

## Article

# A Study on the Mechanism and Pricing of Drainage Rights Trading Based on the Bilateral Call Auction Model and Wealth Utility Function

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**Abstract:** In the context of the high incidence of flooding disputes among neighboring subjects in the basin, exploring the drainage rights trading mechanism and clarifying the pricing method of drainage rights are new ideas to manage flooding disasters. In this study, the trading mechanism of drainage rights is constructed using a bilateral call auction model, in which the price constraint, the control total quantity constraint, the clearing rule, and the matching rule are explained. In addition, we adopt the wealth utility concept to construct a function, apply the social welfare function to associate the wealth utility functions of both sides of the transaction, and draw on the environmental Gini coefficient concept to construct a pricing model for drainage rights transactions based on fairness and efficiency. Finally, we conduct an arithmetic simulation of the drainage rights trading mechanism and transaction pricing for China's Jiangsu section of the Huaihe River Basin. The study's results show that the trading mechanism model can give the optimal trading scheme when multiple subjects are trading drainage rights. This fills the gap in existing studies and can lead to efficient trading of drainage rights. At the same time, the pricing model can give the optimal transaction price that considers efficiency and fairness, providing a more practical pricing theory for drainage rights trading.

**Keywords:** bilateral call auction model; drainage rights trading; wealth utility; social welfare function



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## 1. Introduction

Flood security is an essential part of national security [1–3]. However, the established economic and social pattern and basic national conditions of China are that the population and economic factors are concentrated in the high-risk areas of flooding at the tail of rivers, and losses from floods and droughts account for about 70% of losses from all types of natural disasters [4], so studies related to flood exclusion have been highly valued. Drainage rights management has received more widespread attention as a non-engineering tool to rationalize the planning of each region's drainage authority from the perspective of economic and resource management. Drainage rights are the authority of a subject (e.g., a province or municipality) to remove flood water into a river. Drainage rights can generally be acquired in two ways: by accepting an allocation from a higher level of government (initial allocation of drainage rights) and purchasing drainage rights from other parties (trading of drainage rights). Drainage rights trading is a transaction that relies on the difference in the economic level of neighboring regions to drain flood water from the economically strong to the economically weak. Since the transaction price is between the flood losses of both trading parties, the transaction will not only make both trading parties profit from it but also lower the overall flood losses of the region.

The concept of drainage rights was proposed in 2014 [5]. At present, the research perspectives of drainage rights mainly contain the necessity and feasibility of initial and reallocation, the control total and allocation mechanism of initial allocation, the trading

mechanism of drainage rights, and the pricing of trading [6–13]. The research in this paper focuses on the trading mechanism and pricing of drainage rights. Concerning the trading mechanism of drainage rights, Chinese scholars have used the evolutionary game model to design the operating mechanism of drainage rights trading under “quasi-market” conditions [14]. Scholars in other countries have conducted more case studies, such as flood risk transfer mechanisms in the UK, flood-resilient discharge mechanisms in the Netherlands, and rainwater storage projects in Japanese cities, among others [15–18]. In general, the existing research is relatively poor and fragmented. In addition, these studies all contain a research assumption that the number of buyers and sellers is one. The high incidence of drainage rights trading is a time of flooding, and it is common for multiple entities to need to buy or sell drainage rights at the same time. Therefore, broadening the perspective of research in this area is needed. In terms of pricing the transactions of drainage rights, Chinese scholars have used methods containing the entire cost method, flood loss assessment method, and asymmetric information bargaining model [19,20]. Studies by scholars in other countries still focus on case studies, such as the drainage charging system in the UK and the “stormwater fee” system in Germany [21–25]. In general, the underlying philosophy of these studies remains the use of flood losses as a lower bound on the price of trading drainage rights, which is certainly very reasonable. However, the construction of an asymmetric information bargaining model has triggered scholars to think about it. We should not only focus on the actual cost of the drainage rights but also on the profit game between the two sides of the drainage rights transaction [26]. The asymmetric information bargaining model only gives the game process of drainage rights trading and cannot tell us the optimal trading price after the game.

To overcome the shortcomings of the above study, we explore the optimal matching scheme of transaction subjects when there are multiple sellers and buyers. To this end, we apply the bilateral call auction model to construct a trading mechanism for drainage rights, in which the price constraint, the control total constraint, the clearing rule, and the matching rule are explained. Bilateral call auction models have been applied to pricing wastewater discharge rights, water rights, and electricity [27,28]. It can give an optimal trading scheme when multiple subjects participate in the transaction. Thus, the method applies to the study of drainage rights trading mechanisms. In addition, we adopt the wealth utility concept to construct a function, apply the social welfare function to associate the wealth utility functions of both sides of the transaction, and draw on the environmental Gini coefficient concept to construct a pricing model for drainage rights transactions based on parallel equity and efficiency. The wealth utility function has been applied to financial products and the pricing of wastewater discharge rights, among others [29], and its intrinsic idea is to focus on the satisfaction of buyers and sellers for profits. Thus, we introduce it as a new idea for pricing drainage rights. Finally, we conducted an arithmetic simulation of the drainage rights trading mechanism and transaction pricing for China’s Jiangsu section of the Huaihe River Basin. In summary, the article’s innovation lies first in applying the bilateral call auction model and the wealth utility function. The above two models better solve the subject matching and pricing problems of drainage rights trading. Second, we set the “unfair pullback coefficient” in the pricing model, which significantly improves the fairness of the pricing of drainage rights. Finally, we derive the optimal trading price instead of discussing only the effect of the model’s parameters on the optimal trading price. This provides a more practical pricing theory for drainage rights trading.

