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Modelling and Evaluation of Potato Water Production Functions in a Cold and Arid Environment

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Abstract: This study was conducted at the Yimin Irrigation Experiment Station, Minle County, Zhangye City, Gansu Province, from April to October in 2019 and 2020. The relationship between water consumption and yield of potato at different stages of fertility under deficit-regulated irrigation was analyzed in a field trial study over two growing seasons. The results showed that the average annual water consumption in the tuber bulking stage was the largest, reaching 185.35~239.52 mm, followed by the average annual water consumption in the tuber initiation stage and starch accumulation stage, which were 100.02~132.30 mm and 82.48~112.36 mm, respectively, and the average annual water consumption in the seedling stage was the least, at 49.32~69.81 mm. Simultaneously, the average annual yield of potatoes in the treatment of WD1 was the highest, reaching 47,766.96 kg·hm^{-2}, followed by CK, which was 43,707.6 kg·hm^{-2}, and the yield of WD6 was the smallest in the treatment of moderate water deficit during tuber initiation, which was only 35,721.25 kg·hm^{-2}. Combining the four moisture production function models of Jensen, Minhas, Blank and Stewart, the Jensen and Stewart models were identified as suitable for the potato moisture production function in a cold and arid environment. The water production function model was used to investigate the relationship between water consumption and yield in each growth period of potato, and to provide a theoretical basis for the optimization of the irrigation system under deficit-regulating irrigation conditions for potato in this irrigation area.

Keywords: moisture production function model; water consumption; irrigation; yield; potato

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1. Introduction

The water production function is an intuitive mathematical expression that reflects the relationship between water consumption and harvest yield over the crop growth cycle and is the basis for the study of non-sufficient irrigation. At present, water production functions can be divided into two broad categories: one is a water production function that reflects the relationship between total water consumption and total harvest yield throughout the crop's reproductive life, and the other is a water production function that reflects the relationship between water consumption and yield at each reproductive stage of the crop. The second type of water production function is more widely quoted because the water consumption of a crop at different stages of fertility has different effects on the final yield. International scholars began to study the relationship between crop water consumption and yield in the 1960s, and many classical models emerged during this period, such as: additive models (Howell model [1], Blank model [2], Stewart model [3], Singh model [4]), multiplicative models (Jensen model [5], Rao model [6], Minhas model [7], Hanks model [8]), etc.

David D [9] found a quadratic parabolic function between yield and moisture by studying the relationship between corn yield and crop moisture production functions in the arid northwest of the United States. Debarati D [10] studied the relationship between sweetcorn yield and different irrigation levels and found that yield showed a trend of increasing and then decreasing with increasing irrigation levels. Subsequent studies on yield as a function of moisture were conducted on sugar beet [11], maize [12], sorghum [13], cassava [14], onion [15] and spring wheat [16].

It was only in the late 1980s that agricultural researchers in China began to draw on major classical foreign models to conduct preliminary studies exploring the water production functions between water consumption and yield for major crops such as wheat [17], cotton [18], maize [19], soybean [20] and rice [21], after which more scholars set off a wave of research on the relationship between crop water and yield in different regions and crops. Du Zhendong [22] and others studied the water production function of wheat and maize by drawing on classical foreign models, which laid the theoretical foundation for subsequent studies. Xu Shuzhen [23] et al. fitted a water production function for tomato based on the Jensen model and found that the maximum water deficit sensitivity index at the fruiting stage of tomato was not conducive to non-sufficient irrigation, while the minimum water deficit sensitivity index at the seedling stage allowed for moderate non-sufficient irrigation, and water savings of 63–69.58% were achieved after optimizing the irrigation regime by fitting the model. Han Huiling [24] analyzed the relationship between water consumption and yield of cotton under non-sufficient irrigation conditions and found that the water deficit sensitivity index was the largest at the miner stage and the smallest at the seedling stage, and the irrigation pattern was optimized based on the sensitivity index. Du Yaodong [25] studied the relationship between yield and water consumption in spring wheat by drawing on the Jensen model and found that the maximum sensitivity index was not conducive to deficit-regulating irrigation during the different fertility stages of spring wheat from tassel to mastication, and the minimum sensitivity index was from sowing to tillering, allowing for moderate water stress treatments.

The potato (Solanum tuberosum) belongs to the Solanaceae family and is an annual herb with high nutritional value, adaptability to the environment and high yields, and is widely grown worldwide. To diversify and nutritionally diversify our staple foods and improve our food security, the Ministry of Agriculture officially launched the potato staple food strategy in January 2015, making it the fourth staple food after wheat, maize and rice. However, potato irrigation still uses traditional field irrigation methods, making water use extremely wasteful and inefficient use of irrigation water a serious constraint in the process of potato staple food strategies. Therefore, it is important to study the relationship between potato yield and water for the subsequent achievement of the goal of water saving and yield increases.

2. Materials and Methods

2.1. Experimental Site

Field trials were conducted at the Yimin Irrigation Experiment Station, Minle County, Zhangye City (100°43′ E, 38°39′ N) for two growing seasons (2019 and 2020). The average altitude of the test station is 1970 m, with an average annual temperature of 6.0 °C, an extreme maximum temperature of 37.8 °C and an extreme minimum temperature of −33.3 °C. The average annual rainfall is 285 mm, and the average annual free water evaporation is about 2000 mm, which is a typical semi-arid climate zone. The test field had a flat surface, light loamy soil with a pH of 7.22, a maximum field water holding capacity of 24% in the cultivated layer (determined by the weight method) and an average soil profile capacity of 1.48 t/m^3. The soil organic matter content within the 0–20 cm soil depth was 1.27%, alkaline decomposed nitrogen 60.1 mg/kg, fast-acting potassium 188.5 mg/kg and fast-acting phosphorus 12.9 mg/kg. The test area has a low groundwater table (>10 m) and no salinization effects.

