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Watershed Ecological Compensation Mechanism for Mainstream and Branches Based on Stochastic Evolutionary Game: A Case of the Middle Yellow River

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Abstract: Establishment of a watershed ecological compensation mechanism between multiple subjects is an effective means to realize the collaborative governance of water pollution and maintain the security of water ecology. This paper breaks through the conventional upstream and downstream perspectives of watershed ecological compensation design research and combines them with uncertainty factors. The watershed ecological compensation mechanism for the mainstream and branches was established based on the evolutionary game and the random process. Then, taking the midstream of the Yellow River as an example, some constraint conditions and influencing factors were explored. Results show that: (1) The branch government (i.e., the Shanxi provincial government) is the key to establishing an ecological compensation mechanism between the river mainstream and branches. (2) The proportion of pollution transferred by other branches, the initial probability and the random factors are the main factors affecting the decision-making of branch governments (Shanxi and Shaanxi provincial governments). (3) The compensation and reward of the mainstream government to the branch government and the compensation of the branch government to the mainstream government are the main factors affecting the decision-making of mainstream and branch governments (Shanxi–Henan provincial governments, Shaanxi–Henan provincial governments). The study may provide scientific guidance for the construction of a watershed ecological compensation mechanism between mainstream and multiple branches.

Keywords: mainstream and branches; stochastic evolutionary game; Yellow River Basin; watershed ecological compensation

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1. Introduction

With the growth of the population and industrialization, the amount of waste water discharged into rivers in various regions increases greatly, resulting in frequent water pollution problems in the river basin [1,2]. Due to the migration of water pollutants, the sewage generated in the upstream area affects the downstream area and the sewage generated in the branch area affects the mainstream area [3]. To systematically solve the problem of transboundary water pollution in the basin, collaborative governance is required for the upstream and downstream areas and for the mainstream and branch areas in the basin [4,5]. Additionally, in the context of global climate change, regional water inflow, water intake and ecological environment change in the basin have great uncertainty [6], which will further affect the decision-making of various regional subjects in the basin and cause adverse effects on the basin governance and management. Therefore, it is of great scientific value and practical significance to clarify the decision-making behaviors and influencing factors of all regional governments in the basin under changing conditions and to promote the construction and improvement of a collaborative governance mechanism for water pollution in the upstream and downstream areas, the mainstream and branch areas of the river basin.

The ecological compensation mechanism based on the internalization of public goods space overflow is an effective means to solve the cooperative pollution control of all regions in the basin [7]. Watershed ecological compensation comes from payment for ecosystem services (PES), which mainly coordinates multiple and multilevel intergovernmental interests, promotes regional collaboration and achieves sustainable development [8,9]. At present, research on the watershed ecological compensation mainly includes three parts: compensation concept and theory [10–12], compensation mechanism design [13,14] and evaluation of compensation effects [15,16]. Among them, compensation mechanism design research has always been a frontier of great interest in the field of ecological compensation for river basins, which has been of concern by scholars at home and abroad. The determination of the compensation subject and object and the analysis of its influencing factors is an important part of the watershed compensation mechanism. Taking the Liaohe River Basin as an example, Qu et al. [17] constructed an ecological compensation game model for governments both upstream and downstream of the river basin. It was found through analysis that only by combining the binding agreement between local governments and the vertical financial transfer payment of the central government can the maximum utility of the upstream and downstream ecological compensation mechanism be realized. On this basis, Hu et al. [18] constructed an ecological compensation game model for upstream and downstream governments by comparing two cases with and without the central government's incentive constraint mechanism. In addition, taking the Lijiang River Basin as an example for empirical analysis, they also concluded that the optimal equilibrium strategy (upstream governance, downstream compensation) cannot be achieved only through the evolution of upstream and downstream governments themselves. Wang et al. [19] demonstrated the necessity of combining vertical and horizontal ecological compensation between the upstream government, the downstream government and the central government in the Yellow River Basin; some key elements were analyzed through a simulation which included initial willingness, opportunity costs and punishment. Considering the supervision of the central government, Gao et al. [20] discussed the game relationship between the central government and compensation areas and investigated the influence of the central government's supervision and reward and punishment factors on the strategy choice of upstream and downstream governments. Sheng et al. [21] studied the impact of the central government's incentive mechanism on the ecological compensation of the middle route of China's South-to-North Water Transfer Project. By comparing two cases with and without the incentive mechanism, they found that additional incentive measures could strengthen the pollution control of local governments and effectively improve the quality of water. Jiang et al. [22] adopted an evolutionary game model to analyze the strategy evolution process of upstream and downstream governments and the central government under strong and weak incentive scenarios. On this basis, Yang et al. [23] constructed an evolutionary game model for transboundary water pollution control between the left and right bank areas and the river basin government from the perspective of "left and right banks" and explored differences in the three-party subject decision-making and evolution pattern under the mechanism of reward and punishment, compensation and the combination mechanism (reward, punishment and compensation).

According to the above analysis, most of the previous studies on the design of basin ecological compensation mechanisms are based on the upstream and downstream perspectives. However, the basin is an interactive, complex system composed of the mainstream and multiple multilevel branches [24]. These results from the upstream and downstream perspectives cannot consider the complex pollution control cooperation between the mainstream and multiple branches. Moreover, the majority of existing studies tend to construct the compensation mechanism from the perspective of certainty, which takes insufficient account of the influence of external uncertainty factors, and it is very easy to bias results greatly. Therefore, in order to supplement the existing research and formulate a macro and system mechanism of water pollution control, it is necessary to construct a watershed ecological compensation mechanism between mainstream and multiple branches government under the influence of external uncertainty factors. In addition, it is also of great significance to explore the decision-making behavior, influencing factors and changing conditions in watershed ecological compensation, which has great significance in coordinating the relationships between mainstream and multiple branches of government.

As a result, given the interference of random factors, this paper focuses on the pollution control cooperation of governments between the river mainstream and multiple branches and takes the midstream of the Yellow River as an example for simulation analysis. Specifically, it includes that: (1) For the mechanism design, the compensation and reward mechanism of the downstream government to the upstream government, the compensation mechanism of the upstream government to the downstream government and the punishment mechanism of the central government are integrated to establish an ecological compensation mechanism combining compensation and reward. (2) For the research method, the evolutionary game and the stochastic process are combined, and a I*tô*-type stochastic differential equation is used for model construction and solution. (3) For the numerical simulation, the actual data of Shanxi, Shaanxi and Henan provinces in the midstream of the Yellow River are combined to analyze the influence rule of various influencing factors on the decision-making in terms of initial probability, pollution transferred, reward and random factors, so as to provide a reference for the construction of an ecological compensation mechanism for governments between the river mainstream and multiple branches.

2. Materials and Methods

2.1. Overview of Research Region

Located in Northern China, the Yellow River is one of the longest rivers in the world, with a total length of 5464 km. It flows through China's important core grain-producing area and resource and energy-rich area, covering nine provinces (regions), such as Qinghai Province, and serving as an important economic zone in China [25]. Figure 1 depicts the Yellow River Basin's geographic location. The research area of this paper is the midstream region, including branches of the Fenhe River and the Weihe River and the midstream of the Yellow River. The Fenhe River is the second largest branch of the Yellow River, with a total length of 716 km. The basin area accounts for a quarter of the total area of Shanxi Province and its population accounts for 39% of Shanxi Province. It is the largest river in Shanxi Province. Therefore, the Fenhe River is considered to be Branch 1 and Shanxi Provincial Government is considered to be Branch Government 1. The Weihe River is the largest branch of the Yellow River. The Weihe River Basin in Shaanxi Province covers an area of 67,100 km^2 and it concentrated 63% of the province's population and GDP. It is the political, economic and cultural center of Shaanxi Province [26]. Therefore, the Weihe River is considered to be Branch 2 and Shaanxi Provincial Government is considered to be Branch Government 2. The Fenhe River and the Weihe River merge into the mainstream of the Yellow River and then flows into Henan Province; thus, Henan Province is considered to be Mainstream Government. In recent years, the Fenhe River and the Weihe River have been seriously polluted due to the growth of the population, overexploitation and other reasons. Frequent occurrence of water pollution has seriously affected the ecological environment security and economic and social development of Shanxi, Shaanxi, Henan and even the entire basin [27].