## 2. Study on the Transaction Mechanism of Bilateral Call Auction of Drainage Rights

### 2.1. Basic Assumptions

**Assumption 1.** *There are multiple potential buyers of drainage rights  $B_i$  and sellers  $S_j$  in a given area.  $B_i$  denotes the  $i$ th drainage right purchaser in the region,  $S_j$  denotes the  $j$ th drainage right seller in the region,  $m$  denotes the number of purchasers, and  $n$  denotes the number of sellers. Generally, a subject unit tends to be a seller of drainage rights when its flood losses are low or when the drainage power generates a surplus, and conversely tends to be a purchaser [7].*

**Assumption 2.** Set the unit flood loss values for the *i*th drainage right purchaser and the *j*th drainage right seller as  $C_{bi}$  and  $C_{sj}$ . Unit flood economic loss is abbreviated as unit flood loss (hereinafter), which indicates the direct economic loss formed by a unit of flooding suffered by a trading entity. The unit flood loss is closely related to the economic level of the region [30].

**Assumption 3.** Set the transaction demand of the *i*th drainage right buyer and the *j*th drainage right seller as  $x_i$  and  $y_j$ , and the expected transaction prices as  $o_i$  and  $f_j$ .  $x_i$  and  $y_j$  reflect the difference between the total demand for flood discharge of the subject and the number of drainage rights received in the initial allocation;  $o_i$  reflects the *i*th drainage right purchaser’s assessment of the value of the drainage rights existing in the market (let the *i*th drainage right purchaser consider the lowest value of the drainage rights existing in the market to be  $f_{min}$ ), and  $o_i$  is less than or equal to the unit flood loss ( $C_{bi}$ ) of the *i*th drainage right purchaser, then we have  $f_{min} \leq o_i \leq C_{bi}$ ;  $f_j$  should be greater than or equal to the unit flood loss of the *j*th drainage rights seller ( $C_{sj}$ ) in the case of non-power surplus out of rational economic man assumption, i.e.,  $f_j \geq C_{sj}$ .

**Assumption 4.** Set the expected benefits in the transaction between the *i*th drainage right buyer  $B_i$  and the *j*th drainage right seller  $S_j$  as  $W_i(C_{bi})$  and  $U_j(C_{sj})$ . The revenue expected by the buyer and seller of drainage rights in a transaction can be understood as the product of the expected unit profit and the expected number of transactions. In this study, the expected unit profit of the drainage rights purchaser in the transaction is the difference between the flood losses that the purchaser can avoid through the transaction and the expected purchase price, i.e.,  $C_{bi} - o_i$ ; The expected unit profit of the seller of drainage rights in a transaction is the difference between the seller’s expected sales price and the additional flood losses sustained as a result of the transaction, i.e.,  $f_j - C_{sj}$ ; Then  $W_i(C_{bi}) = x_i * (C_{bi} - o_i)$  and  $U_j(C_{sj}) = y_j * (f_j - C_{sj})$ .

### 2.2. Model Construction

From Assumptions 3 and 4, it can be seen that when  $o_i \geq f_j$ , both parties to the transaction have the expected profit margin and are satisfied with each other’s offers, and the transaction holds, at which time the equilibrium price  $P^0 = (o_i + f_j)/2$ . When  $o_i < f_j$ , the seller fails to obtain the entire expected profit margin, and the transaction does not hold. In the state of incomplete information, multiple solutions of Bayesian Nash equilibrium exist, requiring the design of trading rules and trading systems for bilateral call auctions, in which we consider that market participants expect to maximize not only their returns in centralized bidding but also the social returns. Expressing the above expectations in the functional form will result in the following objective function.

$$\max \sum_{i=1}^m \sum_{j=1}^n [W_i(C_{bi}) + U_j(C_{sj})] \tag{1}$$

$$\max \sum_{i=1}^m W_i(C_{bi}); \max \sum_{j=1}^n U_j(C_{sj}) \tag{2}$$

#### 2.2.1. Rational Price Constraints

After understanding the expectations of the objective function, we give the constraints on the objective function. The construction of the bilateral call auction model is based on the rational economic man’s assumption that rational risk brokers decide to trade and have non-negative returns [31]. This leads to Equations (3) and (4).

$$W_i(C_{bi}) = x_i(C_{bi} - O_i) \geq 0 \tag{3}$$

$$U_j(C_{sj}) = y_j(f_j - C_{sj}) \geq 0 \tag{4}$$

### 2.2.2. Macroscopic Control Total Quantity Constraints

Drainage rights trading is a secondary adjustment to the initial allocation of drainage rights that have been implemented by the higher level of government, and the initial and secondary allocations reflect the joint role of administrative and market instruments for the allocation of resources (disasters) [9]. The primary allocation forms a benchmark pattern for the discharge of flood water and the use of rivers. The secondary allocation is conducive to improving the problems of the initial allocation in which the amount of local allocation does not match the actual demand, and the marketization of drainage rights is conducive to promoting cross-regional water trade. However, excessive secondary adjustment can cause specific hazards, such as flooding that threatens the safety of people's lives and property in local areas or economic speculation that leads to drainage rights trafficking [8–10]. Therefore, the higher government is committed to controlling the final situation of flood distribution by limiting the total number of drainage rights traded in a particular area as a macro-regulatory measure, as shown in Equation (5).

$$Q = \min \left[ \sum_{i=1}^g x_i, \sum_{j=1}^h y_j \right] \quad (5)$$

where  $Q$  is the maximum restricted transaction volume decided by the higher level of government, i.e., the drainage rights (if any) below the volume of  $Q$  will have the power to trade (note that here it is only the power to trade and does not indicate a successful trade).  $g$  and  $h$  are referred to as the number of buyers and sellers included in the trading set, respectively, and  $x_i$  and  $y_j$  denote the trading demand of the  $i$ th drainage right buyer and the  $j$ th drainage right seller, respectively.

