

Development of Simple Formula for Transverse Dispersion Coefficient in Meandering Rivers

Kyong Oh Baek *  and Dong Yeol Lee

Department of Civil and Environmental Engineering, Hankyong National University,
Anseong-si 17579, Republic of Korea; ssuper@hknu.ac.kr

* Correspondence: pko@hknu.ac.kr; Tel.: +82-31-670-5141

Abstract: This study aims to develop a straightforward and practical formula to estimate transverse dispersion coefficients in meandering natural rivers, a critical factor for predicting solute transport. We present a novel expression for the transverse dispersion coefficient based on dispersion and hydraulic data sets obtained from tracer experiments conducted in natural rivers. A distinctive feature of the formula is its reliance on one dimensionless hydraulic parameter, $\frac{\bar{u}}{u_*} \frac{h}{R_c}$. To assess the effectiveness and accuracy of our proposed formula, we compare it with previously established equations commonly employed in the field. Furthermore, we apply the formula to natural river bends situated in the Nakdong River of Korea. This equation serves to estimate the initial value of the dispersion coefficient in two-dimensional solute transport modeling. As a result, the calibrated value of the dimensionless transverse dispersion coefficient is 0.97, which is only a 16% difference between the initial value of 0.81 as obtained from the formula. The formula presented in this study, simplifying and utilizing the dimensionless hydraulic parameter, offers a promising approach to estimating transverse dispersion in natural meandering rivers in cases where tracer and secondary flow data are unavailable. Additionally, the formula can be refined with more recent dispersion data, leading to a clearer, more straightforward, and validated formulation that captures the intricate interplay between topography and transverse dispersion.

Keywords: transverse dispersion coefficient; meandering river; empirical equation; tracer data; two-dimensional solute transport modeling



Citation: Baek, K.O.; Lee, D.Y.

Development of Simple Formula for Transverse Dispersion Coefficient in Meandering Rivers. *Water* **2023**, *15*, 3120. <https://doi.org/10.3390/w15173120>

Academic Editor: Georg Umgiesser

Received: 7 August 2023

Revised: 23 August 2023

Accepted: 29 August 2023

Published: 30 August 2023



Copyright: © 2023 by the authors. Licensee MDPI, Basel, Switzerland. This article is an open access article distributed under the terms and conditions of the Creative Commons Attribution (CC BY) license (<https://creativecommons.org/licenses/by/4.0/>).

1. Introduction

Understanding the dispersion characteristics of rivers is crucial for various environmental and engineering applications. While artificial rivers are designed to follow a specific path, natural rivers tend to meander, making their dispersion characteristics unique. In particular, accurate dispersion coefficients for meandering natural rivers are necessary for predicting and mitigating the effects of pollutants or other materials transported by the river flow. Although sophisticated models exist to estimate these coefficients, they may not always be practical or feasible. Thus, there is a need for a simple formula that can be easily implemented and provide a reasonable estimate of dispersion coefficients as an initial value of numerical models.

Broadly, methods for determining the dispersion coefficient can be divided into two categories: observation methods that use concentration data and estimation methods that use basic hydraulics when concentration data are unavailable [1]. Estimation methods can be further classified into theoretical equations that derive the coefficient of dispersion by considering the effect of shear flow, and empirical equations that obtain the dispersion coefficient through regression analysis based on a large number of concentration experimental data. However, the complexity of theoretical equations can be simplified via empirical methodology, and the empirical equation can be developed based upon a theoretical background, making the two methodologies complementary rather than contradictory [1]. The

hierarchy of methods for determining the dispersion coefficient in open channel flow is schematically drawn in Figure 1.

Of the two parameters of the two-dimensional mass transport equation (longitudinal and transverse dispersion), the transverse dispersion coefficient (TDC) has been studied by many researchers. About 55 years ago, Fischer [2] proposed the following simplified expression for the dimensionless transverse dispersion coefficient in an artificial canal.

$$\frac{D_T}{hu_*} = 0.15 \quad (1)$$

where D_T is the transverse dispersion coefficient, h is the water depth, u_* is the frictional velocity and defined as $\sqrt{ghS_0}$ in an open channel. g is the gravitational acceleration, and S_0 is the bed slope. Fischer et al. [3] proposed a range of values for the transverse dispersion coefficient for slowly meandering rivers with moderate sidewall irregularities, as shown in Equation (2).

$$\frac{D_T}{hu_*} = 0.30 \sim 0.90 \quad (2)$$

Rutherford [4] provided a summary of the range of transverse dispersion coefficients based on the shape of open channels, as

$$\begin{aligned} D_T/hu_* &= 0.15 \sim 0.30 \text{ in straight channel} \\ &= 0.30 \sim 0.90 \text{ in meandering channel} \\ &= 1.00 \sim 3.00 \text{ in sharp channel} \end{aligned} \quad (3)$$

However, these equations are inadequate for application at river bends, as they pertain to reach-scale coefficients and disregard the impact of channel curvature, which contributes to an escalation in the transverse mixing rate. To elucidate the connection between the transverse dispersion coefficient and river bends, multiple theoretical endeavors have been undertaken. Fischer [5] formulated an equation based on the velocity profile and subsequently simplified it via laboratory experiments conducted within a curved channel featuring a constant radius of curvature, as

$$\frac{D_T}{hu_*} = 25 \left(\frac{\bar{u}}{u_*} \right)^2 \left(\frac{h}{R_c} \right)^2 \quad (4)$$

where $\bar{u}$ is the mean velocity, R_c is the radius of curvature. Yotsukura and Sayre [6] made modifications to Equation (4) by incorporating datasets collected from the Missouri River around a bend. They proposed that a more precise transverse dispersion coefficient could be achieved by substituting the channel width (W) for the water depth (h), resulting in Equation (5).

$$\frac{D_T}{hu_*} = 0.4 \left(\frac{\bar{u}}{u_*} \right)^2 \left(\frac{W}{R_c} \right)^2 \quad (5)$$

