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On the Evolution of Different Types of Green Water Events—Part II: Applicability of a Convolution Approach

Jassiel V. H. Fontes ¹ , Edgar Mendoza ^{2,*} , Rodolfo Silva ² , Irving D. Hernández ³ ,
Marcos A. González-Olvera ⁴ and Lizeth Torres ²

¹ Departamento de Engenharia Naval, Escola Superior de Tecnologia, Universidade do Estado do Amazonas, Manaus 69050-020, Brazil; jvfontes@uea.edu.br

² Instituto de Ingeniería, Universidad Nacional Autónoma de México, Mexico City 04510, Mexico; rsilvac@iingen.unam.mx (R.S.); ftorreso@iingen.unam.mx (L.T.)

³ Núcleo de Estruturas Oceânicas—NEO, Programa de Engenharia Oceânica, COPPE, Universidade Federal do Rio de Janeiro, Rio de Janeiro 20945-970, Brazil; irving.david@coppe.ufrj.br

⁴ Colegio de Ciencia y Tecnología, Universidad Autónoma de la Ciudad de México, Mexico City 09790, Mexico; marcos.angel.gonzalez@uacm.edu.mx

* Correspondence: emendozab@iingen.unam.mx

Abstract: Recent research related to the evolution of different types of green water events, generated in wave flume experiments, has shown that some events, such as plunging-dam-break (PDB) and hammer-fist (HF) types, can present multiple-valued water surface elevations during formation at the bow of the structure. However, the applicability of analytical models to capture the evolution (i.e., the spatio-temporal variation of water elevations) of these events has not been tested thoroughly. This could be useful when estimating green water loads in the preliminary design stage of marine structures. The present work extends the research by Fontes et al. (On the evolution of different types of green water events, *Water*, 13, 1148, 2021) to examine the applicability of an analytical convolution approach to represent the variation in time of single-valued water elevations of different types of green water events generated by incident wave trains, particularly PDB and HF types. Detailed experimental measurements using high-speed video in wave flume experiments were used to verify the applicability of the model for single and consecutive green water events of type PDB and HF. The present work is a tentative attempt to compare an analytical approach for HF evolution. Results were also compared with the classic analytical dam-break approach. It was found that the convolution model allows the variation of water elevations in time to be captured better in comparison with the dam-break approach. The convolution model described the trend of water elevations well, particularly at the bow of the structure. The model captured the peak times well in single and consecutive events with multiple-valued water surfaces. Results suggest that this conservative and simplified approach could be a useful engineering tool, if improved and extended, to include the evolution of green water events in time domain simulations. This could be useful in the design stages of marine structures subject to green water events.

Keywords: convolution; dam-break; green water types; water elevations; plunging-dam-break; hammer-fist; wave overtopping

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1. Introduction

The unexpected hydrodynamic loads that can be generated by water volumes shipping on the deck of marine structures are of particular interest in coastal, ocean and offshore engineering [1–16]. These events can cause operational problems or even structural damage [17], sometimes significantly overloading the structures.

Shipping water events can occur in different ways, depending on the wave features and the shape and operational conditions of the structure [17–19]. The study of water

shipping on structures, or green water events, as this is known in ocean and offshore engineering [11,18], requires evaluation for different flooding scenarios.

A simplified experimental approach into different types of shipping water events in ocean engineering was detailed by [17,20,21]. This approach consists of performing wave flume experiments with regular wave trains to generate different types of shipping water events on a barge-type structure. A camera is positioned at the side of the flume, focusing on the bow of the structure, to capture details of the fluid–structure interaction. Certain types of possible shipping water events were proposed by [20]. As discussed by [22], some of these can be obtained with breaking incident waves and others with unbroken waves. The latter could be defined as: dam-break (DB), plunging-dam-break (PDB) and hammer-fist (HF) types of shipping water (see [20,23] for details).

The experimental approach proposed by [20] has been recently extended by [24,25] using a higher capturing velocity in the camera, which has allowed more details of DB, PDB and HF types of events to be identified. From the results of these works and previous concepts presented by [20], it is possible to describe the propagation of these events as shown in Figure 1a–c, respectively. There, the evolution of the free surface for each event type is shown. The red line illustrates the main pattern of each event. In fact, it can be said that if the deck of the structure is sufficiently long and the events are generated by an oscillatory incident flow, such as regular waves, these events will generate finite volumes of water propagating over it. These finite water volumes will not be well identified if the deck length is short, perhaps limited by an obstruction, as shown numerically by [26–28].

Figure 1. Sketches of water surface evolution of possible types of green water events on a barge-type structure generated experimentally with unbroken regular waves, as reported by [20,24,25]. (a) Dam-break (DB), (b) plunging dam-break (PDB), (c) hammer-fist (HF).