### 2.3. Clearance Rules

Drainage rights bilateral call auction participants include higher government, water companies, the two sides of the transaction, and so on (ignoring possible tax collection subjects). The basic process is as follows: the two sides of the transaction on the purchase or sale of drainage rights to register (including basic information, expected transaction price, transaction demand, etc.), submitted to the relevant departments for application, after review, by intermediaries (usually water companies) to Bidding. The target transaction volume and the target transaction price quoted by both sides of the transaction respectively form the buyer's offer set  $b^*$  and the seller's offer set  $s^*$ , and the buyer's offer set  $b^*$  is arranged in descending order to get  $b^* = \{o_1, o_2, o_3, \dots, o_g, \dots, o_m\}$ , the set of seller's offers  $s^*$  is arranged in ascending order to get  $s^* = \{f_1, f_2, f_3, \dots, f_h, \dots, f_n\}$ , where  $o_1 \geq o_2 \geq o_3 \geq o_g \geq o_m$  and  $f_1 \leq f_2 \leq f_3 \leq f_h \leq f_n$ . Assuming  $f_h \leq o_g < f_{h+1}$  and  $o_g < f_h \leq o_{g+1}$ , the top  $g$  subjects in the buyer's offer set and the top  $h$  subjects in the seller's offer set enter the transaction set from the offer set, and this rule is called the clearing rule for bilateral call auctions [32]. At this point, we can write the transaction set of the buying side as  $B^* = \{B_1, B_2, B_3, \dots, B_g\}$  and the transaction set of the selling side as  $S^* = \{S_1, S_2, S_3, \dots, S_h\}$ .

### 2.4. Matching Rules

According to the priority order, the first  $B_1$  in the buyer's transaction set will get the first choice, and its selection process is more complicated, with the main reference factors being the seller's expected sales price, sales demand, related taxes and fees, political considerations, etc. Therefore,  $B_1$  does not necessarily consider only the lowest priced  $S_1$ , so it brings the following picture: (1)  $B_1$  fails to choose  $S_1$  as counterparty,  $B_2$  still enjoys the option to trade with  $S_1$ ; (2)  $B_1$  chooses  $S_1$  as counterparty, but  $S_1$ 's target trading volume does not fully satisfy  $B_1$ 's demand, so  $B_1$  chooses another seller as second counterparty; (3)  $B_1$  chooses  $S_1$  as counterparty, but  $S_1$  still has spare volume for other purchasers to choose after trading with  $B_1$ .

### 3. Transaction Pricing Study of Bilateral Call Auction for Drainage Rights

#### 3.1. Wealth Utility Function Construction

From Assumption 3, the equilibrium price  $P^0 = (o_i + f_j)/2$  for the drainage rights transaction. The equilibrium price at this point is not affected by the parameters and is the transaction price in perfect equilibrium. The transaction price in perfect equilibrium can distribute the difference between the expected transaction prices of the two parties fairly in terms of quantity. However, the pricing of drainage rights transactions should not only take the equal distribution of proceeds between the buyer and the seller as the whole concept, but the satisfaction of both parties to the transaction with the proceeds received and the fairness of resource allocation should also be reflected [33]. Therefore, we adopt the wealth utility concept to construct the function, apply the social welfare function to associate the wealth utility functions of both sides of the transaction, and draw on the environmental Gini coefficient concept to construct a pricing model for drainage rights transactions based on parallel equity and efficiency.

Wealth utility indicates the degree of satisfaction of both parties to the transaction with the benefits obtained from the transaction [34], e.g., given the same volume of benefits for people at different Engel coefficient levels, it is clear that people with higher Engel coefficients receive a higher level of satisfaction. The utility space of the drainage rights transaction (called the “total utility space”) is the difference between the expected transaction prices of the two parties to the transaction, as can be derived from this study. The difference between the buyer’s expected transaction price and the actual transaction price can be called the “buyer’s utility space”, and the difference between the seller’s actual transaction price and its expected transaction price can be called the “seller’s utility space”, and their ratios to the total utility space can be called the “Degree of utility acquisition of the buyer” and the “Degree of utility acquisition of the seller”, respectively.

The wealth utility function construction of the buyer.

$$Z_b(p^*) = \begin{cases} \frac{o_i - p^*}{o_i - f_j} & p^* \leq o_i \\ 0 & p^* > o_i \end{cases} \quad (6)$$

The wealth utility function construction of the seller.

$$Z_s(p^*) = \begin{cases} \frac{p^* - f_j}{o_i - f_j} & p^* \geq f_j \\ 0 & p^* < f_j \end{cases} \quad (7)$$

In Equations (6) and (7),  $p^*$  denotes the actual transaction price, which should be between the desired prices of the two sides of the transaction, i.e.,  $f_j \leq p^* \leq o_i$ ;  $o_i - p^*$  denotes the utility space of the buyer, and  $p^* - f_j$  denotes the utility space of the seller, i.e., the buyer and seller split the total utility space  $o_i - f_j$ ;  $Z_b(p^*)$  denotes the degree of utility acquisition of the buyer, and  $Z_s(p^*)$  denotes the degree of utility acquisition of the seller. When the actual transaction price exceeds the buyer’s expected purchase price, i.e.,  $p^* > o_i$ , the buyer’s willingness to trade is 0. When the actual transaction price is less than the seller’s expected sales price, i.e.,  $p^* < f_j$ , the seller’s willingness to trade is 0. Then in both cases, the transaction is not valid.

#### 3.2. Social Welfare Function Construction

Social welfare is an important element in economics that aims to explore the goals pursued by society [35]. The social welfare function has experienced three expressions: additive, multiplicative, and minimalist. In this study, a two-person Nash multiplier-type social welfare function is selected to combine the utility functions of the two parties to the transaction [36], and the indicator “unfair pullback coefficient” is set according to

the concept of environmental Gini coefficient [37]. A pricing model for drainage rights transactions based on equity and efficiency is constructed as follows.

