &= \frac{1}{\rho_f u_{wq}^2} \left[\left(\mu_{nf} + \frac{1}{\beta_q \delta_q} \right) \frac{\partial u_q}{\partial z_q} - \frac{1}{6\beta_q \delta_q^3} \left(\frac{\partial u_q}{\partial z_q} \right)^3 \right] \Big|_{z_q=0}, \\ C_g &= \frac{1}{\rho_f v_{wq}^2} \left[\left(\mu_{nf} + \frac{1}{\beta_q \delta_q} \right) \frac{\partial v_q}{\partial z_q} - \frac{1}{6\beta_q \delta_q^3} \left(\frac{\partial v_q}{\partial z_q} \right)^3 \right] \Big|_{z_q=0}, \\ Nu_{x_q} &= \frac{x_q}{k_f (T_{wq} - T_\infty)} \left(-k_{nf} \frac{\partial T_q}{\partial z_q} \right) \Big|_{z_q=0}, \\ \text{and} \\ Sh_{x_q} &= \frac{x_q}{D_f (C_{wq} - C_\infty)} \left(-D_f \frac{\partial C_q}{\partial z_q} \right) \Big|_{z_q=0}. \end{aligned} \quad (28)$$

Using Equations (12), (15), (16) and (19) with $S(\xi) = \frac{T_q - T_\infty}{T_{wq} - T_\infty}$, and $H(\xi) = \frac{C_q - C_\infty}{C_{wq} - C_\infty}$ in the aforementioned Equation (28), we obtain the following gradient in dimensionless form

$$\begin{aligned} \text{Re}_{x_q x_q}^{1/2} C_f &= \sqrt{\frac{n+1}{2}} \left[\left(\frac{\mu_{nf}}{\mu_f} + \Sigma_b \right) F''(0) - \left(\frac{n+1}{12} \right) \Sigma_b \Sigma_c F'''(0)^3 \right], \\ \text{Re}_{y_q x_q}^{1/2} C_g &= \sqrt{\frac{n+1}{2}} \frac{1}{\sqrt{E_b}} \left[\left(\frac{\mu_{nf}}{\mu_f} + \Sigma_b \right) G''(0) - \left(\frac{n+1}{12} \right) E_b^2 \Sigma_b \Sigma_c G'''(0)^3 \right], \\ \text{Re}_{x_q x_q}^{-1/2} Nu_{x_q} &= -\sqrt{\frac{n+1}{2}} \frac{k_{nf}}{k_f} S'(0), \\ \text{and} \\ \text{Re}_{x_q x_q}^{-1/2} Sh_{x_q} &= -\sqrt{\frac{n+1}{2}} H'(0). \end{aligned} \quad (29)$$

Hence, $\text{Re}_{x_q x_q} = \frac{u_{wq}(x_q)x_q}{\nu_f}$, and $\text{Re}_{y_q x_q} = \frac{v_{wq}(x_q)x_q}{\nu_f}$ are the local Reynolds number.

3. Graphical Results and Discussion

The symmetrical heat source/sink and suction impact in a three-dimensional Eyring–Powell nanofluid induced by a deformable (expanding/contracting) horizontal plane surface with waste discharge concentrations was considered. The set of similarity equations involved various influential dimensionless parameters, namely, the nanoparticle volume fractions ϕ_{sc} , the material or Eyring–Powell fluid parameters Σ_b and Σ_c , the Prandtl number Pr , the Schmidt number Sc_b , the mass suction/injection parameter f_{wb} , the deformable (expanding/contracting) parameter γ_b , the local external pollutant source parameter δ_a , and the local external pollutant source variation parameter δ_b . The impression of these parameters on shear stress in the streamwise (SMW) and spanwise (SNW) directions, heat transfer rate (HTR), and mass transfer rate (MTR) are bounded via several graphs. In addition, this portion of the work exemplifies the explanation of the acquired graphs in terms of physical viewpoints as well as the validation of the considered approach for a limiting situation. For dual results, the prominent bvp4c was employed to work out the retrieved set of dimensionless similarity equations. The three-stage Lobatto formula is specifically executed by this built-in code, which is part of the MATLAB package and is based on a finite difference procedure (refer to Shampine et al. [44]).

The authentication/validation and accuracy of the considered study scheme are quantitatively shown in Table 3, for a limiting example. This computational table shows a comparison of the up-to-date HTR values with prior research works of Devi and Devi [45],

Wang [46], and Khan and Pop [47], owing to the sundry values of Pr when $f_{wb} = 0$, $\Sigma_b = 0.0$, $\Sigma_c = 0.0$, $\varphi_{sc} = 0$, $Sc_b = 0.0$, $\delta_b = 0.0$, $\delta_a = 0.0$, $A_b^* = 0.0$, $B_b^* = 0.0$, $n = 1.0$ and $\gamma_b = 1.0$. From these tabular values, it is proved that the existing work is accurately aligned with prior research works. Further, this exceptional matching gives us excellent confidence that the located solutions for both (stable and unstable) branches are perfectly acceptable. Figures 2–7 are exclusively constructed to sightsee the influence of several controlling parameters on gradients for the stable branch solution (SBS) and unstable branch solution (UBS). In general, the SBS and UBS exist for the cases of suction and shrinking horizontal plane surface. The current problem raises dual (stable branch (SB) and unstable branch (UB)) solutions for the variance in the different distinct parameters, where the SB is indicated by the solid black lines and the UB solution is indicated by the dashed red lines. However, the point where the SB and UB merge is essentially referred to as a bifurcation point (BP).

Table 3. Comparison values of heat transfer rate for regular base fluid ($\varphi_{sc} = 0$) with several values of Pr when $f_{wb} = 0$, $\Sigma_b = 0.0$, $\Sigma_c = 0.0$, $Sc_b = 0.0$, $\delta_b = 0.0$, $\delta_a = 0.0$, $A_b^* = 0.0$, $B_b^* = 0.0$, $n = 1.0$ and $\gamma_b = 1.0$.

Pr	Devi and Devi [45]	Wang [46]	Khan and Pop [47]	Present
2.0	0.91135	0.9114	0.9113	0.9113543
6.13	1.75968	-	-	1.7596836
7.0	1.89540	1.8954	1.8954	1.8954001
20.0	3.35390	3.3539	3.3539	3.3539025

Figure 2a–d display the consequence of the power-law index n on shear stress coefficients (SSCs) in both the SMW and SNW directions, and HTR and MTR versus f_{wb} of the (water/Ag) nanofluid for the SBS and UBS, respectively. From Figure 2a,b, it is acknowledged that the shear stress in both the SMW and SNW directions increases for the SBS owing to the superior impacts of n , but a contradictory pattern is perceived near the points of bifurcation. However, the SSCs in both the SMW and SNW directions near the BP decline and then incline, with superior impacts of n for the UBS. In addition, the SSC in the SNW direction is higher for the UBS as compared to the outcome obtained in the direction of SMW because of the mounting values of the power-law index parameter. On the other hand, the HTR and MTR profiles were boosted up in both SBS and UBS, owing to the amplified values of n , see Figure 2c,d. Additionally, the SBS and UBS merged at a single position which is called the bifurcation or critical point (BP or CP). This BP is represented in all the graphs via a solid blue ball. The BP is not unique in these diagrams, because it is changed with the changing values of n . Meanwhile, the BP is mathematically represented by $f_{wb}C$; where the outcome is unique, then it is expressed as $f_{wb} = f_{wb}C$. Also, the outcomes are not unique for the situation when $f_{wb} > f_{wb}C$, and no single and multiple outcome possibility arises with $f_{wb} < f_{wb}C$. The magnitude of the bifurcation values (BVs) escalates due to the superior impressions of n , and hence the flow separation of the boundary layer declines from the horizontal plane surface.