Knowing the evolution of the masses of water that propagate over the deck of structures (i.e., the spatio-temporal variation in water elevations) is relevant in the practical estimation of hydrodynamic loads acting over the deck [5,18,29,30]. This is particularly important in project stages where the use of analytical models could give acceptable estima-

tions in reasonable computational time (e.g., [18,29]). Often, the most practical analytical assessments carried out to represent green water evolution are performed using models based on hydraulic approaches, such as dam-break [18,31,32] and flood wave [33–35]. Recently, Hernández-Fontes et al. [36] proposed the use of a simplified convolution analytical model, a solution of the advection–diffusion equation, to represent the green water evolution in isolated green water events generated experimentally with the wet dam-break approach (see wet dam-break in [37–39]). Although the model proposed by [36] is simple and its application to green water still requires more research regarding advection and diffusion parameters, with convolution the assessment of the propagation of finite volumes of water is possible with acceptable approximation in events generated with incident wave trains [24]. In fact, recent works have demonstrated the potential of using the advection–diffusion approach to represent green water evolution, either by identification of parameters [40] or by a simplified fractional analysis [41]. In modelling green water evolution on a marine structure, where the shipping water mass is considered as an external force acting on it, analytical models can be useful alternatives as they may offer faster and more practical time domain simulations than numerical approaches based on computational fluid dynamics (CFD). Until now, the applicability of most common analytical models has been tested against single-valued water surface (SVWS) elevation measurements, obtained from experiments (using cameras or wave gauge sensors) or numerical simulations (e.g., [18,24,30,32,36]). However, Fontes et al. [19] showed that in PDB and HF events, multiple-valued water surface (MVWS) elevations occur at a specific position over the deck (see stage 4 in Figure 1b,c). More information about the SVWS and MVWS definitions can be found in [19,42].

Comparisons between analytical models and experiments have been performed, considering time series of SVWS elevations at equidistant positions over the deck of the structures. However, it has recently been reported [19,25] that interesting features are found in the initial stages of shipping water, where MVWS elevations occur. The performance of analytical convolution models to capture shipping water elevations of different types of green water events in these initial stages had not been tested—a possibility for future research [24]. This paper, therefore, aims to explore the applicability of the advection–diffusion analytical convolution approach proposed by [36] to represent the temporal variation of SVWS elevations found in the initial stages of different types of green water events (mainly PDB and HF types). The paper is an extension of previous work by [19], who reported details of the initial stages of different types of green water events experimentally. The present study is a first attempt to identify the possibilities of the practical implementation of an analytical model that is able to capture SVWS elevations, as well as to identify challenges for further research to capture MVWS elevations.

The applicability of the model was tested with SVWS and MVWS elevation measurements obtained from experiments of shipping water events on a fixed structure generated by regular wave trains [25]. Moreover, the results were compared with the analytical dam-break approach, which is a conservative method to represent green water propagation. A practical procedure to choose the convolution parameters is described, and then the model was applied to different types of events, including single and consecutive events.

The paper is organized as follows: Section 2 shows the main concepts of the advection–diffusion convolution approach and the implementation procedure used in this work. Section 3 describes the experimental data and introduces the study cases. The selection of the input data for the model is described in Section 4, and the results and discussion of model implementation are presented in Sections 5 and 6, respectively. Main conclusions and future works are summarized in Section 7.

2. Analytical Approach

In this study, the applicability of a simplified analytical model, based on convolution operations, was tested with different types of green water events. The model was introduced to green water research by [36] to capture the spatio-temporal variation of SVWS

elevations of isolated shipping water events generated with incident wet dam-break bores. The model consists of an analytical solution of the advection–diffusion (A–D) equation, which was employed to model green water, considering simplifications of the governing Navier–Stokes equations. Although the equation represents the green water physics in a simplified way, by considering advection (A) and diffusion (D) coefficients, it has the advantage of keeping the water elevation (η) as the only dependent variable.

It is important to mention that although the A–D equation was simplified, in assuming a constant flow propagation [36], it has the advantage that it can be solved by mathematical procedures, such as Laplace transform or Green function methods. Practical analytical convolution solutions for different initial conditions can be obtained by assuming constant A and D coefficients [43], thus allowing several possible practical applications, as documented in various research areas [43,44]. Details of green water modelling using this approach can be found in [36,45], while the most representative equations are shown in this section.