$$T = Z_b(p^*)^\alpha * Z_s(p^*)^{1-\alpha} = \left(\frac{o_i - p^*}{o_i - f_j}\right)^\alpha * \left(\frac{p^* - f_j}{o_i - f_j}\right)^{1-\alpha} \tag{8}$$

where  $Z_b(p^*)$  denotes the degree of utility acquisition of the buyer,  $Z_s(p^*)$  denotes the degree of utility acquisition of the seller,  $T$  denotes the value of the social welfare function, and  $\alpha$  denotes the inequity pullback coefficient. The unfair pullback coefficient reflects the loss of flooding carried per unit of a given indicator. Based on existing literature, we argue that such a flood distribution is equitable when the loss of flooding per unit of GDP, area, and population carried by two regions is close [38]. Thus, this study is based on the three dimensions of economic development level, land area, and population size to calculate the unfair pullback coefficient. Considering that the direct economic loss of flooding in the study year of the drainage rights seller may be zero, it will lead to the unfair pullback coefficient of the drainage rights buyer is 1, which loses the significance of the setting, so we chose to extend the data for three years. In summary, the inequitable pullback factor is calculated as follows.

$$\alpha = \sum_{e=1}^3 \left(1 - \frac{y_a/x_{ae}}{\sum_{a=1}^2 (y_a/x_{ae})}\right) / 3 \tag{9}$$

where  $x_{ae}$  represents the arithmetic mean of the number of indicators in region  $e$  in the  $a$ th three previous years,  $y_{ae}$  represents the arithmetic mean of the direct economic losses from flooding in the  $a$ th region in the previous three years, and  $e$  represents the number of the three indicators: GDP, land area, and population.

### 3.3. Solving for the Transaction Price

In addition, a series of derivations from Equation (8) leads to a conclusion that the social welfare function  $T$  achieves its maximum value when  $p^* = \partial((f_j - o_i) + o_i)$ , i.e., the formula for the optimal transaction price  $p^*$  is derived. The derivation process is as follows. First, organize Equation (8) to derive  $T = \frac{(o_i - p^*)^\alpha}{(o_i - f_j)^\alpha} * \frac{(p^* - f_j)^{1-\alpha}}{(o_i - f_j)^{1-\alpha}} = \frac{(o_i - p^*)^\alpha (p^* - f_j)^{1-\alpha}}{(o_i - f_j)^\alpha}$ ; second, find the derivative  $T' = -\alpha(o_i - p^*)^{\alpha-1} (p^* - f_j)^{1-\alpha} + (1 - \alpha)(o_i - p^*)^\alpha (p^* - f_j)^{-\alpha}$  of the social welfare function  $T$ ; third, extract the common factorization  $(o_i - p^*)^\alpha (p^* - f_j)^{-\alpha}$  to obtain  $T' = (o_i - p^*)^\alpha (p^* - f_j)^{-\alpha} \left[\frac{-\alpha(p^* - f_j)}{o_i - p^*} + 1 - \alpha\right]$ ; fourth, in order to find the extreme value point of the social welfare function  $T$ , so that the derivative function  $T'$  is zero, i.e.,  $T' = (o_i - p^*)^\alpha (p^* - f_j)^{-\alpha} \left[\frac{-\alpha(p^* - f_j)}{o_i - p^*} + 1 - \alpha\right] = 0$ , and because in general  $p^*$  is not equal to  $o_i$  and  $f_j$  (the expected transaction price between the two sides of the transaction), there exists the equation  $\frac{-\alpha(p^* - f_j)}{o_i - p^*} + 1 - \alpha = 0$ , by which  $p^* = \partial((f_j - o_i) + o_i)$  can be found; finally, as  $\frac{-\alpha(p^* - f_j)}{o_i - p^*} + 1 - \alpha = 0$  is a monotonically decreasing primary linear function, indicating that there is only one solution to the equation, and the original function first increases and then decreases, that is, to find the  $p^*$  for the original function of the only extreme value point and the extreme value of the maximum.

## 4. Simulation of Calculations for Drainage Rights Trading in Jiangsu Section of Huaihe River Basin

### 4.1. Study Area

The Jiangsu section of the Huaihe River Basin, mainly flowing through the north-central region of Jiangsu Province, China, involves eight prefecture-level cities, namely Xuzhou, Nantong, Lianyungang, Huaian, Yancheng, Yangzhou, Taizhou and Suqian, located at 116°22'–121°00' E and 32°23'–35°07' N. It is the easternmost section of the Huaihe

River Basin, connecting the Tong Yang Canal and the Yangtze River Basin to the south, reaching the Yiliu hilly mountains and the Yellow River Basin to the north, the Yimeng Mountains. There are many lakes and rivers in the Jiangsu section of the watershed, including Hongze Lake, the Beijing–Hangzhou Grand Canal, and the Huaishu New River, among which the Hongze Lake Wetland is an important freshwater wetland reserve in China with a good environment and a variety of biological and plant resources.

#### 4.2. Simulation of Drainage Rights Trading Mechanism

At present, drainage rights trading has not yet entered the pilot phase. In this context, we performed numerical simulations based on existing research results (Buuren et al., 2016; Wang, H.-F. et al., 2014). Let there be eight participants in the drainage rights trading market in the Jiangsu section of the Huaihe River Basin at a specific moment, including four drainage rights buyers and four drainage rights sellers, and both parties make independent offers. The quoted and traded quantities on the buying side are  $B_1(8.14, 100)$ ,  $B_2(5.38, 50)$ ,  $B_3(7.21, 100)$ , and  $B_4(5.2, 62.5)$ , and the quoted and traded quantities on the selling side are  $S_1(9.45, 100)$ ,  $S_2(6.8, 75)$ ,  $S_3(6.47, 50)$ , and  $S_4(4.38, 37.5)$ , so the buying and selling total demand is  $312.5 \text{ m}^3$  and  $262.5 \text{ m}^3$  respectively. Let the total limit of drainage rights trading by the higher government be  $250 \text{ m}^3$ , which indicates that there will be  $62.5 \text{ m}^3$  of total demand for drainage rights purchase and  $12.5 \text{ m}^3$  of total demand for drainage rights sale that cannot be satisfied.