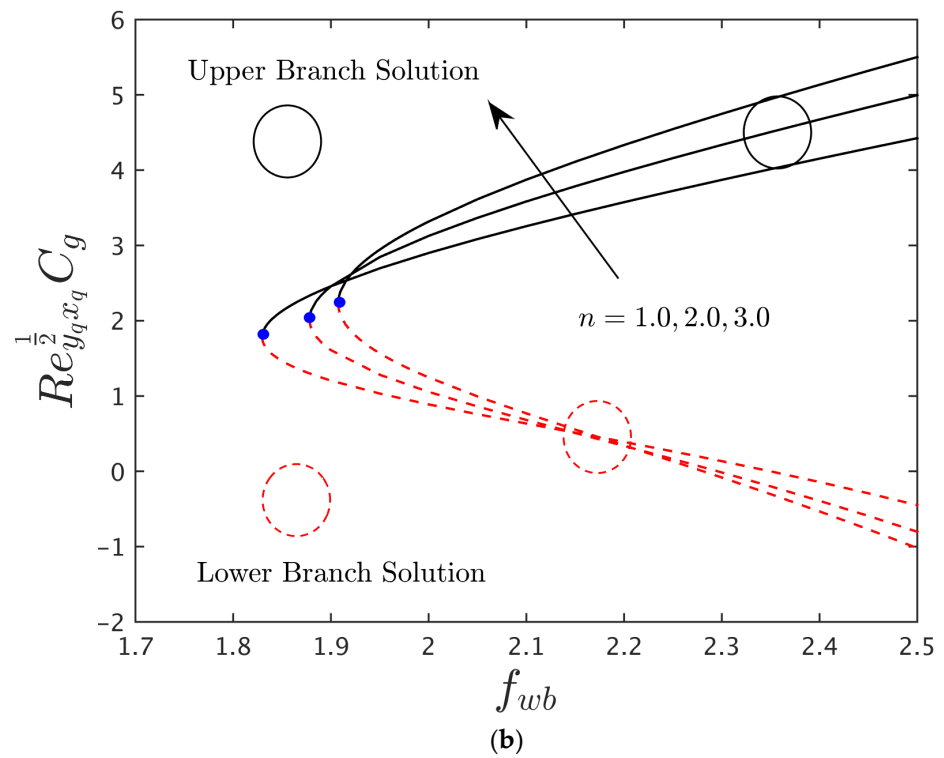
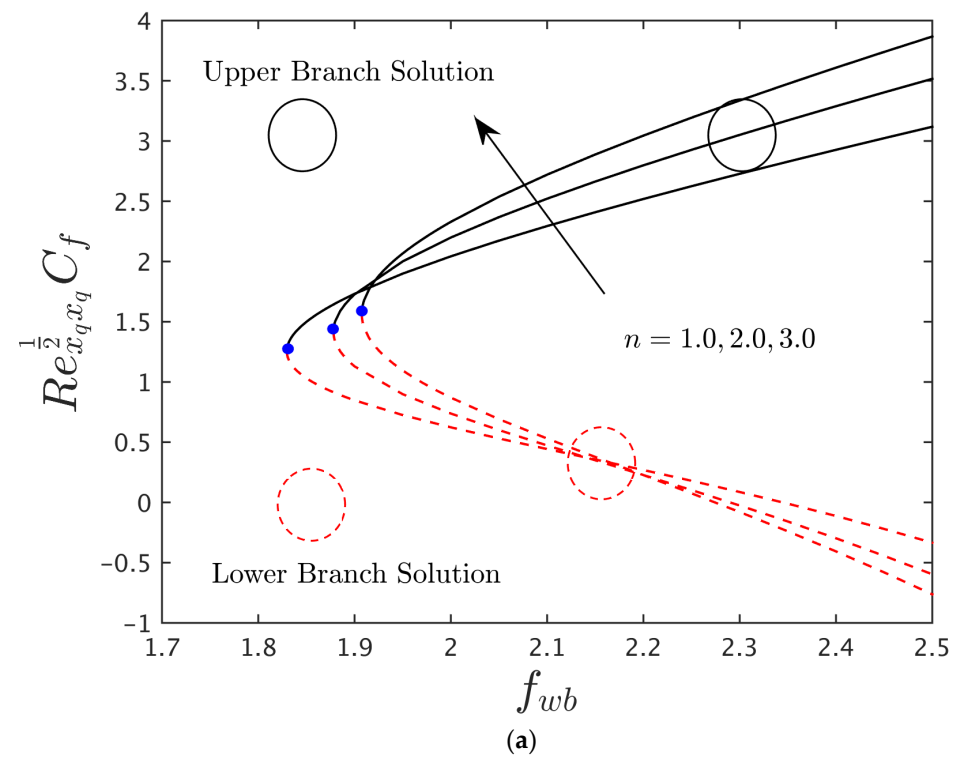


Figure 2. Cont.

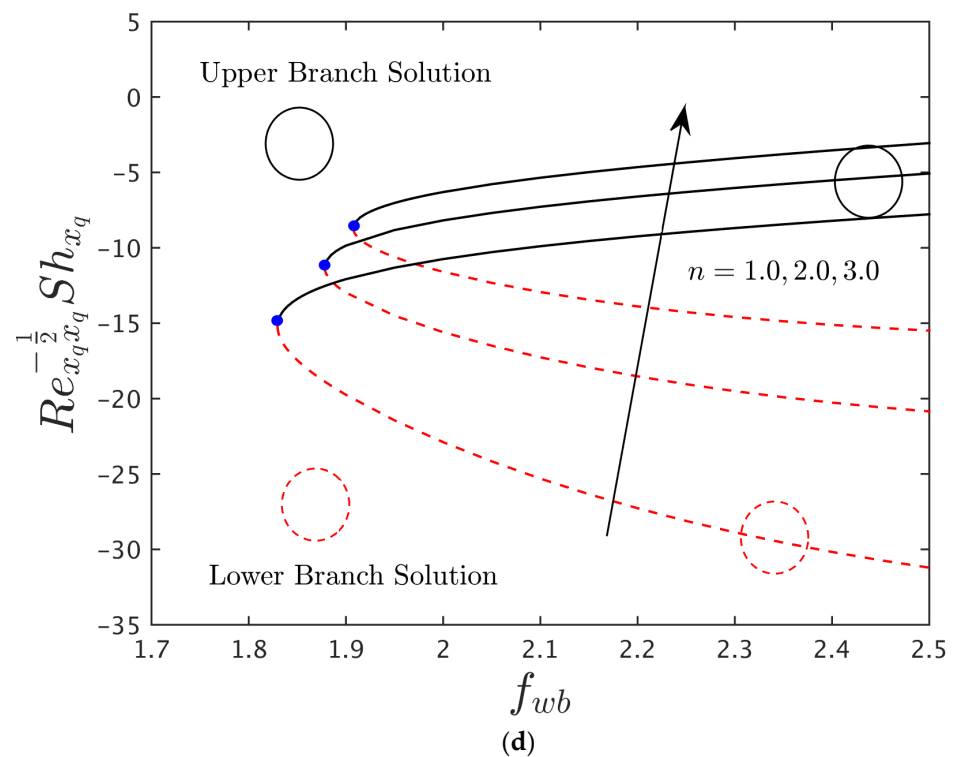
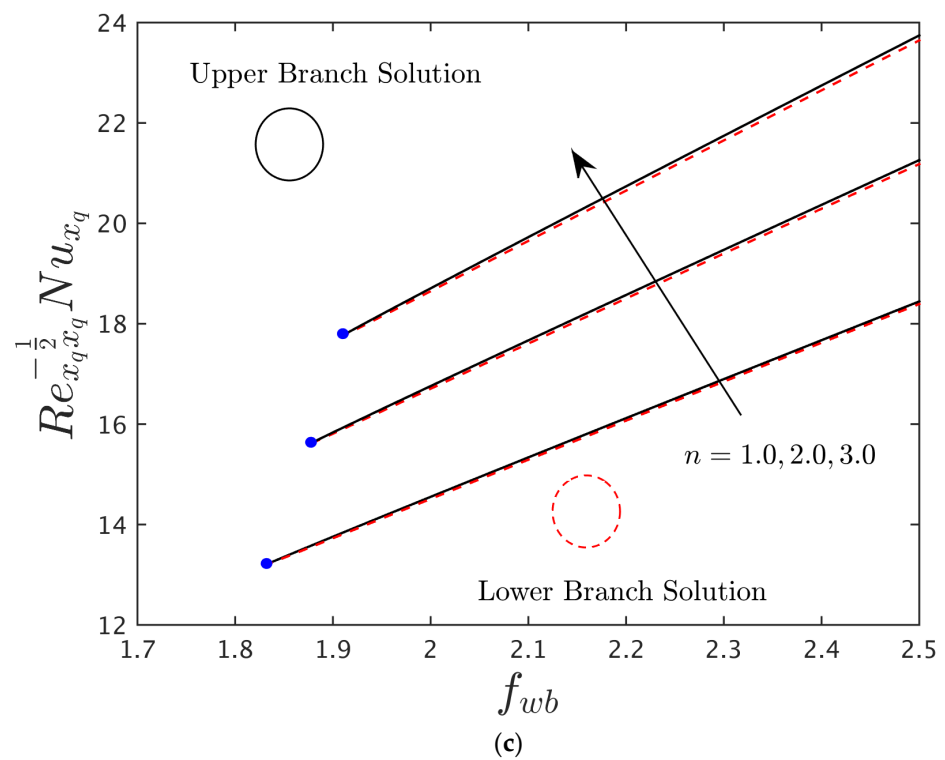


