

Supplementary Information

# A Simple Time-Varying Sensitivity Analysis (TVSA) for Assessment of Temporal Variability of Hydrological Processes

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```
function TVSA(datei,pars,sim,obs,strpars)
% ----- %
% This function performs Time-Varying Sensitivity Analysis based on
% Regional Sensitivity Analysis and plots the sensitivity index and
% parameter values.
% The index is the Maximum Vertical Distance (MVD) between the two
% CDFs (behavioral and non-behavioral).
% ----- %
% datei : initial date - format [yyyy,mm,dd]
% pars : parameter matrix [N,P]
% sim : output time series matrix [T,N]
% obs : observed time series vector [T,1]
% strpars: cell with parameter names {1,P} {'A','B',...}
% ----- %
% N: sample size (simulations)
% P: number of parameters
% T: size of the time series
% ----- %

%% Check inputs
if ~isnumeric(datei);error('input ''datei'' must be a vector of size
(1,3)');end
if ~isnumeric(pars);error('input ''pars'' must be a matrix of size
(N,P)');end
if ~isnumeric(sim);error('input ''sim'' must be a matrix of size (T,N)');end
if ~isnumeric(obs);error('input ''obs'' must be a vector of size (T,1)');end
if ~iscell(strpars);error('input ''strpars'' must be a cell array of size
{1,P}');end
[N,P]=size(pars);[T,Nn]=size(sim);[Tt,x]=size(obs);[x,Pp]=size(strpars);
if T~=Tt;error('input ''sim'' and ''obs'' must have the same number of
rows');end
if N~=Nn;error('Columns of input ''sim'' must be equal to rows of input
''pars'');end
if P~=Pp;error('input ''pars'' and ''strpars'' must have the same number of
columns');end
```

```

%% Input Window
d = dialog('Position',[500 300 250 200], 'Name','Input Window');
txt1 = uicontrol('Parent',d,'Style','text','Position',[10 170 80
20], 'String','Window Step :');
popup1 = uicontrol('Parent',d,'Style','popupmenu','Position',[15 150 80
20], 'String',{'Daily'; 'Monthly'; 'Annual'});
txt2 = uicontrol('Parent',d,'Style','text','Position',[10 115 80
20], 'String','WSize(2n+1) n:');
edit1 = uicontrol('Parent',d,'Style','edit','Position',[15 95 80
20], 'String','1');
txt3 = uicontrol('Parent',d,'Style','text','Position',[140 170 80
20], 'String','Criteria :');
popup2 = uicontrol('Parent',d,'Style','popupmenu','Position',[155 150 80
20], 'String',{'KGE'; 'NSE'; 'RMSE'; 'NSEIQ'});
txt4 = uicontrol('Parent',d,'Style','text','Position',[142 115 80
20], 'String','Threshold :');
edit2 = uicontrol('Parent',d,'Style','edit','Position',[155 95 80
20], 'String','0');
btn1 = uicontrol('Parent',d,'Style','pushbutton','Position',[40 40 70
25], 'String','Ok', 'Callback',@btn1_Callback);
btn2 = uicontrol('Parent',d,'Position',[140 40 70
25], 'String','Close', 'Callback','delete(gcf)');

%% Time-Varying Sensitivity Analysis
function btn1_Callback(hObject, eventdata)
    windowstep = popup1.Value;
    window = str2num(get(edit1,'string'));
    ObjectiveFunction = popup2.Value;
    threshold = str2num(get(edit2,'string'));
    close(gcf)
    %%%%%%%%% Check input window %%%%%%%%%
    if ~isnumeric(threshold)
        error('input ''threshold'' must be a number')
    end
    if ~isnumeric(window) || window<1 || window~=round(window)
        error('input ''window'' must be real positive integer')
    end
    %%%%%%%%%
    for i=1:size(obs,1)
        datev(i,1)=datetime(datei(1),datei(2),datei(3)+i-1);
    end
    dateLb=datenum(datev);
    switch windowstep
        case 1
            t=size(sim,1);
            Temp='Days';
        case 2
            mnths=unique([year(datev) month(datev)], 'rows');
            t=length(mnths);
            Temp='Months';
        case 3
            yrs=unique([year(datev)], 'rows');
            t=length(yrs);
            Temp='Years';
    end
    %%%%%%%%% Objective Function %%%%%%%%%
    h_w=waitbar(0, 'Running Time-Varying Sensitivity Analysis Algorithm');