Equations (1) and (2) represent the x-direction shallow water equations (continuity and momentum equations, respectively) that consider the shear stress produced only by the bottom boundary (τ):

$$\frac{\partial \eta}{\partial t} + \frac{\partial(\bar{u}\eta)}{\partial x} = 0 \quad (1)$$

$$\frac{\partial \bar{u}\eta}{\partial t} + \frac{\partial(\bar{u}^2\eta)}{\partial x} = -gH\frac{\partial \eta}{\partial x} + \frac{1}{\rho}\tau \quad (2)$$

where x is the coordinate in the horizontal direction, t is time, η is the water surface, g is the acceleration due to gravity, ρ is water density, and τ is the bottom shear stress—that is, the stress due to bottom friction. In these Equations, $\bar{u}$ represents the x-direction depth-averaged flow velocity. Using a subcritical flow assumption [36], Equation (1) remains unaltered, while Equation (2) can be simplified as follows:

$$\frac{\partial \eta}{\partial x} = -\frac{\tau}{\rho g \eta} \quad (3)$$

The bottom shear stress τ is considered through resistance approaches, such as the Manning approach, which is a typical practice in hydraulic research [46–48] to consider frictional effects in a practical way. With this assumption, Equation (2) can be rewritten as

$$\frac{\partial \eta}{\partial x} = -\left(\frac{\bar{u}}{\gamma \eta^a}\right)^2 \quad (4)$$

where $\gamma = 1/M_r$ and $a = 2/3$. M_r can be considered as the Manning roughness coefficient (dimension $L^{1/3}T$).

After some algebra (see [45] for details), Equations (1) and (4) can be combined to obtain a single partial differential Equation—that is, the A–D Equation [36,45]:

$$\frac{\partial \eta}{\partial t} + A\frac{\partial \eta}{\partial x} = D\frac{\partial^2 \eta}{\partial x^2} \quad (5)$$

where $A = \bar{u}(a + 1)$ and $D = \bar{u}\eta/2S_f$ are known as the advection and diffusion coefficients, respectively. For green water research [24,36], $\bar{u}$ is considered as the mean horizontal shipping flow velocity at the bow edge, and η in the D coefficient is considered the maximum freeboard exceedance—that is, the maximum possible elevation at the bow edge, during the shipping water event. S_f is a resistance coefficient that empirically represents the shear stress on the deck.

Knowledge of the magnitude of the A and D coefficients in shipping water research is still limited since they can vary during the propagation of the water. For this reason, in previous research, methods of identification to estimate typical A and D coefficients for green water events propagating over small friction surfaces, such as acrylic, were proposed [40]. Consequently, in this work a similar structure for implementing the model (see Section 3)

was employed, focusing on the implementation of the model using parameters applied in [40]. With this, the aim was to verify the practical implementation of the model for different events, considering approximated A and D values. These values were considered through a reference study case, similar to that employed in [40]. Then, to demonstrate the possible applications of the model, various green water scenarios were evaluated using the coefficients from the reference case.

An analytical convolution solution of Equation (5), considering Dirichlet-type boundary value problem in semi-infinite domain, obtained through the Green function method, is shown by Equation (6). This solution was obtained with the boundary condition $\eta(0,t) = F(t)$ upstream of the domain (i.e., at the deck edge) and an initial condition $\eta(x,0) = 0$ (see [36,43]).

$$\eta(x,t) = \int_0^t F(\tau) \left[\frac{[x-A(t-\tau)]}{\sqrt{16\pi B(t-\tau)^3}} \exp\left\{-\frac{[x-A(t-\tau)]^2}{4B(t-\tau)}\right\} + \frac{[x+A(t-\tau)]}{\sqrt{16\pi B(t-\tau)^3}} \exp\left\{\frac{Ax}{B} - \frac{[x+A(t-\tau)]^2}{4B(t-\tau)}\right\} \right] d\tau \quad (6)$$

where $F(t)$ is an input function that is assumed in green water as the time series of water elevations at the start of the deck (freeboard exceedance at the bow edge), and the term next to $F(t)$ is known as the convolution kernel. As mentioned above, to apply Equation (6), the kernel parameters (time step and A and B coefficients) are kept constant to represent different green water scenarios, whereas the input function represents the finite volume of water flooding and propagating over the deck, and it changes for each shipping water case.

Implementation Procedure

Here, the procedure followed to analyze the applicability of the convolution model in representing the temporal variation of water elevations in different types of green water events is described. The aim was to see whether the approach is of practical use in providing approximate representations of this variation. Therefore, the kernel parameters were obtained from a reference case and subsequently used to analyze different green water scenarios. Although green water phenomena include variations in different flow parameters in time and space, this could be a practical strategy in real applications, where parameters identified from a case, considering a specific deck-type (e.g., with a specific roughness), could be assumed constant for different applications (i.e., to different green water events occurring in the same deck). This strategy has been simplified and a preliminary analysis is intended. Therefore, it is important to verify whether it is useful for calculating rough estimates to capture the main trends of water propagation.