Figure 2. (a) Shear stress in the streamwise direction, (b) shear stress in the spanwise direction, (c) heat transfer rate, and (d) mass transfer rate versus f_{wb} for various values of n .

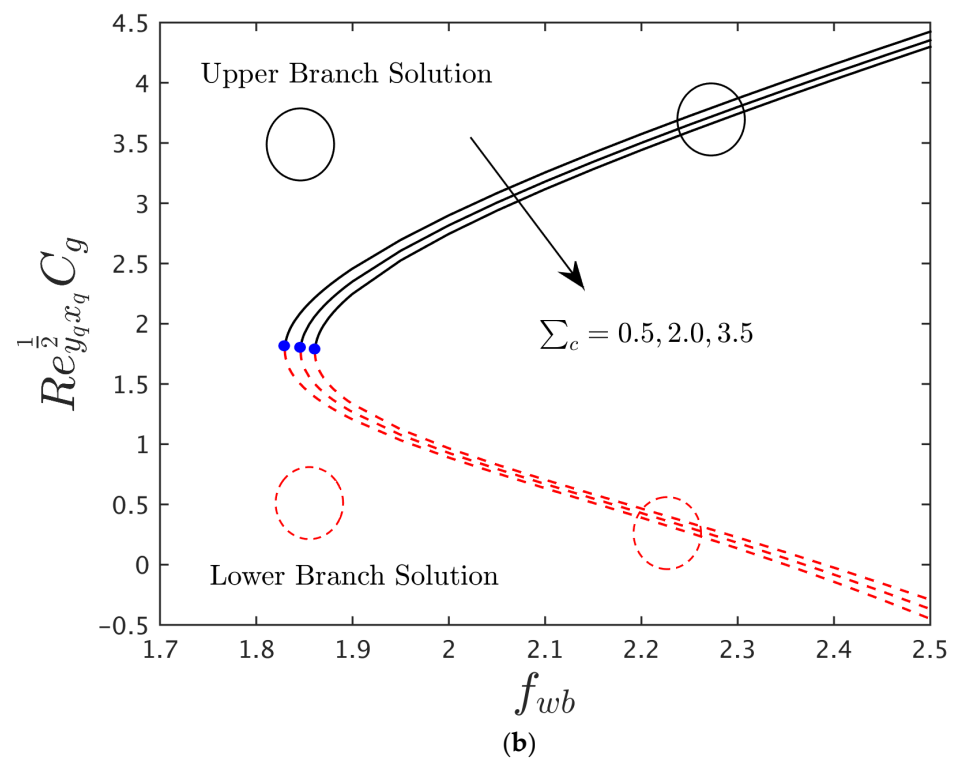
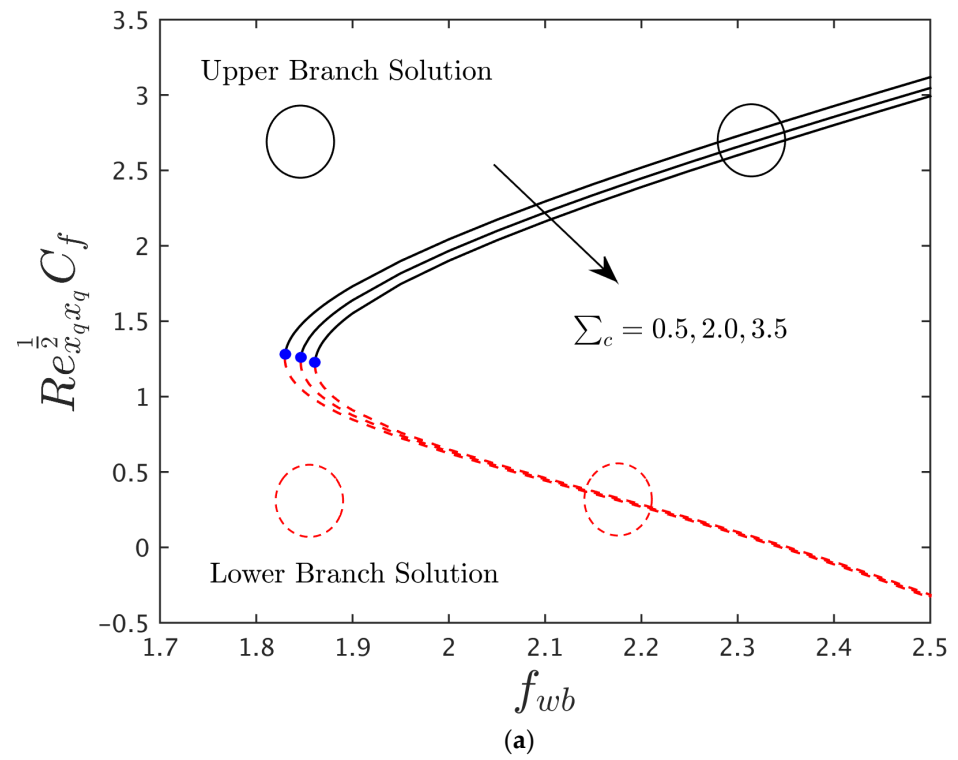


Figure 3. Cont.