Sayre [7] later revised Equation (5) based on further experiments conducted in the Missouri River, as shown in Equation (6).

$$\frac{D_T}{hu_*} = (0.3 \sim 0.9) \left(\frac{\bar{u}}{u_*} \right)^2 \left(\frac{W}{R_c} \right)^2 \quad (6)$$

In the 2000s, Jeon et al. [8] formulated an empirical equation through the utilization of dimensional analysis and a regression method, employing dispersion datasets gathered from various field tracer tests. In their equation, they used sinuosity, which is defined as the ratio between the thalweg length and the down-valley distance, to account for the

influences of multiple bends in natural streams instead of relying solely on the radius of curvature.

$$\frac{D_T}{hu_*} = 0.029 \left(\frac{\bar{u}}{u_*} \right)^{0.463} \left(\frac{W}{h} \right)^{0.299} (S_n)^{0.733} \quad (7)$$

where S_n is the sinuosity. Aghababaei et al. [9] also suggested a formula using the sinuosity.

$$\frac{D_T}{hu_*} = 0.159 \left(\frac{W}{h} \right)^{0.126} \left(\frac{\bar{u}}{u_*} \right)^{-0.148} \left[1 + \left(0.501 \left(\frac{\bar{u}}{u_*} \right)^{0.447} (S_n - 1)^{0.275} \right) \right] \quad (8)$$

The aim of this study is to empirically propose a simplified expression for the transverse dispersion coefficient. This expression is based solely on dispersion and fundamental hydraulic data collected from tracer experiments conducted in natural rivers. The formulated equation presented in this study pertains to the estimation method applicable when secondary flow data are unavailable, as depicted in Figure 1. To assess the performance of the proposed equation, a comparison is conducted against existing equations, evaluating its accuracy and effectiveness via discrepancy analysis. Furthermore, the suggested formula is applied to natural river bends situated in the Nakdong River of Korea. The formula can be used to estimate the transverse dispersion coefficient's initial value in two-dimensional solute transport modeling.

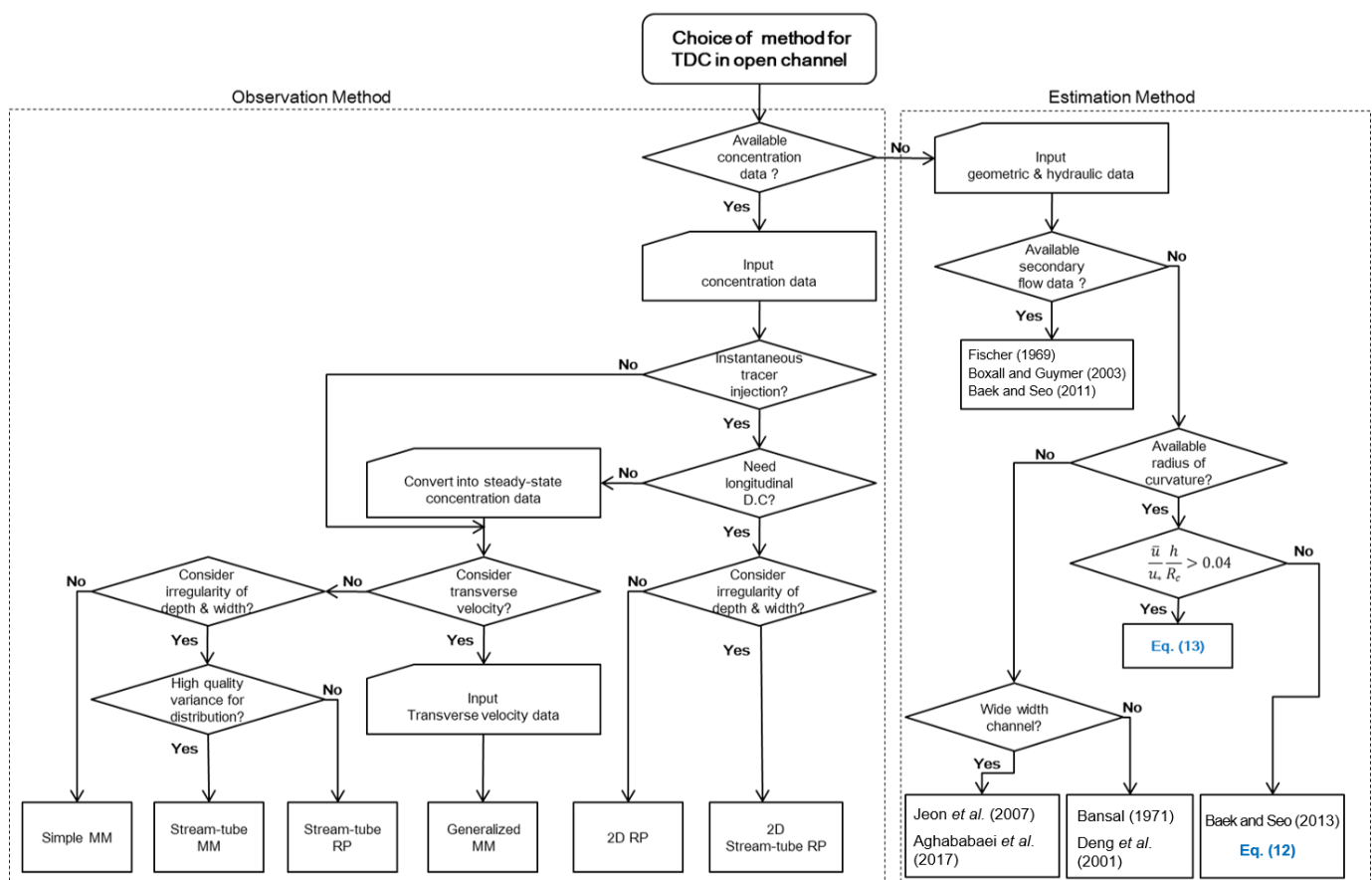


Figure 1. Choice of method for transverse dispersion coefficient with both “observation method” and “estimation method” in open channel flow [1,5,8–13].

