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Modeling of Distributed Control System for Network of Mineral Water Wells

Ivan M. Pershin [1,2], Elena G. Papush [2], Tatyana V. Kukharova [3,*] and Vladimir A. Utkin [4]

1 Department of Automation and Control Processes, Saint Petersburg Electrotechnical University "LETI", 197022 Saint Petersburg, Russia; pershiv_im@rambler.ru or ivmp@yandex.ru
2 Department of Control Systems and Information Technologies, Pyatigorsk Institute (Branch), North Caucasus Federal University, 357500 Pyatigorsk, Russia; elenapapush@rambler.ru
3 Department of System Analysis and Control, Saint-Petersburg Mining University, 199106 Saint Petersburg, Russia
4 Pyatigorsk State Research Institute of Balneology by the Federal Medical and Biological Agency, 357500 Pyatigorsk, Russia; vladuk@mail.ru
* Correspondence: unit-4@yandex.ru

Abstract: The article is devoted to solving the problem of designing a distributed control system for a network of production wells on the example of mineral water deposits in the Caucasus Mineral Waters region, Russia. The purpose was to determine the set of parameters of the control system to ensure technologically effective and safe operating modes of mineral water deposits. A mathematical model of the deposit was developed taking into account the given configuration and production rate of the network of the wells. The detailed algorithm is presented for designing the control system under consideration based on the frequency concept of analysis and synthesis for distributed control systems. The experimental tests and model validation were performed at the production wells facility of "Narzan", Kislovodsk, Russia. The results of modeling and field experiments confirmed the adequacy of the mathematical model and the effectiveness of the proposed algorithm. The authors came to the conclusion that the adapted mathematical model can be used to create a regional automated field cluster management system for monitoring, operational management and forecasting the nature of real hydrogeological processes and ensuring their stability.

Keywords: distributed control system; operating modes of mineral water deposits; controller parameters

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Citation: Pershin, I.M.; Papush, E.G.; Kukharova, T.V.; Utkin, V.A. Modeling of Distributed Control System for Network of Mineral Water Wells. *Water* **2023**, *15*, 2289. https://doi.org/10.3390/w15122289

Academic Editors: Andrea Zanini and Fulvio Celico

Received: 14 May 2023
Revised: 15 June 2023
Accepted: 16 June 2023
Published: 19 June 2023

1. Introduction

More than 200 years ago, mineral water (MW) deposits were explored in the south of Russia, where the largest resort region called the Caucasus Mineral Waters (CMW) was formed. This area includes such world famous resort towns as Kislovodsk, Essentuki, Pyatigorsk, Zheleznovodsk. The economic and ecological well-being of the CMW region is largely determined by balneological resources, including the state of mineral water deposits used for health spa medical treatment and industrial MW bottling.

The growth of technological and technogenic load on the hydrogeological system naturally causes negative consequences for the local resorts. The construction of large sanatorium buildings in the 50–70th of the 20th century, the increase in production of mineral water, environmental degradation together led to degradation of quality indicators of the mineral composition for a number of MW springs. Over the last 25 years, the transition to a market economy has been accompanied by the replenishment of the list of subsoil users and a significant increase in the production of mineral waters. The volume of production has more than doubled, and in some cases such an increase has been made without any serious justification of operating modes. As an example, we can consider well No. 5/0-bis of Dolomitny Narzan mineral water, which cannot be used for drinking both due to bacteriological contamination and due to the complete loss of conditions (by 2012, average annual values of mineralization amounted to 1.16 g/dm^3 (norm is 2–3 g/dm^3), dissolved carbon dioxide-to 0.40 g/dm^3 (norm is 1.0–2.5 g/dm^3)) [1]. The dynamics of changes in mineralization and dissolved carbon dioxide in Dolomite Narzan and in Sulfate Narzan is given in [1]. Existing problems and ways to solve them in relation to groundwater in sanatorium-resort areas are described in detail in [1–3]. There were also studies of the industrial facilities influence at water quality, in particular gravel pits [4] and waste disposal sites [5]. Works [6–8] are devoted to the study of the factors that determine the quality and change in groundwater reserves. Works [9,10] are devoted to the problems of the risk of polluted water for health. The problems of transboundary groundwater resources management are considered in [11,12].

Currently, there are local control systems for individual wells, as well as a reference network of control wells for monitoring the state of the hydrogeological system [13,14]. An important practical and ecological task is to create a unified system of operational monitoring [15–17], forecasting [18,19] and control of processes in the hydrogeological system of the CMW region.

The procedure for developing a system for operational management of MW wells operation modes includes the following stages:

- determination of permissible technologically safe modes during the exploitation of MW deposits (formation of target functions for well network control systems) [20,21];
- determination of the optimal number of production wells;
- calculation of parameters of distributed controllers for local control systems [1,22,23];
- monitoring the current state of the hydrogeological system;
- forecasting processes in the hydrogeological system for short-term (up to 10 years) and long-term (up to 100 years) perspectives.

To date, a number of methods have been developed for the modeling [24–26], analysis [27–29] and synthesis of control systems for objects with distributed parameters [30–32], including hydrogeological processes, as well as technical means for their implementation [1,33,34]. An algorithm for management decision-making on such projects implementation was proposed in [35]. Evaluation methods for hydrogeological parameters are described in [36]. A simulation/optimization model for the identification of unknown groundwater well locations and pumping rates for two-dimensional aquifer systems was proposed in [37]. A way to implement process performance monitoring is described in [38].

When designing a distributed control system for a network of MW production wells, it is necessary to take into account the effect of pumping rate from each individual well on the lowering level in neighboring wells. Obviously, pumping rate impact can't be similar for all wells and it must be calculated for each well, taking into account their relative position. The solution for a linear wells location is given in [33,39].