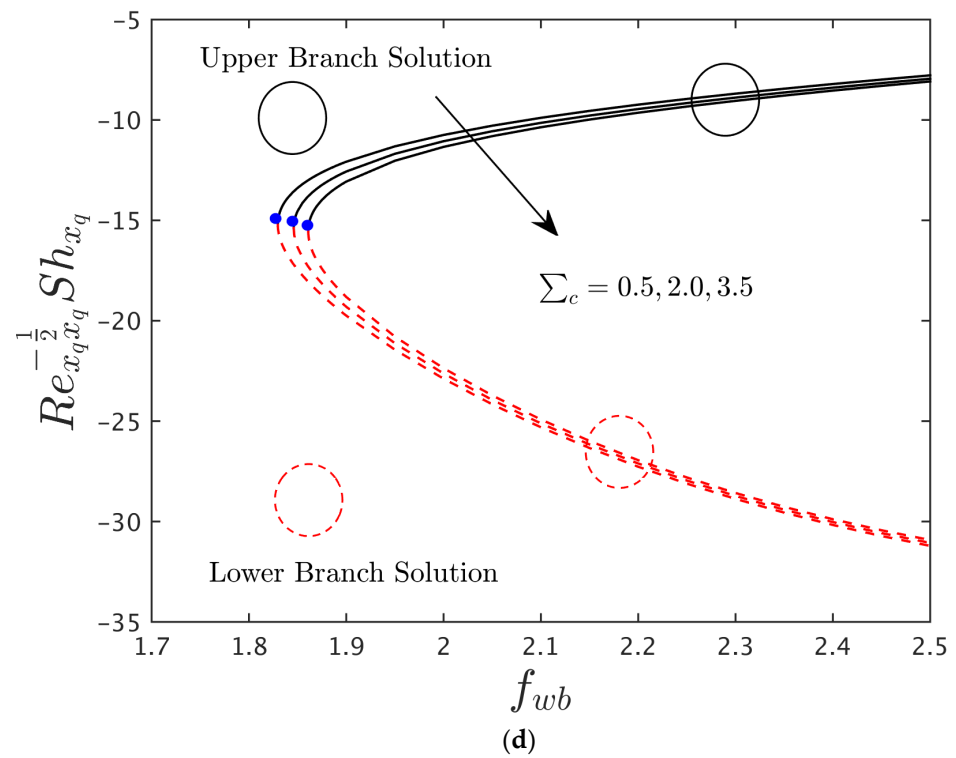
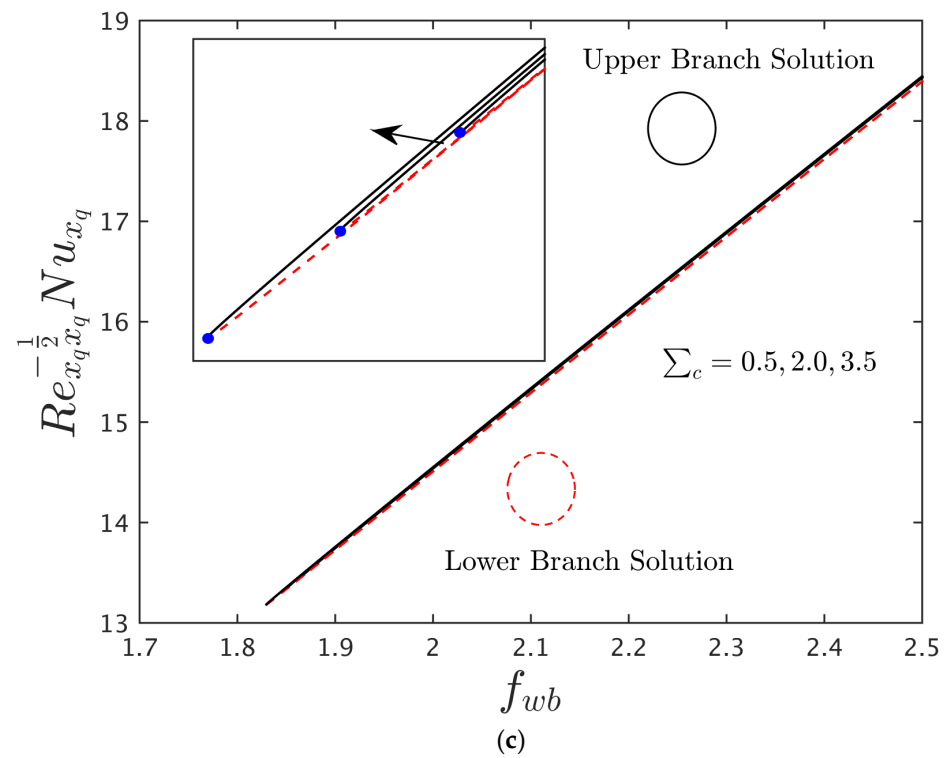


Figure 3. (a) Shear stress in the streamwise direction, (b) shear stress in the spanwise direction, (c) heat transfer rate, and (d) mass transfer rate versus f_{wb} for various values of Σ_c .

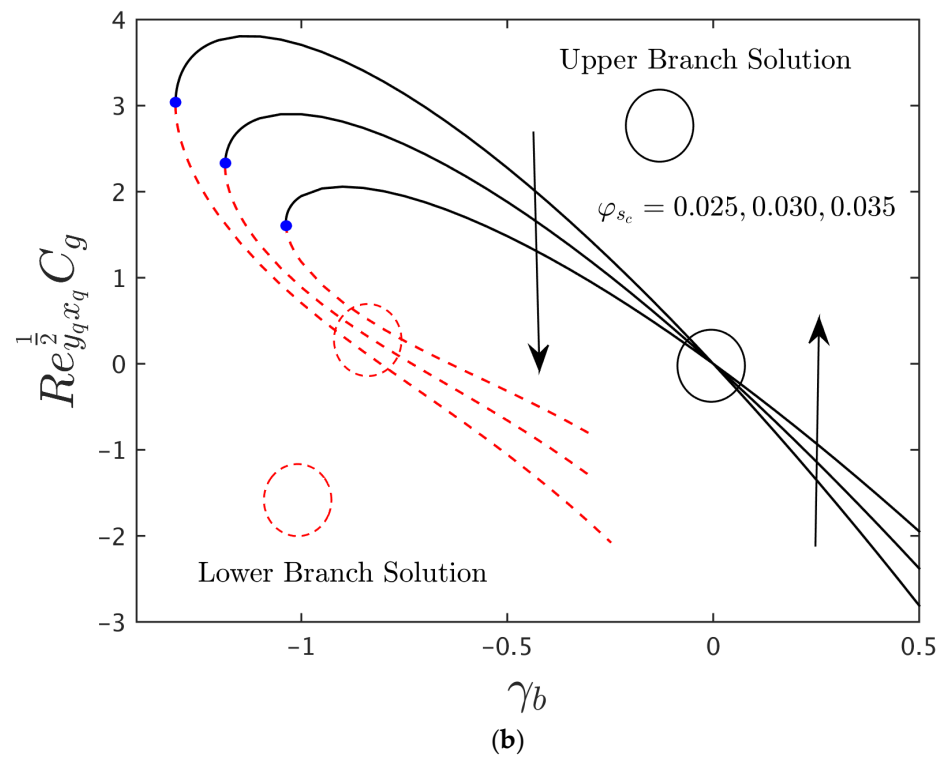
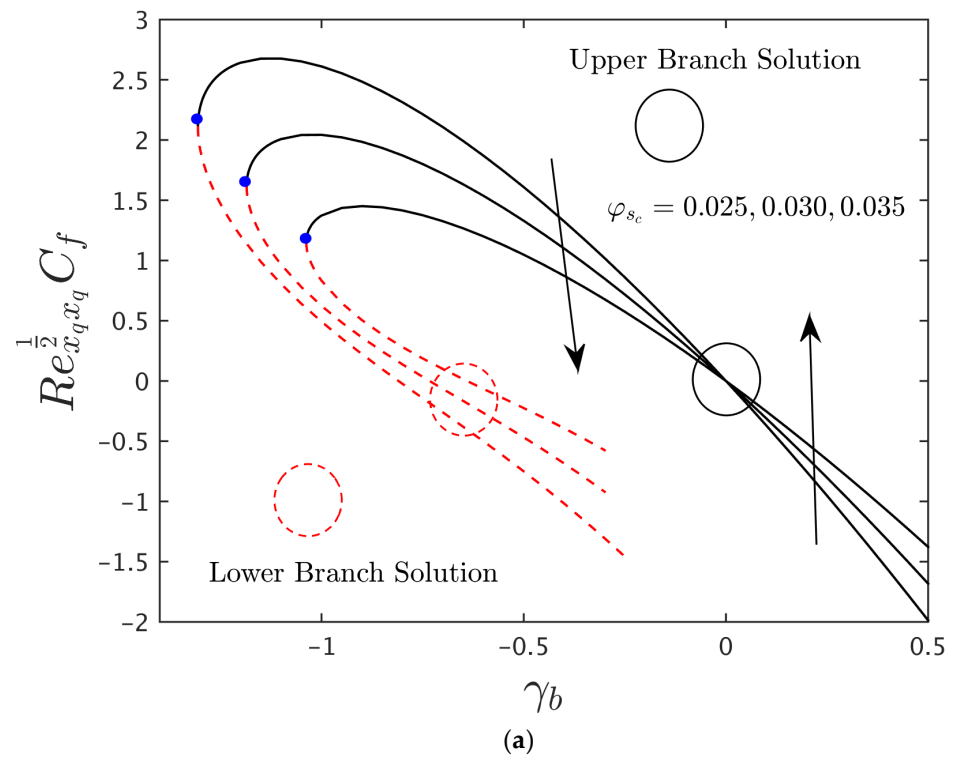