2.2. Experimental Design

The potato variety 'Qingshu 168' was provided by the Qinghai Agricultural Science Research Institute of China Qinghai Agricultural Research Institute. In 2019, potatoes were sown on 14 April and harvested on 9 October; and in 2020, they were sown on 7 April and harvested on 15 September. The row spacing was 40 cm, and the plant spacing was 20 cm (Figure 1). A white plastic film (140 cm wide, 0.01 mm-thick; *China Dongguan Shuotai Industrial* Co., Ltd., Dongguan, China) covered two rows of potatoes with a planting density of 77,000 plants/ha. Drip irrigation was applied under the film with the irrigation pipe placed between two rows. Each treatment and control were repeated three times, and 140 potato plants were sown in each test plot. Each section covered 33.6 m^2 (7 m $\times$ 4.8 m). There were two levels of water deficit: mild, with soil moisture at 55% to 65% of field capacity, and moderate, with soil moisture at 45% to 55% of field capacity. The soil moisture with conventional irrigation (CK) was 65% to 75% of field capacity. Each level of deficit was applied in each of the four growth stages of potato: seedling, tuber initiation, tuber bulking, and starch accumulation stages. Thus, there were eight total treatments: WD1: mild, seedling; WD2: mild, tuber initiation; WD3: mild, tuber bulking; WD4: mild, starch accumulation; WD5: moderate, seedling; WD6: moderate, tuber initiation; WD7: tuber bulking; WD8: starch accumulation (Table 1).

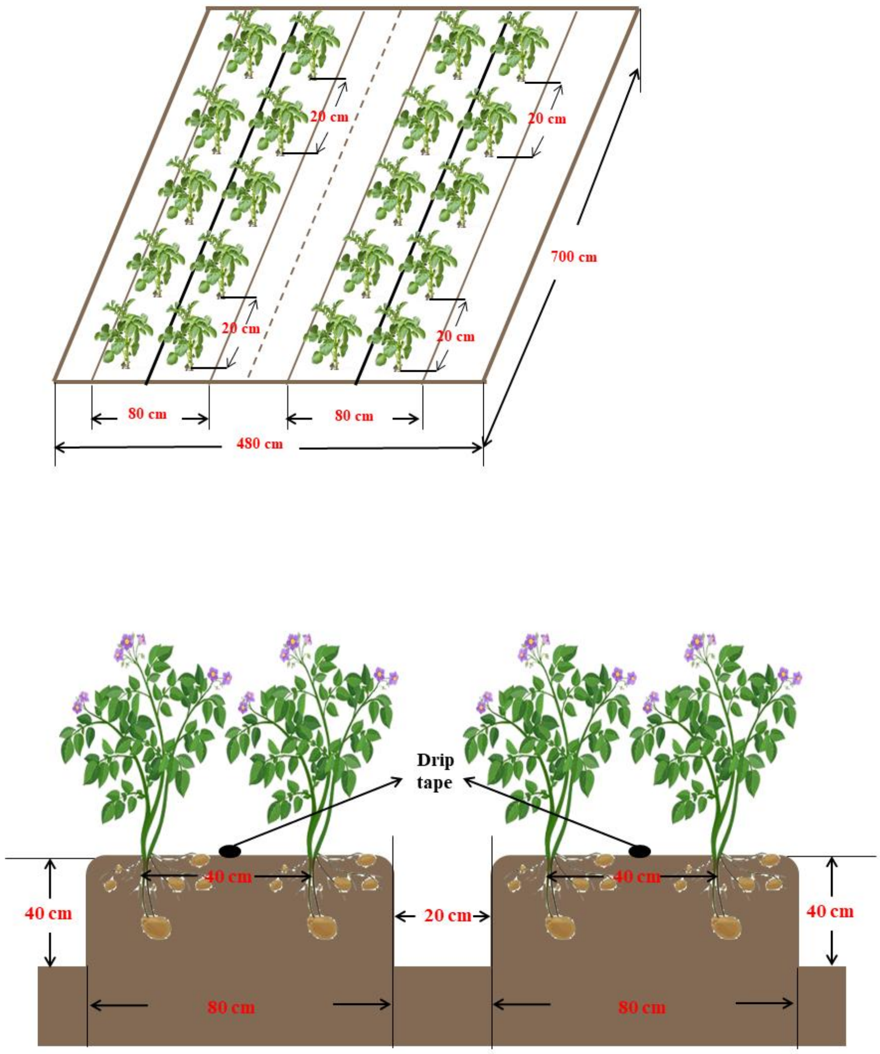

Figure 1. Cultivation of potatoes with regulated deficit drip irrigation on ridges under plastic film mulching.

Table 1. Experimental design.

Treatment	Deficit	Seedling Stage	Tuber Initiation Stage	Tuber Bulking Stage	Starch Accumulation Stage
WD1	Slight water deficit during seedling stage	55%~65%	65%~75%	65%~75%	65%~75%
WD2	Slight water deficit during tuber initiation	65%~75%	55%~65%	65%~75%	65%~75%
WD3	Slight water deficit during tuber bulking	65%~75%	65%~75%	55%~65%	65%~75%
WD4	Slight water deficit during starch accumulation	65%~75%	65%~75%	65%~75%	55%~65%
WD5	Moderate water deficit during seedling	45%~55%	65%~75%	65%~75%	65%~75%
WD6	Moderate water deficit during tuber initiation	65%~75%	45%~55%	65%~75%	65%~75%
WD7	Moderate water deficit during tuber bulking	65%~75%	65%~75%	45%~55%	65%~75%
WD8	Moderate water deficit during starch accumulation	65%~75%	65%~75%	65%~75%	45%~55%
CK	Full irrigation throughout the growth period	65%~75%	65%~75%	65%~75%	65%~75%

Note: Soil water content (% field capacity) in conventional irrigation and regulated deficit drip irrigation treatments during potato growth. The treatments were two levels of water deficit that occurred in each of four growth stages.

