# How to Select the Number of Active Pumps during the Operation of a Pumping Station: The Convex Hyperbola Charts 

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#### Abstract

This research aims to identify the number of pumps that should be working at any moment during the operation of a pumping station in order to provide the desired volume of water whilst consuming the least amount of energy. This is typically done by complex iterative algorithms that require much computational effort. The pumping station should pump the desired volume of water $V^{*}$ using the least specific energy $e^{*}$ (energy per volume). In the methodology of this article, the shape of the curves $\mathrm{e}^{*}-\mathrm{V}^{*}$ was analyzed. The result is that such curves present a convex hyperbola shape. This is a straightforward analytic solution that does not require any iterations. The representation of the Convex Hyperbolas Charts will indicate the best pump combination during the operation of a pumping station. Therefore, this is a straightforward resource for practitioners: the curves immediately tell engineers the number of pumps that should be turned on, depending on the desired volume of water to pump. The elaboration of such charts only requires the use of any calculation sheet, only once, and it is a permanent resource that can be used at any time during the operation. In addition, the Convex Hyperbolas Charts are completely compatible and complementary with any other operation control technique.


Keywords: water supply systems; operation; pumping station; energy

## 1. Introduction

Once a water supply system has been designed, the operation of the utility becomes the protagonist. When the water supply system requires pumping, it is essential to properly manage the number of pumps that are working at a time, in order to minimize the energy expenditure whilst ensuring the water supply. The optimization of the operation should be able to be quickly and easily performed for any pumping situation and at any time of the functioning of the station. This research study will focus on finding the best pumping configuration (e.g., the number of pumps that should be working) to optimize the operation and minimize energy costs.

In the matter of optimizing the operation of a pumping station, many various factors contribute to the performance of the facility, and different approaches can be considered to make the best of the installation: starting at the design stage, the operation costs should be considered from the beginning of the project. This will ensure a better selection of the pumps. Pumping schedules highly rely on the water regulation capacity, and for this matter, the aid of tanks comes in handy. Another popular approach is to use a speed controller; however, this is not always available at all stations. Finally, the latest advances in optimization of pump operation rely on Monte Carlo procedures that explore all the solution space by calculating the objective function for each possible combination of variables. All of these different approaches are analyzed hereafter.

## Optimization at the Design Stage.

Helena Mala-Jetmarova (2018) [1] provides a remarkable summary of the state of the art when it comes to the design of water supply systems. In her work, a brief analysis
of the pumping station schedule is covered. Lansey and Awumah (1994) [2] present a methodology to determine the pumping schedule. They use a dynamic programming optimization algorithm. One of the most interesting aspects of this work is that they limit the number of times pumps can be switched on and off. The reason for this is to minimize the weathering and aim for better maintenance of the station. Even though the station may better adjust the energy consumption by switching pumps on and off, in the long run, it can be detrimental to the well-being of the pumps. Kang and Lansey (2012) [3] also propose a design methodology for water supply systems. In their work, they consider pipe and pump station construction costs but also operational costs. These operational costs are evaluated using the average demand for the entire planning period assuming a constant energy tariff, which is a fair approach.

Stokes et al. (2015) [4] provide a fantastic integration of all optimization strategies into a single software that can be edited and upgraded by the community (the "Water distribution cost-emissions nexus (WCEN)" computational software). In this software, operational management options are represented as integer-coded decision variables. Pumping operational management options considered include discrete scheduling of pumps. Pump schedule options represent the time at which a pump is turned on and off, using a time-step of 30 min . In these options, the number of pumps' on/off switches made each day is limited by the users' criteria so that the weathering of the pump is accounted for, as Lansey and Awumah (1994) [2] propose. The methodology proposed in this paper can be integrated into this software.

## Storage Tanks for Better Regulation.

Stokes et al. (2015) [5] remind designers of the possibility of optimizing the pumping operation by the construction of storage tanks. The use of storage tanks allows the pumps to operate at those times at which the tariffs are the lowest. The larger the tank, the easier the operation; however, large tanks can have prohibitive costs. In their work, they carry out a sensitivity analysis to evaluate the impact of the size of the tank, finding the right balance between operation and construction costs. The use of the reservoir tank is desirable and perfectly compatible with the methodology proposed in this work.

Vamvakeridou-Lyroudia et al. (2005) [6] propose the use of genetic algorithms for a simulation of tanks as network storage, taking into account the tank shape. In their work, they impose the condition of a full tank at the end of the night, and for that, with fuzzy aggregation operators, they find the number of pumps to use. This procedure is part of a design methodology, and it could be very helpful to obtain an idea of how the station would look. However, it requires a large computational effort that is not suitable for day-to-day operation. For that, this research proposes a complementary methodology that would be more indicated for daily use. Genetic algorithms are also used by Jin et al. (2008) [7], in this case in a rehabilitation model that minimizes the energy cost per year. They consider the number of pumps that need to be rehabilitated, and they find the optimal design solution to meet the pressure in all nodes; however, they do not find the optimal operation regime. Walters et al. (1999) [8] use a different variant of genetic algorithm, which is the "messy genetic algorithm", and they establish four pumping periods during the day. Whilst this helps simplify calculations at a design stage, it can be very limited at the actual operation stage.

## Variable Speed Controllers.

Wu et al. (2011) [9] confirm with a sensitivity analysis an intuitive and important conclusion: the electricity tariffs have a tremendous impact on the design of a water supply system. However, this analysis is carried out with a non-variable pumping situation. Wu et al. (2011) [10] expose the benefits of using variable speed pumps versus fixed speed pumps, since better adequacy to the operating point can be achieved. Nevertheless, this requires, indeed, having a variable speed controller. The proposed methodology is compatible with those stations that have both variable and fixed speed pumps. Shokoohi et al. (2017) [11] affirm that water age is influenced by pumping schedule, since booster pumps are used in order to inject chlorine into the system. Hence, this is another reason to
properly address the pumping operation. Kurek et al. (2013) [12] focus on regulating the pumping flow in combination with the tank storage capacity. However, all pumps function at the same time (using variable speed controllers), and no different pump combinations are evaluated.

## Solutions Inspired by Monte Carlo Methods.

In Babaei et al. (2015) [13], the ant colony algorithm is used to minimize the chlorine dosage as well as the energy consumed during the working of a pump station. It is a two-objective optimization, and the authors choose EPANET as the calculating software. Both the dosage and the pump configuration are calculated for each hour, using an iterative process. Variable speed pumps are considered. Continuing with the variable speed pumps (VSP), Wu et al. (2012) [10] decided to compare the greenhouse emissions (GHG) and energy expenditure using VSP and fixed speed pumps (FSP), using as an optimization tool what they define as the power estimation method. They carry out a fantastic analysis, showing important savings using VSP. Nevertheless, in that study, the number of pumps that should be working at each moment is not considered, since they use a fixed configuration. EPANET is also used in Ostfeld (2005) [14]. In their work, they incorporate a genetic algorithm into the software to optimize minimization of the total cost of designing and operating the system for a selected operational time horizon. The decision variables are the pipe diameters, tank maximum storage, maximum pumping unit power, and maximum removal ratios at the treatment facilities. However, as in the previous case, they do not consider different pump configurations. Oshurbekov et al. (2020) [15] prove that almost $30 \%$ of energy savings can be achieved by the use of variable speed drive. Nevertheless, as Goman et al. (2019) [16] point out, most pumping units are still operating without speed control, although in some countries 20-30\% of them already count on variable speed control, according to Kazakbaev et al. (2019) [17].

The operation needs to ensure the correct water supply. This can be controlled by a series of quality parameters. Vicente et al. (2011) [18] summarize these quality parameters of the operation. Vicente et al. (2015) [19] focus on how to control the pressure provided during the operation, which needs to be sufficient at every node. Pérez-Sánchez et al. (2018) [20] change the perspective into valve control to analyze the correct operation of the system.