Figure 4. Cont.

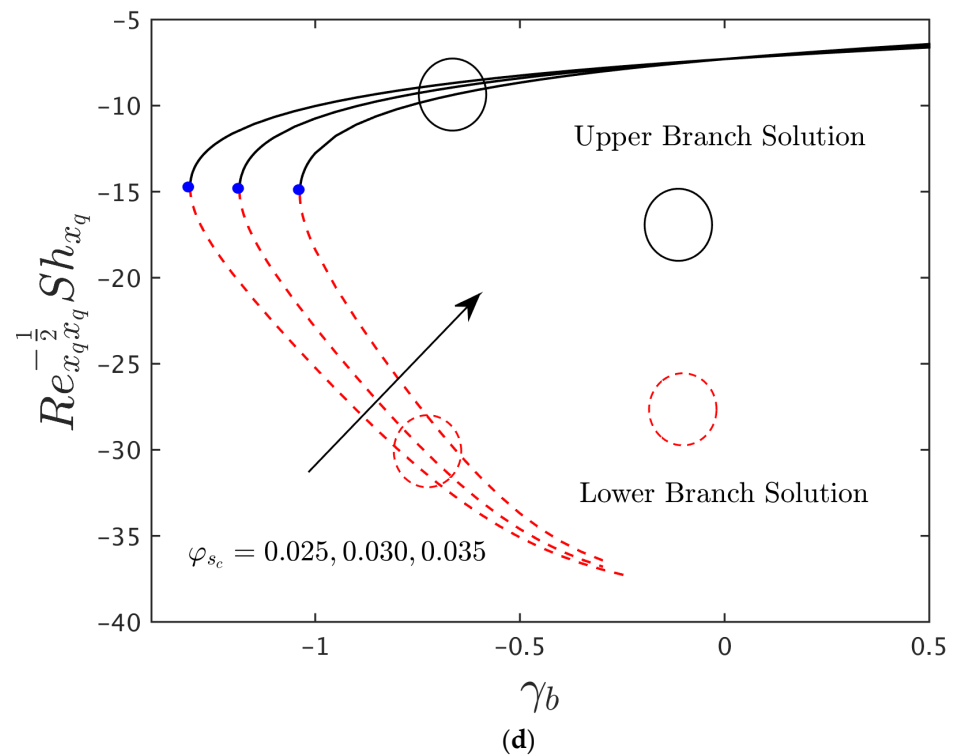
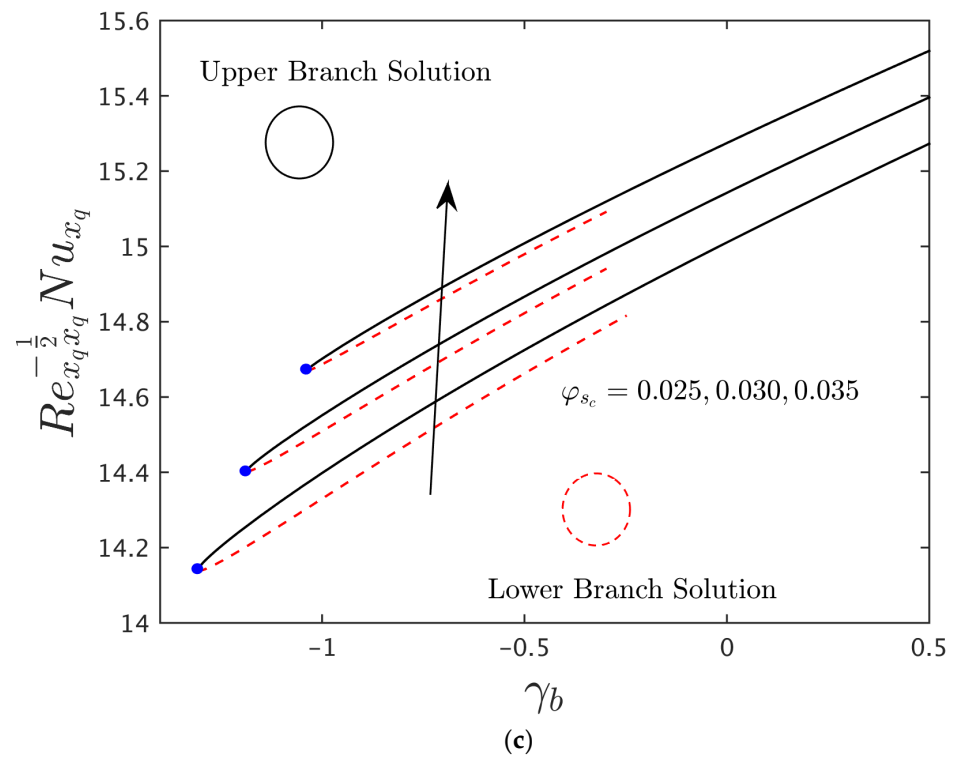


Figure 4. (a) Shear stress in the streamwise direction, (b) shear stress in the spanwise direction, (c) heat transfer rate, and (d) mass transfer rate versus γ_b for various values of φ_{sc} .