The CMS region is located in the foothills of the Caucasus Mountains. The terrain of the region in the northeastern part is flat with the presence of laccolithic mountains. The southwestern part of the region is characterized by features of a mountainous landscape. The features of the region terrain determine the fact that the isolines of the piezometric levels and levels of distribution of groundwater mineralization have a shape close to elliptical [1]. So, in practice, there is a problem when it is necessary to create a distributed control system for an existing deposit, in which the production wells are located in an ellipse or circle of specified dimensions. This configuration corresponds to the natural boundaries of many explored mineral and artesian water deposits.

The difference between this mathematical model and the previously considered models is the configuration of the network of production MW wells located on the ground surface in an elliptical shape.

As a rule, the choice of the target function is determined by economic interests, and the specified restrictions are dictated by environmental requirements. The purpose of this study was to determine the optimal number of production MW wells for a network of elliptical shape that provide maximum profit during a given deposit lifetime. The solution to this problem is limited by the permissible lowering of the water level in wells (less than

2 m). The presented adapted model can also be used for development of a regional network control system production MW wells.

2. Materials and Methods

The development of the production MW wells network control system is based on the frequency analysis and synthesis concept for distributed systems. The theoretical provisions of this method are described in details in [1]. The mathematical model of the hydrogeological process for the MW deposit can be expressed as a partial differential transport equation [1,33]. The boundary conditions are selected based on the analysis of the structure of aquifers, according to the results of geological surveys [33,40,41]. The physical characteristics of aquifers can be set using the method of estimating parameters for aquifers based on experimental filtration operations under hydraulic connection conditions [1].

In this study, the dynamic characteristics of the control object were determined based on the results of experiments at production MW wells (Joint stock company "Narzan", Kislovodsk, Russia). The hydrogeological profile of the Kislovodsk deposit is given in [1,39]. The estimation of variation in the dynamic characteristics of distributed objects during approximation is given in [1]. An approximating link was used to describe the static characteristics of the object under consideration [42]. The input effect on the object (pumping rate) is formed in the form of spatial modes [13].

The hydrogeologicalprocess for the MW deposit is considered as a control object, the scheme of which illustrated in Figure 1.

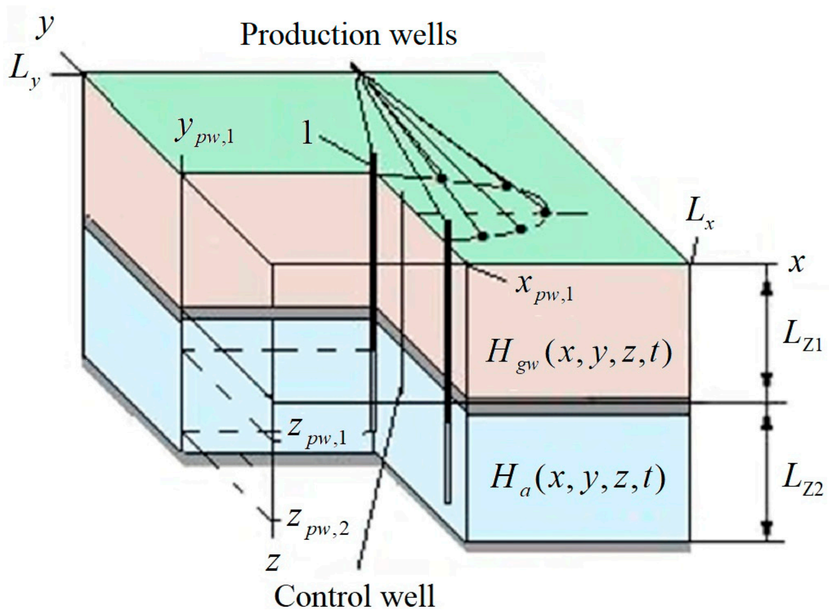

Figure 1. Mineral water (MW) deposit scheme with control and production wells.

The cross section in Figure 1 shows free aquifer and confined aquifer separated by impervious layer. Production wells are located along the perimeter of the ellipse. The control well is used to monitor the pressure reduction in the confined aquifer.

Let us describe the groundwater flows in the free aquifer that lies on the first impervious layer from the surface by Equation (1) and in the confined aquifer by Equation (2) according to [1,33]:

$$\frac{\partial H_{gw}(x,y,z,t)}{\partial t} = k_{gw,x} \times \frac{\partial^2 H_{gw}(x,y,z,t)}{\partial x^2} + k_{gw,y} \times \frac{\partial^2 H_{gw}(x,y,z,t)}{\partial y^2} + k_{gw,z} \times \frac{\partial^2 H_{gw}(x,y,z,t)}{\partial z^2}, \quad 0 < x < L_x;\ 0 < y < L_y;\ 0 < z < L_{Z1}, \tag{1}$$

$$\frac{\partial H_a(x,y,z,t)}{\partial t} = \frac{1}{\eta_a} \times \left(k_{a,x} \times \frac{\partial^2 H_a(x,y,z,t)}{\partial x^2} + k_{a,y} \times \frac{\partial^2 H_a(x,y,z,t)}{\partial y^2} + k_{a,z} \times \frac{\partial^2 H_a(x,y,z,t)}{\partial z^2} \right) + \\ + V(x_{pw,i}, y_{pw,i}, t) \times \delta(x_{pw,i}, y_{pw,i}, z_{pw,1} \le z \le z_{pw,2}), \\ V(x_{pw,i}, y_{pw,i}, t) = K_i \times Q_i(t), 0 < x < L_x;\ 0 < y < L_y;\ L_{Z1} < z < L_{Z2}, \quad (2)$$