2.3. Measurements and Calculations

Soil moisture: The trial mainly followed the traditional soil drying and weighing method of soil auger extraction to determine soil moisture. According to previous research, the potato root system is mainly concentrated in the soil layer between 0 and 40 cm, so soil is taken every 10 days throughout the potato's reproductive life, at a depth of 80 cm, divided into five gradients: 0–10 cm, 10–20 cm, 20–40 cm, 40–60 cm, and 60–80 cm. Additional measurements were taken to a depth of 100 cm at the beginning and at the end of the test, and once before and after flooding and after rainfall. The soil samples obtained from each plot are loaded into the corresponding aluminum boxes, weighed fresh and placed in an oven at 105 °C until they reach a constant weight, then weighed dry and finally combined with the calculation formula to calculate the mass water content of the soil. The calculation Formula (1) is as follows:

$$\mathrm{SWC}\% = \frac{W_1 - W_2}{W_2 - W_3} \cdot 100\% \tag{1}$$

where SWC is the mass water content of the soil (%); W_1 is the total mass of the fresh soil sample and the aluminum box (g); W_2 is the total mass of the dried soil sample and the aluminum box (g); W_3 is the mass of the empty aluminum box (g).

Irrigation water: The test was carried out using PVC pipelines and drip irrigation under the film. A gate valve and a water meter were installed in each treatment area, the corresponding irrigation water amount will be controlled quantitatively by the water meter, and the time and amount of water used for each irrigation will be recorded in a timely and accurate manner. When the soil moisture content is as low as the lower limit of the design value, then irrigation should be carried out in a timely manner and the amount of irrigation required is obtained from the irrigation quota formula.

$$\mathrm{m} = 10\rho\mathrm{bH}\left(\beta_\mathrm{i} - \beta_\mathrm{j}\right) \tag{2}$$

where m is the volume of irrigation water in mm; ρb is the bulk density of the soil in the planned wetted layer, in $\mathrm{g/cm^3}$; H is the depth of the planned wetted layer of soil, based on the experimental design limit of 60, unit: cm; β_i is the target water content (field water holding capacity multiplied by the design target relative water content limit); β_j is the water content of the soil before irrigation.

Yield: After the potatoes have matured, each plot is harvested individually to measure the yield, and the average of three replicates is the actual yield of each treatment. Weighing was carried out using an electronic scale with an accuracy of 0.01 g and converted to a standard yield of kg/hm^2.

2.4. Water Consumption

Plant water consumption of each treatment is calculated by the water balance equation [26]:

$$ET = 10\sum_{a=1}^{b} r_a h_a (SWC_{a1} - SWC_{a2}) + I + R \quad (3)$$

where ET(mm) means crop evapotranspiration; b means the overall number of soil layers; r(g/cm^3) means the bulk density of the ath layer of soil; h means the thickness of the ath layer of soil; $SWC_{a1} - SWC_{a2}$ means the change in mass soil water content between two measurement dates ($a1$, $a2$). Soil moisture in each treatment plot was measured every 7 days by soil drilling and the drying weighing method. The soil sampling depth was 100 cm, which was divided into 6 parts: 0–10 cm, 10–20 cm, 20–40 cm, 40–60 cm, 60–80 cm and 80–100 cm. I means the amount of irrigation water (mm) during the growth period and R means the rainfall (mm) during the growth period.

2.5. Selection of Models

The purpose of crop water production functions is to study how water resources can be used scientifically, efficiently and rationally to achieve high crop yields in places where water resources are in short supply and demand. In this experiment, four common models were used to investigate the water production function of potatoes according to the current situation of crop water production function research in the cold irrigation area to the west of the river, and the models used were the multiplicative model Jensen model (1968) and the Minhas model (1974). Additive models: Blank model (1975), Stewart model (1977).

Jensen model (1968):

$$\frac{Y_a}{Y_m} = \prod_{i=1}^{n} (ET_a/ET_m)_i^{\lambda_i} \quad (4)$$

Minhas model (1974):

$$\frac{Y_a}{Y_m} = a_0 \prod_{i=1}^{n} \left[1 - (1 - ET_a/ET_m)^{b_0}\right]_i^{B_i} \quad (5)$$

Blank model (1975):

$$\frac{Y_a}{Y_m} = \sum_{i=1}^{n} A_i (ET_a/ET_m)_i \quad (6)$$

Stewart model (1977):

$$1 - \frac{Y_a}{Y_m} = \sum_{i=1}^{n} K_{yi}[(ET_{mi} - ET_i)/ET_{mi}] \quad (7)$$