##### 4.2.1. Clearance Rule

According to the clearing rule, the set of buyers' offers  $b^*$  is arranged in descending order to get  $b^* = \{8.14, 7.21, 5.83, 5.20\}$ , the set of sellers' offers  $s^*$  is arranged in ascending order to get  $s^* = \{4.38, 6.47, 6.80, 9.45\}$ , and the final results are shown in Table 1. Since only  $250 \text{ m}^3$  of drainage rights are allowed to be traded, purchaser  $B_4$  will be strictly excluded from the transaction set, and the expected sales price  $f_1$  of seller  $S_1$  is greater than any value in the purchaser's offer set  $b^*$ , indicating that it is not possible to match and reach a deal with either purchaser. Therefore, we can write the transaction set for the buyer as  $B^* = \{B_1, B_3, B_2\}$  and the transaction set as  $S^* = \{S_4, S_3, S_2\}$ .

Table 1. Set of quotes ranked by expected transaction price.

Buyer $B_i$			Seller $S_j$		
Buyer	Expected Acquisition Price $o_i$ (yuan/ $\text{m}^3$ )	Expected Acquisition Volume $x_i$ ( $\text{m}^3$ )	Seller	Expected Sales Price $f_j$ (yuan/ $\text{m}^3$ )	Expected Sales Volume $y_j$ ( $\text{m}^3$ )
$B_1$	8.14	100	$S_4$	4.38	37.5
$B_3$	7.21	100	$S_3$	6.47	50
$B_2$	5.83	50	$S_2$	6.80	75
$B_4$	5.20	62.5	$S_1$	9.45	100

##### 4.2.2. Matching Transaction Sets

First, in order of priority,  $B_1$  in the buyer's transaction set will get the first choice, and its selection process is more complicated, with the main reference factors being the seller's offer, the seller's expected transaction volume, and relevant taxes, political considerations, etc. In this case, only the expected sales price of the seller and the expected transaction volume are used as reference factors. The consideration process is: (1)  $B_1$ 's acquisition demand  $x_1 = 100$ , which is a large amount, gives priority to the full drainage rights of seller  $S_4$  and considers the remaining  $62.5 \text{ m}^3$ . However, the expected sales price of seller  $S_3$  is slightly smaller than the expected sales price of seller  $S_4$ , but it does not fully satisfy  $B_1$ 's remaining demand, and  $B_1$  tends to choose seller  $S_2$  for the  $62.5 \text{ m}^3$  transaction. (2) After all of  $B_1$ 's acquisition demand is satisfied,  $B_3$  will be given the preference, and  $B_3$ 's acquisition

demand  $x_3 = 100$ , which is also a large amount. At this time, the total expected sales volume remaining on the selling side is  $62.5 \text{ m}^3$  ( $50 \text{ m}^3$  remaining on the selling side  $S_3$  and  $12.5 \text{ m}^3$  remaining on the selling side  $S_2$ ), then the purchasing side  $B_3$  will purchase the entire remaining volume of  $62.5 \text{ m}^3$ . By observing Table 1, it can be found that the transaction demands of both drainage right buyer  $B_2$  and drainage right buyer  $B_4$  are not satisfied due to the low bids.

In summary, the final results are shown in Table 2, with a total transaction volume of  $162.5 \text{ m}^3$  (less than the total restricted amount of drainage rights trading), and four transactions were formed, as follows: (1) buyer  $B_1$  and seller  $S_4$  find the optimal transaction price between  $4.38 \text{ RMB/m}^3$  and  $8.14 \text{ RMB/m}^3$  for  $37.5 \text{ m}^3$  of flooding and reach a deal; (2) purchaser  $B_1$  and seller  $S_2$  find the optimal transaction price between  $6.80 \text{ RMB/m}^3$  and  $8.14 \text{ RMB/m}^3$  for  $62.5 \text{ m}^3$  of flood discharge and reach a deal; (3) purchaser  $B_3$  and seller  $S_3$  find the optimal transaction price between  $6.47 \text{ RMB/m}^3$  and  $7.21 \text{ RMB/m}^3$  for a  $50 \text{ m}^3$  flood discharge and reach a deal; (4) purchaser  $B_3$  and seller  $S_2$  find the optimal transaction price between  $6.80 \text{ RMB/m}^3$  and  $7.21 \text{ RMB/m}^3$  for a  $12.5 \text{ m}^3$  flood discharge and reach a deal. In the following, the optimal transaction price will be selected for these four transactions.

**Table 2.** List of transactions.

Transaction Serial Number	Buyer $B_i$	Seller $S_j$	Expected Trading Volume ( $\text{m}^3$ )	Expected Acquisition Price $o_i$ (yuan/ $\text{m}^3$ )	Expected Sales Price $f_j$ (yuan/ $\text{m}^3$ )
I	$B_1$	$S_4$	37.5	8.14	4.38
II	$B_1$	$S_2$	62.5	8.14	6.80
III	$B_3$	$S_3$	50	7.21	6.47
IV	$B_3$	$S_2$	12.5	7.21	6.80

#### 4.3. Simulation of the Pricing of Drainage Rights Transactions

The simulation of the pricing of the drainage rights transaction includes the construction of the wealth utility function, the construction of the welfare function, the unfair pullback coefficient solution, and the optimal transaction price solution. We only take the first transaction as an example for detailed analysis and explanation.

##### 4.3.1. Wealth Utility Function

From the equilibrium price formula of the transaction mentioned in 3.1, the equilibrium prices of transactions 1 to 4 can be obtained as  $6.26 \text{ RMB/m}^3$ ,  $7.47 \text{ RMB/m}^3$ ,  $6.84 \text{ RMB/m}^3$ , and  $7.01 \text{ RMB/m}^3$ , respectively; meanwhile, the wealth utility function of Transaction 1 can be obtained according to Equations (6) and (7).

The wealth utility function for purchaser  $B_1$  is as follows.

$$Z_b(p^*) = \begin{cases} \frac{8.14-p^*}{3.76} & p^* \leq 8.14 \\ 0 & p^* > 8.14 \end{cases}$$

The wealth utility function for seller  $S_4$  is as follows.

$$Z_s(p^*) = \begin{cases} \frac{p^*-4.38}{3.76} & p^* \geq 4.38 \\ 0 & p^* < 4.38 \end{cases}$$

It can be obtained that the two parties to the transaction divide the expected profit (rather than the actual profit) according to a certain philosophy (in this paper, we choose the philosophy of the fairness of the flood distribution) to determine the optimal transaction price.