where: H_{gw} is the water column height in the free aquifer, m; H_a is the water column height in the confined aquifer, m; $k_{gw,x}$, $k_{gw,y}$, $k_{gw,z}$ are the hydraulic conductivities along spatial coordinates in the free aquifer, m/day, $k_{a,x}$, $k_{a,y}$, $k_{a,z}$ are the hydraulic conductivities along spatial coordinates in the confined aquifer, m/day; $x_{pw,i}$, $y_{pw,i}$ are coordinates of the location of production wells, m; η_a is the coefficient of elastic capacity, m^{-1}, in this case $\eta_a = 1.6 \times 10^{-3}$ m^{-1}; x, y, z are the spatial coordinates, m; N is number of wells, [–]; i is an order number of the production well, $i = 1 \ldots N$, [–]; t is time, day; a function $\delta(x_{pw,i}, y_{pw,i}, z_{pw,1} \le z \le z_{pw,2})$ = 1, if $x = x_{pw,i}$, $y = y_{pw,i}$, $z_{pw,1} \le z \le z_{pw,2}$ (that corresponds to an imperfect water intake from an aquifer), and $\delta(x_{pw,i}, y_{pw,i}, z_{pw,i} \le z \le z_{pw,i})$ = 0 in all other cases, [–]; V is a decrease in the water column height due to the impact of the production rate Q_i of the i-th well, m/day; K_i is a coefficient determined experimentally, in this case $K_i = 0.187$ m^{-2}; $Q_i(t)$ are production well rates, m^3/day. In modeling, the condition was accepted that production rates are evenly distributed over the interval $z_{pw,1} \le z \le z_{pw,2}$.

The hydraulic conductivities (m/day) along spatial coordinates are taken on the basis of experimental data: $k_{gw,x} = 0.099$; $k_{gw,y} = 0.097$; $k_{gw,z} = 0.095$; $k_{a,x} = 0.199$; $k_{a,y} = 0.197$; $k_{a,z} = 0.108$.

The leakage processes between free aquifer and the upper boundary of the confined aquifer are given in the following form:

$$\frac{\partial H_{gw}(x,y,L_{Z1},t)}{\partial t} = b_o \times \left(H_a(x,y,L_{Z1},t) - H_{gw}(x,y,L_{Z1},t) \right), \\ \frac{\partial H_a(x,y,L_{Z1},t)}{\partial t} = -b_o \times \left(H_a(x,y,L_{Z1},t) - H_{gw}(x,y,L_{Z1},t) \right), \quad (3)$$

where $b_o = 3 \times 10^{-5}$ day^{-1} is a leakage parameter.

For the lower boundary of the confined aquifer:

$$\frac{\partial H_a(x,y,L_{Z2},t)}{\partial z} = 0. \quad (4)$$

For the lateral boundaries of the area under consideration:

$$H_{gw}(0,y,z,t) = H_{gw}(L_x,y,z,t) = H_{gw}(x,0,z,t) = H_{gw}(x,L_y,z,t) = h_{gw,0}, \\ H_a(0,y,z,t) = H_a(L_x,y,z,t) = H_a(x,0,z,t) = H_a(x,L_y,z,t) = H_{a,0}. \quad (5)$$

When establishing boundary conditions along the y coordinate, it was assumed that the thickness of the confined aquifer is sufficient so that disturbances from production wells do not affect the state of the reservoir at the boundary points.

Initial conditions of undisturbed aquifers:

$$h_{gw,0}(x,y,z,0) = z,\ 0 \le x \le L_x,\ 0 \le y \le L_y,\ 0 \le z \le L_{Z1}, \\ H_{a,0}(x,y,z,0) = 70 - 8 \times x/L_x,\ 0 \le x \le L_x,\ 0 \le y \le L_y,\ L_{Z1} \le z \le L_{Z2}. \quad (6)$$

Geometric data of the MW deposit: $L_x = 150$ m; $L_y = 120$ m; $L_{z1} = 55$ m; $L_{z2} = 85$ m.

Production wells are located along the perimeter of an ellipse, the half-axes of which are equal to $a = 35$ m, $b = 20$ m. The control well is offset along the y axis by a distance of 10 m relative to well 1.

3. Results

3.1. Study of the Characteristics of the Hydrogeological Process

Usually, during a mineral water field development, one production and one control wells are equipped. In this case, the production well 1 and the control well have equipment for measuring the level decrease. At the first stage, the operation of only these two wells was simulated in order to study the functional dependence between the level lowering in one well and the pumping rate of another well taking into account the distance between them.

In the course of experimental studies at the control object, a constant pumping rate in the production well was provided equal to Q_1 = 100 m^3/day. This value was used to determine the static transfer coefficients K_p and K_c, respectively, for the production and control wells. The static transfer coefficient is equal to the steady-state level change in the corresponding well measured at the midpoint $z = (z_{pw,1} + z_{pw,2})/2$ m, divided by the pumping rate (input action) Q_1:

$$K_p = -0.512/(-100) = 5.12 \times 10^{-3}\ \text{m}^{-2};\ K_c = -0.202/(-100) = 2.02 \times 10^{-3}\ \text{m}^{-2}. \quad (7)$$

Structural schemes of approximating links for distributed objects have been considered in [33,42]. In these studies, it is shown that in the case of two wells, when describing the static characteristics of the object, the following structure of the approximating link can be recommended:

$$W_{ap} = \frac{K}{\sqrt{B}} \times \exp\left(-\sqrt{B} \times \Delta y\right), \quad (8)$$

where: W_{ap} is the transfer function of the approximating link; s is differential operator; K, B are parameters whose values are determined using experimental data or numerical modeling.