2. Methods

Almquist and Holley [14] presented a criterion by which the effects of meander on the transverse mixing were discriminated. The criterion can be derived from the theoretical

expression of a transverse dispersion coefficient, as proposed by Fischer [5]. It is described as follows:

$$\frac{\bar{u}}{u_*} \frac{h}{R_c} > 0.04 \quad (9)$$

The criteria expressed by Almquist and Holley [14] are summarized in Figure 2. The depiction presents four distinct regions characterizing various transverse mixing patterns as follows: (1) Bend of insufficient length to induce supplementary mixing, irrespective of secondary circulation intensity (reversible transport); (2) additional mixing driven by secondary circulation, distinct from gradient transport (initial phase); (3) additional mixing characterized by gradient transport due to secondary circulation (dispersive phase). (4) Secondary circulation of insufficient strength to bring about supplementary mixing (bend has no effect). The classification schema depicted in this figure is founded on fundamental mechanisms that have been comprehensively understood to an extent permitting quantification and correlation with relevant dimensionless parameters.

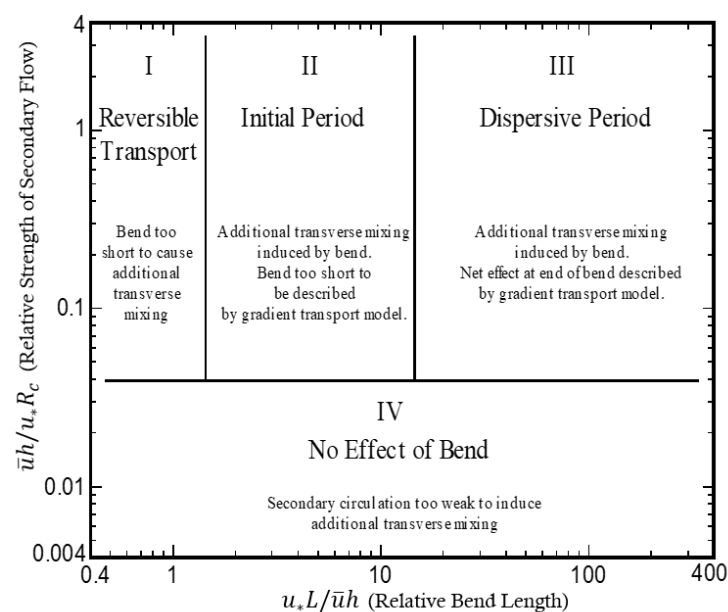


Figure 2. Criteria for transverse mixing induced by bend effects in meandering rivers (modified from Almquist and Holley [14]).

Baek and Seo [1] formulated a hybrid equation that amalgamates theoretical foundations with empirical simplification. Instead of relying on dimensional analysis, they ascertained the functional connection between the dispersion coefficient and hydraulic parameters by employing theoretical equations proposed by Boxall and Guymer [10], as well as their own work [11]. In this approach, the constants in the theoretical equation were treated as regression coefficients, which were established through regression analysis utilizing dispersion datasets obtained from natural streams. The resulting equation is presented as follows:

$$\frac{D_T}{hu_*} = \left(77.88 \frac{\bar{u}}{u_*} \frac{h}{R_c} \right)^2 \left(1 - \exp \left(- \frac{1}{77.88 \frac{\bar{u}}{u_*} \frac{h}{R_c}} \right) \right)^2 \quad (10)$$

Although the functional relationship of Equation (10) has a theoretical background, the equation consists of only hydraulic parameter, $\frac{\bar{u}}{u_*} \frac{h}{R_c}$. In here, Equation (10) is revised as the simplest form of transverse dispersion coefficient equation with the parameter of $\frac{\bar{u}}{u_*} \frac{h}{R_c}$.

$$\frac{D_T}{hu_*} = \alpha \left(\frac{\bar{u}}{u_*} \frac{h}{R_c} \right)^\beta \quad (11)$$

where α and β are regression coefficients. In this study, the regression coefficients are determined by using the dispersion and hydraulic data sets collected from 24 rivers worldwide. The data sets are summarized in Table 1. As shown in this table, the dimensionless transverse dispersion coefficient has large variations from 0.12 to 3.30. This is because the river reaches have various geometric properties, such as the almost straight reach, mild meandering channels, and sharply curved bends.

Table 1. Dispersion and hydraulic data set collected at natural streams.

