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Numerical Simulation of Wave Overtopping on Breakwater with an Armor Layer of Accropode Using SWASH Model

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Abstract: In this paper, a new method for predicting wave overtopping discharges of Accropode armored breakwaters using the non-hydrostatic wave model Simulating WAves till SHore (SWASH) is presented. The apparent friction coefficient concept is proposed to allow the bottom shear stress term calculated in the momentum equation to reasonably represent the effect of comprehensive energy dissipation caused by the roughness and seepage during the wave overtopping process. A large number of wave overtopping cases are simulated with a calibrated SWASH model to determine the values of equivalent roughness coefficients so that the apparent friction coefficients can be estimated to achieve the conditions with good agreement between numerical overtopping discharges and those from the EurOtop neural network model. The relative crest freeboard and the wave steepness are found to be the two main factors affecting the equivalent roughness coefficient. A derived empirical formula for the estimation of an equivalent roughness coefficient is presented. The simulated overtopping discharges by the SWASH model using the values of the equivalent roughness coefficient estimated from the empirical formula are compared with the physical model test results. It is found that the mean error rate from the present model predictions is 0.24, which is slightly better than the mean error rate of 0.26 from the EurOtop neural network model.

Keywords: Accropode armored breakwater; non-hydrostatic wave model; mean overtopping discharge; equivalent roughness coefficient

1. Introduction

The Accropode blocks are the most commonly used armor blocks on the sloping breakwaters in practical projects because of their low engineering cost, good wave dissipation performance and strong wave resistance stability [1]. To design an Accropode armored breakwater, the overtopping discharge needs to be estimated reasonably well because it is an important index to determine the top elevation of the breakwater [2,3]. There are many ways to estimate the overtopping discharge, such as the traditional physical model tests [4] and empirical formulas [5–7]. In recent years, numerical



simulation has become one of the most effective methods to estimate the wave overtopping rate due to the rapid development of computer technology and computational methods.

Early numerical models, including the nonlinear shallow-water equation and Boussinesq equation-based models, have limitations in describing the phenomena of wave overtopping because the dynamic pressure process was not considered [8–11]. The Smoothed Particle Hydrodynamics (SPH) method [12–15], the Finite-Discrete Element Method (FEMDEM) [16], and the Volume of Fluid (VOF) method on solving the Reynolds time averaged Navier–Stokes (N-S) equations (RANS) [1,17,18] for the free-surface elevations are suitable for simulating the strong nonlinear free-surface flow problems, such as wave overturning and breaking, and they have been used for the wave overtopping simulation of seawalls. However, the huge cost and low efficiency of simulations restrict their wide practical engineering applications at present.

The three-dimensional non-hydrostatic wave model, developed rapidly in recent years, may be a compromise between computational cost and accuracy on wave overtopping simulation [19,20]. A non-hydrostatic wave model can simulate the wave propagation by solving the nonlinear shallow-water equations with non-hydrostatic terms [21–23]. In the vertical direction, only two or three layers are needed to model the strong nonlinear and dispersive waves, while both the accuracy requirement and high simulation efficiency can be contented for wave transformation in areas near the shore. Wave overtopping simulations over smooth and non-permeable breakwaters have been successfully carried out by Suzuki et al. [24]. However, in fact, most sloping breakwaters in the field are permeable rubble mound breakwaters with armor layers. For simulations of wave overtopping on those types of breakwaters, the use of a porosity coefficient for a permeable breakwater may lead to wave dissipation without the features of wave climbing and overtopping [25,26]. One feasible method is to treat a permeable breakwater as an impermeable terrain in a numerical model [25,26], where the bottom friction term is used to represent the comprehensive energy dissipation effect caused by the roughness and seepage [25]. In order to distinguish the usual bottom friction coefficients defined in the bottom shear stress terms, those determined with equivalent effects of energy dissipation and roughness on porous breakwaters with layers of armors are called the apparent friction coefficients. Therefore, the determination of a well-represented apparent friction coefficient for a defined breakwater system plays a key role in obtaining results in close agreement between numerical simulations and physical model tests. The objective of this study is to propose a verified empirical formula that can be utilized to determine an equivalent roughness coefficient where the Manning's roughness coefficient that appeared in the bottom shear stress term of the momentum equation can be replaced to reasonably estimate an apparent friction coefficient for an Accropode armored breakwater. In such a way, the non-hydrostatic wave model, SWASH, can be accordingly simulated to calculate with acceptable accuracy the wave overtopping discharge generated by an Accropode armored breakwater without being physically calibrated by a physical model test.

