

Article

A New Multi-Objective Optimization Model of Water Resources Considering Fairness and Water Shortage Risk

Xiaoyu Tang ^{1,2}, Ying He ^{1,*}, Peng Qi ² , Zehua Chang ², Ming Jiang ² and Zhongbin Dai ³

- ¹ College of Water Conservancy and Civil Engineering, Xinjiang Agricultural University, No. 311 Nongda East Road, Urumqi 830052, China; tangxz1995@163.com
- ² Northeast Institute of Geography and Agroecology, Chinese Academy of Sciences, No. 4888 Shengbei Street, Changchun 130102, China; qipeng@iga.ac.cn (P.Q.); changzehua@iga.ac.cn (Z.C.); jiangming@iga.ac.cn (M.J.)
- ³ College of Computer and Software, Nanjing University of Information Technology, No. 219 Ningliu Road, Nanjing 210044, China; zz1157268983@gmail.com
- * Correspondence: xjheying@126.com; Tel.: +86-187-8227-2061

Abstract: Assessing the fairness of water resource allocation and structural water shortage risks is an urgent problem that needs to be solved for the optimal allocation of water resources. In this study, we established a new multi-objective optimization model of water resources based on structural water shortage risks and fairness. We propose an improved NSGA-III based on the reference point selection strategy (ARNSGA-III) to solve the optimization model. The superiority of this method was proven by comparing it with three other methods, namely, NSGA-III, MOSPO, and MOEA/D. The model was applied to optimize the allocation of water resources in Wusu City in China. The results show that the new multi-objective optimization model provides reasonable and feasible solutions for solving water conflicts. The convergence and stability of ARNSGA-III are better than those of the other three algorithms. Allocation schemes of water resources for Wusu City in normal years, dry years, and extremely dry years are proposed. In normal years, the structural water shortage risk index is reduced by 50.1%, economic benefits increased by 0.2%, and fairness is reduced by 60.5%. This study can provide new ideas for solving the multi-objective optimization of regional water resources.

Keywords: water resource allocation; multi-objective optimization; fairness; water shortage risk; NSGA-III



Citation: Tang, X.; He, Y.; Qi, P.; Chang, Z.; Jiang, M.; Dai, Z. A New Multi-Objective Optimization Model of Water Resources Considering Fairness and Water Shortage Risk. *Water* **2021**, *13*, 2648. <https://doi.org/10.3390/w13192648>

Academic Editor: Mashor Housh

Received: 30 August 2021
Accepted: 23 September 2021
Published: 26 September 2021

Publisher's Note: MDPI stays neutral with regard to jurisdictional claims in published maps and institutional affiliations.



Copyright: © 2021 by the authors. Licensee MDPI, Basel, Switzerland. This article is an open access article distributed under the terms and conditions of the Creative Commons Attribution (CC BY) license (<https://creativecommons.org/licenses/by/4.0/>).

1. Introduction

Water shortage is a major socioeconomic and global sustainability issue that imperils human survival and regional development [1]. The overuse and poor management of scarce water resources are exacerbating the impacts of current droughts [2]. Shrinking water resources, the increasing trend of droughts, and their severe damages play a significant role in intensifying the water crisis [3]. The fair and reasonable allocation of water resources is essential to eradicate regional poverty and maintain regional peace and stability [4]. Therefore, determining how to reasonably allocate water resources is a problem that urgently needs to be solved.

Fairness is an important indicator of the optimal allocation of water resources [5]. At present, research on the fairness of water distribution is shifting from qualitative to quantitative analysis. Cullis et al. [6] suggested the Gini coefficient as a temporary method to calculate the fair allocation of water resources. Yang et al. [7] used the Gini coefficient as a constraint to study the fairness of water resource allocation in irrigation areas. Hu et al. [8] used the Gini coefficient to study the relationship between equity and economic benefits in regional water resource allocation. Hu et al. [1] used the Gini coefficient to study the relationship between fairness and economic benefit loss risk in the allocation of water resources in a river basin. Therefore, the fairness of water resource allocation is a current research hotspot. The influence of climate, human activities, etc., increases the uncertainty of the

water resource allocation system, which also increases its risk [9]. Previous studies have seldom considered risks in the process of water resource allocation, and they have ignored the risks of unreasonable water use structure. Therefore, Ma [10] proposed a structural water shortage risk index to guide water resource allocation. Wang et al. [11] used the risk of water shortage to establish an optimization model based on distributed simulation to guide agricultural water management. Gao et al. [12] established an uncertainty-based water shortage risk assessment model (UWSRAM), which is used to analyze the degree of water shortage under uncertain conditions, and it is a convenient method to guide the allocation of water resources. In previous studies, only the fairness of water allocation or structural water shortage risks were considered, which may lead to unfairness or water shortage risks in the results of water resource allocation. However, taking both factors as optimization objectives can help to effectively avoid the above-mentioned problems and make the results of water resource allocation more comprehensive.

The multi-objective optimization allocation model of water resources focuses on solving water resource conflicts from the perspective of optimization objectives to improve the results of water resource allocation [13]. In recent years, with the rapid development of computer technology, a large number of evolutionary algorithms have been proposed. Because of their ability to perform large-scale and complex calculations and because they have the advantage of high versatility, they have been widely used to obtain solutions of water resource optimization models. Such methods include the genetic algorithm [14], particle swarm algorithm [15], non-dominated sorting genetic algorithm [16], and their modified versions [17]. The most popular among them is an evolutionary algorithm using a reference-point-based non-dominated sorting approach (NSGA-III) [18], because it has better convergence and strong practicability when dealing with three or more objectives. However, in the actual problem-solving process, we found that NSGA-III is associated with difficulties in determining the reference point division and poor adaptation to the Pareto frontier of the actual problem. Therefore, we improved NSGA-III by using a quadrant of the population in the decision space. We propose an improved NSGA-III based on the reference point selection strategy (ARNSGA-III), in which information on differential distribution characteristics discriminates the evolution stage of the population, and reference points are selected based on the distribution characteristics of the population in the target space.

The main purpose of this study is to include fairness and the risk of water shortage as factors when determining the allocation of water resources. Wusu City, a typical water-scarce area in China, was selected to explore the best allocation method of water resources. First, a new multi-objective optimization model of regional water resources was established, and then an improved NSGA-III (ARNSGA-III) method was used to solve the optimization model. Finally, water resource allocation schemes for Wusu City in normal years, dry years, and extremely dry years were proposed. This study can provide new ideas for the multi-objective optimization of regional water resources.

2. Materials and Methods

2.1. Optimization Model

Multi-objective optimization of water resources is a complex, large-scale system optimization problem. First, the objectives and decision variables for optimal allocation are determined, and then reasonable objective functions and constraints are constructed, which are then solved by the algorithm.

2.1.1. Objective Function

In the process of water resource allocation, we must not only pay attention to economic benefits, but also consider the regional water shortage risk caused by the unfairness of water distribution and the unbalanced water structure of various sectors. Therefore, we took the minimum structural water shortage risk index, the maximum economic benefit, and the maximum fairness as the objective function.

- (1) Structural water shortage risk

Structural water shortage risk is a risk indicator for the balance of regional water shortage and water use structure. It reflects changes in water shortage and water structure. It is obtained by multiplying the water shortage index and the information entropy of the water use structure. The smaller the value, the safer the water structure and water consumption in the area [10]. The risk is understood as a combination of the probability level of a threat activation and the level of its effects.

$$\min f_1 = W_c e^{-S} = \sum_{i=1}^I \frac{\sum_{j=1}^J W_{ij} - W_{\min_i}}{W_{\max_i} - W_{\min_i}} e^{\sum_{j=1}^J \frac{W_{ij}}{\sum_{j=1}^J W_{ij}} \ln \frac{W_{ij}}{\sum_{j=1}^J W_{ij}}} \tag{1}$$

$$S = -\sum_{j=1}^n p_j \ln p_j \tag{2}$$

$$p_j = \frac{W_j}{\sum_{i=1}^J W_i} \tag{3}$$

$$W_c = \{x : 0 \leq u_w(x) \leq 1\} u_w(x) = \begin{cases} 0 & (0 \leq x \leq W_{\min_i}) \\ \frac{W_{ij} - W_{\min_i}}{W_{\max_i} - W_{\min_i}} & (W_{\min_i} < W_{ij} \leq W_{\max_i}) \\ 1 & (W_{ij} > W_{\max_i}) \end{cases} \tag{4}$$

where f_1 is the structural water shortage risk index, i is the regional coefficient, and j is the water sector coefficient. W_c is the comparative water scarcity index in historical years. S is the information entropy of water use structure. P_j is the water use structure of each water sector in the area. W_{ij} is the water consumption of sector j in region i , m^3 . W_{\min_i} is the minimum value of water consumption in area i in historical years, m^3 . W_{\max_i} is the maximum water consumption of area i in historical years, m^3 .