To find the parameters of the approximating link, the following data must be inserted into the Equation (8): static transfer coefficients K_p, K_c, as well as geometric data: the radius of the well Δy_1 = 0.2 m [1] and the distance between the production well 1 and the control well Δy_2 = 10 m.

$$\begin{cases} 5.012 \times 10^{-3} = \frac{K}{\sqrt{B}} \times \exp\left(-\sqrt{B} \times 0.2\right), \\ 2.02 \times 10^{-3} = \frac{K}{\sqrt{B}} \times \exp\left(-\sqrt{B} \times 10\right). \end{cases} \quad (9)$$

The solution of the system of Equation (9) is that $K = 4.9522 \times 10^{-4}$ m^{-2}; $B = 9.007 \times 10^{-3}$ [–]. Thus, the static transfer coefficient of an object can be written as Equation (10):

$$W_{ap} = \frac{4.9522 \times 10^{-4}}{\sqrt{9.007 \times 10^{-3}}} \cdot \exp\left(-\sqrt{9.007 \times 10^{-3}} \times \Delta y\right) = 5.218 \times 10^{-3} \times \exp(-0.0949 \times \Delta y), \quad (10)$$

where Δy is the distance from the well to the point under consideration, m.

Based on the simulation results, the plot of the static transfer coefficient of the object was presented as a function of the distance between the well and the given point (Figure 2).

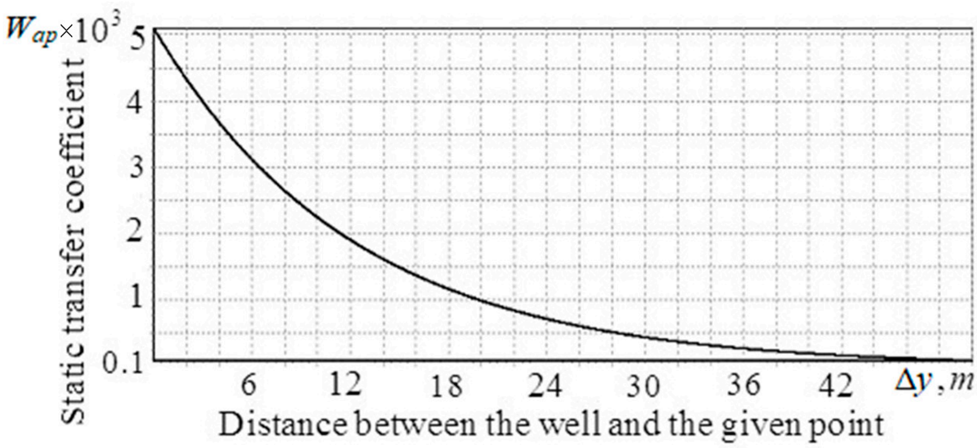

Figure 2. Static transfer coefficient W_{ap} (Δy) as a function of distance.

3.2. Determination of the Optimal Number of Production Wells

In the second stage of the control system development the simulation of the joint operation of the MW wells network in the form of an ellipse was performed. The layout is shown in Figure 3.

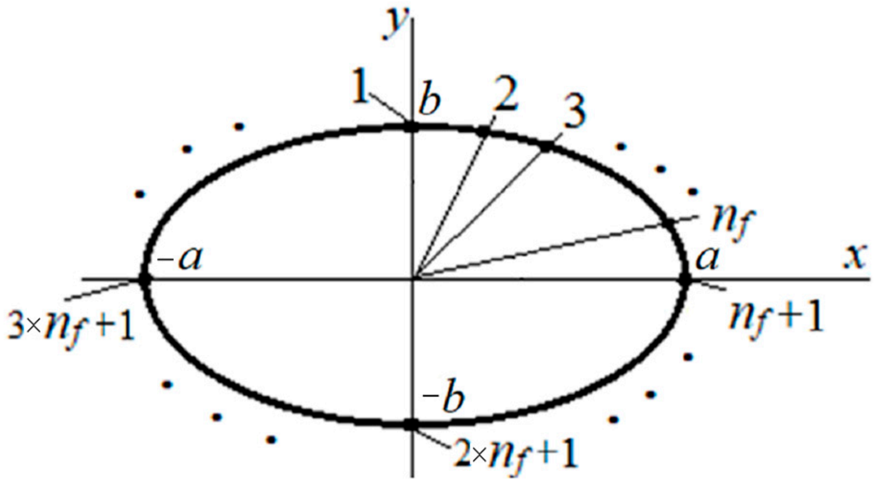

Figure 3. Layout of wells.

The coordinates of production wells are defined as follows: the first well is located at point 1 ($x = x_1 = 0$; $y = y_1 = b$); then the angle $\pi/2$ rad is divided into n_f parts. The canonical ellipse equation and the straight line equation are used to determine the coordinates of the second well:

$$\begin{cases} y_i = x_i \times tg\left(\frac{\pi}{2} \times \left(1 - \frac{(i-1)}{n_f}\right)\right), \\ \frac{x_i^2}{a^2} + \frac{y_i^2}{b^2} = 1. \end{cases} \quad (11)$$

The solution of the system of Equation (11) for the given values a, b, n_f and i provides the coordinates of the point i (x_i, y_i). The coordinates of the other points of the ellipse were defined in the similar way. A total of $4n_f$ points were obtained.