Research	River	h (m)	$\bar{u}$ (m/s)	u_* (m/s)	R_c (m)	W (m)	S_n	$\frac{\bar{u}}{u_*} \frac{h}{R_c}$	$\frac{D_T}{hu_*}$
Yotsukura et al. (1970) [15]	Missouri	2.74	1.75	0.074	3400	183	1.6	0.019	0.60
Yotsukura and Cobb (1972) [16]	Athabasca	2.20	0.95	0.056	-	373	1.0	-	0.76
Fischer (1973) [17]	Atrisco	0.68	0.63	0.063	-	18.3	1.0	-	0.24
	Bernardo	0.70	1.25	0.062	-	20	1.0	-	0.30
	South	0.44	0.18	0.040	-	18.3	1.0	-	0.26
Holley and Abraham (1973) [18]	Waal	4.70	0.82	0.056	3238	266	1.08	0.021	0.29
	Ijssel	4.00	0.97	0.075	1111	69.5	2.01	0.047	0.51
Sayre and Yeh (1973) [19]	Missouri	3.96	5.40	0.085	968	240	2.10	0.260	3.30
Jackman and Yotsukura (1977) [20]	Potomac	1.74	0.58	0.051	1586	350	1.0	0.012	0.65
Sayre (1979) [7]	Missouri	2.94	1.58	0.074	792	214	2.1	0.079	0.73
		1.99	1.39	0.074	792	214	2.1	0.047	0.81
Beltaos (1980) [21]	Athabasca	2.05	0.86	0.078	1875	320	1.2	0.012	0.41
	Beaver	0.96	0.50	0.044	116	42.7	1.3	0.094	1.01
Lau and Krisnappan (1981) [22]	Grand	0.51	0.35	0.069	310	59.2	1.1	0.008	0.26
Somlyody (1982) [23]	Danube	2.90	0.87	0.051	9778	415	1.0	0.005	0.25
		2.90	0.87	0.036	9778	418	1.0	0.007	0.13
		4.20	0.95	0.059	9778	475	1.0	0.007	0.12
Holly and Nerat (1983) [24]	Isere	2.25	1.40	0.059	1612	70	1.25	0.033	0.50
Demetracopoulos and Stefan (1983) [25]	Mississippi	1.00	0.67	0.079	733	178	1.18	0.012	1.26
Seo et al. (2006) [26]	Sum	0.69	0.34	0.049	381	54	1.66	0.013	0.46
		1.02	0.58	0.056	700	65	1.19	0.015	1.21
		0.68	0.31	0.046	-	80.1	1.0	-	0.30
	Cheongmi	0.48	0.34	0.062	397	44.5	1.13	0.007	0.27
	Hongcheon	0.75	0.35	0.047	437.5	58.6	2.38	0.013	0.64
		1.10	0.21	0.057	559	69.9	1.4	0.007	0.23
		0.97	0.20	0.053	355	67	1.54	0.010	0.32
Seo et al. (2016) [27]	Daegok	0.45	0.17	0.019	880.3	12	1.03	0.005	0.32
	Daepo	0.43	0.65	0.061	308.4	9.2	1.03	0.015	0.53
	Gam	0.3	0.53	0.055	316.7	33.5	1.13	0.009	0.43
	Miho	1.27	0.27	0.030	221.3	42.5	1.55	0.052	0.69
		0.49	0.40	0.048	345.6	31	1.54	0.012	0.58
Pouchoulin et al. (2020) [28]	Rhone	9.23	0.60	0.065	1864	274	1.23	0.045	1.34
		9.01	0.63	0.070	1864	275	1.23	0.043	2.21

Consequently, based on the collected dispersion data of Table 1, the following empirical formula is derived as

$$\frac{D_T}{hu_*} = 5.358 \left(\frac{\bar{u}}{u_*} \frac{h}{R_c} \right)^{0.578} \quad (12)$$

The comparison between the observed dispersion coefficient and the derived equation is illustrated in Figure 3.

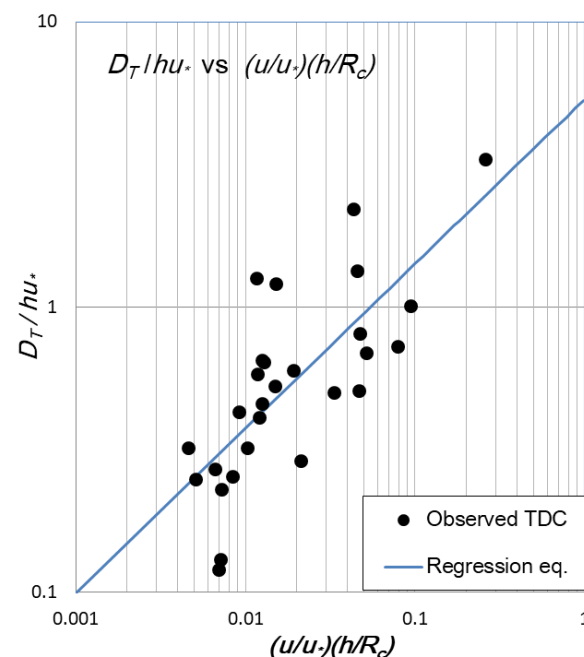


Figure 3. Derivation of simple empirical equation using 33 data sets.

3. Results

The newly derived equation, labeled Equation (12), is subjected to a comparative analysis alongside pre-existing equations that incorporate either the radius of curvature or the sinuosity as hydraulic parameters. In this regard, we specifically consider four empirical equations, namely, Equation (5) (Yotsukura and Sayre [6]), Equation (7) (Jeon et al. [8]), Equation (10) (Baek and Seo [1]), and Equation (8) (Aghababaei et al. [9]). These equations were established using field data obtained from natural river systems and are characterized by specific values rather than a range within their respective formulas, as previously expounded in the introductory chapter. The calculated dispersion coefficients yielded by the aforementioned four existing equations, as well as Equation (12) developed in the present study, are graphically depicted in comparison to the observed dispersion coefficients in Figure 4. A comprehensive error analysis of the outcomes generated by these five equations is succinctly presented in Table 2.

From this figure and table, it can be seen that all equations except for Yotsukura and Sayre's equation are somewhat in good agreement with the observed dispersion coefficient. Yotsukura and Sayre's equation shows some scattering trend and gives high root-mean-squared (RMS) error values. The difference is not significant, but the newly developed formula demonstrates good results in terms of correlation coefficient and RMS error when compared to the existing three formulas, as shown in Table 2.

Yotsukura and Sayre's equation exhibits a tendency to scatter sensitively to variations in the dimensionless hydraulic parameters, with the exponent being squared. The formula by Baek and Seo [1] consisted solely of the dimensionless hydraulic parameter, $u^-/u_*^* h/R_c$ and was derived based on theoretical background. However, it does not demonstrate significant advantages in terms of maintaining its form. Therefore, it seems more favorable to simplify and utilize it, like the formula developed in this study. The formulas by Jeon et al. [8] and Aghababaei et al. [9] incorporate the sinuosity as a parameter instead of the radius of curvature, R_c . These equations are considered suitable alternatives for applying to multiple meandering reaches where a specific R_c cannot be determined.