Luna et al. (2019) [21] present a very complete optimization of the operation of a pumping system. In their work, they use a hybrid optimization method, which is a genetic algorithm with selective mutation mechanisms to boost convergence. The importance of tanks and energy price is once again highlighted. However, most importantly, they prove that the optimization of the pumping schedule can save $15 \%$ of energy costs on average. They show the convergence curve of their optimization, which is a typical metaheuristic convergence curve. In this curve, the minimum number of iterations required to start finding a sign of cost reduction is 15 , and up to 140 to get the best results (as expected, the larger the number of iterations, the better the optimization). Considering the magnitude of the task and the great number of variables counted in the method, these are good results. Torregosa and Capitanescu (2019) [22] compare the efficiency of different algorithm techniques (genetic algorithm, simulated annealing, and particle swarm optimization) obtaining similar values to these. The best performance was reached by the genetic algorithm.

Having to carry out the correspondent iterations is typically beyond the capability of real-time optimization frameworks, as Salomons and Housh (2020) [23] state. However, by reducing the number of decision variables, the process can be shortened, whilst still achieving cost reductions. In their work, they balance accuracy and practicality to make nonlinear programming more suitable for real-time situations. Nevertheless, it would always be better if no iterations had to be made because an analytic solution was possible.

## This Research's Solution: An Analytic Proposal.

The approach of this research is analytic, whilst most of the other popular methodologies are inspired by Monte Carlo Simulations. These methods explore the whole solution
space, trying the optimization function for each combination of variables, and thus, this requires numerous iterations [22]. The advantage of the analytic approach is that it directly provides the solution, and hence, the optimization is immediate, and the methodology does not rely on computational power.

Therefore, in this research study, we propose an analytic methodology to elaborate a series of guidance charts for the operation of a pumping station. These charts will indicate for each pumping situation the number of parallel pumps that should be turned on or off. These charts are compatible with any other control mechanism. They are elaborated to find out the best pump configuration in order to minimize the energy expenditure. Our paper is organized as follows: The methodology will explain the mathematical reasoning behind these charts, and in the results section, the Convex Hyperbola Charts will be fully presented and explained. In the discussion, the main conclusions will be exposed.

## 2. Methodology

## The theory: the starting point.

When pumps are displayed in a parallel configuration, these consume different energy depending on the number of groups working at the time. The aim is to operate at the minimum cost, and therefore, pump the required volume of water using the minimum energy E. The objective of this study is to find the number of groups that should be working to pump a required volume of water using the least amount of energy when pumps are displayed in a parallel configuration. The initial hypothesis is that all pumps are identical and that they work parallel to each other.

The specific energy $e$ is defined as the amount of energy required to pump each cubic meter of water $\mathrm{m}^{3}$, and thus, the objective can be seen from the perspective of operating at the minimum specific energy. Equation (1) shows the definition of e:

$$
\begin{equation*}
\mathrm{e}=\frac{\mathrm{E}}{\mathrm{~V}} \tag{1}
\end{equation*}
$$

### 2.1. The Pumping Variables

When the pump station presents a parallel configuration, the operating points and figures vary depending on the number of pumps that are activated. Figure 1 schematically represents the different operating points of a pump when it is working on its own, along with a second parallel pump, or with also a third parallel pump.


Figure 1. Different operating points of a pump depending on the number of parallel pumps active at the time. $H$ is the pumping head. The pump's flow rate is $q_{B}$, and when there are one, two, and three groups working, it is specified as $q_{B 1 g}, q_{B 2 g}$, and $q_{B 3 g}$, respectively. Finally, $\mu_{B}$ is the pump efficiency, and it is once again specified for the number of active groups.

The power consumed by the pumping station N is defined by Equation (2):

$$
\begin{equation*}
\text { Power: } \mathrm{N}=9.81 \cdot \mathrm{Q} \cdot \mathrm{~h}_{\mathrm{B}} \cdot \frac{1}{\mu_{\mathrm{B}}} \cdot \frac{1}{\mu_{\mathrm{M}}} \tag{2}
\end{equation*}
$$

where Q is the total pumping flow rate pumped by the station. Since this study focuses on a parallel configuration, Q is the sum of the flow rates pumped by each pump (assuming all pumps are equal), and thus $Q=q_{B} \cdot n_{g}$. On the other hand, $h_{B}$ is the pumping head, and it is coincident with the pumping head of the station H , since it is the case of a parallel configuration. $\mu_{\mathrm{B}}$ and $\mu_{\mathrm{M}}$ are the pump and engine efficiency, respectively. $\mu_{\mathrm{B}}$ can be easily obtained by the empiric curve proposed by Martin-Candilejo in [24,25]. With all of the above in mind, the unit power consumed by the pumping station $\frac{\mathrm{N}}{\mathrm{Q}}$, understood as the amount of kW needed for each $\mathrm{m}^{3} / \mathrm{s}$, is calculated with Equation (3):

$$
\begin{equation*}
\text { Unit power: } \mathrm{N} / \mathrm{Q}=9.81 \cdot \mathrm{~h}_{\mathrm{B}} \cdot \frac{1}{\mu_{\mathrm{B}}} \cdot \frac{1}{\mu_{\mathrm{M}}} . \tag{3}
\end{equation*}
$$

On the other hand, the energy E used by the station can be obtained using Equation (5). When the flow rate is expressed in $\mathrm{m}^{3} / \mathrm{s}$, time in hours, the equation of the energy in KWh can be simplified as follows:

$$
\begin{gather*}
\mathrm{E}=\mathrm{N} \cdot \mathrm{t}=\left[9.81 \cdot \mathrm{Q} \cdot \mathrm{~h}_{\mathrm{B}} \cdot \frac{1}{\mu_{\mathrm{B}}} \cdot \frac{1}{\mu_{\mathrm{M}}}\right] \cdot \mathrm{t},  \tag{4}\\
\mathrm{Q}\left[\mathrm{~m}^{3} / \mathrm{s}\right], \mathrm{t}[\mathrm{~h}] \rightarrow \mathrm{E}[\mathrm{KWh}]=9.81 \cdot \frac{\mathrm{~V}}{3600} \cdot \mathrm{~h}_{\mathrm{B}} \cdot \frac{1}{\mu_{\mathrm{B}}} \cdot \frac{1}{\mu_{\mathrm{M}}}, \tag{5}
\end{gather*}
$$

being V the total volume of water pumped in that time period t . Knowing this, the specific energy e used by the station can be calculated as Equation (6):

$$
\begin{equation*}
\mathrm{e}=\frac{\mathrm{E}}{\mathrm{~V}}=9.81 \cdot \frac{1}{3600} \cdot \mathrm{~h}_{\mathrm{B}} \cdot \frac{1}{\mu_{\mathrm{B}}} \cdot \frac{1}{\mu_{\mathrm{M}}} \tag{6}
\end{equation*}
$$

which, written in a different way, is the same as:

$$
\begin{equation*}
\text { Specific energy: } \mathrm{e}=\frac{\mathrm{E}}{\mathrm{~V}}=\frac{\mathrm{N} / \mathrm{Q}}{3600} \tag{7}
\end{equation*}
$$