The purpose of measuring the risk of changes in the water use structure is to show these changes in the year of observation. Equations (2) and (3) show that in the expression of the information entropy of the water use structure, the actual water consumption of each water sector is used as the random variable W_{ij} , and the proportion of water used by each sector (water use structure) represents the probability P_j corresponding to the random variable. The more balanced the water use structure, the greater the information entropy of the water use structure; the more concentrated the proportion of water use in a certain water sector, the lower the information entropy of the water use structure, indicating that water consumption is more evenly distributed among the sectors. Equation (4) shows that the water shortage index uses the maximum and minimum water consumption in historical years to reflect the degree of change in water use during this time. The larger the water shortage index, the greater the water consumption in the area, and the more likely it is to cause regional water shortages.

(2) Economic benefits

Water resources are an important factor affecting agricultural irrigation, industrial production, and people’s lives. Under the premise that the minimum water consumption of each sector is guaranteed, the water allocation of each sector is determined by the economic benefits that it produces. Therefore, the goal is to maximize the direct economic benefits of each user in the development and utilization of regional water resources.

$$\max f_2 = \sum_{i=1}^I \sum_{j=1}^J W_{ij} (b_{ij} - c_{ij}) \tag{5}$$

where f_2 is economic benefits, b_{ij} is the benefit coefficient of sector j in area i , and c_{ij} is the cost coefficient of sector j in area i .

(3) Fairness

Optimizing the fairness of intersectoral water distribution is the main way to resolve intersectoral water use conflicts. The Gini coefficient is used as an indicator to measure the fairness of intersectoral water distribution, and the goal is to obtain the smallest Gini coefficient. The smaller the value, the better the fairness of the area [19].

$$\min f_3 = Gini^{agr} + Gini^{ind} + Gini^{dom} \tag{6}$$

$$Gini^{agr} = \frac{\sum_{i_1=1}^I \sum_{i_2=1}^I \left| \frac{W_{i_1}^{agr}}{AR_{i_1}} - \frac{W_{i_2}^{agr}}{AR_{i_2}} \right|}{2I \sum_{i=1}^I \frac{W_i^{agr}}{AR_i}} \tag{7}$$

$$Gini^{dom} = \frac{\sum_{i_1=1}^I \sum_{i_2=1}^I \left| \frac{W_{i_1}^{dom}}{PE_{i_1}} - \frac{W_{i_2}^{dom}}{PE_{i_2}} \right|}{2I \sum_{i=1}^I \frac{W_i^{ind}}{PE_i}} \tag{8}$$

$$Gini^{dom} = \frac{\sum_{i_1=1}^I \sum_{i_2=1}^I \left| \frac{W_{i_1}^{dom}}{PE_{i_1}} - \frac{W_{i_2}^{dom}}{PE_{i_2}} \right|}{2I \sum_{i=1}^I \frac{W_i^{ind}}{PE_i}} \tag{9}$$

where f_3 is fairness, $Gini^{agr}$ is the Gini coefficient of the agricultural sector, $Gini^{ind}$ is the Gini coefficient of the industrial sector, and $Gini^{dom}$ is the Gini coefficient of the domestic sector. i_1 and i_2 are the same water sector in different regions. $W_{i_1}^{agr}$ is water consumption of the agricultural sector in region i_1 , m^3 ; AR_{i_1} is irrigation area in region i_1 , hm^2 . $W_{i_2}^{agr}$ is water consumption of the agricultural sector in region i_2 , m^3 ; AR_{i_2} is irrigation area in region i_2 , hm^2 . $W_{i_1}^{ind}$ is water consumption of the industrial sector in region i_1 , m^3 ; EC_{i_1} is the economic output value of the industry in region i_1 , yuan; $W_{i_2}^{ind}$ is water consumption of the industrial sector in region i_2 , m^3 . EC_{i_2} is the economic output value of the industry in region i_2 , yuan. $W_{i_1}^{dom}$ is the water consumption of the domestic sector in region i_1 , m^3 ; PE_{i_1} is the number of people in region i_1 ; $W_{i_2}^{dom}$ is the water consumption of the domestic sector in region i_2 , m^3 ; PE_{i_2} is the number of people in region i_2 . The Gini coefficient ranges from 0 to 1, where 0 means absolutely fair and 1 means absolutely unfair. The closer its value is to 0, the fairer the result of water resource allocation. Usually, 0.2–0.3 is regarded as absolute fairness, 0.2–0.3 is regarded as relatively fair, 0.3–0.4 is regarded as relatively fair, 0.4–0.5 is regarded as a large gap, and 0.5 or more is regarded as a large gap [20].

2.1.2. Constraints

(1) Sector water consumption constraints

In order to ensure the normal operation of various water-using sectors, the water distribution of each sector should be greater than the minimum water consumption required to maintain normal activity, and the maximum value should be less than the amount of water available in the sector under total water consumption control.

$$W_{ij}^{\min} \leq W_{ij} \leq W_{ij}^{\max} \tag{10}$$

where W_{ij}^{\min} is the minimum water consumption of sector j in region i necessary to maintain normal operation, m^3 . W_{ij}^{\max} is the available water volume of sector j in region i under the constraints of the “three red lines”, m^3 .

(2) Eco-environmental water consumption constraints

Ecological water use is an important factor to ensure the sustainable development of the ecological environment. Therefore, ecological water use should be greater than the minimum regional ecological water demand.

$$WE^{\min} \leq WE \quad (11)$$

where WE^{\min} is the minimum water requirement of the ecological environment, m^3 , and WE is ecological water consumption, m^3 .

(3) Total water volume constraints

In order to ensure the sustainable development of the region, the total water consumption of all regions and sectors should be less than the available water volume of the region under total water consumption control.

$$WE + \sum_{i=1}^n \sum_{j=1}^m W_{ij} \leq W \quad (12)$$

where W is the available water volume in the region under the constraints of the "three red lines", m^3 .

(4) Water shortage risk constraints

In order to prevent the risk of regional water shortage due to changes in water use structure, the structural water shortage risk index is 0–1.

$$0 < f_1 < 1 \quad (13)$$

2.2. Optimization Algorithm

In order to overcome the shortcomings of the traditional nonlinear programming method, i.e., that the solution speed is slow and that it easily falls into the local optimum, we used the evolutionary algorithm to solve the optimization model.

2.2.1. NSGA-III

Deb et al. [18] proposed an evolutionary algorithm called NSGA-III, which is an improvement of NSGA-II. The reference point mechanism is introduced on the basis of NSGA-II. The calculation formula for the number of reference points is as follows.

$$H(M, p) = \frac{(M + p - 1)!}{p! (M - 1)!} \quad (14)$$

where H is the number of reference points, M is dimension, and p is the number of divisions of each dimension target.

The uniformly distributed reference points constructed in NSGA-III are effective in solving the problem of the uniform Pareto front surface. However, in actual problems, the Pareto front surface is not all uniformly and continuously distributed, resulting in poor calculation results of the algorithm. The number of reference points in NSGA-III determines the running time of the algorithm, and the two are positively correlated. The final evolution of the algorithm will not improve with the increase in reference points. Too many or too few reference points are not conducive to critical non-dominance, and for different problems, the importance of each reference point to individual choice is different. It can be seen that the selection of the reference point has an important impact on the performance of the algorithm.

2.2.2. ARNSGA-III

In order to solve the problems associated with NSGA-III, that is, the difficulty determining the reference point division and the poor Pareto frontier adaptability to actual problems, we proposed an improved NSGA-III based on the reference point selection strategy (ARNSGA-III). Information on the quartile distribution characteristics of the population

in the decision space was used to distinguish the evolution stage of the population, and the reference point was selected based on the distribution characteristics of the population in the target space.

(1) Evolutionary stage decision strategy based on decision space

In the process of population evolution, the population changes from disorder to order and gradually converges, which is a process of entropy reduction. We used the difference between the entropy of two adjacent generations to describe the evolutionary stage of the population [21]. The entropy value e^t was calculated from the standardized quartile inf_i of the population and the standardized median Δmid^t .

$$e^t = -\sum_{h=1}^H inf_i \lg inf_i - \sum_{h=1}^H \Delta mid_i^t \lg \Delta mid_i^t \quad (15)$$

$$\Delta e^t = \left| e^t - e^{t-1} \right| \quad (16)$$

When the population is updated, e^t magnifies the change in the population in the decision space. The larger the value of Δe^t , the greater the change in individuals in the population, indicating that the population is actively exploring the unknown potential solution space. Conversely, the smaller the value of Δe^t , the more stable the population, indicating that the population is converging. Therefore, Δe^t can be used to determine the evolutionary state of the population S^t . In the following, t is the current evolution time of the population, and μ is the threshold.

