Supplementary Materials

Formula Derivation

Equation S1:

Under steady-state conditions, the heat transfer per unit area from the endothermic coating to the outer quartz tube is the same as the heat loss per unit area from the endothermic coating to the ambient environment:

$$U'(T_{abs} - T_a) = (T_{abs} - T_g)(h_{rig} + h_{cig}). \qquad \text{(S1-1)}$$

From formula (S1-1), the temperature of the endothermic coating can be expressed as:

$$T_{abs} = T_g + \frac{U'(T_{abs} - T_a)}{h_{rig} + h_{cig}}. \qquad \text{(S1-2)}$$

Equation (S1-2) represents the relationship between the temperature of the endothermic coating of the tubes, the temperature of the outer quartz tube, and the ambient temperature, indicating the heat balance between the tube system and the ambient environment. As h_{rig} is a function of T_{abs}, equation (S1-2) is an implicit function about T_{abs}.

According to formulas (1) and (S1-1), the net heat increase, Q_{fnet}, of salty water in the vacuum tube can be expressed as:

$$\begin{aligned} Q_{fnet} &= A_{abs}F_c q_{abs} = A_{abs}F_c(\alpha\tau I - q_{loss}) \\ &= A_{abs}F_c[\alpha\tau I - U(T_{abs} - T_a)] \\ &= A_{abs}F_c[\alpha\tau I - 2U'(T_{abs} - T_a)]. \qquad (6) \end{aligned}$$

Equation S2:

The flow in the tubes is steady-state heat transfer, and formula (10) can be transformed to:

$$\alpha\tau I = h_1(T_{abs} - T_g) + h_f(T_{abs} - T_f). \qquad \text{(S2-1)}$$

Formulas (11) and (12) are then substituted into formula (S2-1), which gives:

$$\alpha\tau I = q_{loss} + \dot{m}\Delta H. \qquad \text{(S2-2)}$$

The net heat taken away by the salt water along the tube axis per unit area is recorded as $q_{net} = \alpha\tau I - q_{loss}$ and $\dot{m}\Delta H$ is the expression of the heat energy contained in salty water in the tube unit. The total heat absorbed by the endothermic coating is related to the heat absorption area:

$$Q_{abs} = A_{abs}F_c[\alpha\tau I - U(T_{abs} - T_a)], \qquad \text{(S2-3)}$$

where A_{abs} is the endothermic area and F_c is the heat collecting efficiency factor of tubes. It can be seen that formula (6) is the same as formula (S2-3) and, so, the total heat absorbed by the endothermic coating is the same as the net heat gain of salty water (i.e., $Q_{abs} = Q_{fnet}$). According to formula (10), formula (12), and formula (S2-3), we can establish the basic thermal energy balance equation of the endothermic inner tube and salty water.

Equation S3:

The thermal energy conservation equation of the micro-body is:

$$S\Delta x - U\Delta x(T - T_a) + \left(-k_i\delta\frac{dT}{dx}\right)\bigg|_x - \left(-k_i\delta\frac{dT}{dx}\right)\bigg|_{x+\Delta x} = 0. \qquad (13)$$

When the equation (13) is divided by Δx and the limit as Δx approaches 0 is taken, then:

$$\frac{d^2T}{dx^2} = \frac{U}{k_i\delta}\left(T - T_a - \frac{S}{U}\right). \qquad \text{(S3-1)}$$

The two boundary conditions of the second-order differential equation are as follows: The tube is symmetric about the centerline and the temperature at $x = L$:

$$\frac{dT}{dx}\bigg|_{x=0} = 0, T|_{x=L} = T_L. \qquad \text{(S3-2)}$$

To simplify the equation, we define two variables:

$$m = \sqrt{\frac{U}{k_i \delta}}, \qquad \text{(S3-3)}$$

$$\emptyset = T - T_a - \frac{S}{U}. \qquad \text{(S3-4)}$$

Substituting the variables m and $\emptyset$ into formula (S3-1), the following can be obtained:

$$\frac{d^2\emptyset}{dx^2} - m^2\emptyset = 0. \qquad \text{(S3-5)}$$

The boundary conditions of the formula (S3-5) are:

$$\left.\frac{d\emptyset}{dx}\right|_{x=0} = 0, \emptyset|_{x=L} = T_L - T_a - \frac{S}{U}. \qquad \text{(S3-6)}$$

The general solution of the above formula is:

$$\emptyset = C_1 \sinh mx + C_2 \cosh mx, \qquad \text{(S3-7)}$$

where C_1 and C_2 are constants and the boundary conditions are substituted into the general solution:

$C_1 = 0$, $C_2 = (T_L - T_a - \frac{S}{U})/\cosh mL$.Then, formula (S3-7) can be transformed to:

$$\emptyset = \left(T_L - T_a - \frac{S}{U}\right)\frac{\cosh mx}{\cosh mL}. \qquad \text{(S3-8)}$$