The layout of this paper is as follows. Section 2 introduces the research method. The model verification is described in Section 3. Section 4 details the analysis of the influencing factors related to the equivalent roughness coefficient of an Accropode armored breakwater on the wave overtopping. Section 5 describes the development and verification of an empirical formula established in this study to determine the equivalent roughness coefficient. Finally, the findings of this study are summarized and concluded in Section 6.

2. Research Method

In this study, the non-hydrostatic wave model SWASH is used to set up the numerical flume for simulating the wave overtopping across the breakwaters with an armor layer of Accropode. The SWASH model used in the numerical simulation was developed by the Delft University of Technology, and its source program can be downloaded for free from http://swash.sourceforge.net [27].

The governing equations along the x direction are given as

$$\frac{\partial\zeta}{\partial t} + \frac{\partial hu}{\partial x} = 0 \tag{1}$$

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + g\frac{\partial \zeta}{\partial x} + \frac{1}{h}\int_{-d}^{\zeta} \frac{\partial P}{\partial x}dz + c_f \frac{u|u|}{h} = 0$$
(2)

where ζ is the free-surface elevation, *t* is time, *h* is the total water depth, $h = \zeta + d$, *d* is the still water depth, *u* is the vertically averaged velocity along the *x* direction, *g* denotes the acceleration of gravity, *P* is the non-hydrostatic pressure, and c_f is the dimensionless bottom friction coefficient, which can be related to the Manning's roughness coefficient *n* as

$$c_f = \frac{n^2 g}{h^{\frac{1}{3}}}$$
(3)

The integral expression of the non-hydrostatic pressure gradient term along the water column can then be written as a non-conservative form as

$$\int_{-d}^{\zeta} \frac{\partial P}{\partial x} dz = \frac{1}{2} h \frac{\partial P_b}{\partial x} + \frac{1}{2} P_b \frac{\partial (\zeta - d)}{\partial x}$$
(4)

The surface and bottom velocities along the z axis, w_s and w_b , are shown respectively below:

$$\frac{\partial w_s}{\partial t} = \frac{2P_b}{h} - \frac{\partial w_b}{\partial t}, \ w_b = -u\frac{\partial d}{\partial x}$$
(5)

The final equation of conservation of mass can be written as:

$$\frac{\partial u}{\partial x} + \frac{w_s - w_b}{h} = 0 \tag{6}$$

For more details about the SWASH model, see references of Zijlema et al. (2011) [28].

3. Model Setup and Validation

The SWASH wave model has been proved to be applicable to wave propagation, deformation and overtopping of smooth breakwaters [23,24]. In this section, it will be further applied to the wave overtopping simulation over a breakwater with an Accropode armor. In the numerical simulation, the permeable Accropode armored breakwater is treated as impermeable terrain with apparent friction. c_f in Equation (3) is used to represent the comprehensive energy dissipation effect caused by the roughness and seepage. Since Manning's coefficient *n* in Equation (3) is a parameter that needs to be directly inputted in the numerical model, the main verification work in this study is for the value of *n*.

The numerical simulation verification is based on the physical model test provided by the Crest Level Assessment of coastal Structures by full scale monitoring, neural network prediction and Hazard analysis on permissible wave overtopping (CLASH) database [4]. As shown in Figure 1, water depth (*h*) includes two cases, 0.674 or 0.727 m. The significant wave height (H_{m0}) is between 0.073 and 0.121 m, and the peak period (T_p) is between 1.037 and 1.743 s. G_c is 0 or 0.095 m. The range of relative crest freeboard (R_c/H_{m0}) is 0.75–1.82, the range of wave steepness ($S_{op} = 2\pi H_{m0}/gT_p^2$) is 0.02–0.06, and the mean wave overtopping discharges (*q*) measured by the physical model is between 1.13 × 10⁻⁶ and $3.10 \times 10^{-4} \text{ m}^3/\text{m/s}$.