If $|\Delta e^t| > \Delta \mu$, then the algorithm is in the “exploration” stage, and the population is exploring the solution space. If $|\Delta e^t| < \mu$, then the algorithm enters the “polymerization” stage, and the population may begin to converge.

The judgment basis of S^t in the evolution stage is the threshold μ . The basis for setting the threshold μ in this paper is that for any dimensional finite interval E , the ideal situation is that N individuals in the population are evenly distributed in this interval, and the shortest distance between an individual in the population and the neighboring individual is $|E|/N$, which is $1/N$ after normalization. The formula for calculating the standardized interquartile range when individuals are uniformly distributed is as follows.

$$\overline{inf} = \frac{0.75E - 0.25E}{E} = \frac{1}{2} \quad (17)$$

When the change in \overline{inf} is less than $1/N$ and the population does not shift $\Delta mid \approx 0$, Equation (16) is used to obtain $|\Delta e|$ as the threshold μ , and, as a reference, we further consider the linear space existing between the threshold μ and the dimension of the decision space.

$$\mu = -D \overline{inf} \lg \overline{inf} - D \left(\overline{inf} + \frac{1}{N} \right) \lg \left(\overline{inf} + \frac{1}{N} \right) \quad (18)$$

(2) Reference point selection strategy based on target space

In the proposed reference point strategy, the number of reference points is adaptive to the population size N , and the more important reference points are dynamically selected according to the evolution stage of the population in the target space. This strategy does not require the user to set the parameter p . To ensure the screening effect, the number of eliminated reference points should be greater than or equal to $20\%N$. The steps for selecting reference points are as follows.

1. According to the population size N , select the reference point set Z divided into p in each dimension; the number of reference points is H_p , and p satisfies $H_p \geq 1.2$, $H_{p-1} < 1.2N$.
2. Determine the evolution stage of the population according to the plan in step 1.
3. When the population is in the “exploration” stage, statistical reference point set Z is the sum of the number of associated individuals in each generation Z_{sum} .

4. When the population just enters the “polymerization” stage, the N reference points with the largest number of associations in Z are retained according to Z_{sum} to form a new reference point set Z_n .

(3) ARNSGA-III flowchart

Combining the above two strategies, the calculation process of ARNSGA-III is shown in Figure 1.

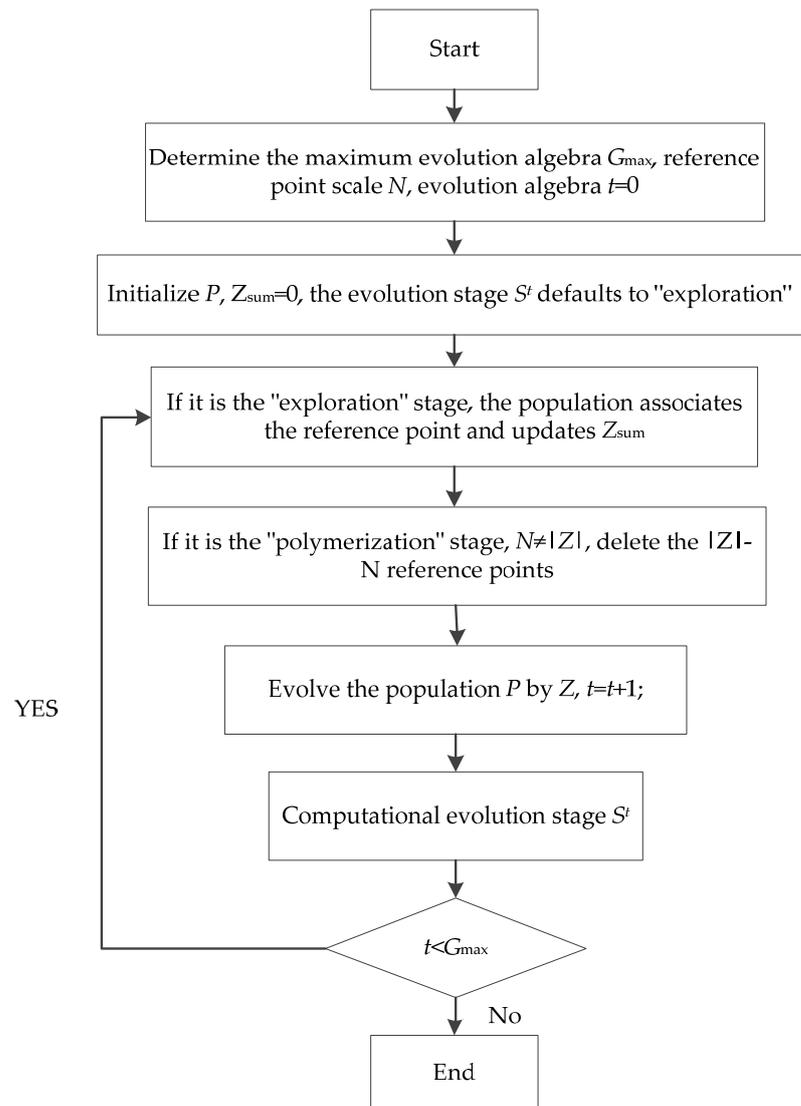


Figure 1. Calculation flowchart of ARNSGA-III.

2.3. HV Algorithm Test Indicators

In order to verify the advantages and disadvantages of ARNSGA-III in solving the multi-objective optimization model, three multi-objective evolutionary algorithms were selected for comparison, namely, NSGA-III [18], MOSPO [22], and MOEA/D [23]. We used four multi-objective evolutionary algorithms to solve the multi-objective optimization model of water resources established in this paper. In order to ensure fairness, real number coding was used for each algorithm. The settings are as follows: population size $N = 100$, maximum evolution algebra $G_{\text{max}} = 1000$. The computing platform is PlatEMO3.0 [24], and the software is Matlab2020a. The specific source code: <https://github.com/1209805090/AR-NSGA-III> accessed on 19 March 2021.

In order to evaluate the effectiveness of the algorithm, Hyper Volume (HV), which can simultaneously reflect the convergence and distribution of the algorithm, was used as the algorithm performance evaluation index. The calculation of the HV index does not need to test the real non-inferior solution frontier of the problem. This approach is suitable for evaluating the performance of algorithms in practical problems [25]. HV represents the volume of the hypercube enclosed by Pareto and the reference point in the target space. The larger the HV value, the better the overall performance of the algorithm.

The calculation method of HV is as follows.

1. Taking (C_1, C_2, C_3) as the reference point in the HV evaluation index, (F_{1j}, F_{2j}, F_{3j}) as each Pareto solution obtained in a certain run of an algorithm, and (C_1, C_2, C_3) and (F_{1j}, F_{2j}, F_{3j}) as the diagonal of the rectangle, the area of the rectangle enclosed by each solution and the reference point is calculated.
2. Taking the union of all the rectangles calculated in step 1, the area of the figure formed is the HV value.

3. Case Study

3.1. Study Area

Wusu City is located in the northwestern part of the Xinjiang Uygur Autonomous Region of China, on the southwestern edge of the Junggar Basin and on the northern slope of the Tianshan Mountains ($83^\circ\sim 86^\circ$ E, $43^\circ\sim 46^\circ$ N), which is a typical arid area. The annual average rainfall is 162.61 mm, and the annual average evaporation is 1208.31 mm. In 2018, the structural water shortage risk index of Wusu City was 1.060, and the fairness was 0.780. There are serious water equity and water structure conflicts among regions and industries. Under the condition of limited available water resources, an unreasonable water use structure among sectors exacerbates the waste of water resources and easily increases the risk of water shortage. Unreasonable distribution of water resources among the same sector in different regions is not conducive to the sustainable development of society and the economy. Therefore, the water use structure of the study area is optimized to realize the fair and sustainable development of the region. Wusu City is mainly composed of four parts, namely, Kuitunhe Area, Sikeshe Area, Chepaizi Area, and Giltui Area. The geographical location and division of Wusu City are shown in Figure 2.

3.2. Data

The data on the permanent population, irrigated area, and economic benefits of Wusu City come from the "Ili Kazakh Autonomous Prefecture Statistical Yearbook 2015–2018". The total economic benefit is 1.10×10^{10} yuan. The agricultural irrigated area, industrial output value, and population of each district are shown in Table 1. The water efficiency coefficient b_{ij} refers to the Local Standards of Xinjiang Uygur Autonomous Region (DB 65/3611-2018), and the water efficiency coefficients for agriculture, industry, and life are 1.77, 330, and 427 yuan/m³, respectively. The cost coefficient c_{ij} was determined by the water fee collection standard in Wusu City in 2018, and the cost coefficients for agriculture, industry, and living are 0.22, 4.69, and 2.35 yuan/m³, respectively. A frequency analysis of water volume in Wusu City over the years identified three different typical years, namely, normal years, dry years, and extremely dry years. The water volume in normal years is 5.32×10^8 m³, the water volume in dry years is 5.12×10^8 m³, and the water volume in extremely dry years is 4.96×10^8 m³. Table 2 shows the minimum and maximum water resource usage in Wusu City in historical years.