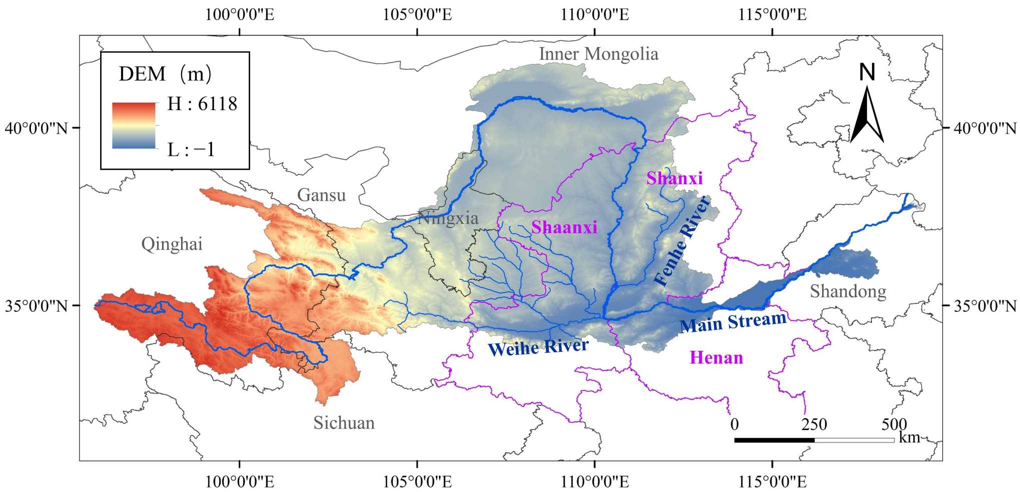


Figure 1. Geographical map of the Yellow River Basin.

2.2. Method

2.2.1. Model Assumptions

The following assumptions shall be set before building the stochastic evolutionary game model for water pollution control among mainstream and branch governments in the midstream of the Yellow River:

1. Shanxi, Shaanxi and Henan provincial governments are all limited rational players, and it is difficult to determine individual optimal strategies in a single game. It takes multiple games to reach a consensus.
2. With the goal of pollution control and emission reduction, Shanxi and Shaanxi governments have two strategies. The first is complete governance: that is, restricting the development of some local industries to achieve the goal of reducing pollutants. The second is incomplete governance: that is, not completely restricting the development of local industries. Thus, the goal of pollution control and emission reduction cannot be completely achieved, and the task of emission reduction will be transferred to the mainstream government and other branch governments.
3. For the mainstream Henan provincial government, there are two strategies: one is compensation and reward. Since the Shanxi and Shaanxi provincial governments have given up on the development of local industries to improve the water environment, this has greatly reduced the pollution control cost of the Henan provincial government. Thus, the provincial government believes that compensation and reward should be given for the opportunities and benefits given up due to pollution control; the other is no compensation or reward. The Henan provincial government believes that it is the obligation of branch governments to eliminate pollutants without compensation or reward.

2.2.2. Mechanism Design

Based on the above assumptions, the design of the basin ecological compensation mechanism among Shanxi, Shaanxi and Henan provinces is as follows: a "compensation" mechanism is adopted between the mainstream government and branch government (Shanxi-Henan, Shaanxi-Henan). This is based on whether water quality indicators for the section at the intersection of mainstream and branch reach the appraisal target. When the water quality indicators reach or exceed the appraisal target, the mainstream government compensates the branch government. When the water quality is worse than the appraisal target, the branch government compensates the mainstream government. Furthermore, if a branch does not achieve complete governance and the emission reduction target is not reached, another branch undertakes this additional task of eliminating pollution transferred by other branches based on achieving the emission reduction target, and also reduces the pollution control cost of the mainstream government. The mainstream government will provide an additional reward to the branch government, building an ecological compensation mechanism of "compensation, reparation and reward". Based on the above mechanism design, the three-party game strategy is shown in Table 1.

Table 1. Three-way game strategy.

Game Subject	Strategy	Selection Probability
Branch Government 1	Complete governance	x
(Shanxi Provincial Government)	Incomplete governance	$1-x$
Branch Government 2	Complete governance	y
(Shaanxi Provincial Government)	Incomplete governance	$1-y$
Mainstream Government	Reward	z
(Henan Provincial Government)	No reward	$1-z$

2.2.3. Model Construction

(1) Definition of parameters

Based on the above analysis, this paper defines the parameters involved as follows:

A_1, A_2, A_3 are the total amounts of pollutants to be governed by Branch Government 1, Branch Government 2 and Mainstream Government, respectively.

C_1, C_2 are the cost of Branch Government 1 and Branch Government 2 to control per unit of pollutants, respectively; C_3 is the cost of the mainstream government to control per unit of pollutants when the two branches failed to govern pollutants completely and discharge them to the mainstream.

S_1, S_2 are the ecological benefits obtained from governing per unit of pollutants by Branch Government 1 and Branch Government 2, respectively.

$P_1(0 \leq P_1 \leq A_1)$ is the amount of pollutants Branch Government 1 transfers to other branches and the mainstream for its own economic development, θ_1 is the proportion of the pollutants transferred to Branch Government 2 for governance to P_1, $1-\theta_1$ is the proportion of pollutants transferred to the mainstream government for governance to P_1.

$P_2(0 \leq P_2 \leq A_2)$ is the amount of pollutants Branch Government 2 transfers to other branches and the mainstream for its own economic development, θ_2 is the proportion of the pollutants transferred to the Branch Government 1 for governance to P_2, $1-\theta_2$ is the proportion of pollutants transferred to the mainstream government for governance to P_2.

L is the benefit (ecological environment improvement brought to the mainstream government) derived from pollutant governance of Branch Government 1 and Branch Government 2.

B_1, B_2 are compensations given by the mainstream government to Branch Government 1 and Branch Government 2 when branch governments achieve complete governance.

F_1, F_2 are compensations given by Branch Government 1 and Branch Government 2 to the mainstream government when branch governments achieve incomplete governance.

J is the reward given by the mainstream government to a branch government for additional governance of water pollution. If Branch Government 1 achieves complete governance and Branch Government 2 achieves incomplete governance, the mainstream government will reward Branch Government 1 with $\theta_2 J$. In the same way, the mainstream government rewards Branch Government 2 with $\theta_1 J$.

W_1, W_2, W_3 are additional penalties given by the central government if any one of Branch Government 1, Branch Government 2 or Mainstream Government unilaterally fails to execute the ecological compensation mechanism.

(2) Certainty evolutionary system

First, the game benefit matrix of water pollution governance among Mainstream Government, Branch Government 1 and Branch Government 2 is formulated (without considering the influence of random factors), as shown in Table 2.

Table 2. Game benefit matrix among Mainstream Government, Branch Government 1 and Branch Government 2.

	Mainstream Government Executes Reward and Compensation (z)		**Mainstream Government Does Not Execute Reward and Compensation ($1-z$)**	
	Branch Government 2 Achieves Complete Governance (y)	**Branch Government 2 Achieves Incomplete Governance ($1-y$)**	**Branch Government 2 Achieves Complete Governance (y)**	**Branch Government 2 Achieves Incomplete Governance ($1-y$)**
Branch Government 1 achieves complete governance (x)	$-C_1A_1+S_1A_1+B_1$ $-C_2A_2+S_2A_2+B_2$ $L(A_1+A_2)-B_1-B_2$	$-C_1(A_1+\theta_2P_2)+S_1(A_1+\theta_2P_2)+B_1+\theta_2J$ $-C_2(A_2-P_2)+S_2(A_2-P_2)-F_2-W_2$ $-C_3(1-\theta_2)P_2+L(A_1+\theta_2P_2+A_2-P_2)-B_1+F_2-\theta_2J$	$-C_1A_1+S_1A_1$ $-C_2A_2+S_2A_2$ $L(A_1+A_2)-W_3$	$-C_1(A_1+\theta_2P_2)+S_1(A_1+\theta_2P_2)$ $-C_2(A_2-P_2)+S_2(A_2-P_2)-W_2$ $-C_3(1-\theta_2)P_2+L(A_1+\theta_2P_2+A_2-P_2)-W_3$
Branch Government 1 achieves incomplete governance ($1-x$)	$-C_1(A_1-P_1)+S_1(A_1-P_1)-F_1-W_1$ $-C_2(A_2+\theta_1P_1)+S_2(A_2+\theta_1P_1)+B_2+\theta_1J$ $-C_3(1-\theta_1)P_1+L(A_1+\theta_1P_1+A_2-P_1)-B_2+F_1-\theta_1J$	$-C_1(A_1-P_1)+S_1(A_1-P_1)-F_1-W_1$ $-C_2(A_2-P_2)+S_2(A_2-P_2)-F_2-W_2$ $-C_3(P_1+P_2)+L(A_1+A_2-P_1-P_2)+F_1+F_2$	$-C_1(A_1-P_1)+S_1(A_1-P_1)-W_1$ $-C_2(A_2+\theta_1P_1)+S_2(A_2+\theta_1P_1)$ $-C_3(1-\theta_1)P_1+L(A_1+\theta_1P_1+A_2-P_1)-W_3$	$-C_1(A_1-P_1)+S_1(A_1-P_1)$ $-C_2(A_2-P_2)+S_2(A_2-P_2)$ $-C_3(P_1+P_2)+L(A_1+A_2-P_1-P_2)$