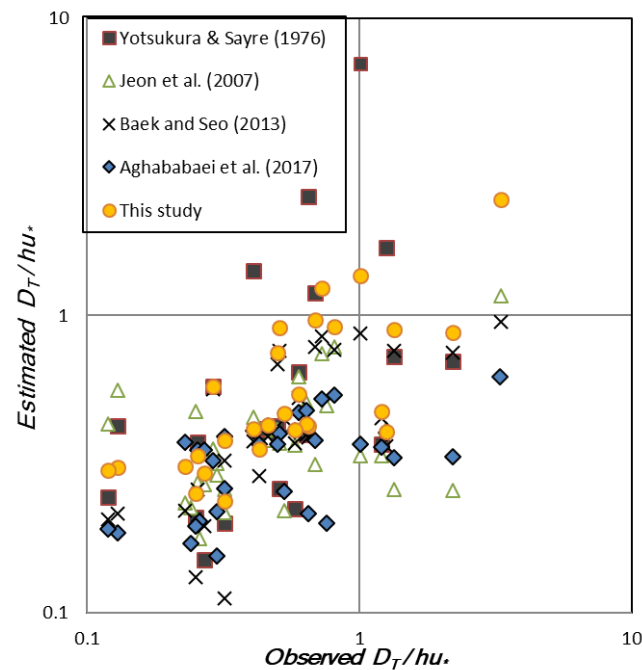


Figure 4. Comparison of dimensionless observed and dimensionless transverse dispersion coefficient including proposed and existing equations [1,6,8,9].

Table 2. Error analysis between observed values and results by five empirical equations.

	Correlation Coefficient	RMS Error
Yotsukura and Sayer (1976) [6]	0.76	19.49
Jeon et al. (2007) [8]	0.53	0.49
Baek and Seo (2013) [1]	0.63	0.54
Aghababaei et al. (2017) [9]	0.50	0.59
This study	0.79	0.34

As shown earlier, Figure 2 presents a criterion by which the effects of meander on the transverse mixing were discriminated. The dispersive period in which is additional transverse mixing induced by bends is $\frac{\bar{u}}{u_*} \frac{h}{R_c} > 0.04$. In this study, another empirical equation is derived based on the form of Equation (12) using the data sets when $\frac{\bar{u}}{u_*} \frac{h}{R_c} > 0.04$ in Table 1. The result is

$$\frac{D_T}{hu_*} = 9.424 \left(\frac{\bar{u}}{u_*} \frac{h}{R_c} \right)^{0.895} \quad (13)$$

The comparison between the observed dispersion coefficient and the derived equation is illustrated in Figure 5.

Although the number of samples is insufficient, the accuracy of the estimation equation improves significantly, with the coefficient of determination (R^2) reaching 0.90 when conducting an error analysis between this equation, Equation (13), and the observed dispersion coefficient. Furthermore, the correlation coefficient and RMS error were 0.98 and 0.23, respectively, demonstrating superior performance compared to other formulas. In natural rivers with pronounced meandering, the dispersion coefficient exhibits greater sensitivity to parameter $\frac{\bar{u}}{u_*} \frac{h}{R_c}$ than any other parameter, indicating a highly robust proportional relationship. To obtain more conclusive results, additional tracer experiments should be conducted for rivers with the condition of $\frac{\bar{u}}{u_*} \frac{h}{R_c} > 0.04$, enabling the accumulation of dispersion data. Subsequently, Equation (13) can be updated based on such data, providing a clearer, simpler, and validated formula that captures the interaction between topography and transverse dispersion.

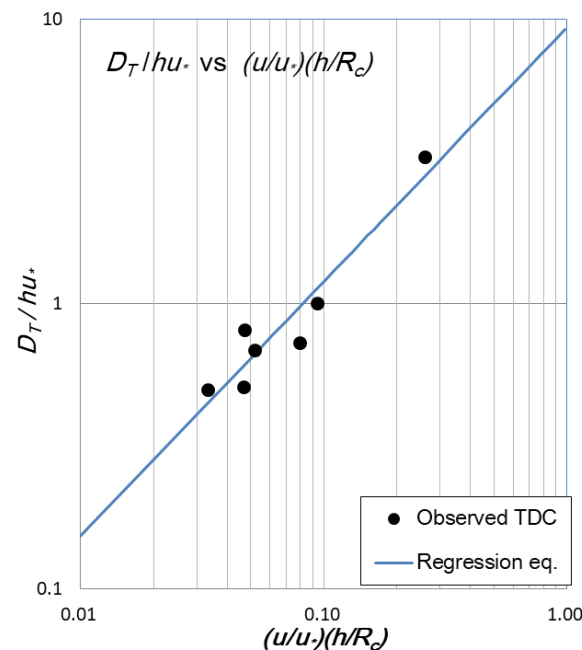


Figure 5. Derivation of another empirical equation in case of $\frac{\bar{u}}{u_*} \frac{h}{R_c} > 0.04$.

4. Application and Discussion

The equations developed in this study were applied to natural river bends located in the Nakdong River of Korea. These equations can be used to estimate an initial value of the dispersion coefficient in two-dimensional solute transport modeling. Of course, the accurate value of the dispersion coefficient can be obtained through a fine-tuning process, which involves achieving the best match between measured concentration data and simulated data from the numerical model. A simple and appropriate equation for the initial value of the dispersion coefficient can significantly enhance simulation efficiency. As the hydrodynamic model, the 2D finite element solver HDM-2D, a depth-averaged numerical model initially developed by Song et al. [29], was utilized in this study. The solute transport model, CTM-2D, initially developed by Seo et al. [30], was employed. The depth-averaged form of the 2D advection–dispersion equation with a reaction term for the non-conservative pollutant is a governing equation of CTM-2D.