The model range of numerical simulation is basically the same as that of the physical model. The length of the numerical flume is 50 m, and the grid resolution in the horizontal direction is 0.01 m. The time step of simulation needs to meet the simulation stability (the Courant number is less than 1), and 0.005 s of the initial time step is applied in this study. The incident spectrum is

the JONSWAP spectrum (γ = 3.3). The weak reflection boundary condition is applied for the wave generating boundary, and the sponge layer with five times wavelength is used at the end of the flume to eliminate the influence of wave reflection. In the simulation process, Manning's coefficient (*n*) needs to be adjusted to make the simulated *q* consistent with that of the physical model. According to Table 1, for different R_c/H_{m0} and S_{op} , *n* changes from 0.02 to 0.122 m^{-1/3}s. Even in the same *h*, H_{m0} and R_c/H_{m0} , the *n* value needs to be adjusted from 0.05 to 0.093 only for changes of T_p , such as case 13 and case 14 in Table 1. Figure 2 shows the time series of wave overtopping of case 13–15. This shows that, when SWASH is applied to simulate the Accropode armored breakwater, it depends heavily on the physical model to obtain the *n* value so that the *q* of numerical simulation is consistent with the physical model.

For the above reason, a formula of the equivalent roughness coefficient (n_A) of the Accropode armor is proposed in next sections, which is used to replace the n in the shear stress term of the momentum equation, so that it can be directly used in the numerical simulation of the wave overtopping on breakwater with an armor layer of Accropode without being calibrated through the physical model tests.

Table 1. The physical model parameters and the comparison of *q* between the numerical simulation and physical model of the CLASH database.

Case No.	<i>h</i> (m)	<i>H_m</i> 0 (n	n) <i>T_p</i> (s)	R_c/H_m) S _{op}	<i>G_c</i> (m)	q-simulated by SWASH (m ³ /m/s)	q-physical model (m ³ /m/s)	n-calibrated (m ^{-1/3} s)
1	0.674	0.118	1.743	1.180	0.025	0.095	8.26×10^{-5}	7.65×10^{-5}	0.085
2	0.674	0.095	1.321	1.458	0.035	0.095	7.58×10^{-6}	$7.16 imes 10^{-6}$	0.072
3	0.674	0.076	1.092	1.817	0.041	0.000	1.05×10^{-6}	1.13×10^{-6}	0.020
4	0.727	0.085	1.037	1.016	0.050	0.000	1.86×10^{-5}	1.85×10^{-5}	0.044
5	0.727	0.099	1.575	0.872	0.025	0.000	1.76×10^{-4}	1.71×10^{-4}	0.122
6	0.727	0.115	1.138	0.745	0.057	0.000	2.34×10^{-4}	2.12×10^{-4}	0.066
7	0.727	0.116	1.365	0.745	0.040	0.000	$3.48 imes 10^{-4}$	$3.10 imes 10^{-4}$	0.101
8	0.727	0.073	1.092	1.176	0.039	0.095	6.34×10^{-6}	5.64×10^{-6}	0.047
9	0.727	0.079	1.092	1.090	0.042	0.000	1.35×10^{-5}	1.35×10^{-5}	0.056
10	0.727	0.091	1.575	0.942	0.024	0.095	1.15×10^{-4}	1.08×10^{-4}	0.109
11	0.674	0.088	1.575	1.588	0.023	0.095	1.46×10^{-5}	1.31×10^{-5}	0.070
12	0.674	0.094	1.092	1.479	0.051	0.000	8.86×10^{-6}	$8.53 imes 10^{-6}$	0.041
13	0.727	0.111	1.820	0.773	0.022	0.095	4.58×10^{-4}	4.43×10^{-4}	0.093
14	0.727	0.106	1.122	0.811	0.054	0.095	1.58×10^{-4}	1.53×10^{-4}	0.050
15	0.674	0.095	1.092	1.471	0.051	0.095	3.59×10^{-6}	3.60×10^{-6}	0.044
16	0.727	0.107	1.365	0.805	0.037	0.095	2.17×10^{-4}	2.04×10^{-4}	0.085
17	0.674	0.103	1.138	1.355	0.051	0.095	2.88×10^{-5}	2.74×10^{-5}	0.035
18	0.674	0.111	1.365	1.257	0.038	0.000	5.22×10^{-5}	4.78×10^{-5}	0.090
19	0.674	0.111	1.138	1.248	0.055	0.000	3.98×10^{-5}	3.95×10^{-5}	0.038
20	0.674	0.121	1.743	1.149	0.026	0.000	1.38×10^{-4}	1.35×10^{-4}	0.089