According to the game benefit matrix in Table 2, the benefits π_{11} and π_{12} from complete governance and incomplete governance by Branch Government 1 are as follows:

$$\begin{aligned}\pi_{11}= \mathrm{yz}(-C_1A_1+S_1A_1+B_1)+(1-\mathrm{y})\mathrm{z}[-C_1(A_1+\theta_2P_2)+S_1(A_1+\theta_2P_2)+B_1+\theta_2J]\\ +y(1-z)(-C_1A_1+S_1A_1)+(1-y)(1-z)[-C_1(A_1+\theta_2P_2)+S_1(A_1+\theta_2P_2)]\end{aligned} \tag{1}$$

$$\begin{aligned}\pi_{12}=yz[-C_1(A_1-P_1)+S_1(A_1-P_1)-F_1-W_1]+(1-y)z\\ [-C_1(A_1-P_1)+S_1(A_1-P_1)-F_1-W_1]+y(1-z)[-C_1(A_1-P_1)+S_1(A_1-P_1)-W_1]\\ +(1-y)(1-z)[-C_1(A_1-P_1)+S_1(A_1-P_1)]\end{aligned} \tag{2}$$

The average benefit of Branch Government 1 $\overline{\pi}_1$ is:

$$\overline{\pi}_1=x\pi_{11}+(1-x)\pi_{12} \tag{3}$$

The replicated dynamic equation of Branch Government 1 is:

$$F(x)=\frac{dx}{dt}=x(\pi_{11}-\overline{\pi_1})=x(1-x)(\pi_{11}-\pi_{12}) \tag{4}$$

In the same way, the benefits π_{21} and π_{22} from complete governance and incomplete governance by Branch Government 2 are:

$$\begin{aligned}\pi 21=xz(-C_2A_2+S_2A_2+B_2)+(1-x)z[-C_2(A_2+\theta_1P_1)+S_2(A_2+\theta_1P_1)+B_2+\theta_1J]\\ +x(1-z)(-C_2A_2+S_2A_2)+(1-x)(1-z)[-C_2(A_2+\theta_1P_1)+S_2(A_2+\theta_1P_1)]\end{aligned} \tag{5}$$

$$\begin{aligned}\pi_{22}=xz[-C_2(A_2-P_2)+S_2(A_2-P_2)-F_2-W_2]+(1-x)z[-C_2(A_2-P_2)+S_2(A_2-P_2)-F_2-W_2]\\ +x(1-z)[-C_2(A_2-P_2)+S_2(A_2-P_2)-W_2]+(1-x)(1-z)[-C_2(A_2-P_2)+S_2(A_2-P_2)]\end{aligned} \tag{6}$$

The average benefit of Branch Government 2 $\overline{\pi}_2$ is:

$$\overline{\pi}_2=y\pi_{21}+(1-y)\pi_{22} \tag{7}$$

The replicated dynamic equation of Branch Government 2 is:

$$F(y)=\frac{dy}{dt}=y(\pi_{21}-\overline{\pi}_2)=y(1-y)(\pi_{21}-\pi_{22}) \tag{8}$$

In the same way, the benefits π_{31} and π_{32} from reward and compensation, no reward and compensation by the mainstream government are:

$$\begin{aligned}\pi_{31} = & xy[L(A_1 + A_2) - B_1 - B_2] + x(1-y)[-C_3(1-\theta_2)P_2 + L(A_1 + \theta_2 P_2 + A_2 - P_2) - B_1 + F_2 - \theta_2 J] \\ & +(1-x)y[-C_3(1-\theta_1)P_1 + L(A_1 + \theta_1 P_1 + A_2 - P_1) - B_2 + F_1 - \theta_1 J] \\ & +(1-x)(1-y)[-C_3(P_1 + P_2) + L(A_1 + A_2 - P_1 - P_2) + F_1 + F_2]\end{aligned} \tag{9}$$

$$\begin{aligned}\pi_{32} = & xy[L(A_1 + A_2) - W_3] + (1-x)y[-C_3(1-\theta_1)P_1 + L(A_1 + \theta_1 P_1 + A_2 - P_1) - W_3] \\ & +x(1-y)[-C_3(1-\theta_2)P_2 + L(A_1 + \theta_2 P_2 + A_2 - P_2) - W_3] + \\ & (1-x)(1-y)[-C_3(P_1 + P_2) + L(A_1 + A_2 - P_1 - P_2)]\end{aligned} \tag{10}$$

The average benefit of Mainstream Government $\overline{\pi}_3$ is:

$$\overline{\pi}_3 = z\pi_{31} + (1-z)\pi_{32} \tag{11}$$

Replicated dynamic equation of Mainstream Government is:

$$F(z) = \frac{dz}{dt} = z(\pi_{31} - \overline{\pi}_3) = z(1-z)(\pi_{31} - \pi_{32}) \tag{12}$$

After simplification, replicated dynamic equations of Branch Government 1 and Branch Government 2 and Mainstream Government are as shown in Equations (13)–(15). They form the certainty evolutionary system of the game model.

$$F(x) = x(1-x)\left\{\begin{array}{l}(S_1 - C_1)(P_1 + \theta_2 P_2) + y[\theta_2 P_2(C_1 - S_1) + W_1] + z(B_1 + \theta_2 J + F_1 + W_1) \\ -yz(\theta_2 P_2 J + W_1)\end{array}\right\} \tag{13}$$

$$F(y) = y(1-y)\left\{\begin{array}{l}z(B_2 + \theta_1 P_1 J + F_2 + W_2) - xz(\theta_1 J + W_2) + x[W_2 + (C_2 - S_2)\theta_1 P_1] + \\ (S_2 - C_2)(P_2 + \theta_1 P_1)\end{array}\right\} \tag{14}$$

$$F(z) = z(1-z)\left\{\begin{array}{l}F_1 + F_2 + xy(\theta_2 J + \theta_1 J - W_3) + x(-B_1 - \theta_2 J - F_1 + W_3) + \\ y(-B_2 - \theta_1 J - F_2 + W_3)\end{array}\right\} \tag{15}$$

(3) Stochastic evolutionary game model

Based on the certainty evolutionary system, a stochastic process is introduced to consider the influence of uncertainty of various factors in the midstream of the Yellow River on the decision-making subject. White Gaussian noise is a nonlinear normal distribution function, which describes random factors well. Therefore, this paper introduces white Gaussian noise to describe the random interference of the game system. After the improvement of Equations (13)–(15), the following equations are obtained:

$$dx(t) = \left\{\begin{array}{l}(S_1 - C_1)(P_1 + \theta_2 P_2) + y[\theta_2 P_2(C_1 - S_1) + W_1] \\ +z(B_1 + \theta_2 J + F_1 + W_1) - yz(\theta_2 J + W_1)\end{array}\right\} x(t)[1 - x(t)]dt + \sigma x(t)d\omega(t) \tag{16}$$

$$dy(t) = \left\{\begin{array}{l}z(B_2 + \theta_1 J + F_2 + W_2) - xz(\theta_1 J + W_2) + \\ x[W_2 + (C_2 - S_2)\theta_1 P_1] + (S_2 - C_2)(P_2 + \theta_1 P_1)\end{array}\right\} y(t)[1 - y(t)]dt + \sigma y(t)d\omega(t) \tag{17}$$

$$dz(t) = \left\{\begin{array}{l}xy(\theta_2 J + \theta_1 J - W_3) + x(-B_1 - \theta_2 J - F_1 + W_3) + \\ y(-B_2 - \theta_1 J - F_2 + W_3) + F_1 + F_2\end{array}\right\} z(t)[1 - z(t)]dt + \sigma z(t)d\omega(t) \tag{18}$$

Replicated dynamic equations after being randomly interfered with of Branch Government 1, Branch Government 2 and Mainstream Government are as shown in Equations (16)–(18), respectively. σ refers to the random interference strength. $\omega(t)$ is a standard one-dimensional Brown motion. Brown motion is an irregular random fluctuation, which can describe the influence of random interference factors well, and $d\omega(t)$ represents Gaussian white noise. When $t > 0$, the step size $h > 0$, and its increment $\triangle\omega(t) = \omega(t+h) - \omega(t)$ obeys normal distribution $N\left(0, \sqrt{h}\right)$.