3. Results

3.1. Effect of Different Fertility Deficit Regulation Treatments on Water Consumption in Potato

As can be seen from Table 2, the stage water consumption of all water-regulated treatments of potato in 2019 was significantly lower ($p < 0.05$) than that of the CK treatment during the water-stressed reproductive period. The water consumption of each treatment at the seedling stage ranged from 58.86 to 77.59 mm, and its deficit-regulating treatments WD1 and WD5 were 11.74% and 22.89% lower, respectively, compared to the control CK, with significant differences. The water consumption during tuber initiation ranged from 109.42 to 137.43 mm, with no significant difference ($p > 0.05$) between the regulating deficit treatments WD1 and WD5 compared to the control after rehydration at this reproductive stage, while the regulating deficit treatments WD2 and WD6 decreased by 9.60% and 18.64%, respectively, compared to CK, with significant differences. At the tuber bulking stage, WD1, WD2 and WD6 treatments increased by 0.59%, 1.87% and 0.94%, respectively, compared to the control CK, with no significant difference, while the deficit adjustment treatments WD3 and WD7 decreased by 8.19% and 16.91%, respectively, compared to the control CK, with significant differences. During the starch accumulation period, WD1, WD2, WD3, WD5, WD6 and WD7 treatments were not significantly different from the control, while their regulating deficit treatments WD4 and WD8 were 11.01% and 23.33% lower than the control CK, respectively, with significant differences.

Table 2. Water consumption of potato at different growth stages.

Year	Treatment	Water Consumption/mm				
		Seedling	Tuber Formation	Tuber Bulking	Starch Accumulation	Water Consumption of Whole Growth Period
2019	CK	76.33 ab	134.49 ab	224.43 ab	117.68 ab	552.93 a
	WD1	67.37 b	134.16 ab	225.76 ab	116.55 ab	543.83 ab
	WD2	76.41 ab	121.58 b	228.62 a	119.69 ab	546.3 ab
	WD3	74.92 ab	135.19 ab	206.05 b	120.08 ab	536.24 ab
	WD4	75.83 ab	135.88 ab	225.82 ab	104.72 b	542.25 ab
	WD5	58.86 c	137.43 a	223.39 ab	115.37 ab	535.05 ab
	WD6	74.18 ab	109.42 c	226.53 ab	119.13 ab	529.26 b
	WD7	77.59 a	133.96 ab	186.47 c	120.81 a	518.83 c
	WD8	74.76 ab	133.53 ab	227.95 ab	90.22 c	526.46 b
2020	CK	56.51 bc	128.43 ab	236.31 ab	92.47 bc	513.72 ab
	WD1	45.54 c	116.39 b	247.96 ab	96.15 ab	506.04 ab
	WD2	60.10 ab	105.17 c	250.42 a	85.14 c	500.83 ab
	WD3	64.70 ab	129.40 a	214.02 bc	89.59 bc	497.71 ab
	WD4	62.35 ab	119.50 ab	244.20 ab	93.52 bc	519.58 a
	WD5	39.78 d	119.35 ab	243.68 ab	94.49 b	497.30 ab
	WD6	62.00 ab	90.61 d	224.15 b	100.15 ab	476.91 ab
	WD7	56.68 b	127.54 ab	184.22 c	103.92 a	472.36 b
	WD8	64.77 a	114.59 bc	244.13 ab	74.73 d	498.23 ab
Average	CK	66.42 ab	131.46 ab	230.37 ab	105.07 ab	533.33 a
	WD1	56.46 b	125.27 ab	236.86 ab	106.35 ab	524.94 ab
	WD2	68.25 ab	113.38 b	239.52 a	102.42 b	523.57 ab
	WD3	69.81 a	132.30 a	210.03 b	104.83 ab	516.98 ab
	WD4	69.09 ab	127.69 ab	235.01 ab	99.12 bc	530.92 ab
	WD5	49.32 c	128.39 ab	233.53 ab	104.93 ab	516.18 ab
	WD6	68.09 ab	100.02 c	225.34 ab	109.64 ab	503.09 ab
	WD7	67.14 ab	130.75 ab	185.35 c	112.36 a	495.60 b
	WD8	69.76 ab	124.06 ab	236.04 ab	82.48 c	512.35 ab
ANOVA	Treatment (T)	**	**	**	**	**
	Year (Y)	ns	ns	ns	ns	ns
	T × Y	ns	ns	ns	ns	ns

Note: Different lowercase letters within a column for a year or the average indicate significant differences among treatments ($p < 0.05$). The irrigation treatments were conventional irrigation (CK) and mild or moderate water deficit during each of four stages of potato growth. Mild deficit was in treatments WD1 (seedling), WD2 (tuber initiation), WD3 (tuber bulking), and WD4 (starch accumulation); moderate deficit was in treatments WD5 (seedling), WD6 (tuber initiation), WD7 (tuber bulking), and WD8 (starch accumulation). Values followed by the same lowercase letters within each year are not significantly different at the $p < 0.05$ level. *, **, and *** are significant at the $p < 0.05$, 0.01, and 0.001 levels, respectively; ns, not significant.

Water consumption in 2020 is similar to that of 2019 for all fertility stages of potato, in the following order of magnitude: tuber bulking, tuber initiation, starch accumulation and seedling stage. Water consumption at the seedling stage ranged from 39.78 to 64.77 mm for each treatment, with the deficit-regulating treatments WD1 and WD5 reducing by 10.97 mm and 16.73 mm, respectively, compared to the control CK, with significant differences ($p < 0.05$). During the tuber initiation period, the water consumption of each treatment ranged from 90.61 to 129.40 mm, with no significant difference between the WD1 and WD5 treatments compared to the control after rehydration, while the deficit-regulating treatments WD2 and WD6 were 18.11% and 29.45% lower than CK, respectively, with significant differences. During tuber bulking, water consumption peaked in all potato treatments, with values ranging from 184.22 to 250.42 mm, and the deficit-regulating treatments WD3 and WD7 were 9.43% and 22.04% lower, respectively, than the control CK. During the starch accumulation period, water consumption began to fall between treatments, with values ranging from 74.73 to 103.92 mm, with the mild water loss treatment WD4 increasing by 1.14% compared to the control CK, with no significant difference ($p > 0.05$); the moderate water loss WD8 decreased by 19.18% compared to CK, with a significant difference. The analysis shows that the degree of water deficit seriously affects water consumption at each stage of fertility, and as the degree of deficit increases, the amount of water consumed at that stage of fertility decreases more significantly.