Figure 1. Schematic diagram of the breakwater with an armor layer of Accropode.



Figure 2. The time series of wave overtopping simulated by the SWASH model. (**a**–**c**) represent Case 13, Case 14 and Case 15 in Table 1, respectively.

4. Influencing Factors of Equivalent Roughness Coefficient

Section 3 shows that the wave overtopping of the Accropode armored breakwater can be accurately simulated by the SWASH model through the appropriate apparent friction coefficient c_f (or Manning's coefficient n). In this section, we try to find the factors that affect the apparent friction coefficient. It provides a basis for fitting the empirical formula.

To obtain the c_f (or *n*) mentioned above, it is firstly necessary to collect a large number of data to determine its main influencing factors. The artificial neural network model, EurOtop is chosen instead of the CLASH physical model in the analysis of influencing factors, because the data of the Accropode armored breakwater in the EurOtop output is large enough to support this work. The EurOtop neural network model was developed based on a large amount of physical model data of the CLASH project by Delft Hydraulics [29], and the training values for the EurOtop neural network are basically from the CLASH database. Therefore, the mean wave overtopping discharges of Accropode armored breakwater for analysis of influencing factors was provided by the EurOtop neural network model.

Firstly, the correlation between the three parameters (R_c/H_{m0} , S_{op} , and slope angle) and dimensionless equivalent friction coefficient (C_{fA}) is analyzed. C_{fA} is the apparent friction coefficient calibrated by the physical model, and expressed as a function of n_A according to Equation (3). R_c/H_{m0} and slope angle are two dimensionless influence factors, which respectively represent the breakwater parameters, and S_{op} represents incident wave parameters. To analyze the influencing factors, numerical simulation examples need to be designed. The designed breakwater cross section with an armor layer of Accropode is shown in Figure 1, and the corresponding wave parameters are given in Table 2. The G_c values are 0 and 0.096. When $G_c = 0$, there is no crown wall, and when $G_c = 0.096$, the crown wall exists. The case where G_c is equal to 0 is discussed in this section. The ranges of R_c/H_{m0} and S_{op} given in Table 2 are 0.8–1.5 and 0.02–0.05, respectively, which is consistent with the scope of the CLASH physical model [29,30].

4.1. Data of Overtopping Discharges

Before conducting the study mentioned above, it is necessary to obtain the *q* of the corresponding cases to calibrate n_A . Among the methods for estimating the *q* based on the wave parameters and the critical dimensions of the breakwater, the EurOtop formula and the artificial neural network method are recognized as the two most applicable approaches [29–31].

To select a more suitable method, the mean error rate $(E_M = \frac{1}{N} \sum_{i=1}^{N} |S_i - M_i| / M_i)$, where S_i is the fitted value, M_i is the measured value from the physical model test) between the two methods and the physical model data are compared. The physical model data for breakwaters with an armor layer of Accropode were obtained from the CLASH database [4,29]. Figure 3 and Table 3 show the comparisons of results and E_M , respectively, for the approaches from both the neural network and the EurOtop empirical formula [29]. Generally, good matches when compared to a large number of examples and physical model tests can be noticed. However, the neural network produces a relatively better performance with $E_M = 0.26$, while the E_M of the EurOtop empirical formula is 0.62. Therefore, the neural network method is adopted to estimate the q in this study.