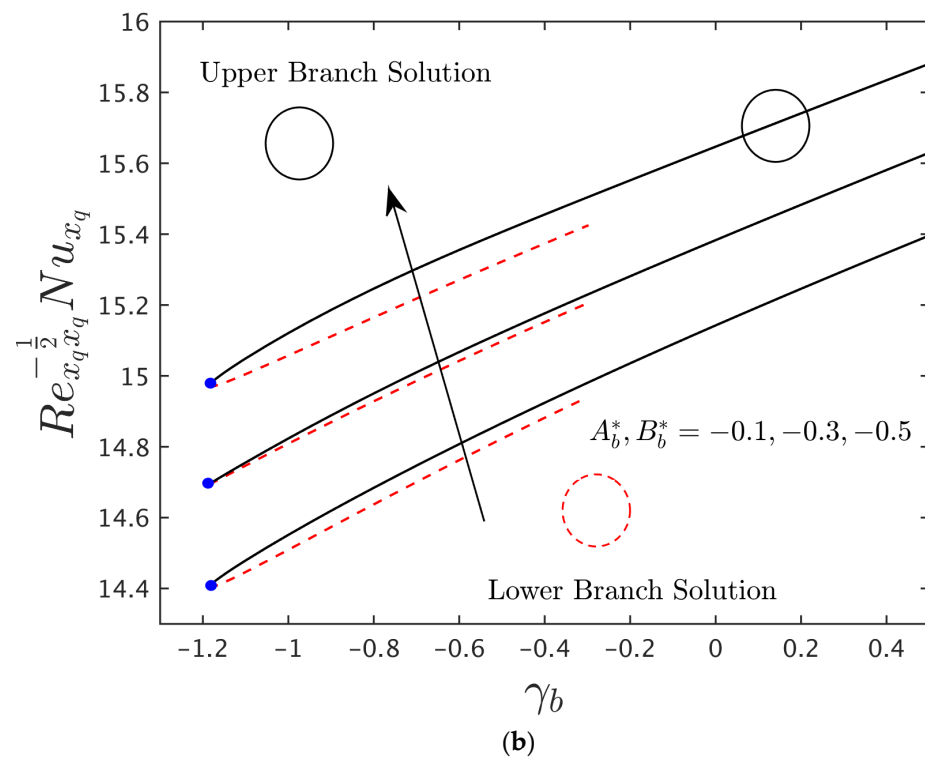
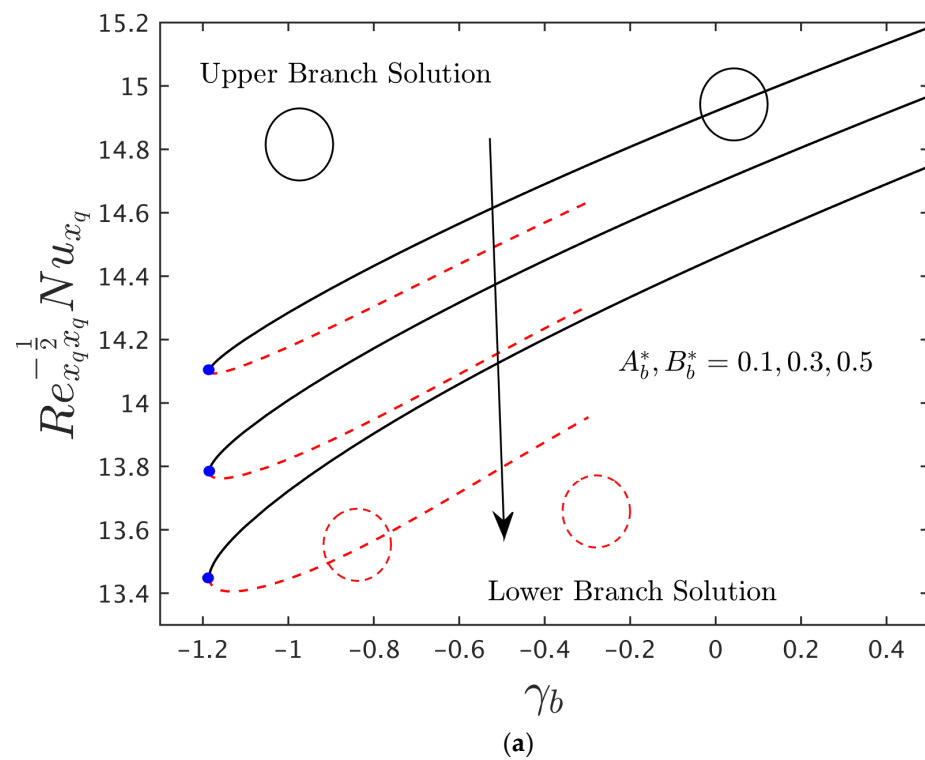


Figure 5. (a) Heat transfer rate versus γ_b for the various values of the internal heat source parameter and (b) for various values of the internal heat sink parameter.

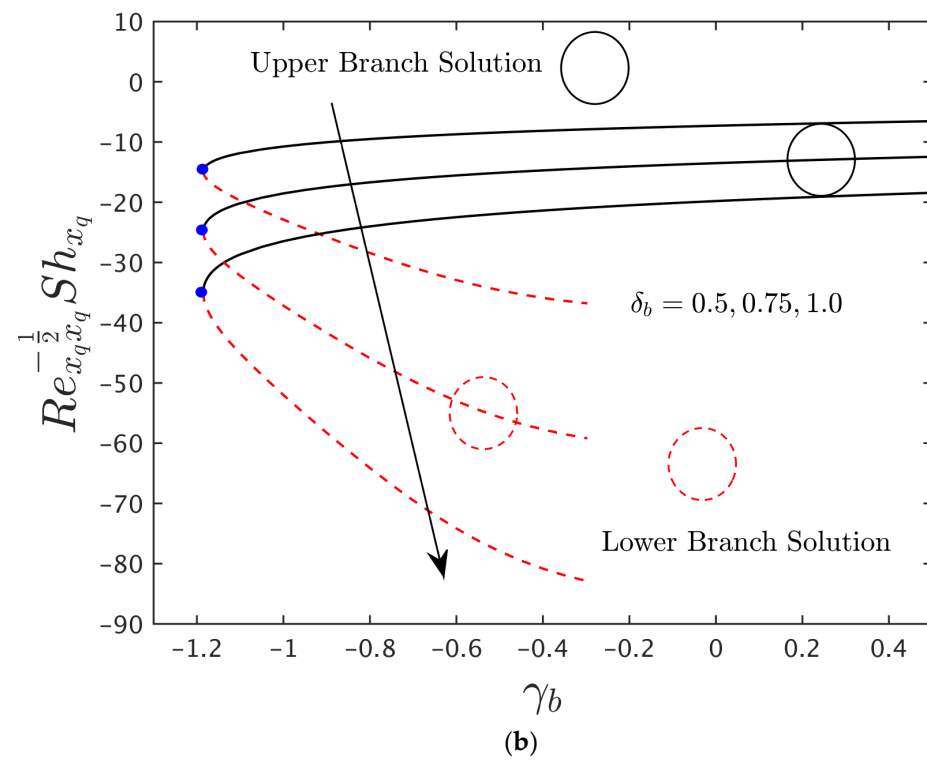
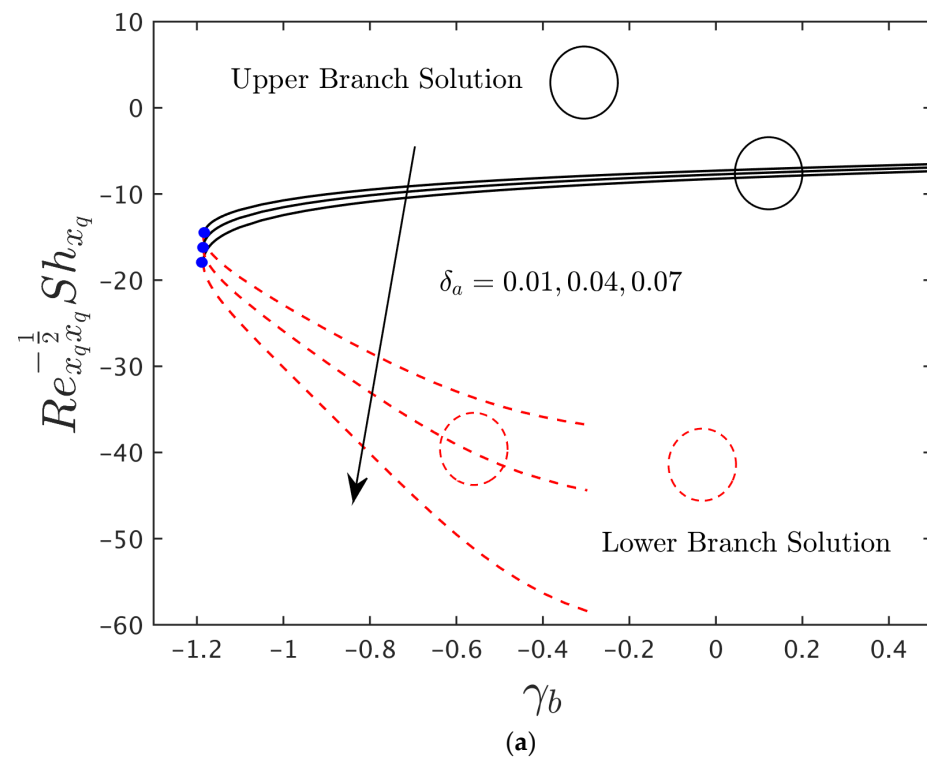


Figure 6. (a) Mass transfer rate versus γ_b for the different values of δ_a and (b) for various values of δ_b .