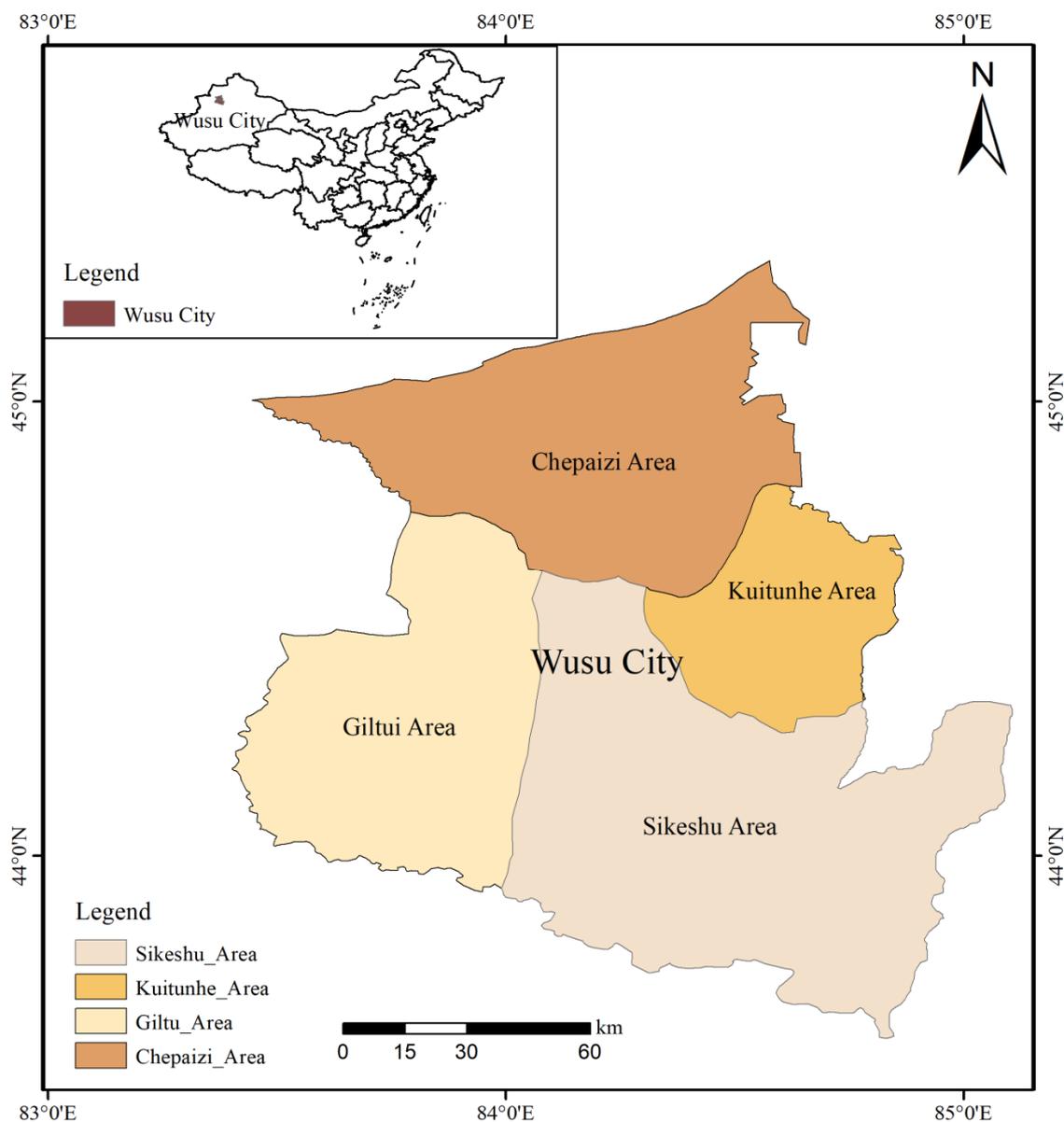


Figure 2. Geographical location and division of Wusu City.

Table 1. Irrigated area, industrial output value, and population in Wusu City in 2018.

Category	Irrigation Area (hm ²)	Industrial Output Value (10 ⁸ Yuan)	Population (10 ⁴)
Kuitunhe Area	4.78	47.81	14.71
Sikeshu Area	6.03	11.92	7.45
Chepaizi Area	1.73	-	1.67
Jiertuhe Area	1.27	-	1.22
Total	13.81	59.73	25.05

Table 2. The minimum and maximum water demand of each sector in Wusu City in 2018.

Category	Agricultural (10 ⁴ m ³)		Industrial (10 ⁴ m ³)		Domestic (10 ⁴ m ³)	
	Minimum	Maximum	Minimum	Maximum	Minimum	Maximum
Kuitunhe Area	16,116	17,895	1303	1789	670	827
Sikeshu Area	20,031	21,681	180	422	284	310
Chepaizi Area	5636	6735	-	-	56	73
Jiertuhe Area.	4146	4904	-	-	35	52

Minimum expresses the minimum water resources usage in Wusu City in historical years; Maximum expresses the maximum water resources usage in Wusu City in historical years.

4. Results and Discussion

4.1. ARNSGA-III Instance Test

Using ARNSGA-III, NSGA-III, MOSPO, and MOEA/D to solve the multi-objective optimization model of water resources established in this paper, the Pareto solution set calculated by each algorithm was put into the HV index calculation formula. By adjusting the reference points (C_1 , C_2 , C_3), the calculation results of each algorithm in the HV index were obtained. When the result of each algorithm was greater than 0 at the same time, the size of the reference point was determined [26]. After repeated tests, the reference points (C_1 , C_2 , C_3) of this analysis were finally determined to be (1, -9.45×10^8 , 1).

As can be seen from Table 3, ARNSGA-III has the largest average value and the smallest standard deviation, followed by NSGA-III, indicating that ARNSGA-III has better performance than NSGA-III in solving the multi-objective optimization model established in this paper. Among the other two algorithms, the effect of MOPSO is intermediate, and that of MOEA/D is the worst.

Table 3. HV index value test of ARNSGA-III, NSGA-III, MOMOP, and MOEA/D.

Algorithm	N	D	Average Value of HV	Standard Deviation of HV
ARNSGAIII	3	12	44.10	0.33
NSGAIII	3	12	43.97	0.39
MOPSO	3	12	35.13	0.78
MOEA/D	3	12	25.56	1.02

As can be seen from Figure 3, the Pareto solution sets of ARNSGA-III and NSGA-III are evenly distributed, and a wider range of solutions are searched in each objective function. The values of the objective functions are quite different among the methods, and the Pareto solution set of ARNSGA-III has the most uniform distribution. The Pareto solution sets of MOPSO and MOEA/D are relatively densely distributed; there are discrete points, and the values of the objective functions are not very different. Taking MOEA/D as an example, the structural water shortage risk index is mainly concentrated in 0.687320–0.689321, economic benefits are mainly concentrated in 1.138180–1.138181, and fairness is mainly concentrated in 0.350414–0.350415. The algorithm has poor convergence and easily falls into the local optimum. In the calculated results, the values of the decision variables of each group are relatively close to the value of the objective function, and the decision space provided to the decision maker is small, which cannot meet the needs of actual practice. In terms of the Pareto solution search, in order to ensure fairness, the number of populations of each algorithm was set to 100. From the analysis of the actual results obtained, ARNSGA-III and MOPSO both achieved 100 Pareto solutions, while NSGA-III and MOEA/D only obtained 91 Pareto solutions. This shows that ARNSGA-III and MOPSO were better than the other two algorithms in the Pareto solution search. In summary, we used ARNSGA-III as the solution algorithm for the optimization model in this paper.