As the study area to which the planar two-dimensional transport model was applied, the reach from Gangjeong Weir to Dalseong Weir in the Nakdong River was selected. In this area, as shown in Figure 6, the Geumho River (tributary) joins from the left bank to the mainstream of the Nakdong River, and the radius of curvature (R_c) is about 1480 m. The model parameters, the longitudinal and transverse dispersion coefficients, were calibrated based on the electrical conductivity (EC) acquired from field measurements. The transverse distribution of EC for each transect was measured in seven transects of the study area on 27 August 2014. The averaged hydraulics, such as flow rate, velocity, channel width, and water depth, observed in this area are summarized in Table 3.

Table 3. Summary of hydraulic parameters in study area of Nakdong River.

Date	River	Q (m^3/s)	h (m)	W (m)	$\bar{u}$ (m/s)	R_c (m)
27 August 2014	Nakdong	312.3	5.47	436.6	0.154	1480

The initial value of the dimensionless transverse dispersion coefficient was calculated using Equation (12) proposed in this study, resulting in a value of 0.81. Given that the parameter $\frac{\bar{u}}{u_*} \frac{h}{R_c}$ is 0.037 within this reach, Equation (12) was selected due to the condition

$\frac{\bar{u}}{u_*} \frac{h}{R_c} < 0.04$. After assigning initial values to the numerical model, the precise values of the longitudinal and transverse dispersion coefficients were determined by optimizing the fit between measured concentration data and simulated data. Consequently, the calibrated value of the transverse dispersion coefficient was found to be 0.97. This value is relatively close to the initial value of 0.81, obtained from Equation (12). A comparison between the measured concentration distribution and the simulated distribution is presented in Figure 7. Additionally, an outline of the field application of proposed equations is summarized in Figure 8.

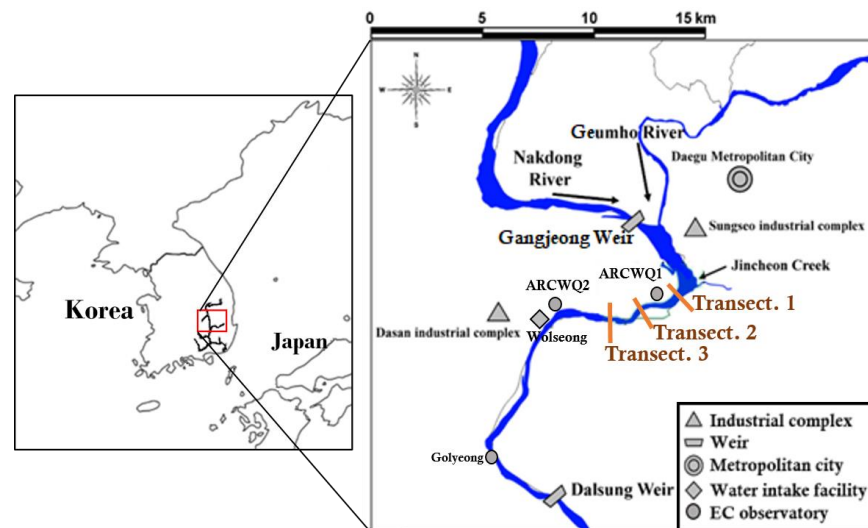


Figure 6. Schematic diagram of the field site of model application.

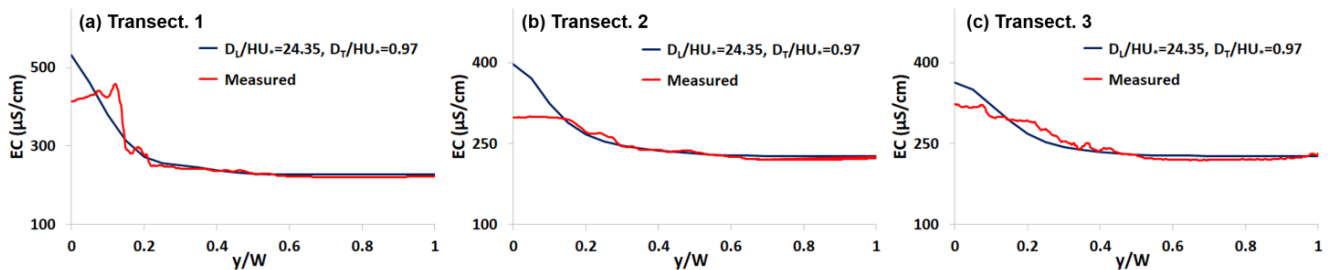


Figure 7. Comparison results between measured and simulated concentration.

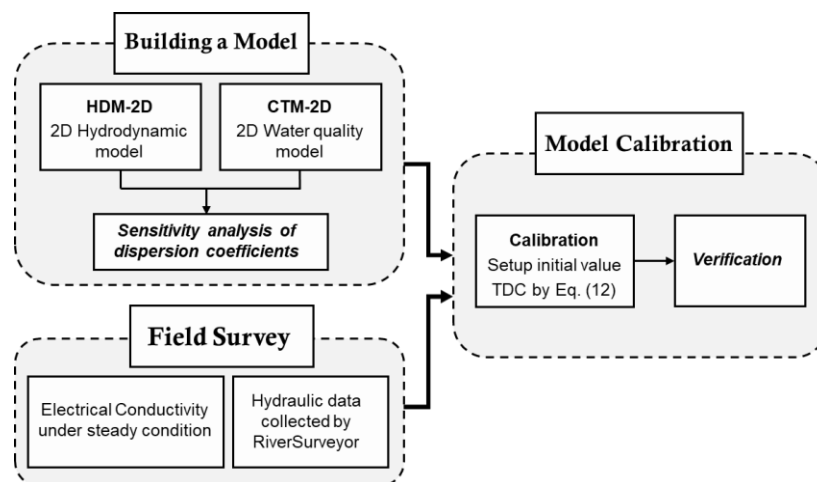


Figure 8. Outline of the field application of proposed equations.