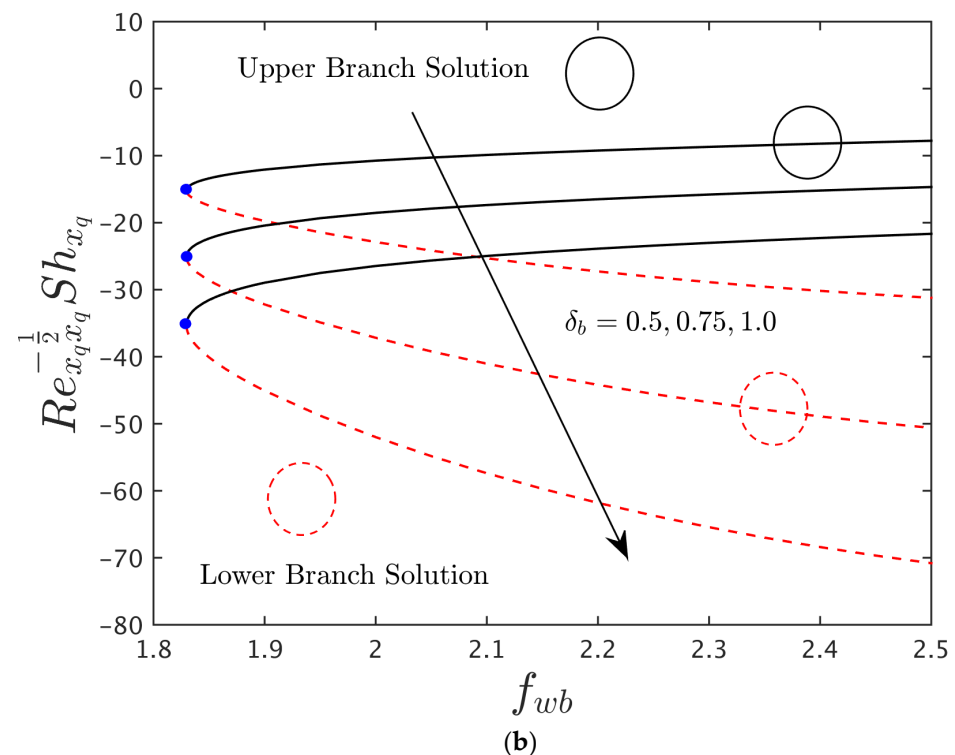
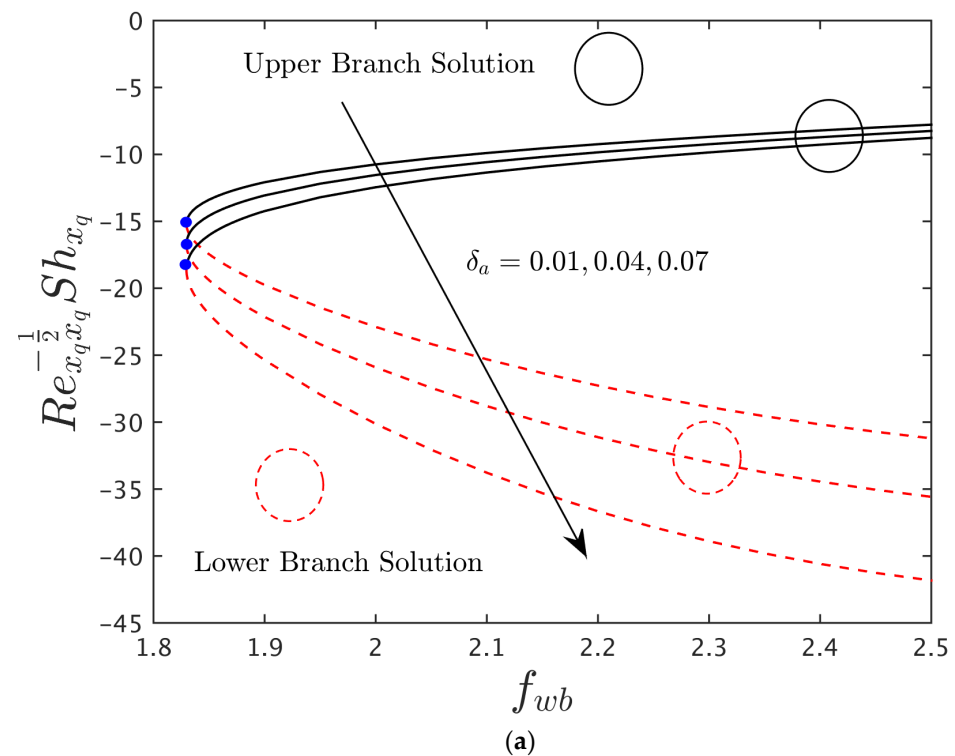


Figure 7. (a) Mass transfer rate versus f_{wb} for the different values of δ_a and (b) for various values of δ_b .

The effects of the material or EPF parameter Σ_c on SSCs in both (SMW and SNW) directions, and HTR and MTR of the (water / Ag) nanofluid for the SBS and UBS are depicted in Figure 3a–d, respectively. Advancing values of Σ_c lead to a remarkable enhancement in the SSCs in both (SMW and SNW) directions and MTR for the UBS, but the HTR substantially reduces. However, the heat transfer rate upsurges for the SBS with the higher impacts of the non-Newtonian parameter Σ_c , while the SSCs in both directions and the

MTR fall for the SBS. In general, the material or non-Newtonian fluid parameter Σ_c is inversely proportional to μ_f of the fluid. The significant rise in the constraint Σ_c generates a reduction in the dynamic viscosity of the fluid, and, as a result, the velocity enhances. Due to this circumstance, a higher velocity of the fluid strongly recommends that the friction factor coefficient in both paths for the Ag nanoparticles should decline. Moreover, the absolute properties of the BVs are uplifted with larger values of Σ_c . This behavior graphically illustrates that the flow separation of the boundary layer from the surface of the plane decays successively, due to the larger consequences of Σ_c .

Figure 4a–d show the effect of the solid nanoparticle volume fractions φ_{sc} on the SSCs in both (SMW and SNW) directions, and HTR and MTR for the SBS and UBS, respectively. It is professed that for a fixed φ_{sc} , two distinct (SB and UB) outcomes of each SSCs in both (SMW and SNW) directions, HTR and MTR, originated. It is seen in Figure 4a,b that the SSCs in both (SMW and SNW) directions are lower for the case of shrinking and higher for the case of stretching, due to the superior impacts of φ_{sc} . Meanwhile, they both perform distinctly for the UBS with higher effects of φ_{sc} . In addition, the SSC in the SNW direction is slightly better and stronger as compared to the value obtained in the case of the SMW direction. Also, the dual SBS and UBS exist in the case of shrinking, while for the case of stretching a single solution has been calculated. On the other hand, the MTR inclines for the UBS and declines for the SBS, due to the higher impacts of φ_{sc} while the heat transfer rate was remarkably boosted up in both solution branches (see Figure 4c,d). Physically, the TCN of the non-Newtonian fluid enriches with superior consequences of φ_{sc} , and thus the rate of heat transfer enlarges. Furthermore, the outcome of two distinct (SB and UB) branches is met at a single point called bifurcation. At this position, the fallouts are exceptional (or unique) and scientifically expressed as $\gamma_b = \gamma_b C$, where $\gamma_b C$ corresponds to the BP or CP. For the specific domain areas such as $\gamma_b C < \gamma_b < \infty$, and $-\infty < \gamma_b < \gamma_b C$, the dual outcomes and no outcomes existed, respectively. Intensifying values of φ_{sc} decreases $|\gamma_b C|$, that is, it raises the flow separation of the boundary layer from the HPS.