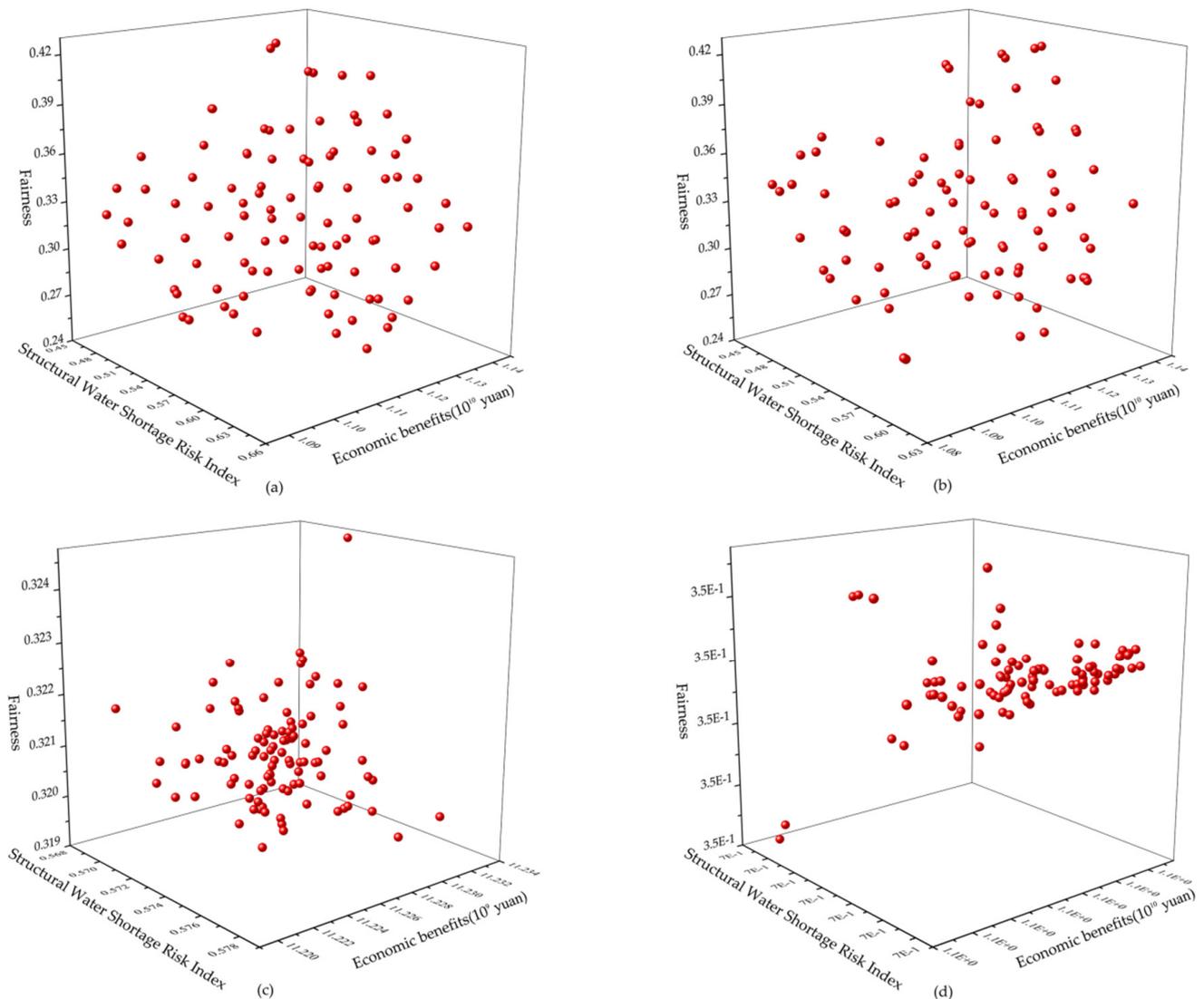


Figure 3. (a) Pareto solution set of ARNSGA-III; (b) Pareto solution set of NSGA-III; (c) Pareto solution set of MOSPO; (d) Pareto solution set of MOEA/D.

4.2. Typical Year Analysis

In order to reduce the impact of extreme weather [27], based on the inflow of water in Wusu City in normal years, dry years, and extremely dry years, and by taking the smallest structural water shortage risk index, the largest economic benefit, and the largest fairness as the objective function, a multi-objective optimization model of regional water resources was established, and we used ARNSGA-III to solve it. The Pareto solution sets of normal years, dry years, and extremely dry years are shown in Figure 4.

As can be seen from Figure 4, the fairness of water distribution in the three different typical years is less than 0.5. Among them, there are schemes with fairness less than 0.3 in normal years and dry years, indicating that the distribution of water resources is fair [28]. The range of the structural water shortage risk index is 0.45–0.65 in normal years and 0.66–0.78 in drought years. The main reason for this difference is the decrease in water and precipitation during the drought years, which lead to an increase in water demand in various regions, increased competition for water resources, and changes in the structure of water use in various regions. In normal years and extremely dry years, the economic benefits change slightly. Both fairness and the structural water shortage risk index changed significantly. The fairness in normal years ranges from 0.25 to 0.40, and the

fairness in extreme drought years ranges from 0.28 to 0.44. The main reason for the fairness change is that the amount of water in extremely dry years is small. In order to ensure regional economic development, areas with higher economic output value will receive more water, reducing the distribution of water in areas with lower economic output value, and a chain reaction will lead to changes in the regional structure of water use and will increase the structural water shortage risk index. In summary, considering the optimization of water resources in different typical years will have a positive impact on maintaining regional economic development, adjusting the structure of regional industrial water use, and improving the equity of regional water use. It is also a necessary means to ensure regional sustainable development.

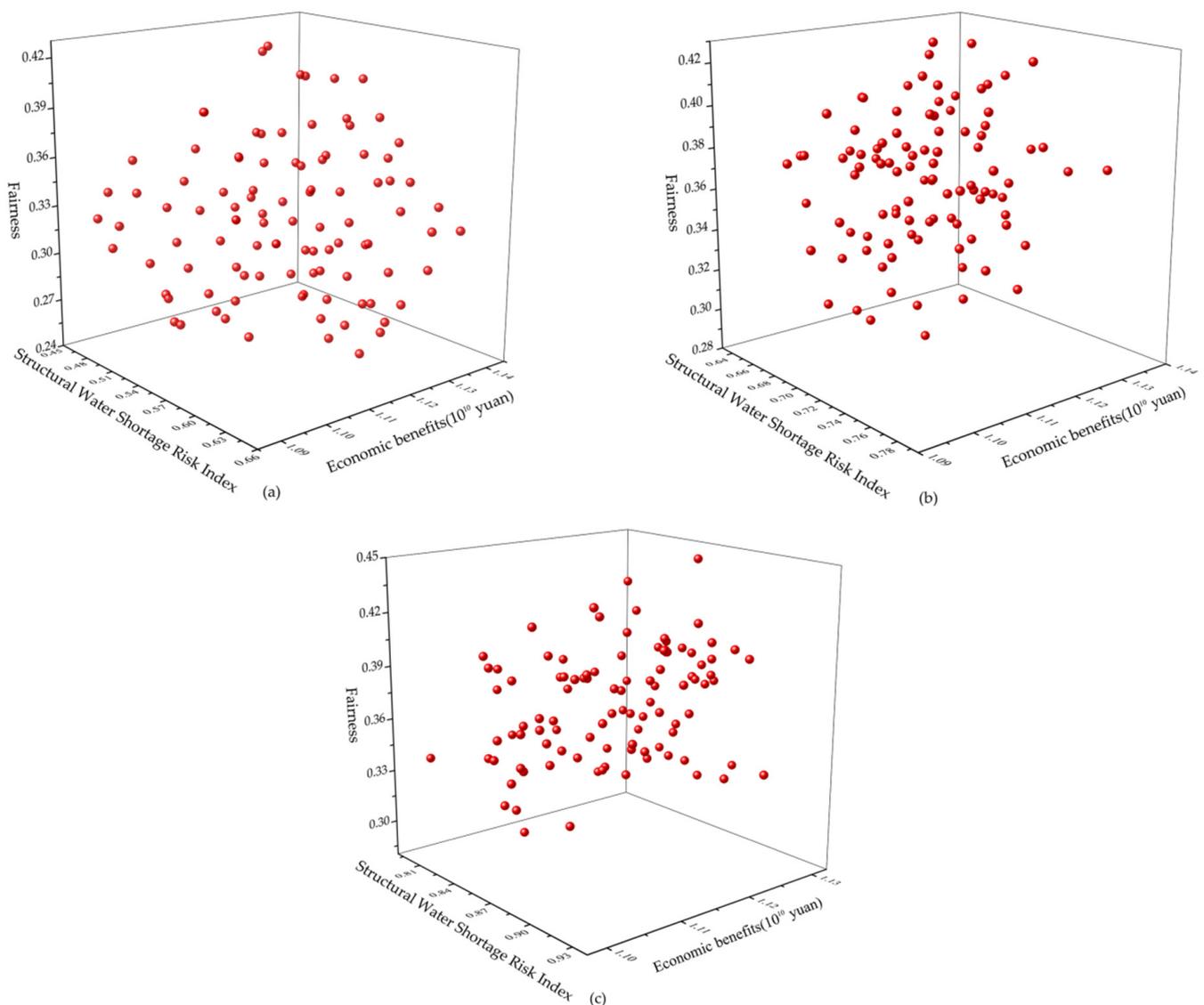


Figure 4. (a) Pareto solution set of normal years; (b) Pareto solution set of dry years; (c) Pareto solution set of extremely dry years.