Depending on the number of parallel groups $\mathrm{n}_{\mathrm{g}}$ that are working at the same time, these variables may change, but all of them are straightforward and can be calculated with the above equations. All values are known. For instance, when there are two groups of parallel pumps working at the same time $n_{g}=2$, the flow rate of each pump $q_{B 2 g}$ is obtained graphically from the operating point as shown in Figure 1, as well as the pump efficiency $\mu_{\mathrm{B} 2 \mathrm{~g}}$. Knowing this, the flow rate pumped by the whole station is calculated as $Q_{2 g}=q_{B} \cdot n_{g}=q_{B 2 g} \cdot 2$, the power $N$ and the unit power $N / Q$ consumed by the station would, respectively, be:

$$
\begin{align*}
& N_{2 g}=9.81 \cdot Q_{2 g} \cdot h_{B 2 g} \cdot \frac{1}{\mu_{\mathrm{B} 2 \mathrm{~g}}} \cdot \frac{1}{\mu_{\mathrm{M} 2 \mathrm{~g}}}  \tag{8}\\
& \mathrm{~N}_{2 \mathrm{~g}} / \mathrm{Q}_{2 \mathrm{~g}}=9.81 \cdot \mathrm{~h}_{\mathrm{B} 2 \mathrm{~g}} \cdot \frac{1}{\mu_{\mathrm{B} 2 \mathrm{~g}}} \cdot \frac{1}{\mu_{\mathrm{M} 2 \mathrm{~g}}} \tag{9}
\end{align*}
$$

In that same example, the total volume of water that the pumping station would supply $V_{2 g}$ if those two groups were pumping the whole time $t_{t}$ would be obtained as:

$$
\begin{equation*}
\mathrm{V}_{2 \mathrm{~g}}=\mathrm{Q}_{2 \mathrm{~g}} \cdot \mathrm{t}_{\mathrm{t}} \tag{10}
\end{equation*}
$$

$t_{t}$ would be the desired amount of time for which operators want to make the pumps work with a specific configuration. This could typically be $1 \mathrm{~h}, 24 \mathrm{~h}$, etc.

The specific energy consumed by the station $\mathrm{e}_{2 \mathrm{~g}}$ could be calculated as:

$$
\begin{equation*}
\text { Specific energy: } e_{2 g}=\frac{E_{2 g}}{V_{2 g}}=\frac{N_{2 g} / Q_{2 g}}{3600} \tag{11}
\end{equation*}
$$

For better visualization of the terminology that will be used in the upcoming mathematical deduction, variables are compiled in the following Table 1.

Table 1. Key variables of the pump station.

| Pumping Variables |  |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Group Number | Pump's Flow Rate | Pumping <br> Height | Pump Efficiency | Station Flow Rate | Station Power | Station Unit Power | Station Volume ${ }^{1}$ | Station Specific Energy |
| $\mathrm{n}_{\mathrm{g}}$ | $\mathrm{q}_{\mathrm{B}}$ | $\mathrm{h}_{\mathrm{B}}=\mathrm{H}$ | $\mu_{\text {B }}$ | $\mathrm{Q}=\mathrm{q}_{\mathrm{B}} \cdot \mathrm{n}_{\mathrm{g}}$ | N | N/Q | V | e |
|  | From pump's characteristic curve |  |  |  | Equation (2) | Equation (3) | $\mathrm{V}=\mathrm{Q} \cdot \mathrm{t}_{\mathrm{t}}$ | $\mathrm{e}=\frac{\mathrm{N} / \mathrm{Q}}{3600}$ |
| 1 g | $\mathrm{q}_{\mathrm{B} 1 \mathrm{~g}}$ | $\mathrm{h}_{\mathrm{B} 1 \mathrm{~g}}=\mathrm{H}_{1 \mathrm{~g}}$ | $\mu_{\mathrm{B} 1 \mathrm{~g}}$ | $\mathrm{Q}_{1 \mathrm{~g}}$ | $\mathrm{N}_{1 \mathrm{~g}}$ | $\mathrm{N}_{1 \mathrm{~g}} / \mathrm{Q}_{1 \mathrm{~g}}$ | $\mathrm{V}_{1 \mathrm{~g}}$ | $\mathrm{e}_{1 \mathrm{~g}}=\frac{\mathrm{N}_{1 \mathrm{~g}} / \mathrm{Q}_{1 \mathrm{~g}}}{3600}$ |
| $\mathrm{n}_{\mathrm{g}}$ | $\mathrm{q}_{\mathrm{B} \mathrm{ng}}$ | $\mathrm{h}_{\mathrm{Bng}}=\mathrm{H}_{\mathrm{ng}}$ | $\mu_{\mathrm{B} \text { ng }}$ | $\mathrm{Q}_{\mathrm{ng}}{ }^{\cdots}$ | $\mathrm{N}_{\mathrm{ng}}$ | $\mathrm{N}_{\mathrm{ng}} / \mathrm{Qng}^{\text {g }}$ | $\mathrm{V}_{\mathrm{ng}}$ | $\mathrm{e}_{\mathrm{ng}}=\frac{\mathrm{N}_{\mathrm{ng}} / \mathrm{Q}_{\mathrm{ng}}}{3600}$ |

${ }^{1}$ If the station was pumping the whole time $t_{t}$ using $n_{g}$.

### 2.2. The Problem to Be Solved—The General Case

As a general rule, the more pumps are working, the more energy is consumed. This occurs because the station is providing a greater total flow rate $Q$ that translates into bigger head losses $\Delta \mathrm{h}$, which are directly related to the energy use. If the specific energy is graphed against the volume of pumped water, as Figure 2 illustrates, $e_{j}$ should generally be drawn above $e_{i}$ ( $e_{i}$ being the specific energy spent when i number of groups are working $n g=i ; e_{j}$ is the specific energy spent with a greater number of groups working $n g=j>i$ (not necessarily the immediately superior number)). Therefore, picking an example, $\mathrm{e}_{2 \mathrm{~g}}$ is typically higher than $\mathrm{e}_{1 \mathrm{~g}}$. Additionally, from Figure 2, the energy E consumed by the system is directly obtained as the area delimited by the rectangle $V_{n g}$ and $e_{n g}$, when $n_{g}$ groups of pumps are working the full-time $t_{t}$.


Figure 2. General case. Specific energy and the volume of pumped water. The energy consumed by the system when 1 group of pumps $\mathrm{E}_{1 \mathrm{~g}}$ is working the whole time $\mathrm{t}_{\mathrm{t}}$ is obtained as the blue area.

It should be noted that the volume pumped in Figure 2 is a discrete variable: when $n_{g}$ groups of pumps are working the full-time $t_{t}$, the operating point is determined in the
characteristic pump and the volume $\mathrm{V}_{\mathrm{ng}}$ and specific energy $\mathrm{e}_{\mathrm{ng}}$ define a discrete series of points (the blue points in Figure 2), as Table 1 states. Therefore, the main question this research study wants to answer is what to do when the desired volume of water $\mathrm{V}^{*}$ is other than those predetermined $\mathrm{V}_{\mathrm{ng}}$. How should the desired water volume $\mathrm{V}^{*}$ before a certain time $t_{t}$ be pumped to consume the least amount of energy? How many groups of pumps should be activated? For the mathematical deduction, the following theoretical example represented in Figure 3 will be used: the desired volume of water $V^{*}$ is somewhere in between $V_{1 g}$ and $V_{2 g}$.


Figure 3. Problem to be solved: how many groups should be activated to pump $V^{*}$ before a certain time $t_{t}$. Illustration theoretical example with $V_{1 g}<V^{*}<V_{2 g}$.

### 2.2.1. Solution A: Use Only One Pump Configuration

Solution A consists of pumping with two groups the whole time, consuming a specific energy $e_{2 g}$ continuously until the desired volume $V^{*}$ is fully supplied. The total energy consumed with this solution $\mathrm{E}_{2 \mathrm{~g}}^{*}$ can be obtained, as Figure 4 illustrates, from the blue area delimited by $\mathrm{e}_{2 \mathrm{~g}}$ and $\mathrm{V}^{*}$.


Figure 4. Solution A. Pump all V* with two pumps.
It should be noted that pumping the desired volume $\mathrm{V}^{*}$ continuously with only one group of pumps could not be possible, because it would require a longer time than the available $t_{t}$ to fulfill the task. Using only one pump would be insufficient to provide the full $V^{*}$ on time. However, by using two pumps, $V^{*}$ would be provided before $t_{t}$ finishes (remember that $V_{2 g}$ would be the volume of water obtained after pumping with two groups during the whole $t_{t}$ ).