The problem of optimizing the number of production wells has the following statement: for the deposit under consideration (Figures 1 and 3), it is necessary to determine the number of production wells ($4n_f$) that provide the highest profit for the period of operation of the deposit, taking into account the restrictions described below.

In this case, the following conditions must be met: the maximum permissible lowering of the level in the production wells $\Delta h = 2$ m; well radius $r_w = 0.2$ m; MW wells operate 24 h per day for 10 years (3650 days); the cost of 1 m^3 of extracted mineral water p_1 is 350 rubles; expenses for the maintenance of buildings and equipment and personnel for 10 years of operation C_p is 520 million rubles, the cost of drilling, construction, equipment and maintenance of each well for 10 years C is 45 million rubles; subsoil use tax is 7.5% [43].

The static transfer coefficient of the object is written by Equation (10). The effect of production wells pumping rate on lowering of the level in i-th production well in accordance with [42] can be represented as Equation (12):

$$\Delta h = \frac{K_i \times Q_i}{\sqrt{B}} \times \exp(-\beta \times \Delta r_w) + \sum_{j=1, j\neq i}^{4nk} \frac{K_j \times Q_j}{\sqrt{B}} \times \exp\left(-\beta \times \Delta r_{i,j}\right),\ i = 1, \ldots 4n_f, \quad (12)$$

where: Δh is the lowering of the level in the i-th well ($i = 1 \ldots 4n_k$), m; r_w is the radius of the well, m; $\Delta r_{i,j}$ is the distance from the i-th to the j-th interacting wells, m; Q_i, Q_j are the pumping rates of the i-th and j-th wells, m^3/day.

In the mathematical model, Equation (7) was presented in matrix form. The total production rate Q can be determined as the sum of the n-well production rates:

$$Q = \sum_{i=1}^{4n_f} Q_i. \tag{13}$$

Knowing total production rate, the highest profit for 10 years of deposit operation can be estimated using Equation (14):

$$P = 3650 \times Q \times p_1 \, (1 - 0.075) - C_p - 4 \times C \times n_f. \tag{14}$$

Based on the results of calculations, the changes in the total pumping rate Q and profit P depending on the number of wells are shown in Figure 4.

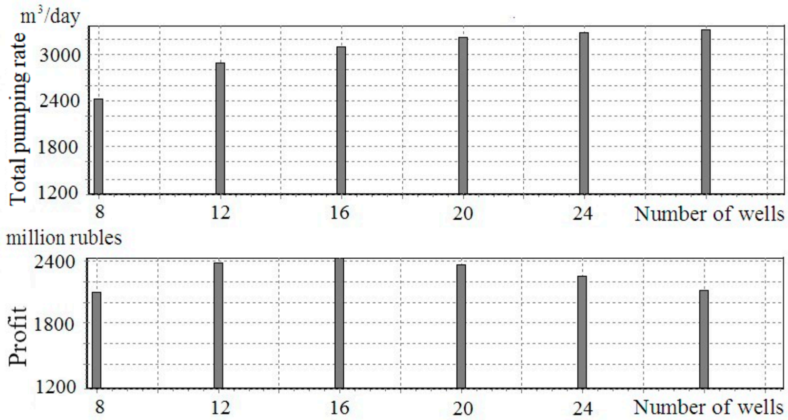

Figure 4. Changes in the total pumping rate Q and profit P depending on the number of wells.

As follows from the graphs, the maximum profit is achieved when using sixteen production wells. The projected income from the operation of the MW deposit for 10 years will amount to 2428.846 million rubles, at the total production rate of Q = 3104 m^3/day.

3.3. Synthesis of a Distributed Controller

Method of synthesis of distributed controllers is reported in [1]. These scientific studies describe a number of dynamic distributed links by analogy with typical dynamic links. These links can be used to control objects with distributed parameters. Also in study [1] characteristics of dynamic distributed links are investigated, rationale for their choice and application is presented. Methods of modeling and transition from concentrated systems to systems with distributed parameters was shown in [44].

The development of the control system for the linear location of wells is described in [33]. In the case the production wells that are positioned in the shape of the ellipse, it is necessary to make the following modifications.

The spatial modes of the input action are formed as follows:

$$\begin{aligned} &U_i = Q \times \cos(\psi_j \times \gamma_i\ ); \ \psi_j = 2 \times \pi \times j/L; \\ &\gamma_i = \Delta\gamma \times (i-1); \ i = 1,\ 2\ , \ldots 16; \ j = 1,\ 2,\ \ldots, \end{aligned} \tag{15}$$

where: ψ_j is the spatial frequency; γ_i is the distance between two considered wells; $\Delta\gamma$ is the distance between adjacent wells.

The spatial mode of the input action on the considered control object is distributed along the perimeter of the ellipse L, which is equal to 176.5 m. The formation of the first mode (j = 1) of the input action is shown in Figure 5. Other spatial modes are formed in a similar way (j = 2, 3, . . .).

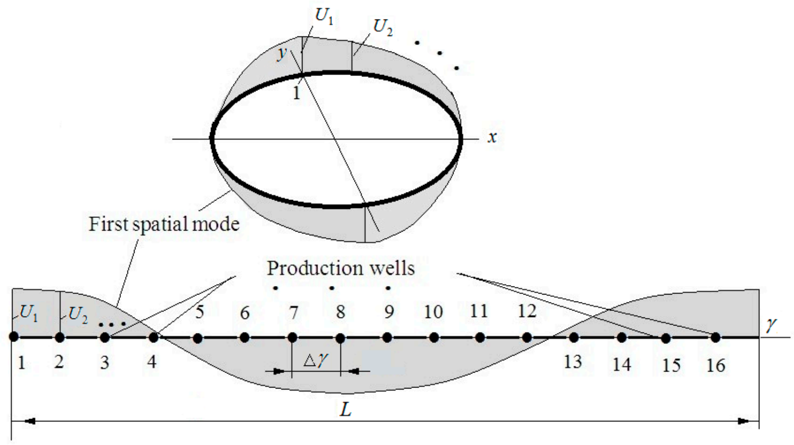

Figure 5. Formation of the first spatial mode.