Figure 5b shows the relationship between the structural water shortage risk index and economic benefits when fairness takes different values. When fairness is fixed, as the structural water scarcity risk index increases, economic benefits also increase. When the fairness value increases, the numerical value of the structural water shortage risk index and the economic benefit shift to the left, showing a trend, in which the structural water shortage risk index decreases and the economic benefit increases. Figure 5c shows the

relationship between the structural water shortage risk index and fairness when economic benefits take different values. When the structural water shortage risk index increases, fairness shows a decreasing trend. Figure 5d shows the relationship between fairness and economic benefits when the structural water shortage risk index takes different values. When the structural water shortage risk index is fixed, fairness increases with the increase in economic benefits. When the water scarcity risk index increases, the values of fairness and economic benefits move to the left, showing a trend of increasing fairness and decreasing economic benefits. Compared with Figure 5c,d, the relationship between target pairs in Figure 5b is more distinct, and the intensity of change in Figure 5b is greater than that in Figure 5c,d.

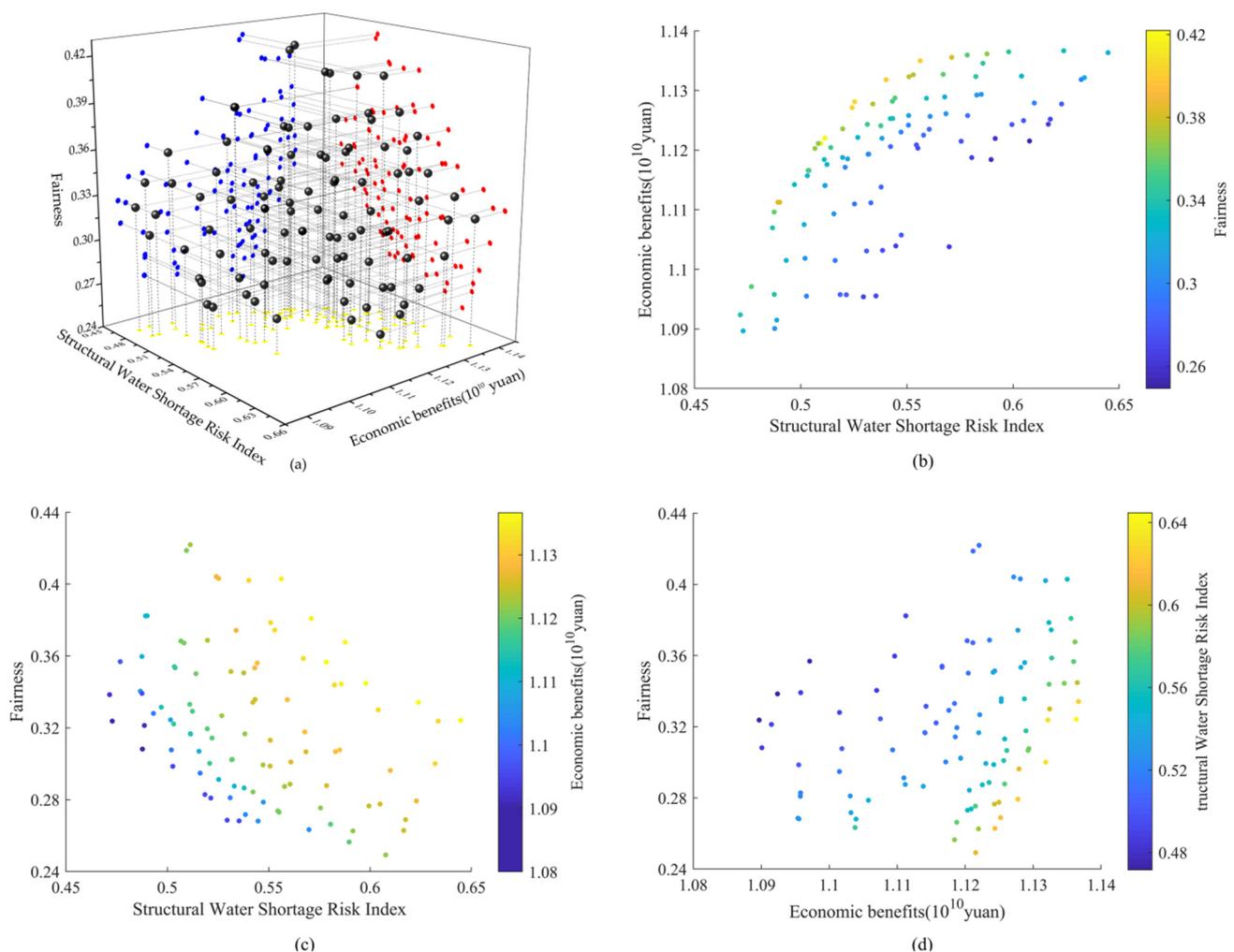


Figure 5. (a) Three-dimensional projection of two-dimensional relationships; (b) the relationship between the structural water shortage risk index and economic benefits; (c) the relationship between the structural water shortage risk index and fairness; (d) the relationship between fairness and economic benefits.

It can be seen from Figure 5 that there is a competitive, restrictive relationship between the three goals. Among them, the structural water shortage risk index has a strong negative relationship with economic benefits. In contrast, the negative relationships between structural water shortage risk index and fairness and between fairness and economic benefits are weaker. This is because, under the condition of a certain amount of water supply, regional development depends on economic benefits. In order to improve economic benefits by adjusting the water use structure of various industries, water resources are allocated to industries with greater economic benefits, and the water available for other industries is

reduced. This leads to increased conflicts between economic benefits and water use structure, and it strengthens the negative relationship between the structural water shortage risk index and economic benefits.

4.3. Water Resource Allocation

In order to analyze the impact of a single goal on other goals when it reaches its optimum potential in normal years, dry years, and extremely dry years, nine groups of different objective function values, with the smallest structural water shortage risk index f_1 , the largest economic benefit f_2 , and the largest fairness f_3 , were selected for comparative analysis.

As can be seen from Table 4, when an objective function value is optimal, other objective function values change. Taking a normal year as an example, in the scheme with the smallest structural water shortage risk index, the structural water shortage risk index is 0.47, the economic benefit is 1.09, and the fairness is 0.34. In the scheme with the largest fairness value, the structural water shortage risk index is 0.61, the economic benefit is 1.12, and the fairness is 0.25. In the two different target schemes, the rate of change of each target value is as follows: the structural water shortage risk index increases by 30%, the economic benefit increases by 3%, and the fairness decreases by 24%. The results showed that when choosing two different goal schemes, one of the goals was the best, which had a greater impact on the other goal, which may have had the worst value as a result. In summary, analyzing the relationship between optimal values among different objective functions can reduce the subjective preferences of decision makers and have a positive impact on ensuring local sustainable development.

Table 4. The optimal value of each objective function in different typical years.

Category	Normal Years			Dry Years			Extremely Dry Years		
	f_1	f_2	f_3	f_1	f_2	f_3	f_1	f_2	f_3
Min f_1	0.47	1.09	0.34	0.65	1.10	0.35	0.80	1.10	0.34
Max f_2	0.62	1.14	0.33	0.77	1.13	0.37	0.89	1.13	0.33
Max f_3	0.61	1.12	0.25	0.71	1.11	0.29	0.84	1.11	0.30

f_1 is structural water shortage risk index; f_2 is economic benefit; f_3 is fairness; Min f_1 is the minimum value of structural water shortage risk index; Max f_2 is the maximum value of economic benefits; Max f_3 is the minimum value of fairness.

In order to select a water resource allocation plan suitable for local development, we adopted the ideal point method. According to the ideal point method of multi-objective planning, the non-inferior solution with the smallest Euclidean distance from the ideal point in the Pareto solution set is the best water resource allocation plan [29]. We consulted experts and local decision makers to consider the fairness of local water resource distribution and the greater risk of water shortage in the water structure, and we finally determined the ideal point coordinates for typical normal years (0.500, 1.100, 0.310), dry years (0.700, 1.100, 0.340), and extremely dry years (0.850, 1.100, 0.370).

The ideal point method was used to calculate the distance between the non-inferior solution and the ideal point in the Pareto solution set [30]. The shortest distance in a normal year is 0.003, and the coordinates are (0.502, 1.102, 0.308); the shortest distance in a dry year is 0.010, and the coordinates are (0.691, 1.095, 0.341); and the shortest distance in an extremely dry year is 0.009, and the coordinates are (0.855, 1.107, 0.368). In 2018, the structural water shortage risk index of Wusu City was 1.06, the economic benefit was 1.100×10^{10} yuan, and the fairness was 0.780. After optimization, the structural water shortage risk index in normal year was reduced by 0.540, economic benefits increased by 0.002×10^{10} yuan, and fairness was reduced by 0.472. In dry years, the structural water shortage risk index was reduced by 0.369, economic benefits were reduced by 0.005×10^{10} yuan, and fairness was reduced by 0.439. In order to reduce the structural water shortage risk index and fairness, part of the economic benefits was abandoned. In

extremely dry years, the structural water shortage risk index decreased by 0.205, economic benefits increased by 0.007×10^{10} yuan, and fairness decreased by 0.412. The structural water shortage risk index and fairness had the smallest decline, but the economic benefits improved.