### 2.2.2. Solution B. The Right Approach. Combine Different Numbers of Pumps

Solution B consists of combining the number of groups working during the available time $t_{t}$. This means that the desired volume $V^{*}$ would be pumped using one group of pumps part of the time, and two groups of pumps the rest of the time. Solution B is represented in Figure 5.


Figure 5. Solution B. Combine the number of pumps.
One portion of the $V^{*}$ is pumped using one group of pumps. This portion is $V^{*} 1 g$, and it consumes $E^{*}{ }_{1 g}$ energy, represented by the area of the rectangle surrounded by the dark blue intermittent lines). The reaming portion of $\mathrm{V}^{*}$, which is $\mathrm{V}^{*}{ }_{2 g}$, is pumped using two groups of pumps, meaning an energy consumption of $\mathrm{E}^{*}{ }_{2 \mathrm{~g}}$, and on this occasion, it is graphed using the color pink. Altogether, the energy consumed to pump $\mathrm{V}^{*}$ would be the light blue area, resulting from the sum of the dark blue and pink rectangles. The time the system is working with one pump has been called $\mathrm{t}^{*}{ }_{1 \mathrm{~g}}$, and the time used by the station with two groups of pumps was called $\mathrm{t}^{*}{ }_{2 \mathrm{~g}}$. Together, $\mathrm{t}^{*}{ }_{1 \mathrm{~g}}$ and $\mathrm{t}^{*}{ }_{2 \mathrm{~g}}$ complete the full amount of available time $t_{t}$. Equations (12) and (13) summarize the statements that govern Solution B:

$$
\begin{align*}
\mathrm{t}_{\mathrm{t}} & =\mathrm{t}^{*}{ }_{1 \mathrm{~g}}+\mathrm{t}^{*}{ }_{2 \mathrm{~g}}  \tag{12}\\
\mathrm{~V}^{*} & =\mathrm{V}^{*}{ }_{1 \mathrm{~g}}+\mathrm{V}^{*}{ }_{2 \mathrm{~g}} \tag{13}
\end{align*}
$$

### 2.2.3. Solution Comparison

To the problem "How to pump the desired water volume $V^{*}$ before a certain time $t_{t}$, consuming the least amount of energy? How many groups of pumps should be activated?", two different solutions have been proposed: Solution A and Solution B. It is time now to analyze which solution uses the least energy. For that, Figure 6 compiles the characteristics of each solution to help visualize the comparison.

The total energy $E^{*}$ consumed by each solution is represented by the blue areas of the graphs. As can be seen, Solution B consumes less energy than Solution A, because it does not include the striped orange rectangle. Therefore, it can be concluded that less energy is spent when combining the number of active groups to pump the desired water volume $\mathrm{V}^{*}$.

| How to pump $\mathrm{V}^{*}$ ? Better solution: Combine number of groups. |  |  |
| :---: | :---: | :---: |
| $\mathrm{e}_{2 g}$ $\mathrm{e}_{19}$ | Solution A: <br> Pump with only one group | Solution B: <br> Combine the number of groups |
| Energy saved by Solution B, respect with Solution $A$ <br> Less energy is spent when combining the number of groups |  |  |

Figure 6. Solution comparison.

### 2.3. The Problem to Be Solved—The Anomaly Case

Both Solutions A and B are formulated regarding a General Case problem in which $e_{1 g}<e_{2 g}$. As a reminder, this is generally the case because when an additional group is activated, the total flow rate consumed by the station $Q$ increases, and so do the head losses. To compensate for this increase in head losses, the pumps have to pump at a higher head, consuming more energy to do so. Therefore, typically $\mathrm{e}_{1 \mathrm{~g}}<\mathrm{e}_{2 \mathrm{~g}}$. Nevertheless, it may be the case that $e_{1 g}>e_{2 g}$. This may occur if the pump efficiency $\mu_{B}$ is significantly better when an additional pump is working, that is to say, when $\mu_{\mathrm{B} 1 \mathrm{~g}} \lll \mu_{\mathrm{B} 2 \mathrm{~g}}$. When this happens, such improvement in $\mu_{\mathrm{B}}$ with an additional pump may compensate for the increase in energy use due to the greater head losses, and it can translate into $\mathrm{e}_{2 \mathrm{~g}}<\mathrm{e}_{1 \mathrm{~g}}$. This is feasible, although it is an anomaly case. Figure 7 represents this anomaly.


Figure 7. Anomaly case, when solutions A and B do not apply.

For these situations, it is always better to pump with an additional pump $n_{g}=2$, since it consumes less specific energy $e_{2 g}$ and covers up $V_{2 g}$ to a greater volume in the available time $t_{t}$. This can be seen in the two examples on the right of Figure 7. Even when the desired volume $V^{*}$ is equal to $V_{1 g}$, it would be better to activate $n_{g}=2$ instead of $n_{g}=1$.

## The theory: deducing the optimal group combination.

Even though it is proven that pumping part-time with $n_{g}=1$ and $n_{g}=2$ is a better solution than only pumping with $\mathrm{n}_{\mathrm{g}}=1$, it is possible that it could be better with $\mathrm{n}_{\mathrm{g}}=1$ and $n_{g}=3$. Therefore, the next question is to find out, for the general case, "What group combination is the best?" and also "How long should each group combination be working for?" The mathematical deduction will be covered in this section.

### 2.4. Deduction of $e^{*}$

When the desired volume $\mathrm{V}^{*}$ is pumped using a combination of groups consuming $\mathrm{E}^{*}$ energy, $\mathrm{e}^{*}$ will be called the average specific energy at which the station works. The objective is to find the expression of $\mathrm{e}^{*}$ to afterward determine the combination of groups that mean the least $\mathrm{e}^{*}$. The same theoretical example used in Solution B will be used to find the equation of $\mathrm{e}^{*}$, and it is represented in the following Figure 8:


Figure 8. Finding the average specific energy $\mathrm{e}^{*}$.
From Solution $B$, the following equations can be stated. Equation (12) combined with Equations (13) and (14) are the expressions that govern the solution. To summarize them all, they are rewritten here:

$$
\begin{gather*}
\text { Equation (12) } \rightarrow t_{t}=\mathrm{t}^{*}{ }_{1 g}+\mathrm{t}^{*}{ }_{2 g}, \\
\text { Equation (13) } \rightarrow \mathrm{V}^{*}=\mathrm{V}^{*}{ }_{1 g}+\mathrm{V}^{*}{ }_{2 g}, \\
\mathrm{E}^{*}=\mathrm{E}_{1 \mathrm{~g}}{ }_{1 g}+\mathrm{E}^{*}{ }_{2 g} \rightarrow \mathrm{~V}^{*} \cdot \mathrm{e}^{*}=\mathrm{V}^{*}{ }_{1 g} \cdot \mathrm{e}_{1 \mathrm{~g}}+\mathrm{V}^{*}{ }_{2 g} \cdot \mathrm{e}_{2 g} \rightarrow \mathrm{e}^{*}=\frac{\mathrm{V}^{*}{ }_{1 g} \cdot \mathrm{e}_{1 \mathrm{~g}}+\mathrm{V}^{*}{ }_{2 g} \cdot \mathrm{e}_{2 g}}{\mathrm{~V}^{*}} . \tag{14}
\end{gather*}
$$

It is important to remember which of these values are known and which are not yet. The known values are:

- The desired volume $\mathrm{V}^{*}$. It is the independent variable.
- The available time $t_{t}$.
- The specific energy the station would consume if one pump was working the full $t_{t}$, $\mathrm{e}_{1 \mathrm{~g}}$. See the last column from Table 1.
- The specific energy the station would consume if two pumps were working the full $t_{t}$, $\mathrm{e}_{2 \mathrm{~g}}$. See the last column from Table 1.
- All other values from Table 1, including $\mathrm{V}_{1 \mathrm{~g}}$ and $\mathrm{V}_{2 \mathrm{~g}}$.