2.2.4. Model Solution

(1) Stability judgment of stochastic evolution system

Regarding Equations (16)–(18), when the initial game $t = 0$, $x(0) = 0, y(0) = 0, z(0) = 0$; the equation has at least the zero solution. This indicates that the system will always be in this stable state without external interference. However, in reality, the decision-making subject is disturbed by environmental changes, which will inevitably affect the stability of the system. According to the study of Baker and Buckwar [28], the stability of the game system can be judged based on the stability judgment theorem of stochastic differential equations.

Lemma 1. *Giving a Stochastic Differential Equation:*

$$dx(t) = f(t, x(t))dt + g(t, x(t))d\omega(t), x(t_0) = x_0 \tag{19}$$

Supposing there exists a function $V(t, x)$ and constants c_1, c_2, such that $c_1|x|^p \leq V(t, x) \leq c_2|x|^p$.

(1) If there is a positive constant γ, such that $LV(t, x) \leq \gamma V(t, x), t \geq 0$, the zero solution of Equation (19) p-order moment is exponentially stable and $E|x(t, x)|^p < (c_2/c_1)|x_0|^p e^{\gamma t}, t \geq 0$ is achieved.
(2) If there is a positive constant γ, such that $LV(t, x) \geq \gamma V(t, x), t \geq 0$, the zero solution of Equation (19) p-order moment is exponentially stable and $E|x(t, x)|^p \geq (c_2/c_1)|x_0|^p e^{\gamma t}, t \geq 0$ is achieved.

Wherein, $LV(t, x) = V_t(t, x) + V_x(t, x)f(t, x) + 1/2g^2(t, x)V_{xx}(t, x)$.

According to Lemma 1, for Equations (16)–(18), take $V_t(t, x) = x, V_t(t, y) = y, V_t(t, z) = z$, $c_1 = c_2 = 1$, $p = 1$, $\gamma = 1$, then:

$$LV(t, x) = f(t, x) = x\left\{ \begin{array}{l} (S_1 - C_1)(P_1 + \theta_2 P_2) + y[\theta_2 P_2(C_1 - S_1) + W_1] \\ +z(B_1 + \theta_2 J + F_1 + W_1) - yz(\theta_2 J + W_1) \end{array} \right\} \tag{20}$$

$$LV(t, y) = f(t, y) = y\left\{ \begin{array}{l} z(B_2 + \theta_1 J + F_2 + W_2) - xz(\theta_1 J + W_2) + \\ x[W_2 + (C_2 - S_2)\theta_1 P_1] + (S_2 - C_2)(P_2 + \theta_1 P_1) \end{array} \right\} \tag{21}$$

$$LV(t, z) = z\left\{ \begin{array}{l} xy(\theta_2 J + \theta_1 J - W_3) + x(-B_1 - \theta_2 J - F_1 + W_3) + \\ y(-B_2 - \theta_1 J - F_2 + W_3) + F1 + F2 \end{array} \right\} \tag{22}$$

If the zero solution of Equations (16)–(18) is exponentially stable, the following conditions should be met:

$$y[\theta_2 P_2(C_1 - S_1) + W_1] + z(B_1 + \theta_2 J + F_1 + W_1) - yz(\theta_2 J + W_1) \leq -1 - (S_1 - C_1)(P_1 + \theta_2 P_2) \tag{23}$$

$$z(B_2 + \theta_1 J + F_2 + W_2) - xz(\theta_1 J + W_2) + x[W_2 + (C_2 - S_2)\theta_1 P_1] \leq -1 - (S_2 - C_2)(P_2 + \theta_1 P_1) \tag{24}$$

$$xy(\theta_2 J + \theta_1 J - W_3) + x(-B_1 - \theta_2 J - F_1 + W_3) + y(-B_2 - \theta_1 J - F_2 + W_3) \leq -1 - F_1 - F_2 \tag{25}$$

(2) System equilibrium

As Equations (16)–(18) are nonlinear I*tô* stochastic differential equations, their analytical solutions cannot be directly obtained. Thus, the numeric solutions will be calculated. The range $[0, T]$ is divided into N subranges and $0 = t_0 < t_1 < t_2 < \ldots < t_{N-1} < t_N = T$, wherein the length of each subrange is $h = T/N$, node $t_n = nh$, $n = 1, 2, 3 \ldots N$. First, Taylor expansion is performed on I*tô* type stochastic differential equation, and then it is corrected by the Milstein method [29]. The results are shown in Equations (26)–(28). Numerical solutions can be obtained by numerical simulation of these formulas. When the numerical simulation is conducted, the parameters are assigned according to the Shanxi Statistical Yearbook, Shaanxi Statistical Yearbook and related research [5,9], and the initial values of each parameter satisfy the constraints of Equations (23)–(25), as shown in Table 3. Numerical simulations are performed using Matlab2019a software.

$$x(t_n+1) = x(t_n) + h\left\{ \begin{array}{l} (S_1 - C_1)(P_1 + \theta_2 P_2) + y[\theta_2 P_2 (C_1 - S_1) + W_1] + \\ z(B_1 + \theta_2 J + F_1 + W_1) - yz(\theta_2 J + W_1) \end{array} \right\} + \Delta\omega n(\sigma x(t_n)) + \frac{1}{2}\left[(\Delta\omega n)^2 - h\right](\sigma x(t_n)) \quad (26)$$

$$y(t_{n+1}) = y(t_n) + h\left\{ \begin{array}{l} z(B_2 + \theta_1 J + F_2 + W_2) - xz(\theta_1 J + W_2) + \\ x[W_2 + (C_2 - S_2)\theta_1 P_1] + (S_2 - C_2)(P_2 + \theta_1 P_1) \end{array} \right\} + \Delta\omega n(\sigma y(t_n)) + \frac{1}{2}\left[(\Delta\omega n)^2 - h\right](\sigma y(t_n)) \quad (27)$$

$$z(t_{n+1}) = z(t_n) + h\left\{ \begin{array}{l} xy(\theta_2 J + \theta_1 J - W_3) + x(-B_1 - \theta_2 J - F_1 + W_3) + \\ y(-B_2 - \theta_1 J - F_2 + W_3) + F_1 + F_2 \end{array} \right\} + \Delta\omega n(\sigma z(t_n)) + \frac{1}{2}\left[(\Delta\omega n)^2 - h\right](\sigma z(t_n)) \quad (28)$$

Table 3. Initial values of each parameter.

P_1	P_2	C_1	C_2	S_1	S_2	S_3	B_1	B_2
15	10	7	9	6	8	8	3	4
F_1	F_2	W_1	W_2	W_3				
3	2	5	5	3				

3. Results

The influence rule of the simulation on the pollution control cost of the mainstream government and the branch government, the pollution control benefits, the ecological compensation from the mainstream government to the branch government and the ecological compensation from the branch government to the mainstream government is almost in line with the results of Gao et al. [9,20]. This similarity can be explained by how under the analytical framework, the ecological compensation game relationship between the mainstream government and a branch government is similar to that between the upstream and downstream governments. Moreover, the current outcomes of the ecological compensation mechanism of the upstream and downstream governments are directly extended to the game analysis framework between the mainstream government and a branch government. Accordingly, it is not described in detail since the influence rule of the relevant parameters on the mainstream government and a branch government is similar to that of the upstream and downstream governments. In the following sections, the initial probability, the proportion of pollution transferred by the branch government, the reward of the mainstream government and the random factors are analyzed.

3.1. Influence Rule of the Initial Probability

The initial probability refers to the initial willingness of each subject to choose different strategies. In other words, it can indicate whether the mainstream government and the branch government are willing to implement ecological compensation. The initial probabilities of the three governments are set as 0.2, 0.4, 0.6 and 0.8, respectively, and the results are shown in Figure 2.