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for dt=1:t
    for no=1:size(sim,2)
        if dt<window+1
            switch windowstep
                case 1 % Daily

step=find(datev==datetime(datei(1),datei(2),datei(3)+dt+window-1));
                case 2 % Monthly
                    ms=datetime(datei(1),datei(2)+dt+window-1,1);

step=find(datev==datetime(datei(1),datei(2)+dt+window-
1,eomday(year(ms),month(ms)))));
                case 3 % Annual
                    yr=datetime(datei(1)+dt+window-1,datei(2)-1,1);
                    step=find(datev==datetime(datei(1)+dt+window-
1,datei(2)-1,eomday(year(yr),month(yr)))));
                end
                obse=obs(1:step);
                sim0=sim(1:step,no);
                NN=size(sim0,1);
                switch ObjectiveFunction
                    case 1 % KGE
                        r=corrcoef(sim0,obse,'rows','pairwise');
                        r=r(1,2);
                        alfag=sqrt(nanvar(sim0)/nanvar(obse));
                        betag=nanmean(sim0)/nanmean(obse);
                        ED=sqrt((r-1)^2+(alfag-1)^2+(betag-1)^2);
                        OF(no,dt)=1-ED;
                    case 2 % NSE
                        OF(no,dt)=1-(nansum((obse-
sim0).^2))/(nansum((obse-nanmean(obse)).^2));
                    case 3 % RMSE
                        OF(no,dt)=sqrt(1/NN*nansum((sim0-obse).^2));
                    case 4 % NSEiQ
                        for i=1:length(obse)
                            if obse(i)==0
                                e=nanmean(obse)/100;
                                break
                            else
                                e=0;
                            end
                        end
                        OF(no,dt)=1-(nansum((1/(obse+e)-
1/(sim0+e)).^2))/(nansum((1/(obse+e)-nanmean(1/(obse+e))).^2));
                    end
                elseif t-dt<window+1
                    switch windowstep
                        case 1 % Daily

step=find(datev==datetime(datei(1),datei(2),datei(3)+dt-window-1));
                        case 2 % Monthly
                            step=find(datev==datetime(datei(1),datei(2)+dt-
window-1,1));
                        case 3 % Annual
                            step=find(datev==datetime(datei(1)+dt-window-
1,datei(2),1));
                    end

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obse=obs(step:size(sim,1));
sim0=sim(step:size(sim,1),no);
NN=size(sim0,1);
switch ObjectiveFunction
    case 1 % KGE
        r=corrcoef(sim0,obse,'rows','pairwise');
        r=r(1,2);
        alfag=sqrt(nanvar(sim0)/nanvar(obse));
        betag=nanmean(sim0)/nanmean(obse);
        ED=sqrt((r-1)^2+(alfag-1)^2+(betag-1)^2);
        OF(no,dt)=1-ED;
    case 2 % NSE
        OF(no,dt)=1-(nansum((obse-
sim0).^2))/(nansum((obse-nanmean(obse)).^2));
    case 3 % RMSE
        OF(no,dt)=sqrt(1/NN*nansum((sim0-obse).^2));
    case 4 % NSEiQ
        for i=1:length(obse)
            if obse==0
                e=0.3;
                break
            else
                e=0;
            end
        end
        OF(no,dt)=1-(nansum((1/(obse+e)-
1/(sim0+e)).^2))/(nansum((1/(obse+e)-nanmean(1/(obse+e))).^2));
    end
else
    switch windowstep
        case 1 % Daily

stepa=find(datev==datetime(datei(1),datei(2),datei(3)+dt+window-1));

stepb=find(datev==datetime(datei(1),datei(2),datei(3)+dt-window-1));
        case 2 % Monthly
            msa=datetime(datei(1),datei(2)+dt+window-1,1);

stepa=find(datev==datetime(datei(1),datei(2)+dt+window-
1,eomday(year(msa),month(msa))));
            stepb=find(datev==datetime(datei(1),datei(2)+dt-
window-1,1));
        case 3 % Annual
            yra=datetime(datei(1)+dt+window-1,datei(2)-1,1);
            stepa=find(datev==datetime(datei(1)+dt+window-
1,datei(2)-1,eomday(year(yra),month(yra))));
            stepb=find(datev==datetime(datei(1)+dt-window-
1,datei(2),1));
    end
    obse=obs(stepb:stepa);
    sim0=sim(stepb:stepa,no);
    NN=size(sim0,1);
    switch ObjectiveFunction
        case 1 % KGE
            r=corrcoef(sim0,obse,'rows','pairwise');
            r=r(1,2);
            alfag=sqrt(nanvar(sim0)/nanvar(obse));

```