The unknown values are:

- The time that the station would be working with only $\mathrm{n}_{\mathrm{g}}=1, \mathrm{t}^{*}{ }_{\mathrm{lg}}$.
- The time that the station would be working with only $n_{g}=2, t^{*}{ }_{2 g}$.
- The partition volume of $\mathrm{V}^{*}$ that would be supplied with $\mathrm{n}_{\mathrm{g}}=1, \mathrm{~V}^{*}{ }_{1 \mathrm{~g}}$.
- The partition volume of $\mathrm{V}^{*}$ that would be supplied with $\mathrm{n}_{\mathrm{g}}=2, \mathrm{~V}^{*}{ }_{2 g}$.
- The average specific energy $\mathrm{e}^{*}$. That is the key variable of this mathematical deduction. In Equation (14), $\mathrm{e}^{*}$ is expressed in terms of unknown terms, since:

$$
\text { Equation }(14) \rightarrow e^{*}=\frac{\mathrm{V}^{*} \mathrm{~g} \cdot \mathrm{e}_{1 \mathrm{~g}}+\mathrm{V}^{*} 2 \mathrm{~g} \cdot \mathrm{e}_{2 \mathrm{~g}}}{\mathrm{~V}^{*}}=\mathrm{f}\left(\mathrm{~V}^{*}{ }_{1 \mathrm{~g}}, \mathrm{~V}^{*}{ }_{2 \mathrm{~g}}, \mathrm{~V}^{*}\right)
$$

The idea is to obtain $e^{*}$ only as a function of $V^{*}, e^{*}=f\left(V^{*}\right)$. For that, $V^{*} 1 g$ and $V^{*} 2 g$ need to be removed from the equation.
2.4.1. Solving the Deduction of $\mathrm{e}^{*}$. Removing $\mathrm{V}^{*}{ }_{2 g}$

From Equation (13), $\mathrm{V}^{*} 2 \mathrm{~g}$ is cleared away, and then introduced in Equation (14), leading to:
$\mathrm{V}^{*}{ }_{2 g}=\mathrm{V}^{*}-\mathrm{V}^{*}{ }_{1 g} \rightarrow \mathrm{e}^{*}=\frac{\mathrm{V}^{*}{ }_{1 g} \cdot e_{1 g}+\left(\mathrm{V}^{*}-\mathrm{V}^{*}{ }_{1 g}\right) \cdot \mathrm{e}_{2 \mathrm{~g}}}{\mathrm{~V}^{*}} \rightarrow \mathrm{e}^{*}=\frac{\mathrm{V}^{*}{ }_{1 g} \cdot\left(\mathrm{e}_{1 \mathrm{~g}}-\mathrm{e}_{2 \mathrm{~g}}\right)+\mathrm{V}^{*} \cdot \mathrm{e}_{2 \mathrm{~g}}}{\mathrm{~V}^{*}} \rightarrow$

$$
\begin{equation*}
e^{*}=e_{2 g}-\frac{V^{*} 1 g}{V^{*}} \cdot\left(e_{2 g}-e_{1 g}\right)=f\left(V^{*} 1 g, V^{*}\right) \tag{15}
\end{equation*}
$$

2.4.2. Solving the Deduction of $\mathrm{e}^{*}$, Removing $\mathrm{V}^{*}{ }_{1 g}$

In order to remove $V^{*}{ }_{1 g}$ from Equation (15), the following relations will be refreshed from Table 1:

$$
\left\{\begin{array}{l}
\mathrm{Q}_{1 \mathrm{~g}}=\frac{\mathrm{V}_{1 g}}{\mathrm{t}_{\mathrm{t}}}  \tag{16}\\
\mathrm{Q}_{2 \mathrm{~g}}=\frac{\mathrm{V}_{2 g}}{\mathrm{t}_{\mathrm{t}}}
\end{array}\right.
$$

$$
\left\{\begin{array} { l } 
{ \mathrm { V } _ { 1 \mathrm { g } } { } ^ { * } = \mathrm { Q } _ { 1 \mathrm { g } } \cdot \mathrm { t } _ { 1 \mathrm { g } } { } ^ { * } }  \tag{17}\\
{ \mathrm { V } _ { 2 \mathrm { g } } ^ { * } = \mathrm { Q } _ { 2 \mathrm { g } } \cdot \mathrm { t } _ { 2 \mathrm { g } } { } ^ { * } }
\end{array} \rightarrow \left\{\begin{array} { l } 
{ \mathrm { t } _ { 1 \mathrm { g } } { } ^ { * } = \frac { \mathrm { V } _ { 1 \mathrm { g } } { } ^ { * } } { \mathrm { Q } _ { 2 \mathrm { g } } } } \\
{ \mathrm { t } _ { 2 \mathrm { g } } { } ^ { * } = \frac { \mathrm { V } _ { 2 \mathrm { g } } } { \mathrm { Q } _ { 2 g } } }
\end{array} \rightarrow \text { Introducing Equation (16) } \rightarrow \left\{\begin{array}{l}
\mathrm{t}_{1 \mathrm{~g}^{*}}=\frac{\mathrm{V}_{1 \mathrm{~g}^{*}}}{\mathrm{~V}_{1 \mathrm{~g}}} \cdot \mathrm{t}_{\mathrm{t}} \\
\mathrm{t}_{2 \mathrm{~g}}{ }^{*}=\frac{\mathrm{V}_{2}{ }^{*}}{\mathrm{~V}_{2 g}} \cdot \mathrm{t}_{\mathrm{t}}
\end{array} .\right.\right.\right.
$$

Equation (12) is rewritten with Equation (17) as:

$$
\begin{gather*}
t_{t}=t^{*}{ }_{1 g}+t^{*} 2 g=\frac{V_{1 g}^{*}}{V_{1 g}} \cdot t_{t}+\frac{V_{2 g}{ }^{*}}{V_{2 g}} \cdot t_{t} \rightarrow \\
\frac{V_{1 g}{ }^{*}}{V_{1 g}}+\frac{V_{2 g}^{*}}{V_{2 g}}-1=0 \rightarrow \\
V_{1 g^{*}} \cdot V_{2 g}+V_{2 g} \cdot V_{1 g}-V_{1 g} \cdot V_{2 g}=0 . \tag{18}
\end{gather*}
$$

In Equation (18), Equation (13) is introduced:

$$
\begin{align*}
\text { Equation (13) } \mathrm{V}^{*}{ }_{2 g} & =\mathrm{V}^{*} 1 \mathrm{~g}-\mathrm{V}^{*} \rightarrow \text { Equation (18) } \\
& \rightarrow \mathrm{V}_{1 \mathrm{~g}}{ }^{*} \cdot \mathrm{~V}_{2 \mathrm{~g}}+\left(\mathrm{V}^{*}{ }_{1 \mathrm{~g}}-\mathrm{V}^{*}\right) \cdot \mathrm{V}_{1 \mathrm{~g}}-\mathrm{V}_{1 \mathrm{~g}} \cdot \mathrm{~V}_{2 \mathrm{~g}}=0 \tag{19}
\end{align*}
$$

Finally, $\mathrm{V}^{*}{ }_{1 \mathrm{~g}}$ is cleared out from Equation (19):

$$
\begin{gather*}
V_{1 g}^{*} \cdot\left(V_{2 g}-V_{1 g}\right)+V^{*} \cdot V_{1 g}-V_{1 g} \cdot V_{2 g}=0 \rightarrow \\
V_{1 g^{*}}=\frac{V_{1 g} \cdot V_{2 g}-V^{*} \cdot V_{1 g}}{\left(V_{2 g}-V_{1 g}\right)} \tag{20}
\end{gather*}
$$