The water consumption of the two growing seasons in 2019 and 2020 was found to be reduced to varying degrees compared to the control after different levels of water regulation treatments at different fertility stages. The loss-adjusting treatments WD1, WD2, WD3, WD4, WD5, WD6, WD7 and WD8 were 1.57%, 1.83%, 3.07%, 0.45%, 3.22%, 5.67%, 7.07% and 3.93% lower than the control, respectively.

3.2. Effect of Different Fertility Loss-Adjustment Treatments on Potato Yield

As a macroscopic effect of water regulation deficit on potato plants' yield, combined with the average annual potato yield for both growing seasons (Figure 2), it was found that all treatments had an effect, compared to the control. The average annual yield of the seedling regulation deficit treatment WD1 was 47,766.96 kg·hm^{-2}, an increase of 9.29% compared to the control, a significant difference ($p < 0.05$). The average annual yield of WD5 in the moderate water regulation deficit treatment at seedling stage was 39,981.89 kg·hm^{-2}, a significant difference of 8.52% compared to the control. At the tuber initiation stage, the average annual yield of WD2 in the deficit treatment was 42,057.35 kg·hm^{-2}, which was not significantly different from the control ($p > 0.05$), but the average annual yield of WD6 in the moderate water deficit treatment was 35,721.25 kg·hm^{-2}, which was 18.27% lower than the control, a significant difference. At the tuber bulking stage, the average yields of WD3 and WD7 were 38,870.26 kg·hm^{-2} and 36,689.46 kg·hm^{-2}, respectively, which were 11.07% and 16.06% lower than the control, with significant differences. During the starch accumulation period, the average annual yield of WD4 was 43,135.46 kg·hm^{-2}, which was not significantly different from the control, while the average annual yield of WD8 was 40,080.83 kg·hm^{-2}, which was significantly different from the control. The analysis of the effect of water loss irrigation on potato yields over two growing seasons found that water loss affected potato yields in all four growing seasons, but the extent of the effect varied between treatments. The degree of influence is in the following order: WD6 > WD7 > WD3 > WD1 > WD5 > WD8 > WD2 > WD4.

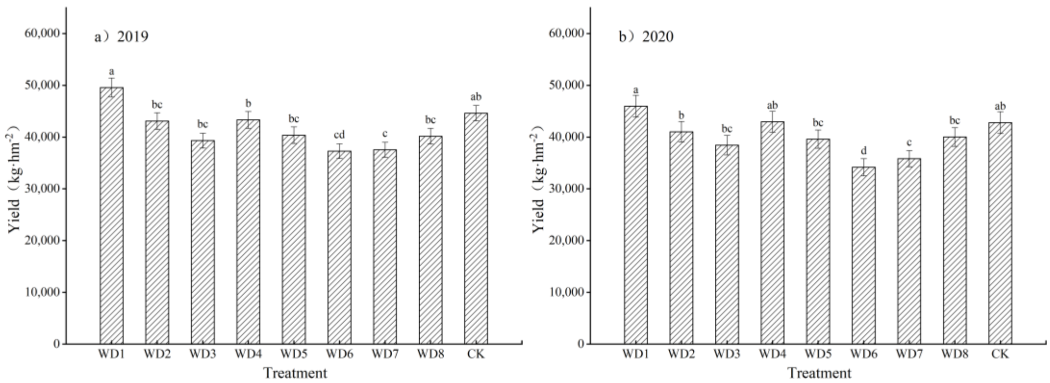

Figure 2. Potato production for the two growing seasons 2019 and 2020.

The above indicates that a mild moisture deficit treatment at the seedling stage will help to increase potato tuber yield, while a moderate moisture deficit at the tuber initiation, tuber bulking and starch accumulation stages will reduce potato yield, with a moderate moisture deficit at the tuber initiation stage having the greatest effect on potato yield, followed by a moderate moisture deficit treatment at the tuber bulking stage.

3.3. Relationship between Water Consumption and Impact on Yield under Different Fertility Deficit Regulation Treatments

As can be seen from Figure 3, the relationship between potato yield and water consumption under sub-membrane drip irrigation for the two growing seasons in 2019 and 2020 can be represented by a quadratic parabola: y = $-2.90411 \times 10^{-6}x^2 + 0.27109x - 847.33681$, ($R^2$ = 0.7548, 2019) and y = $-3.05939 \times 10^{-6}x^2 + 0.2804\,x - 1306.8679$, ($R^2$ = 0.82768, 2020), with more similar fitted curves for the two growing seasons. Potato yields showed a trend of increasing and then decreasing with increasing water consumption in both growing seasons. In 2019, the highest yield was 49,572.00 kg·hm^{-2} when water consumption reached 5438.4 m^3·hm^{-2} and 45,961.91 kg·hm^{-2} when water consumption reached 5060.4 m^3·hm^{-2} in 2020, after which the yield gradually decreases with increasing water consumption. Therefore, by studying the relationship between potato yield and water consumption, it is possible to control the amount of water used throughout the reproductive period, thus achieving the goal of regulating deficits and saving water.