### 4.3.2. Social Welfare Function

We demonstrate the process of calculating the inequity pullback coefficient using the data of purchaser  $B_1$  and seller  $S_4$  in Transaction 1 (Table 3) as an example. According to Equation (9), the inequity pullback coefficient of purchaser  $B_1$  at the gross regional product level is calculated as follows.

$$\partial = [(1 - \frac{1.44/5911.45}{(1.44/5911.45+0.03/2392.48)/2}) + (1 - \frac{1.44/3063}{(1.44/3063+0.03/3012)/2}) + (1 - \frac{1.44/336.2}{(1.44/336.2+0.03/222.01)/2})]/3 = 0.05$$

The inequitable pullback coefficient of the selling side at the gross regional product level is the difference between 1 and this coefficient of the buying side  $B_1$ , which can also be calculated by Equation (9), as follows.

$$1 - \partial = [(1 - \frac{0.03/2392.48}{(0.03/2392.48+1.44/5911.45)/2}) + (1 - \frac{0.03/3012}{(0.03/3012+1.44/3063)/2}) + (1 - \frac{0.03/222.01}{(0.03/222.01+1.44/336.2)/2})]/3 = 0.95$$

The remaining dimensions of the first transaction and the unfair pullback coefficients involved in the second to fourth transactions were calculated using the same approach, as shown in Table 4.

**Table 3.** Data on the unfair pullback coefficient of GDP for  $B_1$  and  $S_4$ .

Indicators (Arithmetic Average of the Previous Three Years)	$B_1$	$S_4$
GDP (100 million yuan)	5911.45	2392.48
Land area (km <sup>2</sup> )	3063	3012
Population (10,000 people)	336.2	222.01
Flooding direct economic loss (100 million yuan)	1.44	0.03

**Table 4.** Calculation results of unfair pullback coefficient.

Indicators	Transaction I		Transaction II		Transaction III		Transaction IV	
	$B_1$	$S_4$	$B_1$	$S_2$	$B_3$	$S_3$	$B_3$	$S_2$
GDP	0.05	0.95	0.36	0.64	0.58	0.42	0.1	0.9
Land	0.02	0.98	0.37	0.63	0.80	0.20	0.73	0.27
Population	0.03	0.97	0.39	0.61	0.65	0.35	0.56	0.44
Average	0.03	0.97	0.37	0.63	0.67	0.33	0.46	0.54

From the above calculation process, it can be seen that the direct economic loss per GDP, land area, and population carried by purchaser  $B_1$  in the past three years is much larger than that of seller  $S_4$ . The inequity pullback coefficient of purchaser  $B_1$  in these three aspects is much smaller than that of seller  $S_4$ , as shown in Table 4. The inequity pullback coefficients of purchaser  $B_1$  and seller  $S_4$  are 0.03 and 0.97, indicating that the market focuses more on the seller’s utility in the process of trading drainage rights, i.e., the increase in social welfare per unit increase in utility on the seller’s side is much greater than that on the buyer’s side.

$$T = Z_b(p^*)^{0.03} * Z_s(p^*)^{0.97} = (\frac{8.14 - p^*}{3.76})^{0.03} * (\frac{p^* - 4.38}{3.76})^{0.97}$$

### 4.3.3. Optimal Transaction Price

The optimal transaction price for the first transaction should be between 4.38 RMB/m<sup>3</sup> and 8.14 RMB/m<sup>3</sup> (as shown in Table 2). According to the previous section, the arithmetic means of the fair pullback coefficients of purchaser B<sub>1</sub> and seller S<sub>4</sub> are 0.03 and 0.97, respectively, indicating that the market pays more attention to the utility of the seller, i.e., expanding the expected profit margin of the seller, in the process of drainage rights trading. The optimal transaction price will occur well above the equilibrium trading price (6.26 RMB/m<sup>3</sup>).

As shown in Figure 1, the social welfare function for the first transaction shows an increasing and then decreasing trend, achieving a maximum social welfare value of 0.87 at 8.03. The optimal transaction price of 8.03 RMB/m<sup>3</sup> is 0.07 RMB/m<sup>3</sup> lower than the purchaser’s expected transaction price of 8.14 RMB/m<sup>3</sup> and 3.65 RMB/m<sup>3</sup> higher than the seller’s expected transaction price of 4.38 RMB/m<sup>3</sup>. From the perspective of the wealth utility function, it can be understood that the total wealth utility space of both sides of the transaction is 3.76 RMB/m<sup>3</sup>, among which the wealth utility space of the buyer is 0.07 RMB/m<sup>3</sup>, and the wealth utility space of the seller is 3.69 RMB/m<sup>3</sup>. When the optimal transaction price is 8.03 RMB/m<sup>3</sup>, the wealth utility acquisition degree of the buyer is 1.86% and the wealth utility acquisition degree of the seller is 98.14%, the wealth utility acquisition degree of the selling side is significantly higher than that of the buying side due to its larger fair pullback coefficient. From the perspective of the transaction itself, it can be understood that the buyer pays 0.07 RMB/m<sup>3</sup> less for 1 m<sup>3</sup> of drainage rights purchased, and the seller gets 3.65 RMB/m<sup>3</sup> more for 1 m<sup>3</sup> of drainage rights sold, and both sides of the transaction obtain certain wealth utility from the transaction and make the social utility at its maximum.