Therefore, $V^{*}{ }_{1 g}$ is ready to be introduced in Equation (15) to find $e^{*}=f\left(V^{*}\right)$ :

$$
\begin{gather*}
\text { Equations (15) and (20): } e^{*}=e_{2 g}-\left[\frac{V_{1 g} \cdot V_{2 g}-V^{*} \cdot V_{1 g}}{\left(V_{2 g}-V_{1 g}\right)}\right] \frac{\left(e_{2 g}-e_{1 g}\right)}{V^{*}} \rightarrow \\
e^{*}=e_{2 g}-\frac{V_{1 g} \cdot\left(V_{2 g}-V^{*}\right)}{V^{*} \cdot\left(V_{2 g}-V_{1 g}\right)} \cdot\left(e_{2 g}-e_{1 g}\right)=f\left(V^{*}\right) \tag{21}
\end{gather*}
$$

### 2.5. Time with Each Number of Groups

The time $\mathrm{t}^{*}{ }_{\mathrm{ig}}$ that the station would be working with $\mathrm{n}_{\mathrm{g}}=1$ and $\mathrm{n}_{\mathrm{g}}=2$ can be calculated from Equation (17):

$$
\text { Equation (17) } \rightarrow \mathrm{t}_{\mathrm{ig}}{ }^{*}=\frac{\mathrm{V}_{\mathrm{ig}}{ }^{*}}{\mathrm{~V}_{\mathrm{ig}}} \cdot \mathrm{t}_{\mathrm{t}}
$$

For that, $\mathrm{V}^{*}{ }_{1 \mathrm{~g}}$ and $\mathrm{V}^{*}{ }_{2 \mathrm{~g}}$ are needed. These can easily be cleared out from the equation system formed by Equations (13) and (14):

$$
\begin{align*}
& \left\{\begin{array}{c}
\text { Equation (13) } \rightarrow V^{*}=V^{*}{ }_{1 g}+V^{*}{ }_{2 g} \\
\text { Equation (14) } \rightarrow V^{*} \cdot e^{*}=V^{*}{ }_{1 g} \cdot e_{1 g}+V^{*}{ }_{2 g} \cdot e_{2 g}
\end{array},\right. \\
& \mathrm{V}^{*}{ }_{2 \mathrm{~g}}=\mathrm{V}^{*}-\mathrm{V}^{*}{ }_{1 \mathrm{~g}} \rightarrow \text { in Equation (14) : } \mathrm{V}^{*} \cdot \mathrm{e}^{*}=\mathrm{V}^{*}{ }_{1 \mathrm{~g}} \cdot \mathrm{e}_{1 \mathrm{~g}}+\left(\mathrm{V}^{*}-\mathrm{V}^{*}{ }_{1 \mathrm{~g}}\right) \cdot \mathrm{e}_{2 \mathrm{~g}} \\
& V^{*} \cdot e^{*}=V^{*}{ }_{1 g} \cdot e_{1 g}+V^{*} \cdot e_{2 g}-V^{*}{ }_{1 g} \cdot e_{2 g} \\
& V^{*} \cdot e^{*}-V^{*} \cdot e_{2 g}=V^{*} 1 g \cdot\left(e_{1 g}-e_{2 g}\right) \\
& \mathrm{V}^{*}{ }_{1 \mathrm{~g}}=\mathrm{V}^{*} \cdot \frac{\left(\mathrm{e}^{*}-\mathrm{e}_{2 \mathrm{~g}}\right)}{\left(\mathrm{e}_{1 \mathrm{~g}}-\mathrm{e}_{2 \mathrm{~g}}\right)} \\
& \mathrm{V}^{*}{ }_{\mathrm{ig}}=\mathrm{V}^{*} \cdot \frac{\left(\mathrm{e}^{*}-\mathrm{e}_{\mathrm{jg}}\right)}{\left(\mathrm{e}_{\mathrm{ig}}-\mathrm{e}_{\mathrm{jg}}\right)}  \tag{22}\\
& \mathrm{t}_{\mathrm{ig}}^{*}=\frac{\mathrm{V}^{*} \cdot t_{\mathrm{t}}}{\mathrm{~V}_{\mathrm{ig}}} \cdot \frac{\left(\mathrm{e}^{*}-\mathrm{e}_{\mathrm{jg}}\right)}{\left(\mathrm{e}_{\mathrm{ig}}-\mathrm{e}_{\mathrm{jg}}\right)} . \tag{23}
\end{align*}
$$

## 3. Results

## Interpretation of the analytic solution.

Equation (21) shows the analytic solution to find the average value of the specific energy e* required to pump the desired volume combining one and two groups of pumps. If instead of one and two groups of pumps, the expression is written for any group combination $\mathrm{n}_{\mathrm{i}}$ and $\mathrm{n}_{\mathrm{j}}$ (e.g., $\mathrm{n}_{\mathrm{g}}=1$ and $\mathrm{n}_{\mathrm{g}}=3$; or $\mathrm{n}_{\mathrm{g}}=2$ and $\mathrm{n}_{\mathrm{g}}=3$, or $\mathrm{n}_{\mathrm{g}}=2$ and $\mathrm{n}_{\mathrm{g}}=4$, etc.), the expression has this generic form:

$$
\begin{equation*}
\mathrm{e}^{*}=\mathrm{e}_{\mathrm{jg}}-\frac{\mathrm{V}_{\mathrm{ig}} \cdot\left(\mathrm{~V}_{\mathrm{jg}}-\mathrm{V}^{*}\right)}{\mathrm{V}^{*} \cdot\left(\mathrm{~V}_{\mathrm{jg}}-\mathrm{V}_{\mathrm{ig}}\right)} \cdot\left(\mathrm{e}_{\mathrm{jg}}-\mathrm{e}_{\mathrm{ig}}\right)=\mathrm{f}\left(\mathrm{~V}^{*}\right) \tag{24}
\end{equation*}
$$

In Equation (24), all values $\mathrm{V}_{\mathrm{ig}}, \mathrm{V}_{\mathrm{jg}}, \mathrm{e}_{\mathrm{ig}}$, and $\mathrm{e}_{\mathrm{jg}}$ are known, because Table 1 is calculated in the first place. Therefore, it should be noted that this expression can be written in terms of constants in the following way:

$$
\begin{equation*}
\mathrm{e}^{*}=\alpha-\frac{\beta}{\mathrm{V}^{*}}=\text { Constant }- \text { Hyperbola } . \tag{25}
\end{equation*}
$$

Being:

- $\quad \alpha=$ Constant $1=\mathrm{e}_{\mathrm{jg}} \rightarrow$ On the example case : $\alpha=\mathrm{e}_{2 \mathrm{~g}}$
- $\beta=$ A different constant $=\frac{\mathrm{V}_{\mathrm{ig}} \cdot\left(\mathrm{V}_{\mathrm{ig}}-\mathrm{V}^{*}\right)}{\left(\mathrm{V}_{\mathrm{ig}}-\mathrm{V}_{\mathrm{ig}}\right)} \cdot\left(\mathrm{e}_{\mathrm{jg}}-\mathrm{e}_{\mathrm{ig}}\right) \rightarrow$

On the example case : $\beta=\frac{V_{1 g} \cdot\left(V_{2 g}-V^{*}\right)}{\left(V_{2 g}-V_{1 g}\right)} \cdot\left(e_{2 g}-e_{1 g}\right)$

Therefore, the most important conclusion extracted from Equation (25) is that the specific energy e* follows an inverted hyperbola shape, otherwise baptized as the Convex Hyperbola. To help visualize this conclusion, Figure 9 is shown:


Figure 9. Generic expression of the specific energy e*: Convex Hyperbola.
The example case that has been used for this deduction can thus be graphically represented as well, as Figure 10 illustrates. It would be the representation of Equation (21):


Figure 10. Solution of the specific energy e* of the example case, where the station would combine $\mathrm{n}_{\mathrm{g}}=1$ and $\mathrm{n}_{\mathrm{g}}=2$.