The implementation of the convolution model here consisted of two main steps:

1. Choose a reference case (a green water event) from the experimental data available from [25] to identify kernel parameters (time step, A and D coefficients). In this work, a green water event with SVWS elevations (CA, Section 3.1) was chosen because the propagation patterns are very similar to those considered in the development of the convolution model [36], i.e., a mass of water invading the deck of a fixed structure almost horizontally, resembling a DB type of green water. The event selected was generated with a regular wave shipping onto an acrylic-type deck similar to that in [40], where approximate A and D parameters were identified for such a case. These parameters were used here. For case CA, a parametric analysis was performed to choose a convenient time step to implement the kernel.
2. Implement and verify the applicability of the convolution for different green water scenarios, using the parameters found in step 1 (time step, A and D coefficients). Note that the input series $F(t)$ will vary for each event considered. Here, the input function of the convolution ($F(t)$, Equation (6)) represents the time series of freeboard exceedance at bow edge. This series can represent a finite amount (volume) of water that ships over the deck, as illustrated in Figure 1. In real applications, perhaps these input data could be predicted roughly, from relative motions between the incident waves and the structure, either by deterministic or statistical methods.

The main objective is to verify the applicability of the model to capture the evolution, $\eta(x,t)$, of different green water scenarios (single and consecutive events), using the constant kernel parameters obtained from the reference case. The scenarios include small single PDB and HF events, large single PDB and HF events, and consecutive PDB events and consecutive HF events (cases CB1 and CB2, CC1 and CC2, and CD1 and CD2, respectively; Section 3.1). Detailed experimental measurements of SVWS and MVWS elevations of the initial stages of PDB and HF events are considered in the comparisons (Section 5), which also include the implementation of the analytical dam-break approach (Appendix A).

3. Experimental Data

To validate and test the applicability of the convolution approach, data from wave flume experiments of green water events occurring in a rectangular fixed structure were used [25]. These were generated by incident wave trains, obtained with regular wave parameters in an experimental setup, as shown in Figure 2a. The complete description of these experiments is presented in [25]. This includes a database of the videos of fifteen study cases, taken at 250 fps, for different wave parameters, and was made freely available (see database in [49]). As the main parameter to be analyzed in the present work is the water surface elevation, image-based techniques were used to process the data and to acquire time series of water elevations, $\eta(t)$, at several positions along the deck of the structure, Figure 2b, following the same procedure as [19]. These methods basically consist in the binarization of the videos [50] in order to obtain constant water surface contours, of constant thickness, through erosion and dilation morphological procedures [42]. Once these contours are obtained, their motion within the regions of interest is defined from virtual wave probes (VWPs) [51]. These measurements are then converted into time series of water surface elevation data and used to compare the results of the model. For comparison with the convolution results, time series of water elevations taken at the above deck positions from $x = 6$ mm to $x = 250$ mm (20 VWP's), defined in Figure 2b, were considered for the case studies shown in Section 3.1. The input function of the convolution, i.e., the maximum freeboard exceedance time series, is taken at $x = 2$ mm (Figure 2b).

Figure 2. Experimental setup. (a) Configuration of the fixed structure inside the wave flume (Adapted from [25]). (b) Positions at which the water surface elevation measurements were taken (Adapted from [19,25]).

It is important to note that SVWS and MVWS elevation measurements were considered in the present work (see [19] for details) to be compared with SVWS elevations given by analytical models. To the best of the authors' knowledge, this is the first time that simplified SVWS analytical models have been compared with MVWS elevations taken at the bow for HF type events in green water research.

3.1. Study Cases

Table 1 shows the study cases of the present work selected to demonstrate the applicability of convolution in assessing the evolution of different types of green water events. The table includes information regarding the experimental data from [25,49] that was employed to implement the study cases. In the experiments, consecutive green water experiments were generated by wave trains. The last column of the table indicates the number of events considered for each train.

Case CA is the shipping water reference case for defining kernel parameters (i.e., time step, A and D coefficients). The applicability of the model was tested to represent the evolution of two PDB-type events, one small and one large (CB1 and CB2, respectively), a small and a large HF-type event (CC1 and CC2, respectively), and four consecutive PDB and HF type events (CD1 and CD2, respectively).

Table 1. Study cases considered in this work, including type of analysis and characteristics of the regular waves used to generate the green water events. More details regarding these data can be found in [25,49].