It examines what position the equations developed in this study occupy within the hierarchy of methodologies for determining the transverse dispersion coefficient in river mixing analysis. As shown earlier in Figure 1, Baek and Seo [31] presented a flowchart that was proposed with the criteria to select a suitable method under specific hydraulic and geometric conditions of the river among the so-called estimation method. Furthermore, Baek [32] proposed a flowchart concerned about the so-called observation method. The observation methods can be employed to compute the transverse dispersion coefficient in rivers when tracer concentration data are available. Among the most commonly utilized techniques is the change of moment method [33–35]. Routing procedures have also been widely employed to evaluate dispersion coefficients for analyzing the mixing characteristics of transient concentration conditions [36,37].

As shown in Figure 1, the proposed formulas in this study belong to the estimation method and are helpful when secondary flow data are unavailable in natural meandering rivers. In particular, when $\frac{\bar{u}}{u_*} \frac{h}{R_c} > 0.04$, it is recommended to utilize Equation (13) proposed in this study instead of relying on existing empirical equations, especially under conditions that are expected to increase transverse dispersion due to severe curvature. However, for many natural river reaches that fall within the conditions of $\frac{\bar{u}}{u_*} \frac{h}{R_c} < 0.04$, it appears more appropriate to employ Equation (12) presented in this study or the existing empirical formula by Baek and Seo [1].

In situations where it is challenging to specify a single value for the radius of curvature (e.g., when there are numerous alternating curves), sinuosity can be used as an alternative parameter. In such cases, it is recommended to consider Jeon et al. [8] or Aghababaei et al. [9] among the existing empirical formulas. Furthermore, for creeks or artificial waterways with a small channel-width, Baek and Seo [31] suggested opting for Bansal [12] or Deng et al. [13], as shown in Figure 1.

5. Conclusions

This study proposes a new expression for the transverse dispersion coefficient, which is based on dispersion and hydraulic data sets obtained from tracer experiments conducted on natural rivers worldwide. The proposed formulas, incorporating a single dimensionless hydraulic parameter, $\frac{\bar{u}}{u_*} \frac{h}{R_c}$, demonstrate promising results in terms of accuracy and effectiveness compared to existing equations. Through a comparison with four commonly used empirical equations in the field, the proposed formulas exhibit superior performance. In contrast, the formula by Baek and Seo [1], derived from a theoretical background, does not present significant advantages in maintaining its form. Therefore, the newly developed formulas in this study, which simplify and utilize the dimensionless hydraulic parameter, represent a favorable option for estimating transverse dispersion in natural meandering rivers when both tracer and secondary flow data are unavailable.

In conclusion, the proposed formulas provide a simple and easily implementable approach for estimating transverse dispersion coefficients in meandering natural rivers. Their effectiveness and accuracy have been demonstrated through comparisons with existing equations, highlighting their potential for practical applications in predicting and mitigating the effects of pollutants or other materials transported by river flow. It is recommended to conduct further research and validation of the formula in various river systems to enhance its robustness and broaden its applicability.

Author Contributions: Conceptualization, K.O.B.; methodology, D.Y.L.; software, D.Y.L.; validation, D.Y.L.; writing—original draft preparation, K.O.B.; writing—review and editing, K.O.B.; visualization, D.Y.L.; supervision, K.O.B.; All authors have read and agreed to the published version of the manuscript.

Funding: This research has been performed as Project Open Innovation R&D (OTSK_2022_016) and supported by K-water.

Data Availability Statement: Please contact the corresponding author for data.

Conflicts of Interest: The authors declare no conflict of interest.

Notation List

α, β	regression coefficients
D_T	the transverse dispersion coefficient
g	the gravitational acceleration
h	the water depth
R_c	the radius of curvature
S_0	the bed slope
S_n	the sinuosity
$\bar{u}$	the mean velocity
u_*	the frictional velocity and defined as $\sqrt{ghS_0}$ in an open channel here
W	the channel width