Case Number	<i>h</i> (m)	<i>R_c</i> (m)	<i>H_{m0}</i> (m)	T_p (s)	R_c/H_{m0}	Sop
1	0.727	0.086	0.108	1.855	0.80	0.02
2	0.727	0.086	0.108	1.515	0.80	0.03
3	0.727	0.086	0.108	1.312	0.80	0.04
4	0.727	0.086	0.108	1.173	0.80	0.05
5	0.727	0.086	0.086	1.659	1.00	0.02
6	0.727	0.086	0.086	1.355	1.00	0.03
7	0.727	0.086	0.086	1.173	1.00	0.04
8	0.727	0.086	0.086	1.049	1.00	0.05
9	0.674	0.139	0.093	1.722	1.50	0.02
10	0.674	0.139	0.093	1.406	1.50	0.03
11	0.674	0.139	0.093	1.218	1.50	0.04
12	0.674	0.139	0.093	1.089	1.50	0.05

Table 2. Designed breakwater parameters and wave elements.

Table 3. E_M between estimated and physical model values.

Empirical Formula	E_M	
EurOtop formula	0.62	
EurOtop neural networks	0.26	



Figure 3. Comparison of different empirical formulas and physical model tests.

4.2. The Influence of the R_c/H_{m0} on C_{fA}

The relationship between the dimensionless equivalent friction coefficient of the Accropode armor layer C_{fA} and R_c/H_{m0} is discussed in this section. From Figure 4a, it is noticed that when R_c/H_{m0} increases from 0.8 to 1.5, C_{fA} is shown to have a decreasing trend. When $S_{op} = 0.02$ and 0.03, C_{fA} decreases linearly with a rate of approximately 35%. When $S_{op} = 0.04$ and 0.05, the changing rate between C_{fA} and R_c/H_{m0} is different when R_c/H_{m0} increases from 0.8 to 1.0, where C_{fA} is reduced by 26% and 53%, respectively. However, when R_c/H_{m0} increases from 1.0 to 1.5, C_{fA} is only reduced by approximately 14.5%. This can be seen in Figure 4b when R_c/H_{m0} varies from 1.0 to 1.5, the *q* value is changed insignificantly. The decrease in C_{fA} is, therefore, not significant. According to the results, there is a certain relationship between R_c/H_{m0} and C_{fA} , and the linear correlation coefficient between them is only -0.56, so there is a strong nonlinear correlation between them.



Figure 4. Relationship between R_c/H_{m0} and C_{fA} (**a**), q (**b**) simulated by the SWASH model.

4.3. The Influence of the S_{op} on C_{fA}

As seen from Figure 5a, when $R_c/H_{m0} = 0.8$, S_{op} changes from 0.02 to 0.05, C_{fA} decreases from 0.107 to 0.049; when $R_c/H_{m0} = 1.0$, S_{op} changes from 0.02 to 0.05, C_{fA} decreases from 0.096 to 0.023; when $R_c/H_{m0} = 1.5$, S_{op} changes from 0.02 to 0.05, C_{fA} decreases from 0.096 to 0.023; when $R_c/H_{m0} = 1.5$, S_{op} changes from 0.02 to 0.05, C_{fA} decreases from 0.066 to 0.020. The reduction rates of C_{fA} range from -0.54 to -0.76, which is close to the reduction rates of q from -0.72 to -0.80, as shown in Figure 5b. According to the results, the linear correlation coefficient between S_{op} and C_{fA} is -0.75, therefore, there is a nonlinear correlation between them.



Figure 5. Relationship between S_{op} and C_{fA} (**a**), q (**b**) simulated by the SWASH model.