It is important to notice that the combination of working groups does not have to be a consecutive number. The convex hyperbola can be graphed between any group combination $n_{i}$ and $n_{j}$, such as. $n_{g}=1$ and $n_{g}=3$, etc. In the following section, this aspect will be covered in depth.

How to use the results: practical steps.

### 3.1. The Optimal Group Combination

In order to pump the desired water volume $\mathrm{V}^{*}$, the pumping station can be working for a portion of time with a certain amount of active groups of pumps $n_{i}$, and the rest of the time with a different number of active groups $n_{j}$. For instance, it could be initially one group of pumps $n_{i}=1$, and after a certain time, switch into two groups $n_{j}=2$, or instead, it could begin with three groups of pumps $n_{i}=3$, and then change to five groups of pumps $n_{j}=5$, etc.

How to choose which pump combination is the key question, and the answer is simple: the combination that consumes the least amount of energy to pump the desired volume $V^{*}$ should be chosen. That is to say, the combination that consumes the least specific energy e*.

### 3.1.1. How to Use the Results for A Certain Volume V*—Specific Case

Equation (21) demonstrates how $\mathrm{e}^{*}$ can be calculated given a combination of pumps. Therefore, the strategy will consist of forming different groups of pump combinations, then calculating $e^{*}$ and seeing which combination works at the minimum $e^{*}$ for that specific $\mathrm{V}^{*}$. This process is illustrated in the following Table 2:

Table 2. Steps to select the group combination for a certain volume $V^{*}$.

| Steps to Select the Group Combination for A Certain Volume $\mathrm{V}^{*}$ |  |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Step 1 |  | Step 2 |  | Step 0: Calculate Table 1 Step 3 |  | Step 4 | Step 5 | Step 6 <br> Choose the group combination that gives the minimum $\mathrm{e}^{*}$ |
|  |  | Obtain th (if th pumping | Volume was ole time $t_{t}$ | Obtain <br> (if th pumpin | Energy was lime $t_{t}$ | Define the desired volume $\mathrm{V}^{*}$ | Calculate the specific energy $\mathrm{e}^{*}$ |  |
| $\begin{aligned} & \mathbf{n}_{\mathbf{i}} \\ & e g \end{aligned}$ | $\begin{aligned} & \mathbf{n}_{\mathrm{j}} \\ & e g \end{aligned}$ | $V_{i g}$ Colum | $V_{\text {ig }}$ <br> Table 1. | $\mathrm{e}_{\mathrm{ig}}$ Colum | $\mathbf{e}_{\text {jg }}$ able 1 | V* | $\begin{gathered} \mathbf{e}^{*} \\ \text { Equation (22) } \end{gathered}$ |  |
| 1 | 2 | $V_{1 g}$ | $V_{2 g}$ | $e_{1 g}$ | $e_{2 g}$ | $V^{*}$ | $e^{*}$ with $1 g$ and $2 g$ |  |
| 1 | 3 | $\ldots$ | $\ldots$ | $\ldots$ | $\ldots$ | $V^{*}$ | ... |  |
| 2 | 3 | $\ldots$ | $\ldots$ | $\ldots$ | $\ldots$ | $V^{*}$ | $\ldots$ |  |
| $\cdots$ | $\ldots$ | $\cdots$ | $\cdots$ | $\cdots$ | $\cdots$ | $V^{*}$ | $\cdots$ |  |

Once identified the best group combination, the final step (7) would consist of using Equation (23) to find out the time each number of groups should be working.

$$
\text { Equation }(23) \rightarrow \mathrm{t}_{\mathrm{ig}}^{*}=\frac{\mathrm{V}^{*} \cdot \mathrm{t}_{\mathrm{t}}}{\mathrm{~V}_{\mathrm{ig}}} \cdot \frac{\left(\mathrm{e}^{*}-\mathrm{e}_{\mathrm{jg}}\right)}{\left(\mathrm{e}_{\mathrm{ig}}-\mathrm{e}_{\mathrm{jg}}\right)}
$$

### 3.1.2. How to Use the Results for Any Desired Volume V*-General Case

The previous section focuses on one specific volume $V^{*}$; however, a more interesting approach is to be able to find the best combination of pumps for any desired volume $\mathrm{V}^{*}$. This can be achieved by creating a chart collection for the pumping station. In this chart, all possible volumes $\mathrm{V}^{*}$ are contemplated, and it offers the best combination of pumps for any situation. To create this chart, engineers may represent the convex hyperbolas of all possible groups' combinations of the station, using Equation (21).

The compiled steps to generate these charts are:

- Step 0: Choose the regulation period (e.g., hourly, daily, etc.) and calculate all values from Table 1.
- Step 1: Form all possible group combinations $n_{i}$ and $n_{j}$.
- Step 2: For each group combination, create a table such as Table 3. This table should include sufficient values of $\mathrm{V}^{*}$ to draw the hyperbolas with accuracy.

Table 3. Calculation of the convex hyperbola for a certain group combination.

|  | Group Combination: $\mathbf{n}_{\mathbf{i}}$ and $\mathbf{n}_{\mathbf{j}}$ |
| :---: | :---: |
| Volume $\mathbf{V}^{*}$ | Specific Energy $\mathbf{e}^{*}$ <br> (Equation (21)) |
| 10 | $\ldots$ |
| 20 | $\cdots$ |
| $\cdots$ | $\cdots$ |

- $\quad$ Step 3: Represent all the previous Table 3 in one chart.
- Step 4: Identify the curves that are the lowest. Those indicate the most convenient pump combinations for any desired volume.
- Step 5: Once the best group combination is identified, use Equation (23) to find out the time each number of groups should be working.

$$
\text { Equation }(23) \rightarrow t_{\mathrm{ig}}^{*}=\frac{\mathrm{V}^{*} \cdot t_{\mathrm{t}}}{\mathrm{~V}_{\mathrm{ig}}} \cdot \frac{\left(\mathrm{e}^{*}-\mathrm{e}_{\mathrm{jg}}\right)}{\left(\mathrm{e}_{\mathrm{ig}}-\mathrm{e}_{\mathrm{jg}}\right)} .
$$

The generation of these charts only requires a calculation sheet, and it could save a lot of energy and money by only knowing which is the least energy-using group combination to use. They do not need to be changed; on the contrary, they are a permanent guide chart for any pumping situation.

## Examples

Figure 11 shows an example of a pumping station that has up to three pumps. Operators have chosen the time $t_{t}$ (this could be e.g., $1,6,24 \mathrm{~h}$, etc.). During that time, they may wish to supply different volumes of water $\mathrm{V}^{*}$. For example, if it is an hourly regulation, from 06:00-07:00, a volume of $\mathrm{V}^{*}=6001$ might be needed, but from 07:00-08:00, they might need $\mathrm{V}^{*}=12.000 \mathrm{l}$. In this example, to pump any desired volume $\mathrm{V}^{*}$, operators could choose to use any of the following configurations:

- One group for a while and then switch to 2 groups for the rest of the time. This is $n_{i}=1$ and then $n_{j}=2$.
- One group for a while and then switch to 3 groups for the rest of the time. This is $\mathrm{n}_{\mathrm{i}}=1$ and then $\mathrm{n}_{\mathrm{j}}=3$.
- Two groups for a while and then switch to 3 groups for the rest of the time. This is $\mathrm{n}_{\mathrm{i}}=2$ and then $\mathrm{n}_{\mathrm{j}}=3$.


Figure 11. Conservation of the energy. Solution of the specific energy e* of the example case, where the station would combine $n_{g}=1$ and $n_{g}=3$. The average specific energy $e^{*}$ will be the sum of the energy spent when the station is working with $n_{g}=1$ and when the station works with $n_{g}=3$.