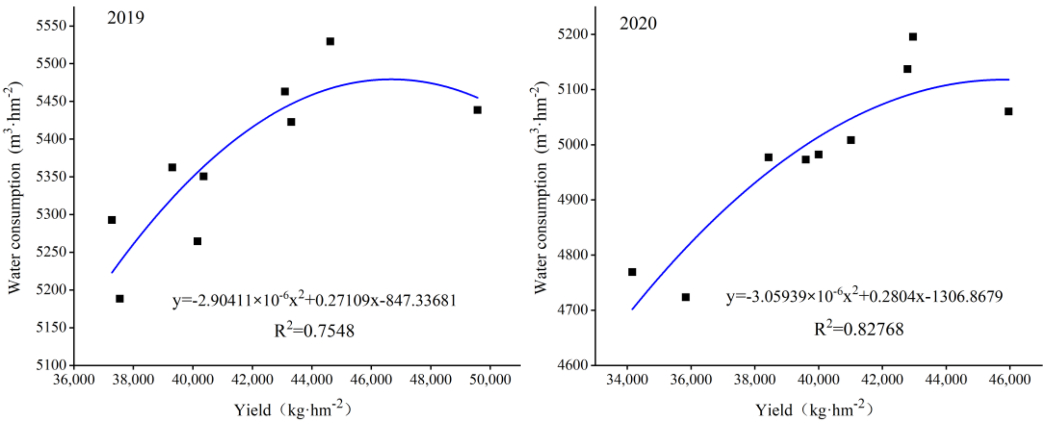

Figure 3. Relationship between potato yield and water consumption under drip irrigation and deficit regulation in the two growing seasons 2019 and 2020.

3.4. Moisture Production Function Model Solving

Taking the logarithm of each side of Equations (4) and (5) yields Equations (8) and (9).

$$Ln\left(\frac{Y_a}{Y_m}\right) = \prod_{i=1}^{n} Ln(ET_a/ET_m)_i^{\lambda_i} \tag{8}$$

$$Ln\left(\frac{Y_a}{Y_m}\right) = a_0 \prod_{i=1}^{n} Ln[1-(1-ET_a/ET_m)^{b_0}]_i^{B_i} \tag{9}$$

Order:
Jensen model:

$$Ln\left(\frac{Y_a}{Y_m}\right) = Z \quad Ln(ET_a/ET_m)_i^{\lambda_i} = X_i \tag{10}$$

Minhas model:

$$Ln\left(\frac{Y_a}{Y_m}\right) = Z \quad Ln\left[1-(1-ET_a/ET_m)^{b_0}\right] \tag{11}$$

Blank model:

$$\frac{Y_a}{Y_m} = Z \quad \frac{ET_a}{ET_{mi}} = X_i \tag{12}$$

Stewart model:

$$1-\frac{Y_a}{Y_m} = Z \quad (ET_{mi}-ET_i)/ET_{mi} = X_i \tag{13}$$

Assuming that the estimates of the dependent variables are linearly related to the multivariate independent variables, the above four models can be transformed into a unified multiple linear regression model.

$$Z = \lambda_1 x_1 + \lambda_2 x_2 + \lambda_3 x_3 + \cdots \lambda_n x_n = \sum_{i=1}^{n} \lambda_i x_i \tag{14}$$

In the potato under-membrane drip irrigation deficit regulation trial, the experimental design had a total of N groups with corresponding treatment numbers: j = 1, 2, K, N, where N denotes the number of multiple linear regression equations. The entire fertility period of the crop is divided into n stages with the stage number: i = 1,2,3 ..., n. n denotes the dimensionality of the parameter λ_i, which will be transformed into an n-dimensional problem of solving N multiple linear regression equations. Among the N group of treatments, there was a control treatment CK (full fertility irrigation) and an N-1 group of under-membrane drip irrigation to adjust the deficit treatment.

We can solve for each relative yield $\frac{Y_a}{Y_m}$ and relative evapotranspiration (ET_a/ET_m) by taking the natural logarithm of the relative yield and relative evapotranspiration to obtain N sets of n-dimensional observations in columns Z_j and X_{ij}.

$$\left\{ \begin{array}{c} Z_1, x_{11}, x_{21}, x_{31}, \Lambda, x_{n1} \\ Z_{2,} x_{12}, x_{22}, x_{32}, \Lambda, x_{n2} \\ Z_3, x_{13}, x_{23}, x_{33}, \Lambda, x_{n3} \\ \Lambda \cdots\cdots\cdots \Lambda \cdots\cdots\cdots \Lambda \\ Z_m, x_{1m}, x_{2m}, x_{3m}, \Lambda, x_{nm} \end{array} \right\} \tag{15}$$

The coefficients L_{ij} and the constant term L_{iz} in the system of linear equations can be found from the above data.

$$L_{ij} = \sum_{k}^{n} X_{ij} \cdot X_{jk} \tag{16}$$

$$L_{iz} = \sum_{k}^{n} X_{ik} \cdot Z_k \tag{17}$$

The coefficients and constant terms are obtained by solving the above equations and constructing a linear system of equations that gives the water scarcity sensitivity index.

$$\left. \begin{array}{c} L_{11}\lambda_1 + L_{12}\lambda_2 + L_{13}\lambda_3 + \cdots + L_{1n}\lambda_n = L_{1Z} \\ L_{21}\lambda_1 + L_{22}\lambda_2 + L_{23}\lambda_3 + \cdots + L_{2n}\lambda_n = L_{2Z} \\ L_{31}\lambda_1 + L_{32}\lambda_2 + L_{33}\lambda_3 + \cdots + L_{3n}\lambda_n = L_{3Z} \\ \Lambda \cdots\cdots\cdots\cdots\cdots\cdots \Lambda \cdots\cdots\cdots\cdots\cdots\cdots \Lambda \\ L_{n1}\lambda_1 + L_{n2}\lambda_2 + L_{n3}\lambda_3 + \cdots + L_{nn}\lambda_n = L_{nZ} \end{array} \right] \tag{18}$$

The Gaussian elimination method combined with Matlab software was used to solve the above system of multivariate equations, which resulted in the moisture deficit sensitivity indices shown in Table 3.