Case Name	Type of Analysis	Case Number in [25] (Frames in the Database)	Wave Length (L_w , in m)	Wave Height (H_w , in m)	Wave Steepness (H_w / L_w)	Event Number
CA	Parameters definition	A1 (632–957)	2.5	0.08	0.032	3
CB1	Evolution of a small PDB-type event	B2 (551–f837)	2	0.12	0.060	3
CB2	Evolution of a large PDB-type event	B3 (818–1092)	2	0.16	0.080	4
CC1	Evolution of a small HF type event	E1 (1–168)	0.75	0.08	0.107	1
CC2	Evolution of a large HF type event	D2 (960–1164)	1	0.12	0.120	6
CD1	Evolution of four PDB-type consecutive events	C2 (474–1452)	1.5	0.12	0.080	3–6
CD2	Evolution of four HF-type consecutive events	D3 (1–783)	1	0.16	0.160	1–4

4. Kernel Data Selection with Reference Case

Green water events are known to exhibit a spatio-temporal variation of flow kinematics [2,9,21,52–54]. Based on the observation of data provided by [25] and experimental results by [55], it is possible to state that in cases where the structure is barge-shaped and there is an available freeboard before the bow run-up, the wave kinematics changes and the water flow on the deck, from the deck edge, accelerates. This can be confirmed by analyzing the events in the video database of [25]. Moreover, the behavior of the water-on-deck distribution will be different if the structure has a finite (restricted) length, as in [5,26,28,56]. However, to demonstrate the practical use of the convolution approach for this work, constant kernel parameters (time step, A and D coefficients) were assumed, taking into account that the input function ($F(t)$) will be the only variable for each case, Table 1. Therefore,

using a single green water event as an application case, some assumptions were carried out to consider them.

To define the kernel parameters, case CA (Table 1) was taken as representing the cases in this work. This is the third green water event, generated by the wave train of case A1 in [25]. It was generated with a wave steepness of 0.032, corresponding to a PDB type of green water, with a very small cavity formed at the bow edge. This event was chosen because a similar event, generated with similar wave parameters and structure, was used in [40]. In that work, parameter identification techniques were employed to estimate mean (constant) A and D coefficients for such a case—0.3752 and 0.0094, respectively—showing a good performance in capturing the elevations of consecutive small green water events of the PDB-type with SVWS elevations.

As little research has been done to determine A and D kernel parameters for green water, and assuming that the structure and incident wave for case CA in this work are similar to the ones in [40], the values used there were employed as reference to test the applicability of the model for different cases. The reader is referred to case C3 in [40] for other comparative details in the estimation of the A and D parameters. These A and D parameters were employed to find an adequate time step for implementing the model, as shown in Figure 3. This figure shows a comparison of convolution kernels for different time steps for CA, considering a short distance from the position at which the freeboard exceedance ($F(t)$) was defined ($x = 0.004$ m), where the kernel peak becomes sharper. Note that the kernel area should be equal to unity to accurately represent the evolution of the water over the deck [36]. Thus, the minimum time step for this was chosen (i.e., 1×10^{-4} s), although the input function (i.e., time series of freeboard exceedance at the start of the deck) will vary for each case.

Figure 3. Time step kernel analysis, considering $x = 0.004$ m and six different time steps for case CA.

5. Results of the Implementation of Convolution Model

In this section a comparison is given of experimental and analytical results to represent the variation in time of water elevations (η) of the six green water scenarios described in Table 1. The green water cases for single and consecutive events of PDB and HF types are shown in Sections 5.1 and 5.2, respectively (see Figures 4–9). The figures in these sections show time series of water elevations taken at the 20 positions on the deck defined

in Figure 2. In the comparisons made at each position, the input function of the convolution, $F(t)$, i.e., the time series of freeboard exceedance measured at $x = 2$ mm (see Figure 2), has been included to indicate the decay trend of water elevations as the water propagates downstream on the deck. The A and D kernel parameters are kept constant while the time step in the kernel and input function are kept the same. Moreover, the comparisons include the well-known dam-break approach (see Appendix A), which is a traditional, conservative approach to analyze water-on-deck propagation [18]. Below the comparisons, snapshots of the events are included to illustrate relevant stages during the events. Some of the differences observed in the comparisons are discussed in Section 5.3.

5.1. Evolution of Single Green Water Events

5.1.1. Single PDB-Type Events

In the first cases (CB1 and CB2), the convolution model was implemented to represent the propagation of PDB types of green water events. Making a qualitative comparison of the size of the cavity formed at the bow edge, these were classified as small PDB (Figure 4) and large PDB (Figure 5).

The features of the small PDB (Figure 4, CB1) resemble those of CA, from which the kernel parameters were selected. For this case, note that convolution represents acceptably the variation in water elevations for the positions closer to the deck edge ($0.006 \text{ m} < x < 0.05 \text{ m}$). While in the experiments, the values of η tended to decrease more than those in the analytical results; however, the time at which peak values occurred was similar. For positions farther away from the deck edge ($x > 0.05 \text{ m}$), the times of the peak water elevations (peak times hereafter) were more significant. The convolution results give an improved approximation of the amplitudes of experiments compared with the classic dam-break approach, especially after the maximum elevations occur. Despite its practical implementation, it is well known that this approach tends to overestimate the amplitudes in experiments and is not able to adequately represent the decay in the experimental curves [18,32,36]. The main reason for this is that the theoretical dam-break approach considers an infinite amount of water entering the deck.