References

- Baek, K.O.; Seo, I.W. Empirical equation for transverse dispersion coefficient based on theoretical background in river bends. *Environ. Fluid Mech.* **2013**, *13*, 465–477. [\[CrossRef\]](#)
- Fischer, H.B. Transverse mixing in a sand-bed channel. *US Geol. Surv. Prof. Pap.* **1967**, *575-D*, 267–272.
- Fischer, H.B.; List, J.E.; Koh, C.R.; Imberger, J.; Brooks, N.H. *Mixing in Inland and Coastal Waters*; Academic Press: San Diego, CA, USA, 1979.
- Rutherford, J.C. *River Mixing*; Wiley: Chichester, UK, 1994.
- Fischer, H.B. The effect of bends on dispersion in streams. *Water Resour. Res.* **1969**, *5*, 496–506. [\[CrossRef\]](#)
- Yotsukura, N.; Sayre, W.W. Transverse mixing in natural channels. *Water Resour. Res.* **1976**, *12*, 695–704. [\[CrossRef\]](#)
- Sayre, W.W. Shore-attached thermal plumes in rivers. In *Modelling in Rivers*; Shen, H.W., Ed.; Wiley-Interscience: London, UK, 1979.
- Jeon, T.M.; Baek, K.O.; Seo, I.W. Development of an empirical equation for the transverse dispersion coefficient in natural streams. *Environ. Fluid Mech.* **2007**, *7*, 317–329. [\[CrossRef\]](#)
- Aghababaei, M.; Etemad-Shahidi, A.; Jabbari, E.; Taghipour, M. Estimation of transverse mixing coefficient in straight and meandering streams. *Water Resour. Manag.* **2017**, *31*, 3809–3827. [\[CrossRef\]](#)
- Boxall, J.B.; Guymer, I. Analysis and prediction of transverse mixing coefficients in natural channels. *J. Hydraul. Eng.* **2003**, *129*, 129–139. [\[CrossRef\]](#)
- Baek, K.O.; Seo, I.W. Transverse dispersion caused by secondary flow in curved channels. *J. Hydraul. Eng.* **2011**, *137*, 1126–1134. [\[CrossRef\]](#)
- Bansal, M.K. Dispersion in natural streams. *J. Hydraul. Div.* **1971**, *97*, 1867–1886. [\[CrossRef\]](#)
- Deng, Z.; Singh, V.P.; Bengtsson, L. Longitudinal dispersion coefficient in straight rivers. *J. Hydraul. Eng.* **2001**, *127*, 919–927. [\[CrossRef\]](#)
- Almquist, C.W.; Holley, E.R. *Transverse Mixing in Meandering Laboratory Channels with Rectangular and Naturally Varying Cross Sections*; Technical Report CRWR-205; University of Texas: Austin, TX, USA, 1985.
- Yotsukura, N.; Fischer, H.B.; Sayre, W.W. *Measurement of Mixing Characteristics of the Missouri River between Sioux City, Iowa and Plattsmouth, Nebraska*; USGS Water, Supply Paper No. 1899-G; USGPO: Washington, DC, USA, 1970. [\[CrossRef\]](#)
- Yotsukura, N.; Cobb, E.D. *Transverse Diffusion of Solutes in Natural Streams*; USGS Water, Supply Paper No. 582-C; US Government Printing Office: Washington, DC, USA, 1972.
- Fischer, H.B. Longitudinal dispersion and turbulent mixing in open channel flow. *Annu. Rev. Fluid Mech.* **1973**, *5*, 59–78. [\[CrossRef\]](#)
- Holley, E.R.; Abraham, G. Field Tests on Transverse Mixing in Rivers. *J. Hydraul. Div.* **1973**, *99*, 2313–2331. [\[CrossRef\]](#)
- Sayre, W.W.; Yeh, T. *Transverse Mixing Characteristics of the Missouri River Downstream from the Cooper Nuclear Station*; Rep. No. 145; University of Iowa: Iowa City, IA, USA, 1973.
- Jackman, A.P.; Yotsukura, N. *Thermal Loading of Natural Streams*; USGS Professional Paper No. 991; US Government Printing Office: Washington, DC, USA, 1977.
- Beltaos, S. Transverse mixing tests in natural streams. *J. Hydraul. Div.* **1980**, *106*, 1607–1625. [\[CrossRef\]](#)
- Lau, Y.L.; Krishnappan, B.G. Modelling transverse mixing in natural streams. *J. Hydraul. Div.* **1981**, *107*, 209–226. [\[CrossRef\]](#)
- Somlyódy, L. An approach to the study of transverse mixing in stream. *J. Hydraul. Res.* **1982**, *20*, 203–220. [\[CrossRef\]](#)
- Holly, F.M.; Nemat, G. Field calibration of stream-tube dispersion model. *J. Hydraul. Eng.* **1983**, *109*, 1455–1470. [\[CrossRef\]](#)
- Demetropoulos, A.C.; Stefan, H.G. Transverse mixing in wide and shallow river: Case study. *J. Hydraul. Eng.* **1983**, *109*, 685–699. [\[CrossRef\]](#)
- Seo, I.W.; Baek, K.O.; Jeon, T.M. Analysis of transverse mixing in natural streams under slug tests. *J. Hydraul. Res.* **2006**, *44*, 350–362. [\[CrossRef\]](#)

27. Seo, I.W.; Choi, H.J.; Kim, Y.D.; Han, E.J. Analysis of Two-Dimensional Mixing in Natural Streams Based on Transient Tracer Tests. *J. Hydraul. Eng.* **2016**, *142*, 4016020. [[CrossRef](#)]
28. Pouchoulin, S.; Le Coz, J.; Mignot, E.; Gond, L.; Riviere, N. Predicting transverse mixing efficiency downstream of a river confluence. *Water Resour. Res.* **2020**, *56*, e2019WR026367. [[CrossRef](#)]
29. Song, C.G.; Seo, I.W.; Kim, Y.D. Analysis of secondary current effect in the modeling of shallow flow in open channels. *Adv. Water Res.* **2012**, *41*, 29–48. [[CrossRef](#)]
30. Seo, I.W.; Lee, M.E.; Baek, K.O. 2D modeling of heterogeneous dispersion in meandering channels. *J. Hydraul. Eng.* **2008**, *134*, 196–204. [[CrossRef](#)]
31. Baek, K.O.; Seo, I.W. Estimation of transverse dispersion coefficient for two-dimensional models of mixing in natural streams. *J. Hydraul. Res.* **2017**, *15*, 67–74. [[CrossRef](#)]
32. Baek, K.O. Flowchart on choosing optimal method of observing transverse dispersion coefficient for solute transport in open channel flow. *Sustainability* **2018**, *10*, 1332. [[CrossRef](#)]
33. Sayre, W.W.; Chang, F.M. *A Laboratory Investigation of Open Channel Dispersion Processes for Dissolved, Suspended, and Floating Dispersants*; Professional Paper No. 433-E.; U.S. Geological Survey: Reston, WV, USA, 1968.
34. Holley, E.R.; Siemons, J.; Abraham, G. Some aspects of analyzing transverse diffusion in rivers. *J. Hydraul. Res.* **1972**, *10*, 27–57. [[CrossRef](#)]
35. Beltaos, S. Evaluation of transverse mixing coefficients from slug tests. *J. Hydraul. Res.* **1975**, *13*, 351–360. [[CrossRef](#)]
36. Baek, K.O.; Seo, I.W. Routing procedures for observed dispersion coefficients in two-dimensional river mixing. *Adv. Water Res.* **2010**, *33*, 1551–1559. [[CrossRef](#)]
37. Baek, K.O.; Seo, W. On the methods for determining the transverse dispersion coefficient in river mixing. *Adv. Water Res.* **2016**, *90*, 1–9. [[CrossRef](#)]

Disclaimer/Publisher's Note: The statements, opinions and data contained in all publications are solely those of the individual author(s) and contributor(s) and not of MDPI and/or the editor(s). MDPI and/or the editor(s) disclaim responsibility for any injury to people or property resulting from any ideas, methods, instructions or products referred to in the content.