Since there are three possible pumping configurations, three possible Convex Hyperbolas can be graphed, Equation (24). The steps to map out these hyperbolas are explained
in the preceding section. Once the Convex Hyperbolas (the green lines) have been drawn (two upper figures on Figure 11), the guidance chart for that regulation period (e.g., hourly regulation) is obtained. For each pumping station, the curves of the hyperbolas can look completely different (e.g., one hyperbola above or under the other, etc.). The curves in Figure 11 are only an example of what they could look like.

In this figure, it is represented that the best solution would be to (always) combine one group $n_{g}=1$ and three groups of pumps $n_{g}=3$ to pump any desired volume $V^{*}$, since it is the lowest convex hyperbola (and therefore, the cheapest pump combination). The energy spent in this combination $E^{*}$ is the sum of the energy consumed while using $\mathrm{n}_{\mathrm{g}}=1$ $\left(\mathrm{E}^{*}{ }_{\mathrm{g}}\right)$ and $\mathrm{n}_{\mathrm{g}}=3\left(\mathrm{E}_{3 \mathrm{~g}}\right)$.

One must always select the lowest Convex Hyperbola, since it indicates the least energy use. The strategy is, therefore, to always look for the lowest curve.

Figure 12 represents different examples of how these charts could look:


Figure 12. Examples of Convex Hyperbola charts for a pump station.
From the different boxes in Figure 12:

- Case (a) and (b): They represent the same pumping station with three pumps. In case (a), the desired volume $V^{*} . V_{1 g}<V^{*}<V_{2 g}$ would be best to pump with one and two groups. However, if the desired volume is greater, $\mathrm{V}_{2 \mathrm{~g}}<\mathrm{V}^{*}<\mathrm{V}_{3 \mathrm{~g}}$, the least-energy-use combination is to use two and three groups of pumps.
- Case (c): The pumping station also counts with three pumps, but the convex hyperbolas show a different shape, and the least-energy-use option is to use one and three groups of pumps, for any desired volume $V_{1 g}<V^{*}<V_{3 g}$.
- Case (d): It illustrates a more complex situation, where the station counts for up to five pumps. In this example, the convex hyperbolas have a shape in which the lowest curves (those that show the least energy use combinations) are only the combina-
tion of one and three groups (when $\mathrm{V}_{1 \mathrm{~g}}<\mathrm{V}^{*}<\mathrm{V}_{3 \mathrm{~g}}$ ), or for bigger volumes (when $\mathrm{V}_{3 \mathrm{~g}}<\mathrm{V}^{*}<\mathrm{V}_{5 \mathrm{~g}}$ ), three and five groups.
Minimizing the vibrations in pumps by turning on and off.
It must be kept in mind that the Convex Hyperbola Charts are drawn for a defined time $t_{t}$ (for example $5 \mathrm{~min}, 1 \mathrm{~h}, 6 \mathrm{~h}, 24 \mathrm{~h}$, etc.). During that time, operators need to provide the demanded volume of water, and the charts will indicate the best configuration (for example, firstly using 2 pumps and then switching to 4 pumps). The shorter $t_{t}$ (the reevaluation period), the better adjustment to variable demands. However, short reevaluation periods implicate that the pumps will be switching on and off very frequently to adjust to the demand. This is never good for the sake of the durability of the machinery, especially when the pumps are large and the vibrations can be strong. Therefore, it is always desirable to have facilities to regulate the demand at the destination point, such as tanks or reservoirs.

The present methodology requires only one change (and no more) in the configuration during each $t_{t}$ (because, as it was proven in the Section 2.2.2. Solution B, this way, the system consumes less energy). The important point is that turning on or off will only happen once during each $t_{t}$. How this one configuration change affects the durability of the pumps only depends on $t_{t}$ (the reevaluation period): one configuration change in, for example, $t_{t}=6 \mathrm{~h}$, is a minimal impact, but one configuration every $t_{t}=5 \mathrm{~min}$, for example, can be damaging for the pumps.

However, this is external to the present method. All other optimization methods have the same restraint: every method needs to reevaluate the configuration every certain time $t_{t}$. It only depends on the capability to regulate the demand, and it is always better and desirable to have some sort of deposit for this task at the destination point.

## 4. Discussion

This research aimed to address the number of pumps that should be working at any moment during the operation of a parallel pumping station to provide the desired volume of water whilst consuming the least amount of energy. This is not only one of the biggest environmental challenges today, but also it goes hand in hand with being economically responsible.

This research proposes the preparation of the Convex Hyperbolas Charts to indicate the best pumping strategy during the operation of a facility. These charts take their name from the shape of the specific energy-volume curves. The specific energy $e^{*}$ is the amount of energy consumed by the station per unit of volume. The pumping station should pump the desired volume of water $\mathrm{V}^{*}$ using the least specific energy $\mathrm{e}^{*}$. This research analyzes the shape of the curves $\mathrm{e}^{*}-\mathrm{V}^{*}$. The result is that such curves have a convex hyperbola shape.

## Strengths and limitations

The elaboration of the Convex Hyperbolas Charts allows engineers to know exactly what is the best number of active pumps for any pumping situation. The making of such charts only requires the use of any calculation sheet, only once, and it is a permanent resource that can be used at any time during the operation. It immediately tells the operator how many pumps should be turned on, depending on the desired volume of water. As simple as they are, these charts could save great quantities of energy and money, in both the short and the long run. The proposed methodology is completely inexpensive, in both the material and the computational aspect. It does not require any heavy complex algorithm, but on the contrary, it could be simply done with a calculation sheet. It simply comes from the mathematical deduction of the specific energy used, and not from any iterative process that is the current trend, all its applicability relying on powerful computers. Hence, the solution is not approximate, but exact. This work offers an analytic solution, which is a great advancement compared to the existing methods.

In addition, the Convex Hyperbolas Charts are completely compatible and complementary with any other operation control algorithm, and they do not need to adapt the installations or machinery. An additional advantage is that our methodology is energy
cost-free: our methodology does not need to be updated with energy price changes, because it optimizes the amount of energy consumed and not its cost.

Practitioners can easily benefit from this tool thanks to its simple application: no programming or computational skills are needed. Once the charts are plotted, it is only a matter of identifying the lowest curve in the graph and selecting the indicated pump configuration. The proposed methodology is very operational-friendly. A powerful and simple tool to be more energetically mindful in the operation of water supply systems was presented.

As a limitation, it should be recalled that, just like any other optimization method, operators will need to reevaluate the pump configuration after a certain time $t_{t}$, depending on the variability of the demand and the water regulation capacity. Different Convex Hyperbolas Charts will need to be elaborated for the same pumping station depending on the period considered for the optimization $t_{t}$ (e.g., hourly regulation, daily regulation, etc.). During the night, longer periods could be considered, whilst during the daytime, the reevaluation periods might be shorter. Nevertheless, this is a minor limitation, since it would be enough to plot a small collection of charts, according to different reevaluation periods. Additionally, all other optimization methods have the same restraint: every method needs to reevaluate the configuration every certain time. This aspect is always improved when regulation in the destination point is available (e.g., tanks, reservoirs, etc.).

## Future research

One of the initial hypotheses of the method is that all pumps in the pumping station are parallel and identical. Future research will be undertaken to analyze the effects of speed controllers. The effect of the speed controller is equivalent to "changing the pumping curve", and in this way, pumps can no longer be considered identical. This is a research line that will be studied in depth in future investigations.

## 5. Conclusions

A new methodology to select the number of pumps during the operation has been exposed. The objective function is to minimize the energy consumed during the operation. The energy function can be represented in the Convex Hyperbolas Charts. With these charts, operators can immediately visualize what the combination of pumps is that means the least energy consumption in order to pump any determined volume of water. These charts are obtained analytically, which means that no iterations are needed. They can be calculated just through a calculation sheet.

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