```

betag=nanmean(sim0)/nanmean(obse);
ED=sqrt((r-1)^2+(alfag-1)^2+(betag-1)^2);
OF(no,dt)=1-ED;
case 2 % NSE
    OF(no,dt)=1-(nansum((obse-
sim0).^2))/(nansum((obse-nanmean(obse)).^2));
case 3 % RMSE
    OF(no,dt)=sqrt(1/NN*nansum((sim0-obse).^2));
case 4 % NSEiQ
    for i=1:length(obse)
        if obse==0
            e=0.3;
            break
        else
            e=0;
        end
    end
    OF(no,dt)=1-(nansum((1/(obse+e)-
1/(sim0+e)).^2))/(nansum((1/(obse+e)-nanmean(1/(obse+e))).^2));
end
end
end
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%% Sensitivity Analysis %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
% Behavioral and Non-Behavioral Models
L=OF(:,dt);
[N,P]=size(L);
for i=1:size(pars,2)
    Pop=pars(:,i);
    if ObjectiveFunction==2 % RMSE
        if threshold==0
            threshold=0.2*nanmean(obse);
        end
        thrs=sum(OF(:,dt)>repmat(threshold,N,1),2)==P;
    else % KGE - NSE - NSEiQ
        thrs=sum(OF(:,dt)<repmat(threshold,N,1),2)==P;
    end
    B=Pop(~thrs,1); % Behavioral Models (B-M)
    nB=size(B,1); % Number of B-M
    NB=Pop(thrs,1); % Non-Behavioral Models (NB-M)
    nNB=size(NB,1); % Number of NB-M
    if nB<=0 || nNB<=0
        mvd(:,dt)=0;
        parB(i,dt)=NaN;
        parNB(i,dt)=NaN;
    else % CDFs of B and NB models
        xx=unique(sort(Pop(:,1)));
        %%% B models %%%
        parB(i,dt)=prctile(B,50); % Parameter values of B models
        tmxB=sort(B);
        tmyB=(1:length(tmxB))/length(tmxB);
        [tmxB,iuB] = unique(tmxB,'last');
        tmyB=tmyB(iuB);
        FiB=ones(size(xx));
        for j=length(tmyB):-1:1
            FiB(xx<=tmxB(j))=tmyB(j);
        end
        FiB(xx<tmxB(1))=0;
    end
end

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```

models
    %%% NB models %%%
    parNB(i,dt)=prctile(NB,50); % Parameter values of NB

    tmxNB=sort(NB);
    tmyNB=(1:length(tmxNB))/length(tmxNB);
    [tmxNB,iuNB] = unique(tmxNB,'last');
    tmyNB=tmyNB(iuNB);
    FiNB=ones(size(xx));
    for jj=length(tmyNB):-1:1
        FiNB(xx<=tmxNB(jj))=tmyNB(jj);
    end
    FiNB(xx<tmxNB(1))=0;
    % Maximum Vertical Difference (MVD)
    mvd(i,dt) = max(abs(FiB-FiNB));
end
end
clear L
waitbar(dt/t);
end
close(h_w);
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%% PLOT %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
if nNB==0
    warndlg('Cannot find any value below (above-RMSE) the
threshold','Warning!')
elseif nB==0
    warndlg('Cannot find any value above (below-RMSE) the
threshold','Warning!')
else
    if windowstep==1
        mvdf=mvdf; parBf=parB; parNBf=parNB;
    end
    for i=1:t
        for f=1:size(obs,1)
            if windowstep==2 & mnths(i,:)==[year(datev(f)) month(datev(f))]
                mvdf(:,f)=mvdf(:,i);
                parBf(:,f)=parB(:,i);
                parNBf(:,f)=parNB(:,i);
            end
            if windowstep==3 & yrs(i,:)==year(datev(f))
                mvdf(:,f)=mvdf(:,i);
                parBf(:,f)=parB(:,i);
                parNBf(:,f)=parNB(:,i);
            end
        end
    end
    end
    figure
    yyaxis left
    colrs=gray;
    colrs=colrs(end:-1:1,:);
    colormap(colrs)
    imagesc(dateLb,[1:size(pars,2)],mvdf); hold on
    ylabel('Parameters')
    set(gca,'YTick',1:size(pars,2),'YTickLabel',strpars)
    colorbar
    yyaxis right
    plot(dateLb,obs,'r','LineWidth',1)
    ylabel('Flow, m^3/s')

```



*water*



```
datetick('x',28,'keeplimits')
xlabel('Time')
title(['Time-Varying Sensitivity Analysis - Window Size:
',num2str(2*window+1),' ', Temp])

if size(pars,2)<=4
    subp='2,2,';
elseif size(pars,2)>4&&size(pars,2)<=6
    subp='3,2,';
elseif size(pars,2)>6&&size(pars,2)<=9
    subp='3,3,';
elseif size(pars,2)>9&&size(pars,2)<=12
    subp='4,3,';
else %size(pars,2)>12&&size(pars,2)<=16
    subp='4,4,';
end

figure
for i=1:size(pars,2)
    if size(pars,2)>1
        eval(['subplot(' subp num2str(i) ')'])
    end
    scatter(dateLb,parBf(i,:),3,'b','filled');hold on;
    scatter(dateLb,parNBf(i,:),3,'r','filled');hold on;
    plot(dateLb,parBf(i,:),'-b','linewidth',1);hold on;
    plot(dateLb,parNBf(i,:),'-r','linewidth',1);hold on;
    datetick('x',28,'keeplimits')
    xlim([dateLb(1) dateLb(end)])
    ylabel([' ' strpars{1,i} '']);
end
end
end
end
```