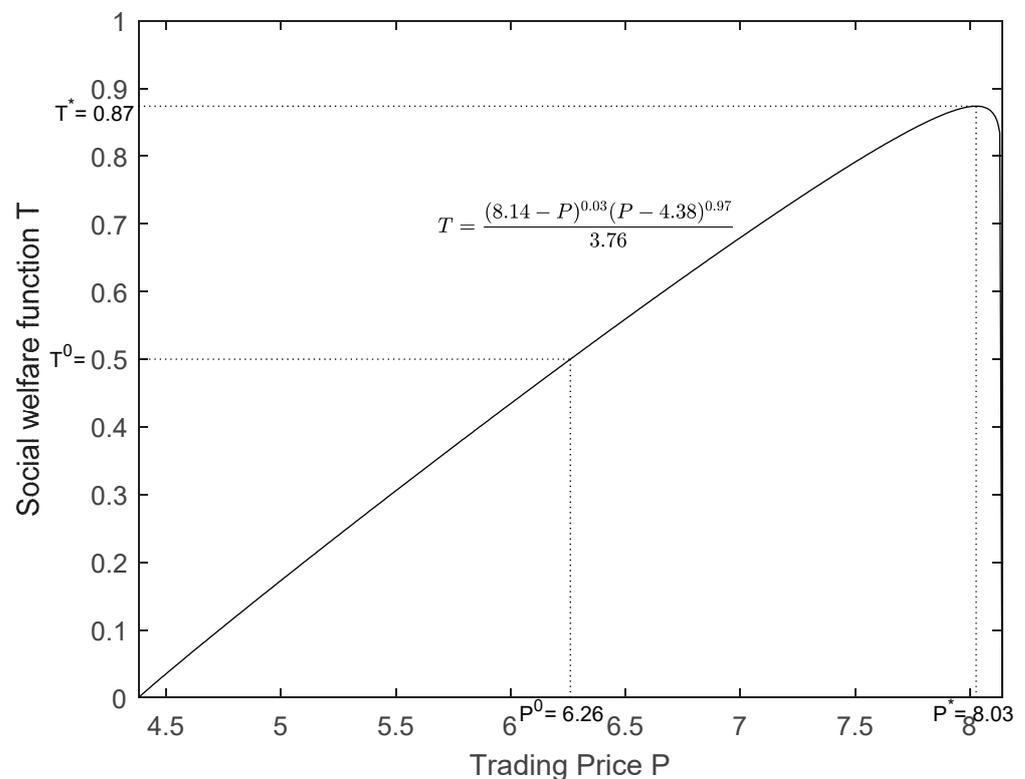


Figure 1. Schematic diagram of the optimal transaction price.

As shown in Figures 2–4, the optimal transaction prices for transactions two to four are 7.64 RMB/m<sup>3</sup>, 6.71 RMB/m<sup>3</sup>, and 7.02 RMB/m<sup>3</sup>, and the optimal social welfare values achieved are 0.52, 0.53, and 0.5016, respectively.

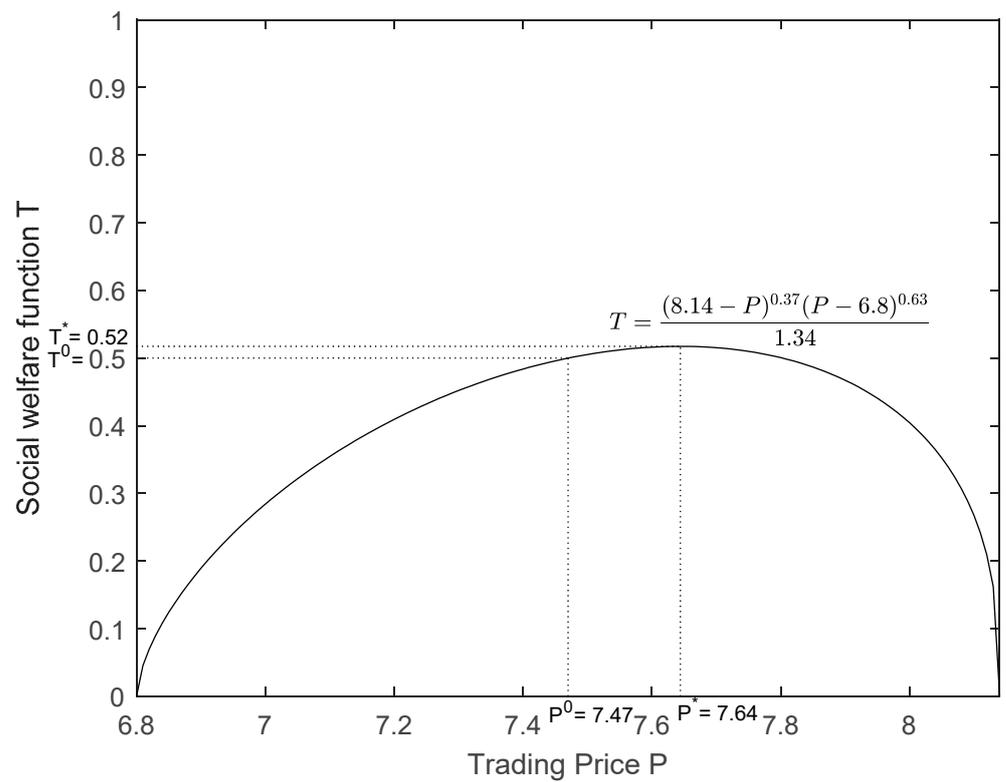


Figure 2. Schematic diagram of the optimal transaction price.