Table 3. Sensitivity indices obtained from the four models for each fertility stage of potato.

Mathematical Models	Sensitivity Index	Stages			
		Seedling	Tuber Initiation	Tuber Bulking	Starch Accumulation
Jensen	λi	0.4517	0.8205	0.9233	0.3748
Minhas	Bi	2.0158	4.9177	4.8805	1.8402
Blank	Ai	0.1170	0.3318	0.4509	0.3700
Stewart	Ki	0.4855	0.8392	0.9524	0.4013

The water sensitivity index λ solved by the model leads to the following water production function for potatoes under the under-membrane drip irrigation deficit regulation treatment in the cool irrigated areas of the west of the river.

Jensen:

$$\frac{Y_a}{Y_m} = \left(\frac{ET_1}{ET_{m1}}\right)^{0.4517} \cdot \left(\frac{ET_2}{ET_{m2}}\right)^{0.8205} \cdot \left(\frac{ET_3}{ET_{m3}}\right)^{0.9233} \cdot \left(\frac{ET_4}{ET_{m4}}\right)^{0.3748} \quad (19)$$

Minhas:

$$\frac{Y_a}{Y_m} = \left[1-\left(1-\frac{ET_1}{ET_{m1}}\right)^2\right]^{2.0158} \cdot \left[1-\left(1-\frac{ET_2}{ET_{m2}}\right)^2\right]^{4.9177} \cdot \left[1-\left(1-\frac{ET_3}{ET_{m3}}\right)^2\right]^{4.8805} \cdot \left[1-\left(1-\frac{ET_4}{ET_{m4}}\right)^2\right]^{1.8402} \quad (20)$$

Blank:

$$\frac{Y_a}{Y_m} = 0.1170\left(\frac{ET_1}{ET_{m1}}\right) + 0.3318\left(\frac{ET_2}{ET_{m2}}\right) + 0.4509\left(\frac{ET_3}{ET_{m3}}\right) + 0.3307\left(\frac{ET_4}{ET_{m4}}\right) \quad (21)$$

Stewart:

$$1 - \frac{Y_a}{Y_m} = 0.4855\left(\frac{ET_{m1}-ET_1}{ET_{m1}}\right) + 0.8329\left(\frac{ET_{m2}-ET_2}{ET_{m2}}\right) + 0.9524\left(\frac{ET_{m3}-ET_3}{ET_{m3}}\right) + 0.4013\left(\frac{ET_{m4}-ET_4}{ET_{m4}}\right) \quad (22)$$

where: 1, 2, 3 and 4 represent the different stages of potato fertility (seedling, tuber initiation, tuber bulking and starch accumulation).

The sensitivity indices of the four models were validated and the Minhas model was found to have a sensitivity index greater than 1, which did not meet the criterion of less than 1 [7], so the model will be discarded. The sensitivity indices of the Jensen model are in the following order throughout the potato reproductive period: tuber bulking (0.9233) > tuber initiation (0.8205) > seedling stage (0.4517) > starch accumulation (0.3748), and there is no significant error between the measured and simulated values as found in Figure 4, thus conforming to the potato water production function. In the Blank model, with a sensitivity index in the following order: tuber bulking stage (0.4509) > starch accumulation stage (0.3700) > tuber initiation stage (0.3318) > seedling stage (0.1170), although the sensitivity index meets the criteria, there is a significant error between the measured and simulated values, so the model will be discarded. The sensitivity indices of the Stewart model for the four potato reproductive stages were in the following order: tuber bulking (0.9524) > tuber initiation (0.8392) > seedling (0.4855) > starch accumulation (0.4013), and there were no significant errors between the measured and simulated values, so they were consistent with the potato water production function. In summary, it was found that of the four models, only the multiplicative Jensen model and the additive Stewart model met the criteria.