For the large PDB (Figure 5, CB2), similar behavior was observed, although the formation a larger cavity was observed, giving MVWS elevations at positions closer to the deck edge (for instance $x = 0.022 \text{ m}$). The convolution model, which only gives SVWS elevations, does not show this tendency, but captures the general trend of the experimental curves, including the decay, thus offering an improvement on the dam-break approach. As for the small PDB in CB1, at positions farther from the deck edge (for example $x > 0.15 \text{ m}$), the peak times in the experiments tended to occur earlier than in the convolution results (see an analysis of peak times in Section 5.3), suggesting that flow was propagating faster than the prediction made by convolution.

5.1.2. Single HF-Type Events

Figures 6 and 7 show the results for small (CC1) and large (CC2) HF-types of green water events, respectively. In these events the MVWS elevations at bow of the structure are evident. They are present because a block of water remains suspended at the bow for a while, then falls hitting the deck, to finally propagate over it, as was described by [20]. For CC1 (small HF type, Figure 6), the classic dam-break approach captured the experiments close to the bow edge well, but tended to significantly overestimate them as the water propagated over the deck. The convolution model captured the general trends of experiments closer to the deck edge well (see $x = 0.006$ and 0.10 m), while the block of water remained almost suspended at the bow. Limitations in representing the MVWS elevations are seen. Then, during the propagation of the water over the deck, a thin layer of water remaining on the deck was seen, and the convolution tended to overestimate the magnitude of η . However, the peak times of the experiments were approximated well by convolution for most cases.

Figure 4. Comparison of time series of the water elevations (η) between the experimental and analytical (convolution) results for CB1. The results correspond to a single green water event (small PDB-type). The snapshots were obtained from the database provided by [25,49]. Some text was included in the original snapshots, which were used under a CC BY 4.0 license.

Figure 5. Comparison of the time series of the water elevations (η) between experimental and analytical (convolution) results for CB2. The results correspond to a single green water event (large PDB-type). The snapshots were obtained from the database provided by [25,49]. Some text was included in the original snapshots, which were used under a CC BY 4.0 license.

Figure 6. Comparison of the time series of the water elevations (η) between experimental and analytical (convolution) results for CC1. The results correspond to a single green water event (large HF-type). The snapshots were obtained from the database provided by [25,49]. Some text was included in the original snapshots, which were used under a CC BY 4.0 license.

Figure 7. Comparison of the time series of the water elevations (η) between experimental and analytical (convolution) results for CC2. The results correspond to a single green water event (large HF-type). The snapshots were obtained from the database provided by [25,49]. Some text was included in the original snapshots, which were used under a CC BY 4.0 license.

Figure 8. Comparison of the time series of the water elevations (η) between experimental and analytical (convolution) results for CD1. The results correspond to four consecutive green water events (PDB-type). The snapshots were obtained from the database provided by [25,49]. Some text was included in the original snapshots, which were used under a CC BY 4.0 license.

Figure 9. Comparison of the time series of the water elevations (η) between experimental and analytical (convolution) results for CD2. The results correspond to four consecutive green water events (HF-type). The snapshots were obtained from the database provided by [25,49]. Some text was included in the original snapshots, which were used under a CC BY 4.0 license.

Similar behavior occurred in CC2 (large HF type), although at positions closer to the deck edge ($0.006 \text{ m} < x < 0.018 \text{ m}$); the maximum experimental η values were even higher than the freeboard exceedance and, therefore, overestimated the analytical results. Unlike CC1, in this case a larger amount of water propagated over the deck, which perhaps caused higher flow acceleration at the middle of the deck, which may explain the differences in attaining the peak times between the experiments and the convolution approach (see for instance, $x > 0.15 \text{ m}$).

5.2. Evolution of Consecutive Green Water Events (PDB and HF Types)

Figure 8 shows the results for the four consecutive green water events of the PDB-type (CD1). As the dam-break approach only requires a maximum freeboard exceedance value as input (Appendix A) and no procedure has been established for consecutive events, in this case, it was implemented with input data from the first event to examine its applicability. This approach showed limitations in capturing peak values of the events in the bow region ($x < 0.046 \text{ m}$) but may serve as a conservative approach for water propagating downstream on the deck. As with the individual cases in Section 5.1, the general trends in the experiments are described acceptably by convolution at the beginning of the deck ($x < 0.034 \text{ m}$), including η amplitudes and starting and peak times of the events. Subsequently, for $x > 0.034 \text{ m}$, the convolution results tended to overestimate the maximum experimental values. It is evident that in experiments, η significantly decreases once the flow begins to propagate horizontally over the deck. The convolution results for $x > 0.2 \text{ m}$, when the water propagating over the deck became a layer of water; greater differences with the peak times of experimental data were observed.