4.4. Influence of Slope Angle on C_{fA}

The results of wave overtopping at the 1:1.5 and 1:1.33 slope that are commonly used in the design of breakwaters with an armor layer of Accropode are compared to explore the effect of slope on C_{fA} . The results in Figure 6a show that when $R_c/H_{m0} = 0.8$, $S_{op} = 0.04$ and $C_{fA} = 0.070$, the numerically simulated q are basically consistent with those from EurOtop neural networks, and the incremental rate of the prediction for 1:1.33 slope versus that of 1:1.5 slope is 6%. According to Collins and Weir [32],

a greater critical wave height can be noticed for a case with a steeper slope, which can be confirmed by the results shown in Figure 6b. The critical wave height of the 1:1.33 slope is 6.9% higher than that of the 1:1.5 slope. Therefore, the steeper the slope is, the more waves are expected to propagate across the top of the breakwater.



Figure 6. Relationship between C_{fA} and increment rate of q of 1.33 slope relative to that of the 1.5 slope under R_c/H_{m0} varied from 0.8 to 1.5 (**a**), and the spatial evolution of H_{m0} under 1.33 and 1.5 slope (**b**) (S_{op} is 0.04).

In addition, for the case of $S_{op} = 0.04$, the incremental rate of the predicted q for the slope of 1:1.33 relative to that of the 1:1.5 slope decreases gradually from 6.0% to 2.4% when R_c/H_{m0} increases from 0.8 to 1.5, and the corresponding C_{fA} decreases from 0.070 to 0.044. This further suggests that the difference of q caused by a small change of slope (e.g., 1:1.5 versus 1:1.33) is relatively small. The small difference in the simulations of the q values can be reflected by changing in the terrain slope rather than adjusting the equivalent roughness coefficient.

5. Development of an Empirical Formula of the Equivalent Roughness Coefficient

The analyses described in the previous sections show that the equivalent roughness coefficient of the Accropode armor needs to be calibrated by numerical simulation based on the q of the physical model, which is generally not convenient for practical applications. For this reason, this study is further extended to develop an empirical formula of n_A , so that it can be directly applied to the practical project of estimating the wave overtopping.

5.1. Fitting

From the analyses described in Section 4, C_{fA} has a complex nonlinear relationship with R_c/H_{m0} and S_{op} . For this reason, the Quasi-Newton method [33,34], which has a good advantage for nonlinear fitting, is used to develop the empirical formula. The parameter directly used in the numerical simulation is the equivalent roughness coefficient n_A , which is related to the dimensionless equivalent friction coefficient C_{fA} in Equation (3). Therefore, n_A also has a complex nonlinear relationship with R_c/H_{m0} and S_{op} as C_{fA} according to its expression. The fitted empirical formula is defined as an expression of $f\left(\begin{array}{c} R_c/H_{m0}, & S_{op}, & n_A \end{array}\right) = 0$ to be more convenient for use in numerical simulations.

In addition to the data summarized in Section 4, more calculations and results for the cases with $R_c/H_{m0} = 1.2$ and 1.4 and the corresponding S_{op} values varying from 0.02 to 0.05 are added in the fitting process. Moreover, the added results also include those from the case with $G_C = 0.096$ m, which indicates that the crown wall is considered. The empirical formula of n_A developed based on the Quasi-Newton method is given as follows:

$$n_{A} = \exp \left(\begin{array}{c} (a_{1} + a_{2} \times \ln(R_{c}/H_{m0}) + a_{3} \times (\ln(R_{c}/H_{m0}))^{2} + a_{4} \times (\ln(R_{c}/H_{m0}))^{3} \\ + a_{5} \times S_{op}) / (1 + a_{6} \times \ln(R_{c}/H_{m0}) + a_{7} \times (\ln(R_{c}/H_{m0}))^{2} \\ + a_{8} \times (\ln(R_{c}/H_{m0}))^{3} + a_{9} \times S_{op} + a_{10} \times S_{op}^{2}) \end{array} \right)$$
(7)

$$a_1 = -3.972 \times 10^3;$$
 $a_2 = 2.235 \times 10^3;$ $a_3 = -3.424 \times 10^5;$ $a_4 = 5.500 \times 10^5;$
 $a_5 = -4.469 \times 10^5;$ $a_6 = -3.684 \times 10^3;$ $a_7 = 1.451 \times 10^5;$ $a_8 = -2.393 \times 10^5;$
 $a_9 = 3.776 \times 10^5;$ $a_{10} = -4.435 \times 10^6;$