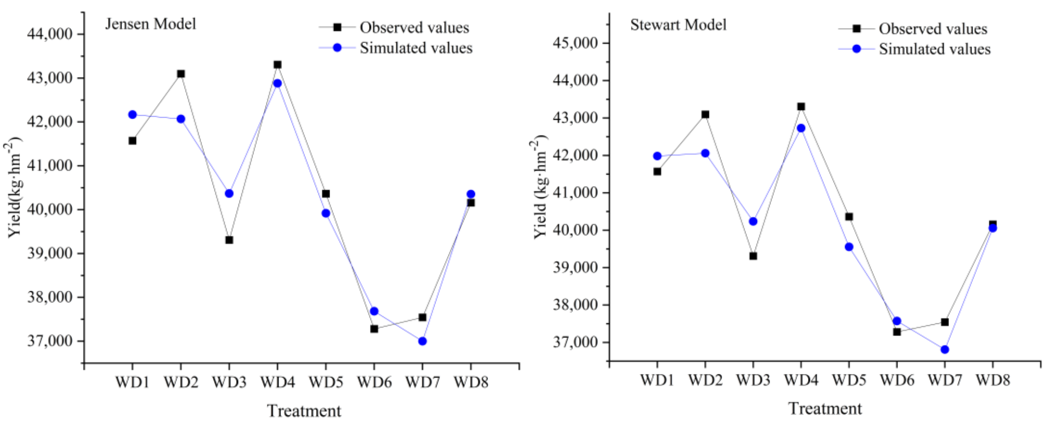

Figure 4. *Cont.*

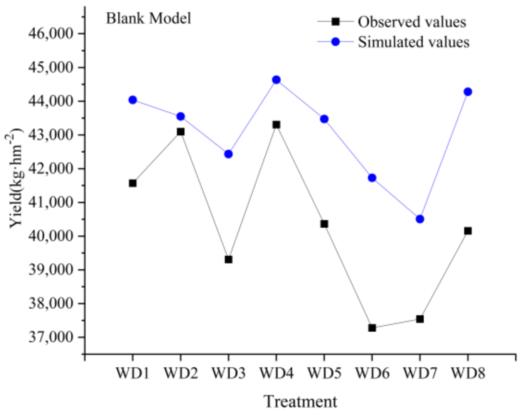

Figure 4. Comparison of observed and simulated values for each model.

4. Discussion

Water is the lifeblood of crop growth, and crop yield is the lifeblood of a country. Therefore, from the 1950s to the present, scholars at home and abroad have conducted in-depth research and exploration on the relationship between water and yield from different regions, crops and perspectives, during which the crop water production function (CWPF) was also applied [27]. The water production function is an intuitive mathematical expression for the effect of water on crop yield throughout the growing process [28]. After a long period of research, the more mature water production functions for bulk crops are divided into two main categories: additive models including the Howell model, Blank model, Stewart model, and Singh model and multiplicative models including the Jensen model, Rao model, Minhas model, and Hanks model.

In this study, we found that firstly, the water consumption at the seedling stage, tuber initiation stage, tuber bulking stage and starch accumulation stage ranged from 49.32 to 69.81 mm, 100.02 to 132.30 mm, 185.35 to 239.52 mm and 82.48 to 112.36 mm, respectively, and the water consumption showed the maximum at the tuber bulking stage, the second at the tuber initiation stage and the minimum at the seedling stage. The water consumption showed a dynamic trend of maximum at the tuber bulking stage, followed by the tuber initiation stage, and it was minimum at the seedling stage. Second, by fitting the curve equation between water consumption and yield, it was found that in 2019, when the water consumption reached 5438.4 $m^3 \cdot hm^{-2}$ during the whole reproductive period, the yield was highest at 49,572.00 $kg \cdot hm^{-2}$, and in 2020, when the water consumption reached 5060.4 $m^3 \cdot hm^{-2}$, the yield was highest at 45,961.91 $kg \cdot hm^{-2}$, the maximum yield was 45,961.91 $kg \cdot hm^{-2}$, and then the yield gradually decreased with the increase in water consumption. This is mainly due to the low temperature of the seedling stage, with inter-tree evaporation being small, and while the potato is in the seedling stage, the growth time is short, the number of leaves is small, and water consumption is low. The tuber initiation stage, which is dominated by nutritional growth, has significantly higher water consumption and is therefore more sensitive to water stress than the seedling stage. During the tuber bulking stage, the temperature rises significantly, the nutritional growth of the crop is stable, and the reproductive growth is in the rapid growth period, so the nutritional and reproductive growth coexist in this reproductive stage, and the crop consumes the most water; during the starch accumulation period, the temperature decreases, and with the weakening of nutritional and reproductive growth, its water requirement also gradually decreases, so the effect of water stress on this reproductive stage is less than other reproductive stages. Zhang [29] et al. found that the response of potato water consumption and yield to water regulation loss at each reproductive stage varied with the reproductive stage, with the highest water consumption at the tuber bulking stage being the most significantly affected by water stress and having the lowest yield in the corresponding treatment, and the lowest water consumption at seedling stage was less responsive to water regulation loss, which is consistent with the findings of this study.

In this experiment, the relationship between water consumption and yield of potatoes at each fertility stage under drip irrigation in the cool irrigated areas to the west of the river was investigated by using the more established multiplicative (Jensen model, Minhas model) and additive (Blank model, Stewart model) models. By fitting the four models, it was found that the sensitivity index of the Minhas model was greater than one, which did not meet the model criterion of sensitivity index less than one [25]. The differences between the simulated and measured values fitted by the Blank model were far from each other, so it also did not meet its fitting criterion, which was different from the results of Wang Yucai [30] et al. The reason for the different results may be the difference in the study crop, fertility division and experimental design, which led to the different final results. The Jensen and Stewart models had a better fit and the magnitude of the water deficit sensitivity index was in the following order: tuber bulking stage > tuber initiation stage > seedling stage > starch accumulation stage, which is consistent with the findings of Jiang, Jiang X [31], Shukla M K i [19], Kuu H [32] et al. This indicates that during the period from tuber formation to tuber expansion, when the potato is in a critical period of excessive reproductive growth from nutritional growth, the crop needs sufficient water to supply the necessary water for growth, and therefore water stress is not appropriate at this stage, while during the seedling and starch accumulation stages the crop is purely nutritional or reproductive in nature and does not require much water, so moderate water stress can be applied.

5. Conclusions

(1) The relationship between water consumption and yield of potato at all stages of fertility under drip irrigation in the cool irrigated areas to the west of the river can be expressed by the Jensen and Stewart models. The water deficit sensitivity indices were in the following order: tuber bulking stage > tuber initiation stage > seedling stage > starch accumulation stage.

(2) Fitting the model revealed that the high water demand of the crop during the potato tuber formation and tuber expansion stages was not conducive to water stress treatments, and that water stress treatments with field water holding capacity (55–65%) could be applied during the seedling and starch accumulation stages.

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