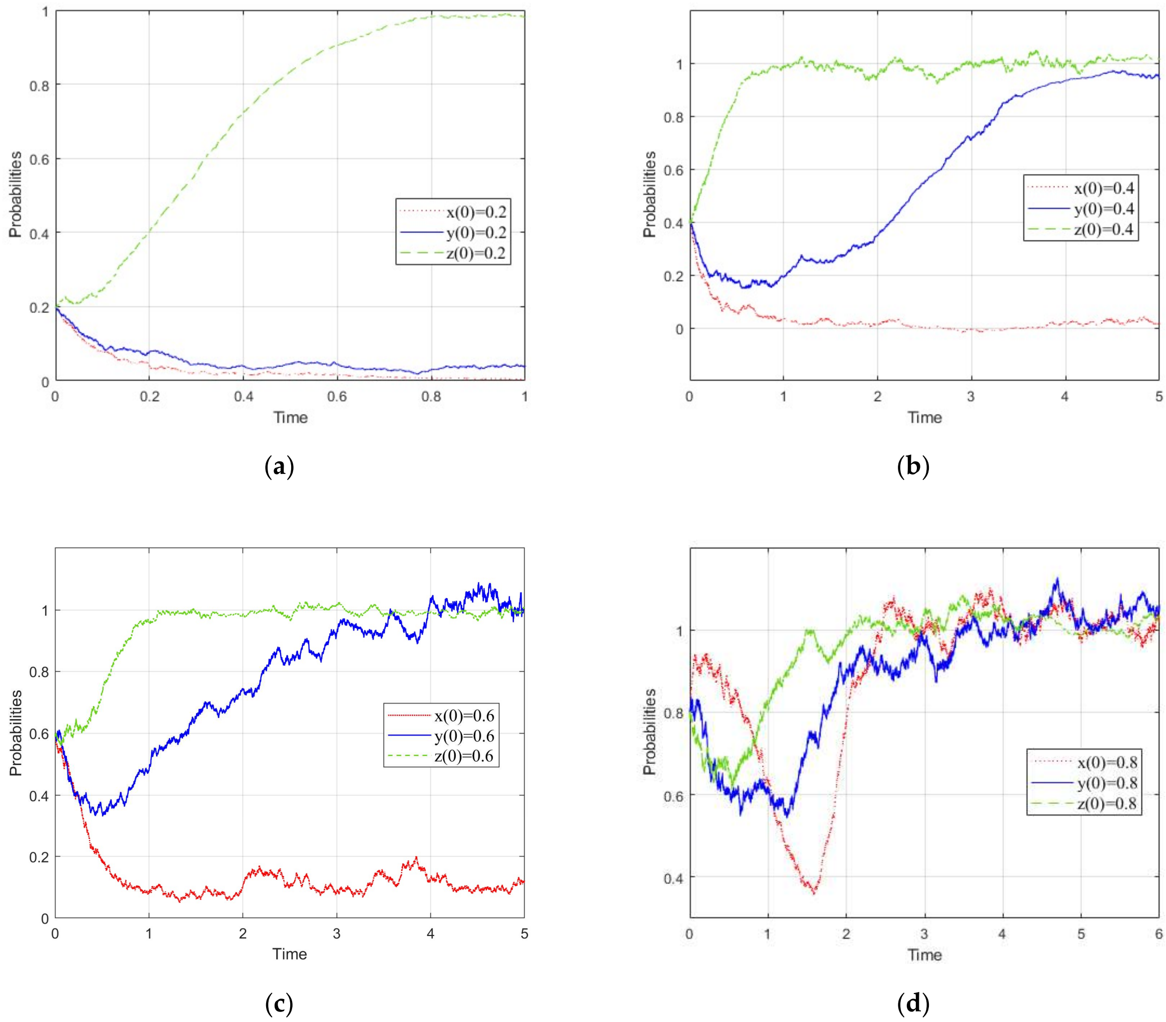


Figure 2. Influence rule of the initial probabilities on three governments: (**a**) influence rule of the initial probabilities on three governments when $x(0) = 0.2$, $y(0) = 0.2$, $z(0) = 0.2$, (**b**) influence rule of the initial probabilities on three governments when $x(0) = 0.4$, $y(0) = 0.4$, $z(0) = 0.4$, (**c**) influence rule of the initial probabilities on three governments when $x(0) = 0.6$, $y(0) = 0.6$, $z(0) = 0.6$, and (**d**) influence rule of the initial probabilities on three governments when $x(0) = 0.8$, $y(0) = 0.8$, $z(0) = 0.8$.

The initial probability affects the strategic choices of the mainstream government and the branch government, in which a higher initial probability promotes the three governments to achieve the optimal situation of complete governance and reward and compensation. To be specific, in Figure 2a, it can be seen that in the case of low initial probability of the three governments, Branch Government 1 and Branch Government 2 will quickly stabilize the incomplete governance strategy whereas the government of the mainstream will stabilize to the reward and compensation strategy. In Figure 2b, it can be seen that if the initial probability of the three governments increases to 0.4, then Branch Government 2 will gradually stabilize to the complete governance strategy whereas Branch Government 1 will still stabilize to the incomplete governance strategy. Hence, the increase of the initial probability can first affect the areas with less pollution treatment. In Figure 2c, it can be seen that when the initial probability continuously increases to 0.6, the time needed by Branch Government 2 to stabilize to the complete governance strategy will decrease whereas Branch Government 1 adopts an unstable strategy. In Figure 2d, it can be seen that with the continuous increase of the initial probability, the two branch governments will stabilize to the complete governance strategy whereas the mainstream government will stabilize to the reward and compensation strategy.

3.2. Influence Rule of θ_1

$\theta 1 = 0.1, 0.3, 0.5, 0.7$ are respectively set. This is designed to explore the influence rule of the proportion change of the pollution transferred by Branch Government 1 to Branch Government 2 in the total pollutant amount $P1$ that needs to be treated by Branch Government 1 by the decision-making of the three governments.

(1) Influence rule of changes of θ_1 on the Branch Government 1

The initial probabilities of the three governments are set as $x(0) = 0.7, y(0) = 0.7, z(0) = 0.7$ and $x(0) = 0.2, y(0) = 0.2, z(0) = 0.2$, respectively. This is designed to analyze the rule of the decision changes in Branch Government 1 with the change of θ_1 under different initial intentions of the three governments. The results are shown in Figure 3.