As a rule, in accordance with the method of synthesis of distributed control systems, it is necessary to determine the reaction of an object to any two spatial modes. The first mode is formed in the space $\{\gamma\}$, and is implemented at the given points ($x_{pw,i}$, $y_{pw,i}$) in the space $\{x, y\}$ (at the points of location of the production wells). If the calculated value U_i ($i = 1 \ldots 16$) is displayed in the space $\{x, y\}$, then the following expression can be obtained:

$$U(x_{pw,i}, y_{pw,i}) = U_i,\ V(x_{pw,i}, y_{pw,i}, t) = K_i \times U(x_{pw,i}, y_{pw,i}),\ i = 1 \ldots 16. \tag{16}$$

Mathematical modeling allows determining the response of the object to the first and second spatial modes (Equation (15)), and calculating the transfer coefficient of the control object for the considered modes of the input action. The calculated values of the transfer coefficients (K_1 and K_2) and the generalized coordinate (G_1 and G_2) were determined as:

$$K_1 = 775.0\ \text{m}^{-2};\ G_1 = \psi_1{}^2 = (2 \times \pi \times 1/L)^2 = 1.003 \times 10^{-3}\ [\text{-}];\ K_2 = 583.3\ \text{m}^{-2};\ G_2 = \psi_2{}^2 = (2 \times \pi \times 2/L)^2 = 4.01 \times 10^{-3}\ [\text{-}]. \tag{17}$$

Based on the modeling, the plots of the input action and the output function were drawn for all wells, which allowed determining the phase shift of the output function relative to the input action. The graphs of the input action and the output function for the production well 8 is shown in Figure 6.

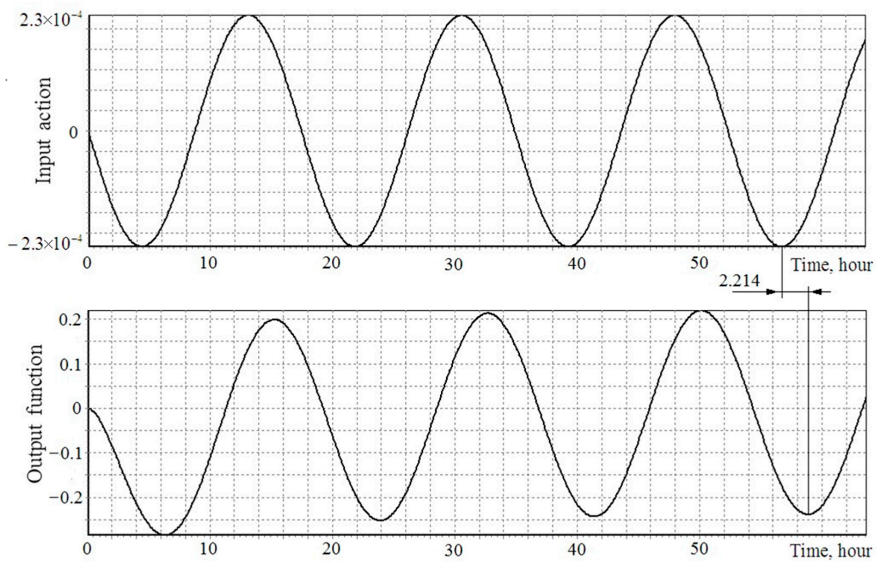

Figure 6. Graphs of input action and output function.

Further, the phase shift of the output function relative to the input action is determined graphically:

$$\Delta\varphi = 2 \times \pi \times (-2.214)/17.45 = -0.797 \text{ rad}. \quad (18)$$

It is shown in [33,39] that the dynamic characteristics of objects with a linear location of the well network can be described by the following structure of approximating links:

$$W_{ap}(s,G) = \frac{K}{\beta+1} \times \exp(-\beta \times D_\gamma),\ \beta = \left(\frac{s}{a} + G\right)^{0.5},\ G_j \le G \le \infty, \quad (19)$$

where: K, a, D_γ are the defined parameters.

Using the previously calculated values K_1 = 775.0 m^{-2}; G_1 = 1.003 × 10^{-3} [–]; K_2 = 583.3 m^{-2}; G_2 = 4.01 × 10^{-3} [–]; Δφ = −0.797 rad, an approximation model of the control object was determined:

$$W_{ap}(s,G) = \frac{1030.67}{1 + \sqrt{\frac{s}{5.916 \times 10^{-3}} + G}} \times \exp\left(-8.02 \times \sqrt{\frac{s}{5.916 \times 10^{-3}} + G}\right),\ G_1 \le G \le \infty. \quad (20)$$

The transfer function of the distributed controller [1,13] can be written as:

$$W(\gamma,s) = E_a \times \left(\frac{n_a - 1}{n_a} - \frac{1}{n_a}\nabla^2\right) + E_i \times \left(\frac{n_i - 1}{n_i} - \frac{1}{n_i}\nabla^2\right) \times \frac{1}{s} + E_d \times \left(\frac{n_d - 1}{n_d} - \frac{1}{n_d}\nabla^2\right) \times s, \quad (21)$$

where: E_a, E_i, E_d are the overall gains of the spatially amplifying, spatially integrating and spatially differentiating links, respectively; γ is the spatial coordinate; s is differential operator; ∇^2 is Laplacian; n_a, n_i, n_d are weighting factors of the spatially amplifying, spatially integrating, and spatially differentiating links.