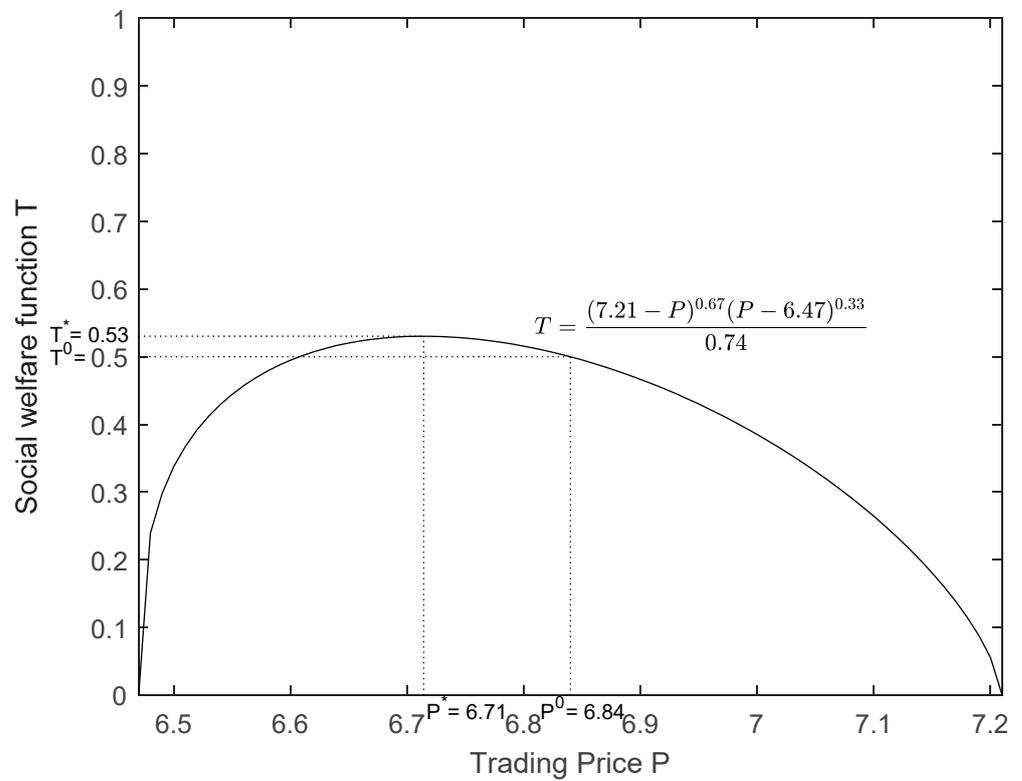


Figure 3. Schematic diagram of the optimal transaction price.

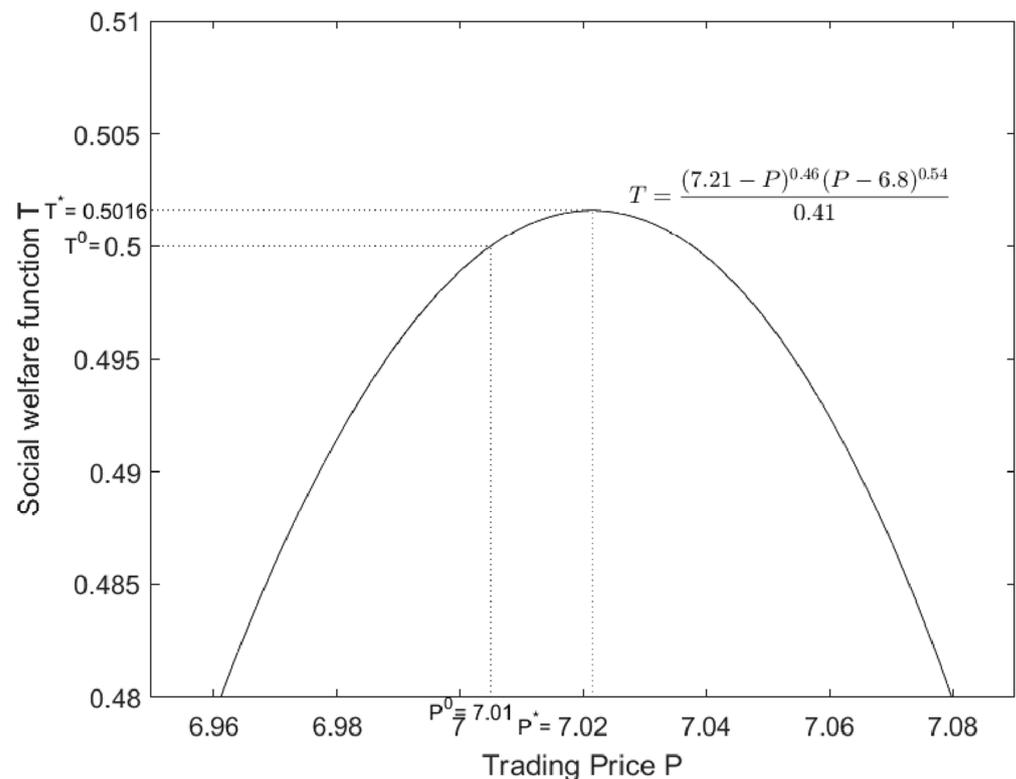


Figure 4. Schematic diagram of the optimal transaction price.

## 5. Conclusions

We found two shortcomings after combing through the existing studies. Firstly, the existing trading mechanism of drainage rights cannot solve the problem of simultaneous trading of multiple subjects, for which we have constructed a trading mechanism of drainage rights using a bilateral call auction model. Second, the existing pricing concept of drainage rights trading still takes the flood loss as the lower bound of the price. We broaden the idea and change the intrinsic concept of the model to focus on the degree of profit satisfaction for both buyers and sellers. Therefore, we constructed the pricing model using the wealth utility and social welfare function and drawing on the environmental Gini coefficient concept. The study's results show that the trading mechanism model can give the optimal trading scheme when multiple subjects are trading drainage rights. This fills the gap in existing studies and can lead to efficient trading of drainage rights. At the same time, the pricing model can give the optimal transaction price that considers efficiency and fairness, providing a more practical pricing theory for drainage rights trading. In response to the above study, we give two policy recommendations. First, the government should establish good clearing rules so that the post-trade drainage pattern remains in the overall interest of the region. Second, the government should dig deeper into the factors that affect the fairness of drainage rights pricing and add more parameters to the pricing model. Of course, we found the following limitations of this study. When both parties are willing to trade and are satisfied with the transaction price, we consider the transaction to be established. In fact, if there is another subject between the geographic locations of the two parties to the transaction, we need to explore whether the parties need to pay a “toll” to this intermediary party. Therefore, we will explore this issue in-depth in our future research.

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