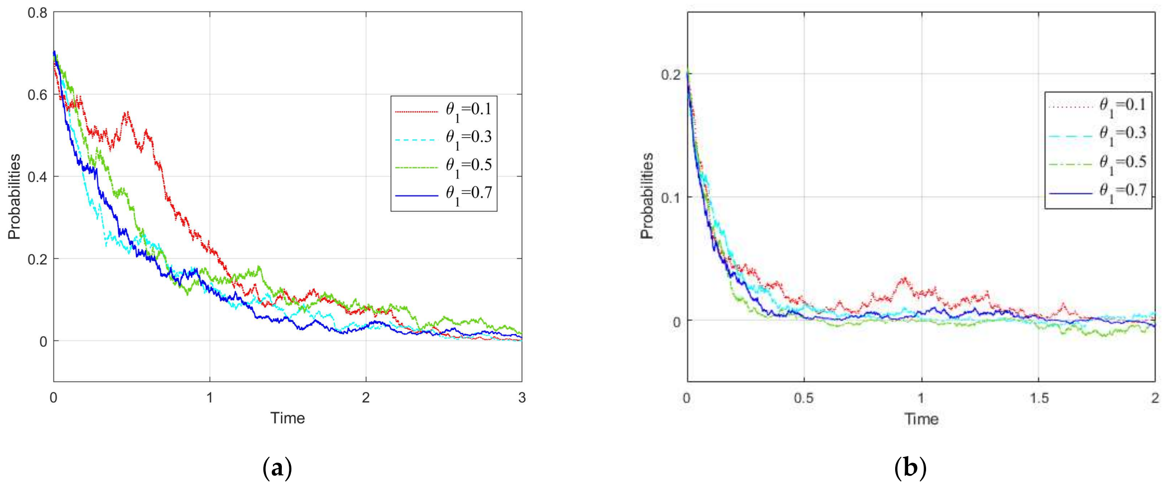


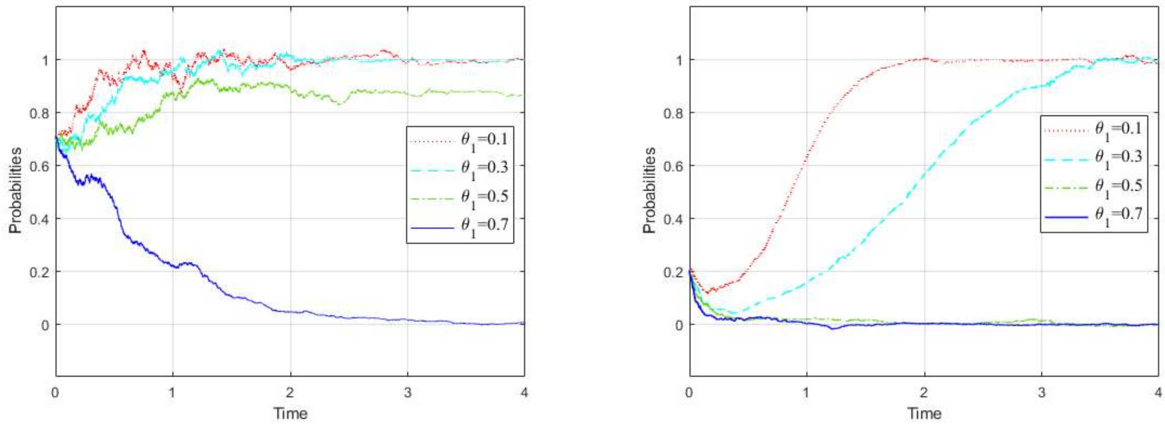
Figure 3. Influence rule of changes of $\theta 1$ on Branch Government 1: (**a**) influence rule of changes of $\theta 1$ on Branch Government 1 when $x(0) = 0.7$, $y(0) = 0.7$, $z(0) = 0.7$, and (**b**) influence rule of changes of $\theta 1$ on Branch Government 1 when $x(0) = 0.2$, $y(0) = 0.2$, $z(0) = 0.2$.

Despite the initial probability being 0.7 or 0.2, it exerts little impact on the strategy choice of Branch Government 1. Accordingly, the change of θ_1 will not change the strategic choice of Branch Government 1. When the initial willingness increases, Branch Government 1 will stabilize to the incomplete governance strategy at a faster speed. When the initial probability decreases, the influence of random factors on the decision-making of Branch Government 1 will also decrease.

(2) Influence rule of the change of θ_1 on Branch Government 2

The initial probabilities of the three governments are set as $x(0) = 0.7, y(0) = 0.7, z(0) = 0.7$ and $x(0) = 0.2, y(0) = 0.2, z(0) = 0.2$, respectively. This is designed to analyze the rule of the decision changes in Branch Government 2 with the change of θ_1 under different initial intentions of the three governments. The results are shown in Figure 4.

θ_1 change exerts a greater impact on the strategy choice of Branch Government 2. To be specific, in Figure 4a, it can be seen that when the initial willingness is 0.7, with the increase of θ_1, the time needed for Branch Government 2 to stabilize to the complete governance strategy will be longer; when θ_1 is 0.5, the strategy of Branch Government 2 is unstable; and when θ_1 is above 0.5, Branch Government 2 will stabilize to the incomplete governance strategy. Accordingly, in the case of high initial willingness, Branch Government 2 will choose the incomplete governance strategy if the pollution proportion transferred by Branch Government 1 is more than half. In Figure 4b, it can be seen that when the initial willingness is 0.2, the probability of Branch Government 2 to choose governance decreases first and then stabilizes to the complete governance strategy. With the increase of θ_1, the time needed to stabilize the governance strategy is longer; in the case of low initial willingness, the critical value of Branch Government 2 to choose the incomplete governance strategy is even smaller.

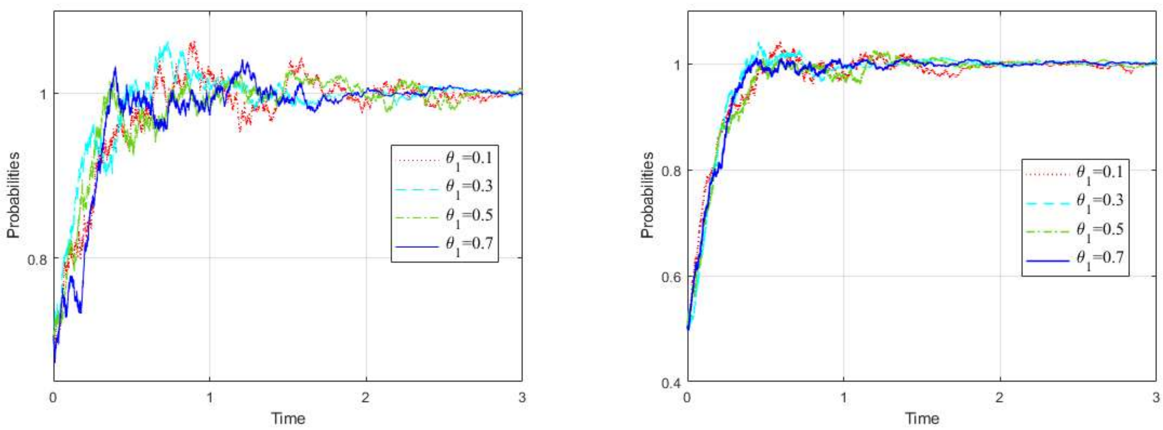


(**a**) (**b**)

Figure 4. Influence rule of changes of $\theta 1$ on Branch Government 2: (**a**) influence rule of changes of $\theta 1$ on Branch Government 2 when $x(0) = 0.7$, $y(0) = 0.7$, $z(0) = 0.7$, and (**b**) influence rule of changes of $\theta 1$ on Branch Government 2 when $x(0) = 0.2$, $y(0) = 0.2$, $z(0) = 0.2$.

(3) Influence rule of change of θ_1 on the mainstream government

The initial probabilities of the three governments are set as $x(0) = 0.7, y(0) = 0.7, z(0) = 0.7$ and $x(0) = 0.5, y(0) = 0.5, z(0) = 0.5$, respectively. This is designed to analyze the rule of the decision change of the mainstream government with the change of θ_1 under different initial intentions of the three governments. The results are shown in Figure 5.



(**a**) (**b**)

Figure 5. Influence rule of changes of $\theta 1$ on Mainstream Government: (**a**) influence rule of changes of $\theta 1$ on Mainstream Government when $x(0) = 0.7$, $y(0) = 0.7$, $z(0) = 0.7$, and (**b**) influence rule of changes of $\theta 1$ on Mainstream Government when $x(0) = 0.5$, $y(0) = 0.5$, $z(0) = 0.5$.

θ_1 change has little impact on the strategic choice of the mainstream government. To be specific, when the initial willingness is 0.7 and 0.2, with the increase of θ_1, the mainstream government quickly stabilizes to the reward and compensation strategy and is less affected by the random factors. This can be explained by how with the increase of θ_1, the Branch Government 2 will stabilize to the no governance strategy. Hence, the mainstream government should actively implement the reward and compensation strategy to encourage the branch government to deal with water pollution.

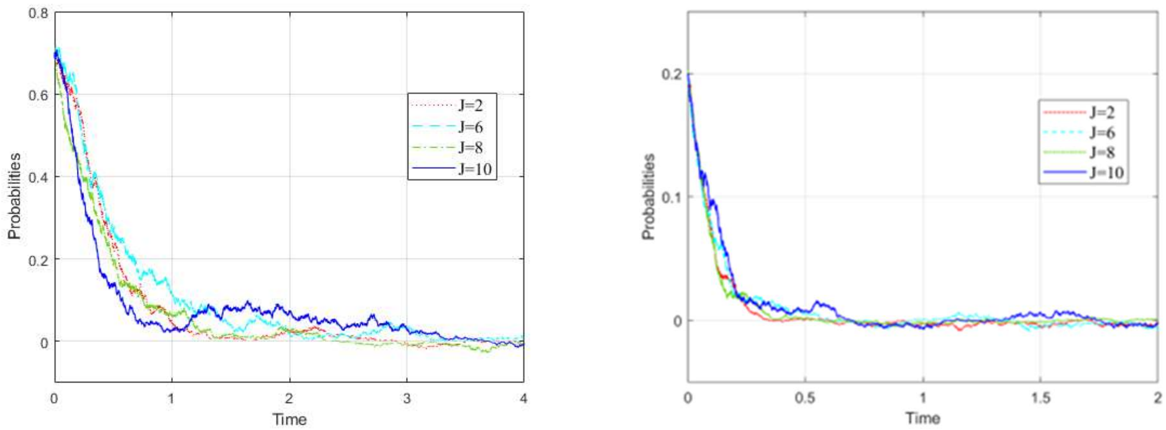
Since the influence rule of the change of θ_2 on the three governments is similar to that of the θ_1 change, this will not be discussed in detail. In the following sections, the influence rule of the change of the mainstream government's reward to the two branch governments on the strategy choice of the three governments is explored.