In Figure 6a, the optimal solution is located to the left of the ideal point. Comparing the ideal point with the optimal solution, the structural water shortage risk index increases by 0.002, the economic benefit increases by 0.002×10^{10} yuan, and the fairness decreases by 0.002. In Figure 6b, the coordinate difference between the optimal solution and the ideal point is $(-0.009, -0.005, 0.001)$. In Figure 6c, the optimal solution is located to the right of the ideal point. Comparing the ideal point with the optimal solution, the structural water shortage risk index increases by 0.005, the economic benefit increases by 0.007×10^{10} yuan, and the fairness decreases by 0.002. In the opinions of decision makers, the optimal solutions in different typical years met the needs of development and satisfy the requirements of local sustainable development. The water resource allocation plans for normal years, dry years, and extremely dry years are shown in Table 5.

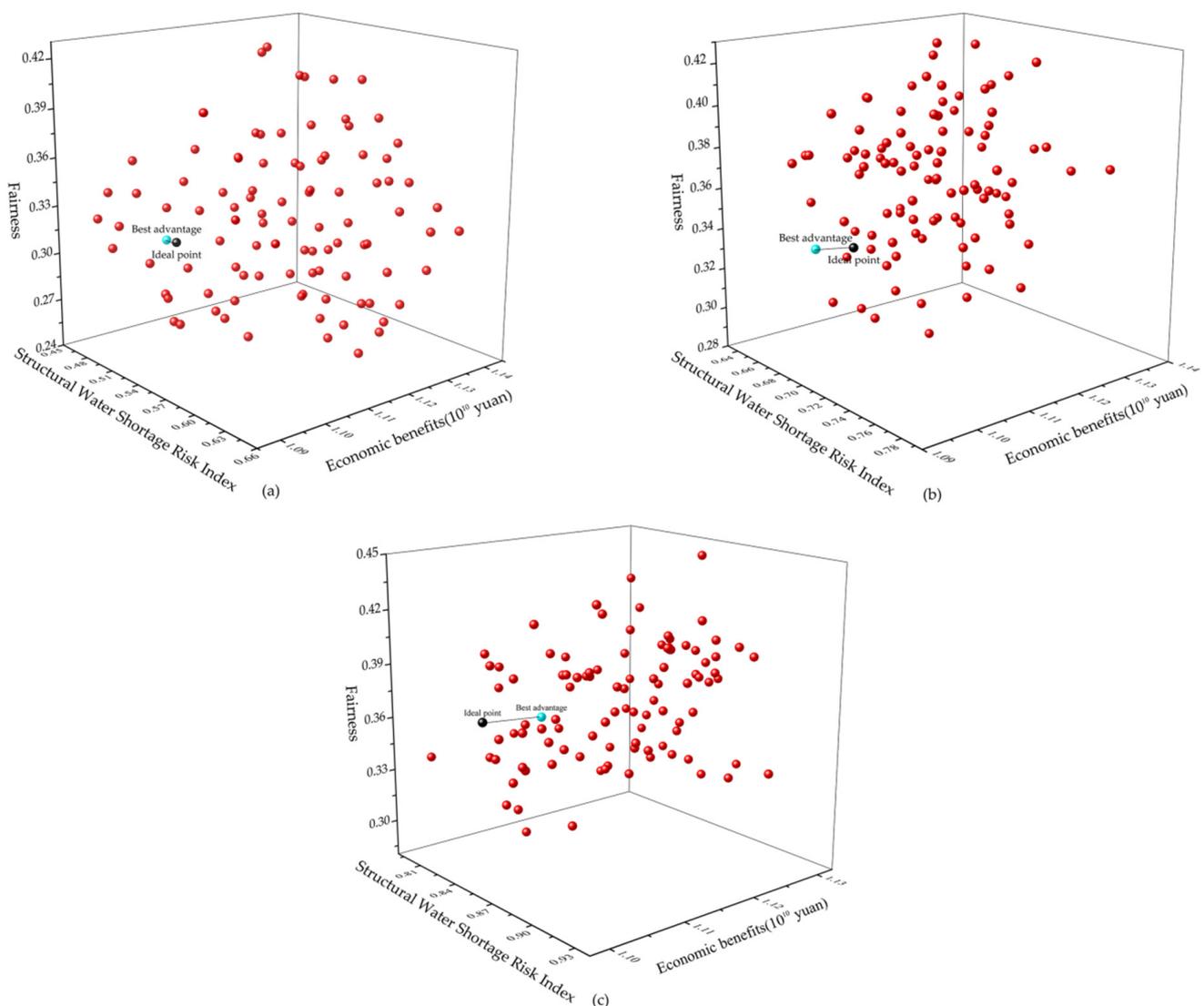


Figure 6. (a) The spatial relationship between ideal points and alternatives in normal years; (b) the spatial relationship between ideal points and alternatives in dry years; (c) the spatial relationship between ideal points and alternatives in extremely dry years.

Table 5. Allocation of water resources for normal years, dry years, and extremely dry years.

Category		Agricultural (10^4 m^3)	Industrial (10^4 m^3)	Domestic (10^4 m^3)
Kuitunhe Area	Normal years	17,017	1376	675
	Dry years	16,680	1387	677
	Extremely dry years	16,349	1392	680
Sikeshu Area	Normal years	21,099	343	284
	Dry years	20,751	344	280
	Extremely dry years	20,076	359	284
Chepaizi Area	Normal years	6090	-	64
	Dry years	6024	-	63
	Extremely dry years	5716	-	63
Jiertuhe Area	Normal years	4465	-	46
	Dry years	4347	-	48
	Extremely dry years	4174	-	48

It can be seen in Table 5 that in different typical years, the water consumption of the domestic sector in each area did not change much, the water consumption of the industrial sector had an upward trend, and the water consumption of the agricultural sector had a downward trend. This is mainly due to the shortage of water resources; in order to ensure local economic benefits and sufficient water for people's domestic use, the water consumption of the industrial sector should be increased, and the water consumption of the agricultural sector should be reduced. Taking the Kuitun River area as an example, the water consumption of the agricultural sector decreased by 3.93% in extremely dry years and by 2.02% in dry years compared with that in normal years, and it decreased by 1.98% in extremely dry years compared with that in dry years. Compared with normal years, the annual water consumption of the industrial sector increased by $11 \times 10^4 \text{ m}^3$ in dry years and by $16 \times 10^4 \text{ m}^3$ in extremely dry years. Water consumption per 10,000 yuan of output value in the industrial sector is lower than that of the agricultural sector. Therefore, the economic benefits of the industrial sector's unit water consumption are greater than those of the agricultural sector, and coordinating the water consumption between the industrial and agricultural sectors can maximize economic benefits. In summary, with reference to the opinions of decision makers, combined with actual local conditions, the optimal water distribution plan selected by the ideal point method can provide a fair and effective reference plan for local water resource allocation.

5. Conclusions

In order to avoid only focusing on economic benefits and ignoring the fairness of water distribution and the water shortage risk caused by an unbalanced water structure in water resource allocation, a new multi-objective optimization model for water resources was established. This optimization model is suitable for water-scarce areas where there are conflicts of water use, but not for areas where supply is based on demand. There is good performance in similar research areas. The main conclusions are as follows.

1. The new multi-objective optimization model, which combines the fairness of water allocation with structural water shortage risks, provides reasonable and feasible solutions for solving water conflicts caused by unfair water distribution and water shortage risks.
2. Analyzing the relationship between the objective functions reveals that there is a competitive, restrictive relationship between the three objective functions, among which the structural water shortage risk index and economic benefits have the strongest negative relationship.
3. The convergence and stability of ARNSGA-III are better than those of NSGA-III, MOSPO, and MOEA/D, which proves that ARNSGA-III has strong practicability for water resources allocation.

4. The new multi-objective optimization model has been applied to the allocation of water resources in Wusu City of China. The optimal allocation schemes of water resources in normal years, dry years, and extremely dry years are proposed, respectively. Taking the normal years as an example, the structural water shortage risk index is reduced by 0.540, economic benefits by 0.002×10^{10} yuan, and fairness is reduced by 0.472. The results show that the model is applicable in the field of water resources allocation.