Substituting formula (S3-6) into formula (S3-8):

$$\frac{T - T_a - S/U}{T_L - T_a - S/U} = \frac{\cosh mx}{\cosh mL}. \qquad \text{(S3-9)}$$

Along the direction of salty water flow, the heat transfer from the center of tubes to the right end is q':

$$q' = -k_i\delta\left.\frac{dT}{dx}\right|_{x=L} = \left(\frac{k_i\delta m}{U}\right)[S - U(T_L - T_a)]\tanh mL, \qquad \text{(S3-10)}$$

where $k_i\delta m/U$ is equal to $1/m$. The formula (S3-10) only considers the heat transfer in the right half of the tube section and, so, the heat loss q for the entire tube section is:

$$q = 2L[S - U(T_L - T_a)]\frac{\tanh mL}{mL}. \qquad \text{(S3-11)}$$

Defining the heat collecting efficiency factor F as

$$F = \frac{\tanh mL}{mL}, \qquad \text{(S3-12)}$$

and solving the equation, we can define the heat collecting efficiency factor F_c of the endothermic coating as:

$$F_c = 1 - \frac{\tanh mL}{mL}. \qquad \text{(14)}$$

Equation S4:

The internal temperature distribution of the tube is

$$U' = \frac{1}{{h_{rig}}^{-1} + \left(h_{rga} + h_{cga}\right)^{-1}} \qquad \text{(16)}$$

$$= \frac{1}{\frac{\left(T_g^2 + T_{abs}^2\right)^{-1}\left(T_g + T_{abs}\right)^{-1}}{\sigma}\left[\frac{1-\varepsilon_p}{\varepsilon_p} + \frac{1}{F_{ig}} + \frac{(1-\varepsilon_g)A_{abs}}{\varepsilon_g A_g}\right] + \left[\varepsilon_g\sigma\left(T_g^2 + 2T_a^2\right)\left(T_g + T_a\right) + (5.7 + 3.8V_{air})\right]^{-1}}.$$

As there is no convective heat transfer between the inner and outer tubes, we have $h_{cig} = 0$. Using the known quantities T_g and T_a (measured), we first assume T_{abs}, calculate U' from formula (16), and then calculate T_{abs} by formula (S1-2). If the calculated value is not equal to the assumed value, we repeat the above iterative steps until the calculated T_{abs} is equal to the assumed value; then, the iteration terminates with the output T_{abs} and U'. The total heat transfer coefficient $U = 2U'$, such that the net heat increment per unit area of each unit q_{net} can be calculated by formula (12). The sum of local heat increments of the tubes Q_{fnet} is calculated by formula (6). Finally, according to the measured initial temperature, T_{fi}, the outlet water temperature T_{fo} of the unit is calculated from the formula (7) as the inlet water temperature of the next unit; that is, $T_{fo,n} = T_{fi,n+1}$. Finally, we can calculate the water temperature distribution throughout the entire tube.

Equation S5:

The average water temperature in the tubes is approximately taken as the temperature of the endothermic coating, then the total heat loss at unit $n + 1$ is obtained as:

$$Q_{loss,n+1} = A_{abs}[U_{n+1}(T_{fm,n+1} - T_a)], \quad (20)$$

where U_{n+1} is the total heat loss coefficient at unit $n + 1$ and $T_{fm,n+1}$ is the average water temperature at unit $n + 1$. U_{n+1} is determined according to the following formula:

$$U_{n+1} = h_{rga,n+1} + h_{cga,n+1}, \quad \text{(S5-1)}$$

where $h_{rga,n+1}$ is the radiative heat transfer coefficient of the quartz tube to the air and $h_{cga,n+1}$ is the convective heat transfer coefficient of the quartz tube to the air. The calculation of $h_{cga,n+1}$ refers to formulas (4) and (5). $T_{fm,n+1}$ is equal to the average of the sum of the inlet water temperature and the outlet water temperature of unit $n + 1$:

$$T_{fm,n+1} = \frac{T_{fi,n+1} + T_{fo,n+1}}{2}. \quad \text{(S5-2)}$$

The relationship between the inlet and outlet water temperature of unit $n + 1$ is:

$$T_{fi,n+1} = \frac{Q_{loss,n+1}}{\dot{m}C_f} + T_{fo,n+1}. \quad \text{(S5-3)}$$

The theoretical efficiency, η, of a single solar vacuum tube system can be determined as follows:

$$\eta = \frac{(\sum_{i=1}^{n} Q_{fnet,i}) - Q_{loss,n+1}}{A_{abs}I} = \frac{\dot{m}C_f(T_{fo} - T_{fi})}{A_{abs}I}. \quad \text{(S5-4)}$$