3.3. Influence Rule of J

$J = 2, 6, 8, 10$ are set, respectively. This is designed to analyze the influence rule of the change of the mainstream government's reward to the two branch governments by the decision-making of the three governments.

(1) Influence rule of J change on Branch Government 1

The initial probabilities of the three governments are set as $x(0) = 0.7, y(0) = 0.7, z(0) = 0.7$ and $x(0) = 0.2, y(0) = 0.2, z(0) = 0.2$, respectively. This is designed to analyze the rule of the change of decisions in Branch Government 1 with the change of J under different initial intentions of the three governments. The results are shown in Figure 6.



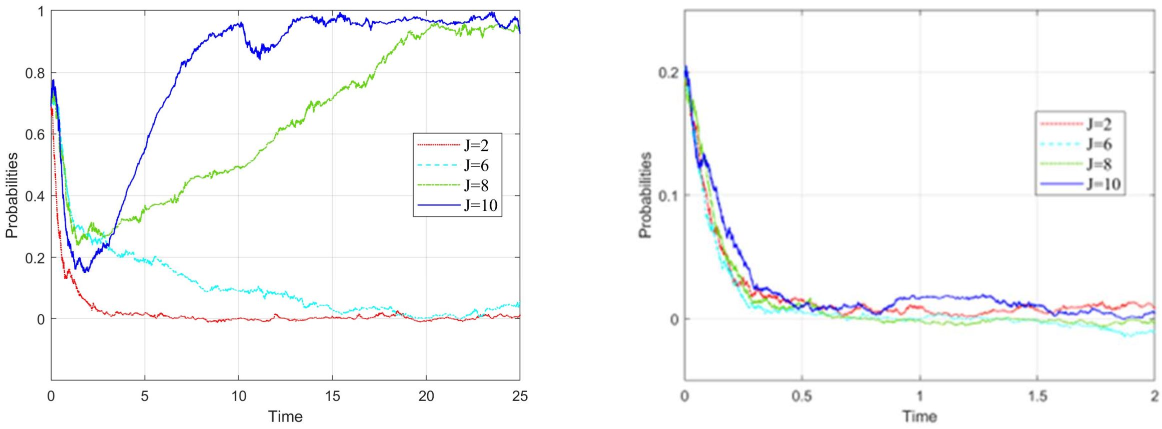
(**a**) (**b**)

Figure 6. Influence rule of J change on Branch Government 1: (**a**) influence rule of J change on Branch Government 1 when $x(0) = 0.7$, $y(0) = 0.7$, $z(0) = 0.7$, and (**b**) influence rule of J change on Branch Government 1 when $x(0) = 0.2$, $y(0) = 0.2$, $z(0) = 0.2$.

When Branch Government 1 chooses the incomplete governance strategy, the reward J of the mainstream government to the branch government does not affect the final decision of Branch Government 1. However, with the increase of J, Branch Government 1 will need a longer time to choose the incomplete governance strategy.

(2) Influence rule of J change on Branch Government 2

The initial probabilities of the three governments are set as $x(0) = 0.7, y(0) = 0.7, z(0) = 0.7$ and $x(0) = 0.2, y(0) = 0.2, z(0) = 0.2$, respectively. This is designed to analyze the rule of the change of decisions in Branch Government 2 under different initial willingness of the three governments. The results are shown in Figure 7.



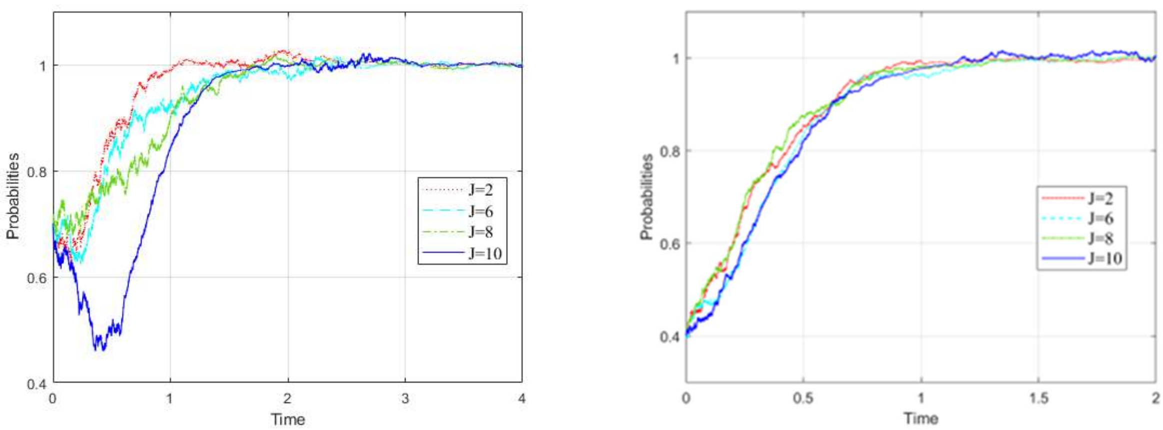
(**a**) (**b**)

Figure 7. Influence rule of J change on Branch Government 2: (**a**) influence rule of J change on Branch Government 2 when $x(0) = 0.7$, $y(0) = 0.7$, $z(0) = 0.7$, and (**b**) influence rule of J change on Branch Government 1 when $x(0) = 0.2$, $y(0) = 0.2$, $z(0) = 0.2$.

In the case of high initial willingness, the size of J influences the strategy choice of Branch Government 2. With the increase of J, Branch Government 2 will gradually transform from an incomplete governance strategy to a complete governance strategy at a faster speed. Therefore, the greater the incentives for additional pollution control and emission reduction by the mainstream government, the more the branch government is motivated to implement pollution control. In the case of low initial willingness, the reward of the mainstream government to the branch government does not affect the strategy choice of the branch government.

(3) Influence rule of J change on the mainstream government

The initial probabilities of the three governments are set as $x(0) = 0.7, y(0) = 0.7, z(0) = 0.7$ and $x(0) = 0.4, y(0) = 0.4, z(0) = 0.4$, respectively. This is designed to analyze the rule of changes in the decisions of Branch Government 2 under different initial intentions of the three governments. The results are shown in Figure 8.



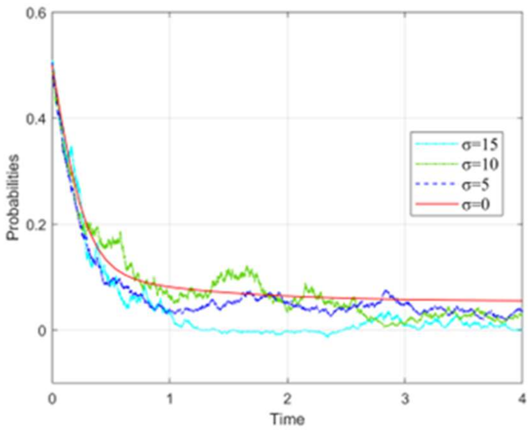
(**a**) (**b**)

Figure 8. Influence rule of J change on Mainstream Government: (**a**) influence rule of J change on Mainstream Government when x(0) = 0.7, y(0) = 0.7, z(0) = 0.7, and (**b**) influence rule of J change on Mainstream Government when x(0) = 0.4, y(0) = 0.4, z(0) = 0.4.

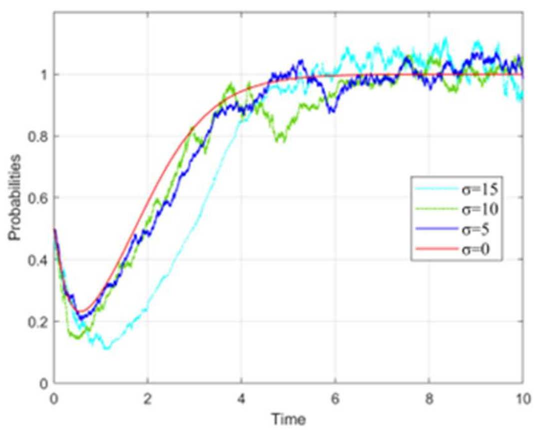
In Figure 8a, it can be seen that in the case of high initial probability, the larger the mainstream government's reward to the branch government, the longer the mainstream government will take to stabilize to the reward and compensation strategy. In Figure 8b, it can be seen that in the case of low initial probability, the mainstream government will stabilize to the reward and compensation strategy at a relatively faster speed and the change of J size exerts less influence on the policy selection of the mainstream government. This can be explained by how in the case of low initial probability, the mainstream will actively offer rewards and compensations to motivate the two branch governments to treat water pollution.