During the synthesis of the distributed controller, the following restrictions must be taken into account: phase stability margin $\Delta\varphi \ge \pi/6$ rad; stability margin modulo $\Delta L \ge 10$ dB; parameter that takes into account parametric disturbances of the control object Δ = 2.5 [–].

As a result of the implementation of the synthesis procedure, parameters of distributed controller (Equation (21)) are obtained: E_a = 0.22999 m^2, E_i = 0.00108 m^2, E_d = 7.71 m^2, n_a = 1.065 [–], n_i = 1.0859 [–], $n_d \to \infty$ [–].

4. Discussion

Thus, when forming the control action for each of the well location points, the mismatch and the accumulated error both at the given and neighboring points will be taken into account, while the differential link will form the corresponding component of the control action only based on the change rate of the mismatch at a given point.

The block diagram of a distributed control system for a MW well network positioned in the form of an ellipse is shown in Figure 7.

To analyze the operation of a closed loop control system, a signal $\Delta H_a(i,t)$ is sent to the input of the controller, this signal is a mismatch between the desired value of the level $H_{ad}(i,t)$ in the well location zones and the current value of the level $H_a(i,t)$:

$$\Delta H_a\,(i,t) = H_{ad}\,(i,t) - H_a(i,t), \quad i = 1, 2, \ldots, 16. \quad (22)$$

The output is the function Q_i (pumping rate of the i-th production well):

$$\begin{gathered} Q_i(t) = 0.22999 \times \left(\tfrac{0.065}{1.065} \times \Delta H_a(i,t) - \tfrac{1}{1.065} \times \nabla^2 \Delta H_a(i,t)\right) + \\ +0.00108 \times \int_0^t \left(\tfrac{0.0859}{1.0859} \times \Delta H_a(i,\tau) - \tfrac{1}{1.0859} \times \nabla^2 \Delta H_a(i,\tau)\right) d\tau + 7.71 \times \tfrac{d\Delta H_a(i,t)}{dt}, \\ i = 1, \ldots 16, \end{gathered} \quad (23)$$

$$\nabla^2\Delta H_a(i,t) = \frac{\Delta H_a(i-1,t) - 2\times\Delta H_a(i,t) + \Delta H_a(i+1,t)}{\Delta\gamma^2},\ i = 2,\ldots 15,$$
$$\nabla^2\Delta H_a(1,t) = \frac{\Delta H_a(16,t) - 2\times\Delta H_a(1,t) + \Delta H_a(2,t)}{\Delta\gamma^2},$$
$$\nabla^2\Delta H_a(16,t) = \frac{\Delta H_a(15,t) - 2\times\Delta H_a(16,t) + \Delta H_a(1,t)}{\Delta\gamma^2}. \quad (24)$$

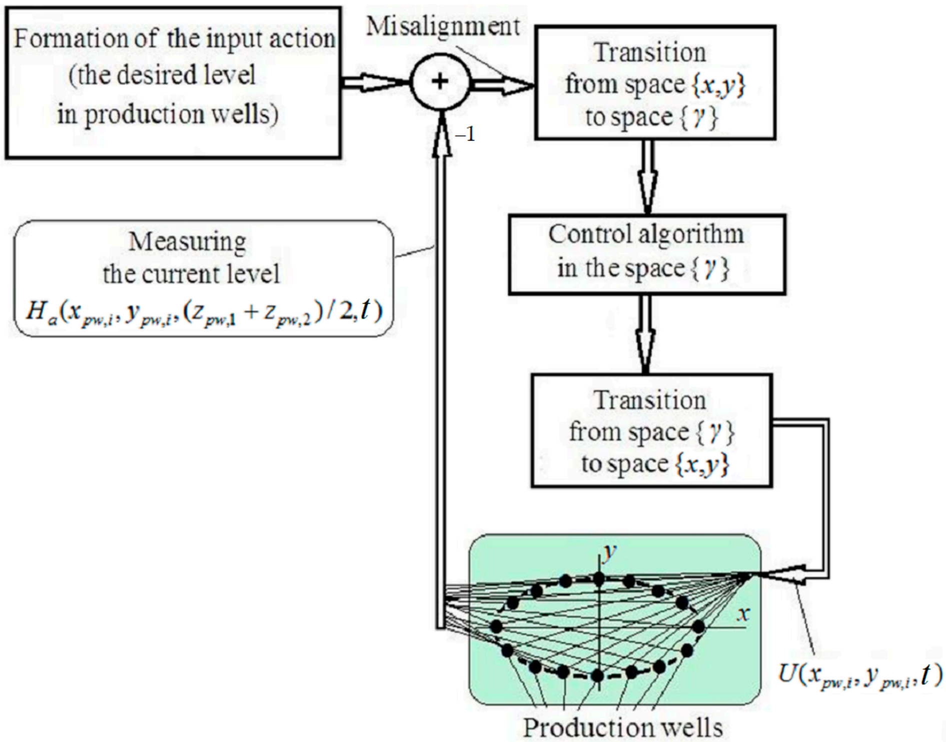

Figure 7. Block diagram of a distributed control system for a MW well network.

Based on the results of modeling the operation of a closed loop control system, transient graphs were obtained for all wells, which allowed us to assess the quality of regulation. Transient graph for the MW well 13 is shown in Figure 8.