The four consecutive events of the HF-type (CD2) are shown in Figure 9. As in CD1, the applicability of the dam-break approach considering data of the first event was tested. The results show that it cannot attain the peak amplitudes of all the events at the bow. On the other hand, the convolution model captured the main trends of η at positions close to the deck edge well (see for instance $x < 0.022 \text{ m}$). In these parts of the deck, the stages when the main patterns of the HF were formed at bow occurred, showing MVWS elevations. The convolution did not follow the exact trend of MVWS elevations. For $x > 0.022 \text{ m}$, the convolution model tended to overestimate the amplitudes of the experimental results; however, the main trends were captured for almost all the first half of the deck.

5.3. Quantitative Analysis of Differences between Convolution and Experiments

To quantify the main differences between the results of Sections 5.1 and 5.2, the maximum water elevations were compared between the time series of experiments and the convolution results.

The error parameter used is $\varepsilon = (\eta_{\text{exp}} - \eta_{\text{con}})/\eta_{\text{exp}}$, where η_{exp} and η_{con} are the maximum elevations observed for the experiments and the convolution results for each case study, respectively. The percentages of error between maximum values, at each position over the deck, x , are shown in Figure 10. This figure also includes a global root mean square error (RMSE) for each case (Equation (7)), calculated to provide a general idea of the overestimation of the convolution results:

$$\text{RMSE} = \sqrt{\frac{\sum_{i=1}^N (\eta_{\text{exp}_i} - \eta_{\text{con}_i})^2}{N}} \quad (7)$$

where the subindex i indicates the position number (x) considered in the comparisons of the convolution and experiments ($i = 1:N$, where $N = 20$ positions).

The results in Figure 10 show that the convolution model is conservative with respect to estimating peak η amplitudes, particularly for positions farther from the bow of the structure. For most cases, the maximum overprediction was $\sim 150\%$, for experiments on single-small HF and consecutive HF-type events (CC1 and CD2, respectively). For the single large HF-type event (CC2), it was observed that the experiments overestimated

the analytical model at the bow because the rising fluid arm was suspended. However, once this fluid became a layer of water propagating over the deck, convection tended to overestimate the experimental results. On the other hand, for small and large PDB events (CB1 and CB2, respectively) and consecutive PDB events (CD1), the overestimation was close to 50% of experiments for $x < 0.05$ m. In these cases, the assessment of η by convection was very acceptable at the bow of the deck.

Figure 10. Quantification of differences between the maximum values of the water elevations found between convection and experimental results for each study case, for the different positions over the deck (x). The differences are defined by an error parameter ε (in %). The figure includes the global RMSE (in m), calculated for each case.

6. Discussion

As explained in the Introduction, it is important to note that when the deck is sufficiently long, the shipping water events can be well identified as finite amounts of water propagating over the deck. These events seem to lose incident wave kinematics when they interact with the freeboard of the structure (see the snapshot database of [25]). From the bow edge, they develop features such as cavities (PDB) of suspended blocks of water (HF). The horizontal momentum then becomes dominant once the water interacts with the deck, tending to accelerate as the wavefront propagates downstream.

The differences observed in Figures 4–9, regarding the peak times of the experiments for positions farther from the deck edge, can be attributed to the acceleration of flow. This effect is not considered in the model, which was formulated under the assumption of subcritical flow. According to [55], in green water events analyzed experimentally, the horizontal flow velocities tend to start from values close to zero at bow, tending to increase as the water propagates over the deck. In a deck domain with no restrictions and little roughness on the acrylic deck, as in the present work, it is expected that the flow accelerates.

It is also worth mentioning that in the videos of [25], the deck is seen to have very little slope. This means that a very thin layer of water stays on the deck after each event. This can cause backflow effects that are not considered in the development of the model. These effects can be more significant for small volumes of water propagating over the deck, such as in cases CB1 and CC1.

Regarding the assessment of HF-type events, it is important to mention that this may be the first time that simplified analytical models are compared with MVWS elevations of HF type of green water. It is evident that the physics involved in this event do not correspond to those considered in the development of the model [36], which uses SVWS elevations in events resembling the DB-type. The main differences are related to the overestimation of the experiments by convection at positions farther from the deck edge, mainly at $x > 0.06$. This occurred because the rising water levels at the bow may indicate a

large overtopping of freeboard, but not necessarily a large amount of water propagating over the deck, as demonstrated in [19].