The effective range of the empirical formula is $0.8 \le R_c/H_{m0} \le 1.5$ and $0.02 \le S_{op} \le 0.05$. The relationship between the calibrated n_A and the n_A calculated by Equation (7) is shown in Figure 7a. The numerically simulated q with n_A calculated by Equation (7) agrees well with the predicted results from the neural network model, as shown in Figure 7b. According to the results, the R^2 value between the calculated n_A and the calibrated n_A is as high as 0.92, and the R^2 value between the simulated q and the predicted results of the neural network reaches a high value of 0.97. This suggests that the established empirical formula (Equation (7)) can be used to produce well fitted results.



Figure 7. Comparison of the calculated n_A and the calibrated n_A (**a**) and the predicted q by the neural network and numerically simulated q (**b**) by the SWASH model.

5.2. Verification

The performance test of the developed empirical formula requires the wave overtopping data measured from the physical model tests. In this study, only part of the *q* of the Accropode armored breakwater measured by the physical model tests of the CLASH project is used to compare with the numerically simulated *q* based on the n_A predicted by the empirical formula (Equation (7)). It can be seen from Table 1 that the verification data contains examples that are considered with $(G_C = 0.095 \text{ m})$ and without $(G_C = 0)$ the effect of a crown wall. Figure 8a shows that the value of n_A is discretely distributed within the effective range of the empirical formula. Therefore, the selection of the verification data is essentially reasonable and effective.

From the verification results in Figure 8b, the *q* simulated based on the n_A estimated from Equation (7) agrees well with the measured values from physical model tests. The standard deviation between them is 0.00012 m³/m/s, and the E_M is 0.24. The *q* values calculated based on the EurOtop neural networks are also compared with the values from physical model tests (See also in Figure 8b). The standard deviation between them is 0.00014 m³/m/s and the E_M is 0.26, which are similar to the comparisons of *q* predictions from the numerical simulations by the SWASH model.



Figure 8. The predicted n_A values (**a**) and the comparisons of *q* from the physical model tests (CLASH project) with the *q* simulated by the SWASH model based on the estimated n_A (Equation (7)) and the *q* values predicted by the neural networks (**b**) of Accropode armor.

6. Conclusions

In this study, a new method with the determination of the defined apparent friction coefficient for bottom shear stress calculation is developed in combination with the numerical simulations of the non-hydrostatic SWASH wave model for wave overtopping on breakwaters with an armor layer of Accropode. The permeable Accropode armored breakwater is treated as impermeable terrain but with the equivalent effect of friction for a description of the comprehensive energy dissipation caused by the roughness and seepage during the wave overtopping process. The conclusions are summarized as follows:

(1) When the SWASH model is applied to simulate the wave overtopping at the Accropode armored breakwaters using the concept of the apparent friction coefficient, it is found that the related equivalent roughness coefficient n_A as depends heavily on the model test results must be obtained with proper calibrations so that the meaningful apparent friction coefficient and the numerically simulated wave overtopping discharges can be consistent with the physical model results.

(2) The analysis of influencing factors on the equivalent roughness coefficient shows that certain negative correlations exist between the apparent friction coefficient and the two dimensionless variables of relative crest freeboard, R_c/H_{m0} , and wave steepness, S_{op} , and the linear correlation coefficients are -0.56 and -0.75, respectively. In addition, the breakwater slope has little effect on the apparent friction coefficient when changing from 1:1.5 to 1:1.33.

(3) The developed equivalent roughness coefficient formula n_A for the estimation of the apparent friction coefficient, which is used directly in the model simulations, can allow the bottom shear stress term calculated in the momentum equation to suitably represent the bottom friction effect on the process of wave overtopping and to determine with reasonable values of overtopping discharges at an Accropode armored breakwater without the generally considered calibration procedure through physical model tests. The recommended applicable ranges of the physical variables from the present study are $0.8 \le R_c/H_{m0} \le 1.5$, $0.02 \le S_{op} \le 0.05$ and for both the 1:1.5 and 1:1.33 breakwater slope.

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