3.4. Influence Rule of Random Factors

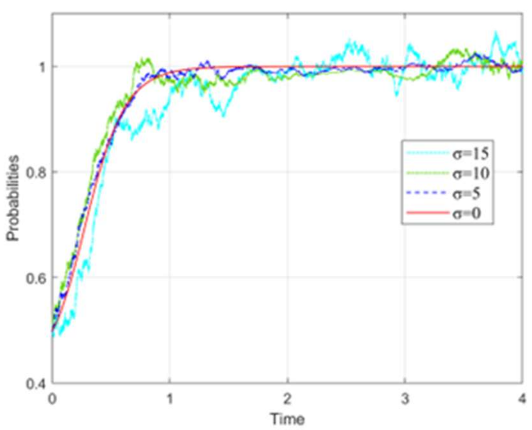
σ is set as 0, 5, 10 and 15, respectively, and the initial probabilities of the three governments are set as $x(0) = 0.5, y(0) = 0.5, z(0) = 0.5$, respectively. This is designed to analyze the rule of the change of the decisions of Branch Government 1, Branch Government 2 and the mainstream government with the change of random factor σ. The results are shown in Figure 9. It can be seen that branch governments are most affected by random factors and the mainstream government is less affected by random factors. For Branch Government 1, the stronger the random interference, the faster it takes for Branch Government 1 to stabilize to the incomplete governance strategy. For Branch Government 2, the stronger the random interference, the longer it takes for Branch Government 2 to stabilize to the complete governance strategy.



(**a**)



(**b**)



(**c**)

Figure 9. Influence rule of random factor σ change on three governments: (**a**) influence rule of random factor σ change on Branch Government 1, (**b**) influence rule of random factor σ change on Branch Government 2, and (**c**) influence rule of random factor σ change on Mainstream Government.

4. Discussion

This paper targets the problem of ecological compensation mechanism design and expands the upstream and downstream governments to the three governments of the mainstream government and the two branch governments.

Based on the analysis of Section 3.1, it can be seen that the initial probability affects the final strategy selection, and a higher initial probability enables the system to stabilize to the optimal strategy (complete governance, complete governance, reward and compensation). When the initial probabilities are high, it means that the mainstream government is willing to choose the strategy of reward and compensation, and the branch governments are willing to choose the strategy of complete governance water pollution. On the contrary, when the initial probabilities are low, the mainstream government is willing to choose the strategy of reward and compensation, but the branch governments are willing to choose the strategy of not complete governance of water pollution. This is because if the branch governments are unwilling to conduct water pollution control, then the pollution control cost of the mainstream government will increase, and ecological benefits obtained from governing pollutants by branch governments will decrease. As a result, the mainstream government will take the initiative to implement the reward and compensation strategy to encourage the water pollution control of the two branch governments. All of these conclusions are similar to those of Jiang et al. [22] and Sheng et al. [21]. Their research showed that the initial probability affects the strategic choices of upstream and downstream governments. It is suitable to expand the research subject and construct the compensation and reward mechanism, which can produce reliable outcomes.

Based on the analysis of Section 3.1, it can be seen that the mainstream government (compared with the downstream government) firstly stabilizes to the compensation policy, and the branch governments play a critical role in the building of the ecological compensation mechanism of the Yellow River Basin. This is different from the conclusion of Gao et al. [30]. Following the stochastic evolutionary game, Gao et al. constructed the basin ecological compensation mechanism of the upstream government, the downstream government and the central government, concluding that the water demand area (the downstream government) played a critical role in the building of the ecological compensation mechanism of the South-to-North Water Diversion Project. This can be explained by the fact that in the South-to-North Water Diversion Project in China, water quality serves as one of the most important indicators. The downstream government does not have high willingness to pay more ecological compensation. This is because the central government strictly supervises the water quality of the upstream water supply area, the upstream government has to pay a huge cost to guarantee the water quality and the downstream government has been using clean water resources free of charge for a long time to ensure its development. However, in the middle reaches of the Yellow River, industrialization for a long time has led to serious pollution of the Weihe River and the Fenhe River. If the river basin ecological compensation agreement is signed, then the branch government should input costs continuously and change the local industrial structure; therefore, a branch government has less willingness than the mainstream government. This is also true for the Shanxi Province government, which has serious pollution.

Regarding the influence of random factors on water pollution control, based on the analysis of Section 3.4, it can be seen that while branch governments are most affected by random factors, the mainstream government is less affected by random factors, and the government decision-making process of Branch Government 2 is affected by randomness to a large extent. The stronger the random interference, the longer it takes for Branch Government 2 to stabilize to the governance strategy. This can be explained by how even though the state actively encourages the establishment of an ecological compensation mechanism along the Yellow River Basin, the ecological compensation mechanism is still in the exploration stage due to the fragile ecological environment and serious water pollution. When Branch Government 1 does not actively govern the pollution, despite the existence of the mainstream government actively implementing the reward and compensation strategy, Branch Government 2 will take a longer time to reach stability in the complete governance strategy under the influence of external random factors because of the governance difficulty and the lack of uniform provisions of the reward and compensation amount. According to a random factors study, Jiang [31] took the Xin'an River Basin as the research object and used the stochastic differential game to study transboundary pollution in adjacent areas under the influence of random factors (including terrain, meteorological conditions, river runoff, etc.); the range of pollutant stock under a 95% confidence interval was also provided. Compared with the stochastic differential game method, the stochastic evolutionary game method can obtain the influence process of random factors on the decision-making agent's strategy selection.

Under the analysis framework in this paper, the government is considered to be the game subject, but the analysis model does not involve the enterprises, farmers and urban residents producing pollution in the Yellow River Basin. This can lead to certain limitations. In subsequent analyses, these polluters can be incorporated into the watershed water pollution control analysis model by considering the internal and external relations of the group. For the internal groups of enterprises, farmers and urban residents, the challenge is considering intra-group influences in pollution control in its groups. For the external relations, the challenge is mechanism design in pollution control between enterprises (or resident) group and region government.

5. Conclusions and Suggestion

By incorporating the evolutionary game and stochastic process, this paper built a stochastic evolutionary game model of ecological compensation in the midstream of the Yellow River with the integration of compensation and reward. Moreover, the influence rules of the main parameters on the decisions of the Shanxi, Shaanxi and Henan governments were analyzed. Below are the conclusions.

(1) The initial probability affects the decisions of the three governments. With the increase of the initial probability, Branch Government 2 (the Shaanxi Provincial government) will first stabilize from the incomplete governance strategy to the complete governance strategy. In the case of high initial willingness, the three governments will stabilize to the optimal strategy (complete governance, complete governance, reward and compensation). As a result, strengthening the publicity of the ecological compensation policy and the guidance of the branch and mainstream governments plays an important role in the implementation of the ecological compensation mechanism.

(2) With the increase of the transferred pollution amount of a branch, other branch governments are more unwilling to govern the pollution. When the proportion of transferred pollution amount exceeds a certain critical value, then other branch governments will choose the incomplete governance strategy. In addition, in the case of the high initial willingness of other branch governments to control pollution, the critical value of the proportion of transferred pollution will also increase. As a result, when carrying out the actual implementation of policies, it is essential to strengthen the supervision of a branch government on water pollution governance. When necessary, compensation mechanisms for watershed ecology can be combined with other policies, including China's "The River Chief System" policy, to avoid a branch government's "free-rider" phenomenon. This means that it is necessary to avoid the situation of a branch government transferring the pollution it shall treat to other branch governments.

(3) The more the mainstream government (Henan provincial government) rewards the branch governments (Shanxi and Shaanxi provincial governments) for more pollution control, the more the branch governments will be encouraged to carry out pollution control. Thus, the mainstream government can increase the rewards to the branch governments within a certain range, to encourage them to increase pollution control efforts. To encourage the effective implementation of the mechanism, the central

government can also offer some incentives to the river basin and branches where the ecological compensation mechanism is well implemented.

(4) Branch governments (Shanxi and Shaanxi provincial governments) are greatly affected by random factors, and the greater random interference, the more unfavorable for the branch governments regarding pollution control. It is necessary to consider the influence of random factors when formulating the ecological compensation mechanism. According to the long time series data, the impact of random factors on branch governments should be quantified, and differentiated compensation standards should be set under different circumstances.

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