In large HF events, the water elevations at positions close to the deck edge tend to surpass the maximum freeboard exceedance (for instance Figure 7, $0.006\text{ m} < x < 0.014\text{ m}$), which could not be achieved by the analytical models considered.

For consecutive events, regarding the practical application of the convolution model, it provided acceptable results. The implementation of the dam-break approach for consecutive events requires imposing the starting time of the model at the beginning of each event.

As each type of green water event has proper features during the wave–bow interaction, assessing the exact water elevations of all the types of green water with a simple analytical approach is difficult. Well known analytical methods to investigate green water propagation, such as dam-break [18,30–32] and flood wave [33,35,57] approaches, can be seen as conservative engineering approaches. All these methods tend to overpredict the water on deck, representing lower distributions of water on the deck than the maximum freeboard exceedance. These characteristics of analytical models are also seen in the convolution approach examined in this work. However, the capability of the convolution approach to practically capture, in space and time, the decay of the input function along the deck was seen. This may allow the representation of finite volumes of water propagating over decks with no restrictions, as illustrated in Figure 1.

In general, the use of convolution can be used to represent the general trends of single and consecutive green water events in a conservative and simplified way, particularly at the bow of the structure. Some overestimations were observed in the evaluation of the peak times, but in general, the peak times of the experiments were acceptably achieved for the first half of the deck.

The implementation of the convolution model, employing constant kernel parameters, can be useful in evaluating the effects of green water in time domain simulations. The simulation of dynamic systems could be performed from a conservative but computationally efficient point of view. Perhaps, this would allow including a security factor in the estimations of water elevations or induced loads.

To investigate different green water scenarios, more convolution solutions of the advection–diffusion equation could be explored, by varying the domain of propagation (infinite, semi-infinite or finite) and the initial and boundary conditions [43]. The applicability of the model for water propagation over decks with different roughness could also be investigated by finding suitable kernel coefficients. However, scale effects should also be evaluated.

7. Conclusions

The present research investigates the applicability of an analytical convolution model to capture the temporal variation of water elevations in green water events of different types (PBD and HF), including detailed MVWS elevations obtained at bow of a fixed structure. The main objective was to verify whether the use of convolution could be a practical alternative for capturing the evolution of isolated and consecutive events, considering input values obtained from a representative event. The convolution model is a simplified solution of the advection–diffusion equation applied to green water research and was proposed in [36,45] to represent the propagation of DB-type events—that is, those events that invade and propagate the deck almost horizontally, giving SVWS elevations. Despite being an analytical and simplified approach, the convolution model gave an acceptable representation of different types of green water events, which have different patterns. Peak times and trends in the time series of water elevation were well approximated, showing improvement over the traditional analytical dam-break approach. Unlike the dam-break model, the use of convolution allows a decay trend on water-on-deck distributions to be considered. This can be useful when analyzing different types of green water propagating along the deck in the form of finite amounts of water.

Although the convolution model is a simplified approach that captures SVWS elevations, it may become a useful tool in green water research after researching some topics, such as alternatives to estimate the input $F(t)$ function (i.e., freeboard exceedance).

On the other hand, little research has been done on finding typical A and D kernel parameters for green water applications, particularly for decks with little roughness, such as acrylic decks. Perhaps, these coefficients may help to characterize the effect produced by rougher decks, as discussed in [26]. The convolution model provides SVWS elevations. As this research is a first attempt to compare with MVWS elevations from experiments, the development of analytical models to capture this type of behavior would be useful.

For the preliminary stages in the design of marine structures, the model was seen as appropriate in representing the decay effect of consecutive events in a simplified way. This can be very useful in time domain simulations with varying loading scenarios caused by consecutive green water events. The approach seemed to be conservative enough to consider a security factor in load predictions using water elevations for its estimation (see [18,29]).

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Appendix A

This appendix summarizes the dam-break model, which is a well-known hydraulic approach, proposed from decades ago to analyze green water type flows. The works of [18,31,36] are useful references to understand the application of the dam-break approach to green water research. The dam-break approach considered in this work is that described by [18], which corresponds to the analytical solution developed by [37]. The analytical equation to obtain the water elevations in space (x) and t can be rewritten as follows:

$$\eta(x, t) = \frac{1}{9g} \left(2\sqrt{gh_1} - \frac{x}{t} \right)^2$$

To apply this equation for a green water event, g is the acceleration due to gravity and h_1 is considered as $h_1 = 9\eta_0/4$, with η_0 corresponding to the maximum freeboard exceedance of the event (for details, see [18,36]).

The implementation of the dam-break approach requires input data of single green water events. Therefore, for implementing this model in the analysis of Section 5, the maximum freeboard exceedance of the single events analyzed was considered as η_0 . It is

important to mention that the starting time of the dam-break results had to be synchronized with the beginning of the green water events in the comparisons.

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