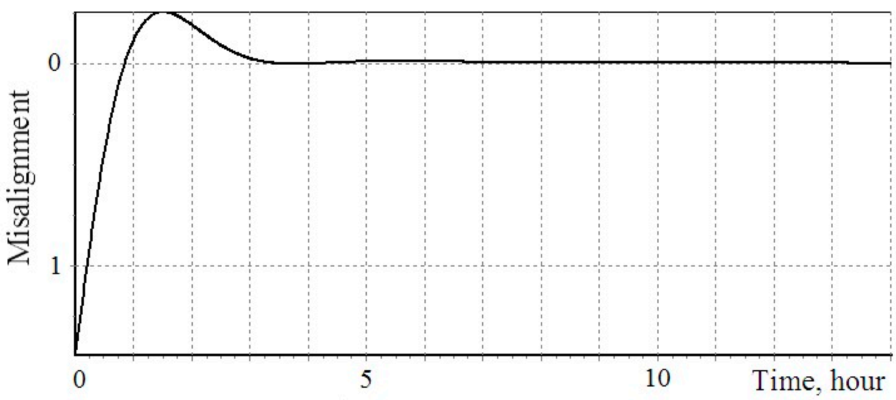

Figure 8. Transient graph for well 13.

According to the graph, it can be seen that the level of stabilization (zero misalignment) can be achieved after 13.25 h when the input action is applied and conclude that the obtained parameters of the controller can ensure stable operation of the control system under the specified restrictions.

Thus, a model of a distributed control system for a network of mineral water production wells has been developed. The novelty of the proposed model lies in the fact that, in comparison with previous models with a linear configuration of the MW network, the elliptical shape of the location of the MW wells was investigated. The elliptical configuration more accurately reflects the natural boundaries of the explored mineral and artesian water deposits [1]. Due to the consideration of another area of the MV field, a quantitative comparison of this model with the model presented in the study [33] is not possible. However, the linear configuration of wells during field development is not always possible due to the peculiarities of the terrain of the region under consideration and the need to preserve natural monuments, as well as objects of historical and cultural heritage. In addition, the elliptical location of production wells will allow equipping the MW field based on the conditions of proximity of mineralization levels and piezometric levels in the wells of the field if the isolines of these levels are close to elliptical, which is common for the region under consideration [1].

For the first time, the article presents an algorithm for developing a distributed control system for the considered well network, as well as solving practical problems, including determining dynamic characteristics for the control object under study and calculating the parameters of a distributed controller. Based on the results of modeling the operation of a closed loop control system, a transition processes schedules were obtained, which makes it possible to evaluate the quality indicators of the control process. When studying the hydrogeological processes, it is necessary to consider the influence of random factors. The issues of control of hydrodynamic processes under random impacts in the aquifers of mineral water deposits considered in [45]. The influence of random factors can be largely leveled by introducing feedback into the system, which is implemented in this work.

The advantages of a feedback system are regulation by deviation from the set value and robustness. Regulation by deviation from the set value (i.e., based on the results of current measurements) allows us to take into account changes in productivity during the operation of the field. Robustness (low sensitivity to parametric disturbances) makes it possible to maintain a given value of the output function (level lowering in production wells) with some changes in the parameters of the object, for example, hydraulic conductivity. The degree of robustness of the synthesized system can be the object of further research.

Also, the optimal number of production wells was found, based on the condition of maximum economical profit for the specified period of the deposit operation (for 10 years). When solving the problem, the restriction in the form of acceptable lowering of of the level in the production wells is taken into account. The projected income from the operation of the deposit for 10 years is approximately 2428.846 ruble for the total production rate of MW of Q = 3104 m^3/day.

The development is based on the frequency analysis and synthesis concept for distributed control systems. An important advantage of the frequency controller synthesis method is the possibility of developing mathematical models based on the dynamic characteristics of the object obtained experimentally.

5. Conclusions

The article presents a solution to the theoretical problem of developing an algorithm for the synthesis of a distributed control system with the location of production wells in the form of an ellipse. The practical problems were also solved. The optimal number and pumping rate of production wells in the well network providing the maximum profit for the enterprise during a given operation period of the deposit have been determined. The dynamic characteristics for the control object under consideration, as well as the parameters of a distributed controller for the control system of the network of mineral water wells have been calculated.

The developed algorithm for the synthesis of a distributed control system for a deposit, in which the production wells are located in an ellipse, can be applied to both free and confined aquifers with different thickness, length, lithology, hydraulic conductivity by making appropriate changes to the original mathematical model. The description of the boundary conditions can also be changed when considering another (or larger) area of the deposit, which may be a direction for further research, but does not entail restrictions in the application of the proposed algorithm for the synthesis of a mineral water well network control system.

The effectiveness of the proposed approach is confirmed by the results of modeling. The proposed model can be used for operational monitoring and forecasting of hydrogeological processes and creating a regional management system for mineral water production in the network of explored deposits.

Author Contributions: Conceptualization, I.M.P. and E.G.P.; methodology, I.M.P.; software, E.G.P. and T.V.K.; validation, E.G.P. and V.A.U.; formal analysis, T.V.K.; investigation, E.G.P.; resources, I.M.P. and E.G.P.; data curation, V.A.U.; writing—original draft preparation, E.G.P.; writing—review and editing, T.V.K.; visualization, E.G.P. and T.V.K.; supervision, I.M.P. and V.A.U.; project administration, I.M.P. and T.V.K. All authors have read and agreed to the published version of the manuscript.

Funding: This research received no external funding.

Data Availability Statement: Not applicable.

Acknowledgments: The experimental part of the study was performed on MW production wells facility that is property of the joint-stock company "Narzan", Kislovodsk, Russia.

Conflicts of Interest: The authors declare no conflict of interest.

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