Author Contributions: Conceptualization, X.T. and Y.H.; methodology, X.T., P.Q. and Y.H.; code, Z.D.; data curation, Y.H.; writing—original draft preparation, X.T.; writing—review and editing, P.Q., Y.H. and M.J.; graphics, X.T. and Z.C. All authors have read and agreed to the published version of the manuscript.

Funding: This research was supported by National Key R&D Program of China (grant number 2019YFC0409104), the National Natural Science Foundation of China (grant number 42001032), and the Xinjiang Uygur Autonomous Region University Scientific Research Project (grant number XJEDU2021Y023).

Institutional Review Board Statement: Not applicable.

Informed Consent Statement: Not applicable.

Data Availability Statement: All data generated or analyzed during this study are included in this published article.

Acknowledgments: We thank the anonymous reviewers for their constructive reviews and the editor and associate editor for their remarks.

Conflicts of Interest: The authors declare no conflict of interest.

References

1. Hu, Z.; Wei, C.; Yao, L.; Li, L.; Li, C. A multi-objective optimization model with conditional value-at-risk constraints for water allocation equality. *J. Hydrol.* **2016**, *542*, 330–342. [[CrossRef](#)]
2. Eliasson, J. The rising pressure of global water shortages. *Nat. News* **2015**, *517*, 6. [[CrossRef](#)]
3. Habibi Davijani, M.; Banihabib, M.E.; Nadjafzadeh Anvar, A.; Hashemi, S.R. Multi-Objective Optimization Model for the Allocation of Water Resources in Arid Regions Based on the Maximization of Socioeconomic Efficiency. *Water Resour. Manag.* **2016**, *30*, 927–946. [[CrossRef](#)]
4. Cosgrove, W.J.; Loucks, D.P. Water management: Current and future challenges and research directions. *Water Resour. Res.* **2015**, *51*, 4823–4839. [[CrossRef](#)]
5. Tian, J.; Guo, S.; Liu, D.; Pan, Z.; Hong, X. A Fair Approach for Multi-Objective Water Resources Allocation. *Water Resour. Manag.* **2019**, *33*, 3633–3653. [[CrossRef](#)]
6. Cullis, J.; Koppen, B.V. *Applying the Gini Coefficient to Measure Inequality of Water Use in the Olifants River Water Management Area, South Africa*; IWMI Research Report 113; IWNI: Colombo, Sri Lanka, 2009; p. 19.
7. Yang, G.; Guo, P.; Huo, L.; Ren, C. Optimization of the irrigation water resources for Shijin irrigation district in north China. *Agric. Water Manag.* **2015**, *158*, 82–98. [[CrossRef](#)]
8. Hu, Z.; Chen, Y.; Yao, L.; Wei, C.; Li, C. Optimal allocation of regional water resources: From a perspective of equity-efficiency tradeoff. *Resour. Conserv. Recycl.* **2016**, *109*, 102–113. [[CrossRef](#)]
9. Zhang, C.; Li, M.; Guo, P. An interval multistage joint-probabilistic chance-constrained programming model with left-hand-side randomness for crop area planning under uncertainty. *J. Clean. Prod.* **2017**, *167*, 1276–1289. [[CrossRef](#)]
10. Ma, L. Data-Driven Model of the Water Use and Water Uniform-Scarcity Risk Analysis in Shiyang River. Ph.D. Thesis, Northwest Agriculture and Forestry University of Science and Technology, Xi'an, China, 2012.
11. Wang, Y.; Guo, P. Irrigation water resources optimization with consideration of the regional agro-hydrological process of crop growth and multiple uncertainties. *Agric. Water Manag.* **2021**, *245*, 106630. [[CrossRef](#)]
12. Gao, X.; Liu, Y.; Sun, B. Water shortage risk assessment considering large-scale regional transfers: A copula-based uncertainty case study in Lunan, China. *Env. Sci. Pollut. Res. Int.* **2018**, *25*, 23328–23341. [[CrossRef](#)]
13. Fu, J.; Zhong, P.-A.; Xu, B.; Zhu, F.; Chen, J.; Li, J. Comparison of Transboundary Water Resources Allocation Models Based on Game Theory and Multi-Objective Optimization. *Water* **2021**, *13*, 1421. [[CrossRef](#)]
14. Louati, M.H.; Benabdallah, S.; Lebdi, F.; Milutin, D. Application of a Genetic Algorithm for the Optimization of a Complex Reservoir System in Tunisia. *Water Resour. Manag.* **2011**, *25*, 2387–2404. [[CrossRef](#)]
15. Shourian, M.; Mousavi, S.J. Performance Assessment of a Coupled Particle Swarm Optimization and Network Flow Programming Model for Optimum Water Allocation. *Water Resour. Manag.* **2017**, *31*, 4835–4853. [[CrossRef](#)]

16. Gao, Y.; Zhang, X.; Zhang, X.; Li, D.; Yang, M.; Tian, J. Application of NSGA-II and Improved Risk Decision Method for Integrated Water Resources Management of Malian River Basin. *Water* **2019**, *11*, 1650. [[CrossRef](#)]
17. Liu, S.; Wang, N.; Xie, J.; Jiang, R.; Zhao, M. Optimal Scale of Urbanization with Scarce Water Resources: A Case Study in an Arid and Semi-Arid Area of China. *Water* **2018**, *10*, 1602. [[CrossRef](#)]
18. Deb, K.; Jain, H. An evolutionary many-objective optimization algorithm using reference-point-based nondominated sorting approach, part I: Solving problems with box constraints. *IEEE Trans. Evol. Comput.* **2013**, *18*, 577–601. [[CrossRef](#)]
19. Dai, C.; Qin, X.S.; Chen, Y.; Guo, H.C. Dealing with equality and benefit for water allocation in a lake watershed: A Gini-coefficient based stochastic optimization approach. *J. Hydrol.* **2018**, *561*, 322–334. [[CrossRef](#)]
20. Wang, Y.; Liu, L.; Guo, S.; Yue, Q.; Guo, P. A bi-level multi-objective linear fractional programming for water consumption structure optimization based on water shortage risk. *J. Clean. Prod.* **2019**, *237*, 117829. [[CrossRef](#)]
21. Chen, H.; Chen, Z.; Chen, Z.; Xu, Y. A self-adaptive multi-objective particles warm optimization algorithm based on swarm distribution characteristic. *Control Decis.* **2017**, *32*, 1386–1394. [[CrossRef](#)]
22. Coello, C.C.; Lechuga, M.S. MOPSO: A proposal for multiple objective particle swarm optimization. In Proceedings of the Proceedings of the 2002 Congress on Evolutionary Computation, Honolulu, HI, USA, 12–17 May 2002; pp. 1051–1056. [[CrossRef](#)]
23. Qingfu, Z.; Hui, L. MOEA/D: A Multiobjective Evolutionary Algorithm Based on Decomposition. *IEEE Trans. Evol. Comput.* **2007**, *11*, 712–731. [[CrossRef](#)]
24. Tian, Y.; Cheng, R.; Zhang, X.; Jin, Y. PlatEMO: A MATLAB platform for evolutionary multi-objective optimization [educational forum]. *IEEE Comput. Intell. Mag.* **2017**, *12*, 73–87. [[CrossRef](#)]
25. Tian, Y.; Cheng, R.; Zhang, X.; Li, M.; Jin, Y. Diversity Assessment of Multi-Objective Evolutionary Algorithms: Performance Metric and Benchmark Problems [Research Frontier]. *IEEE Comput. Intell. Mag.* **2019**, *14*, 61–74. [[CrossRef](#)]
26. He, Y.; Tang, X.; Peng, L.; Ju, J. Optimized selection of the solution for multi-objective optimal allocation of water resources in Fengshou Irrigation Areas of South Xinjiang. *Trans. CSAE* **2021**, *37*, 117–126. [[CrossRef](#)]
27. Guo, Y.; Tian, X.; Fang, G.; Xu, Y.-P. Many-objective optimization with improved shuffled frog leaping algorithm for inter-basin water transfers. *Adv. Water Resour.* **2020**, *138*, 103531. [[CrossRef](#)]
28. Xu, J.; Lv, C.; Yao, L.; Hou, S. Intergenerational equity based optimal water allocation for sustainable development: A case study on the upper reaches of Minjiang River, China. *J. Hydrol.* **2019**, *568*, 835–848. [[CrossRef](#)]
29. Wang, Y.; Li, Z.; Liu, L.; Guo, P. A fuzzy dependent-chance interval multi-objective stochastic expected value programming approach for irrigation water resources management under uncertainty. *Desalin. Water Treat.* **2020**, *212*, 17–30. [[CrossRef](#)]
30. Yu, Z.; Shang, S. Multi-objective optimization method for irrigation scheduling of crop rotation system and its application in North China. *J. Hydraul. Eng.* **2016**, *47*, 1188–1196.