The impression of the internal heat source parameter $A_b^*, B_b^* > 0$, and the internal heat sink parameter, $A_b^*, B_b^* < 0$, on the rate of heat transfer of the H₂O-based Ag particles for the dual (SB and UB) outcomes are depicted in Figure 5a,b, respectively. Growing values of $A_b^*, B_b^* > 0$ (the heat-source parameter) leads to a noteworthy reduction in both the SBS and UBS. However, a contradictory behavior is seen in both outcomes when we upsurge the values of $A_b^*, B_b^* < 0$ (heat-sink parameter). Physically, this propensity is due to the incremental choices of $A_a^* < 0, B_a^* < 0$ which generates the heat in the familiar form of energy, and thus the rate of heat transfer rises. However, the heat produced is much less or negligible because of the augmentation in the sequential values of the heat generation parameter $A_a^* > 0, B_a^* > 0$, and as a response, the rate of heat transfer decelerates. In addition, the critical values are the same if we change the value of A_b^* and B_b^* to simultaneously positive and negative, see Figure 5a,b. Hence, the gap between the LB and UB curves is better for the circumstance of the heat source as compared to the heat sink parameter.

Figure 6a,b describe the influence of the external pollutant parameters δ_a , and δ_b on the MTR of the (water/Ag) nanofluid against the expanding/contracting parameter γ_b for the SBS and UBS, respectively. From the graphs, it is seen that dual solutions are only possible in the case of contracting, while only a single solution is noted for the circumstance of expanding. In addition, the MTR declined for both solution branches due to the continuous improvement in the external pollutant parameters δ_a , and δ_b . Generally, by increasing the external pollutant parameters, a more significant influx of pollutants into the deformable horizontal plane surface increases pollutant concentrations. The fluid flow patterns may be affected, as the higher pollutant concentration alters the velocity distribution and may cause flow instabilities and increase temperature and concentration but reduce the rates of HT and MT. More significant improvement in the pollutant parameter will improve the pollutant load, which challenges the efficiency of pollution control techniques. Also, the critical point (CP) will remain the same if we change the values of δ_a and δ_b . Meanwhile,

the gap between the solution curves for the MTR is better for the variations in the value of δ_b as compared to δ_a .

The consequences of the external pollutant parameters δ_a and δ_b on the MTR of the (water/Ag) nanofluid against the suction parameter f_{wb} for the SBS and UBS are presented in Figure 7a,b, respectively. Here, the complete dual (SB and UB) solutions are found against the suction parameter for varying the influence of the parameters δ_a and δ_b . It is seen that the MTR declined continuously in both SBS and UBS with the sophisticated external pollutant parameters δ_a and δ_b . By intensifying the values of external pollutant parameters δ_a and δ_b , the bifurcation values $|f_{wb}C|$ will remain fixed and constant. Moreover, the mass transfer rate solution gap between the two curves (SB and UB) for the varying values of δ_b is higher than the outcomes obtained for the changing values of δ_a . Hence, these parameters can significantly impact the system's behavior and the effectiveness of waste treatment and pollution control.

4. Conclusions

In this manuscript, the separations of the boundary layer and multiple (SB and UB) solutions were explored for a 3D Eyring–Powell nanofluid flow past a porous deformable (expanding/contracting) horizontal plane surface. The impacts of erratic heat source/sink and pollutant concentration for the heat and mass transfer analysis were also taken into account. The new set of variable ansatz was used to transform the problem from PDEs to ODEs. The substantial outcomes of this model are listed below.

- The dual (SB and UB) outcomes occur only for the limiting cases like shrinking surface and mass suction parameters, with variation of the other influential parameters.
- The boundary layer separations lessened with the higher influence of n and Σ_c but were augmented with superior impacts of φ_{sc} .
- The inclusion of the silver nanoparticles φ_{sc} improves the heat transfer rate for both (SB and UB) consequences, whereas the rate of mass transfer declines for the SBS and rises for the UBS.
- The mass transfer rate declined for both (SB and UB) solutions with the superior impact of the local external pollutant concentration parameters.
- The shear stress coefficients in both (SMW and SNW) directions initially incline and then decline for the SBS owing to the higher impacts of φ_{sc} , while the contrary behaviors are found for the UBS.
- The mass transfer rate values increases with the significant impact of pollutant concentration parameters δ_a , and δ_b .

Further, this effort can be expanded by investigating an unsteady flow or buoyancy flow, due to their significant applications like nuclear reactors, flows in the atmosphere, and the ocean due to seasonal change, and varying densities along the vertical direction of a lake causing circulation.

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Nomenclature

A, B	Constant stretching rates
A_b^*	Exponentially decaying space coefficient
B_b^*	Time-dependent heat absorption/generation
c_p	Specific heat (J/Kg. K)
E_b	Stretching-rate ratio parameter
K_b, D_b	Arbitrary constants
$u_{wq}(x_q)$	Velocity of the deformable surface (m/s)
C_{wq}	Wall Concentration
T_{wq}	Temperature of wall (K)
T_∞	Ambient temperature (K)
$F(\xi), G(\xi)$	Dimensionless velocities in streamwise and spanwise direction
$S(\xi)$	Dimensionless temperature
$H(\xi)$	Dimensionless concentration
Pr	Prandtl number
f_{wb}	Mass transfer factor
Nu_{x_q}	Heat transfer rate
Sh_{x_q}	Mass transfer rate
C_f, C_g	Skin friction coefficients in streamwise and spanwise direction
$Re_{x_q x_q}, Re_{y_q y_q}$	Local Reynolds numbers
D_f	Concentration molecular diffusivity (m ² /s)
n	Power-law index
k	Thermal conductivity (W/m K)
Q_q, b^*	External pollutant strengths
w_0	Mass transpiration(m/s) velocity
u_q, v_q, w_q	Velocity components along x_q –, y_q and z_q – axes (m/s)
(x_q, y_q, z_q)	Coordinates (m)
<i>Greek symbols</i>	
β_q, δ_q	Eyring–Powell fluid characteristics
δ_a	Pollutant parameter of external source
δ_b	Pollutant parameter external source variation
τ_{ij}	Shear stress (kg/m ² s)
φ	Solid fraction volume of nanoparticles
ν_f	Kinematic viscosity (m ² /s)
γ_b	Deformable horizontal plane surface
μ	Viscosity (N. s/m ²)
ξ	Pseudo-similarity variable
$(\rho c_p)_{nf}$	Specific heat capacity (J kg ^{−1} K ^{−1})
ρ	Density (kg m ^{−3})
ψ	Stream function
Σ_b, Σ_c	Eyring–Powell fluid parameters
<i>Acronyms</i>	
Ag	Silver
EPF	Eyring–Powell fluid
SMW	Streamwise
SNW	Spanwise
HTR	Heat transfer rate
MTR	Mass transfer rate
H ₂ O	Water
SBS	Stable branch solution
UBS	Unstable branch solution
SB	Stable branch
UB	Unstable branch
BP	Bifurcation point
CP	Critical point
SSCs	Shear stress coefficients
TCN	Thermal conductivity
HPS	Horizontal plane surface

3D	Three-dimensional flows
BC	Boundary condition
<i>Subscripts</i>	
sc	Solid nanoparticles
nf	Nanoliquid
f	Base fluid
w	Boundary condition at wall
∞	Far-field condition
<i>Superscript</i>	
$(\prime)$	Derivative with respect to ξ

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