



Physical Parameterization of IDF Curves Based on Short-Duration Storms

Alfonso Gutierrez-Lopez ^{1,*}, Sergio Bernardo Jimenez Hernandez ² and Carlos Escalante Sandoval ³

- ¹ Water Research Center, Centro de Investigaciones del Agua-Queretaro (CIAQ), International Flood Initiative, Latin-American and the Caribbean Region (IFI-LAC), International Hydrological
- Programme (IHP-UNESCO), Universidad Autonoma de Queretaro, 76010 Queretaro, Mexico
 ² Facultad de Ingenieria, Centro de Investigacion y Desarrollo en Ingenieria Portuaria, Maritima y Costera (CIDIPORT), Universidad Autonoma de Tamaulipas, 87000 Tamaulipas, Mexico
- ³ Facultad de Ingenieria, Universidad Nacional Autonoma de Mexico Cd. Universitaria, Coyoacan, 04510 Ciudad de Mexico, Mexico
- * Correspondence: alfonso.gutierrez@uaq.mx; Tel.: +52-442-192-1200 (ext. 6401)

Received: 1 April 2019; Accepted: 8 August 2019; Published: 30 August 2019



Abstract: Intensity–duration–frequency (IDF) curves are empirical mathematical formulations that have been used for years in engineering for planning, design, and operation of hydraulic projects. The expression proposed by Sherman (1931) has been validated and used largely by many researchers. In all cases, the four parameters of this formulation are obtained through a numerical procedure. Although these parameters are obtained from historical rainfall observations, the optimization of these parameters implies an infinite combination between them and all those solutions would be valid. Of the four parameters, only one of them (C) has units, and for this reason, a physical sense of parameter C is searched for. Having certainty that some of them can be measured in situ would represent a great advance for modern hydrology. With data from 523 storms monitored every minute, a parametric adjustment was made to the Sherman equation and the typical duration of storms at each site was also obtained. To demonstrate how rainfall intensities vary with the change in C value, rainfall intensities calculations for of 5, 10, 15, and 20 min rainfall duration are used to validate the proposed methodology. The results show that typical storm duration is correlated with the additive parameter of Sherman's formula.

Keywords: IDF curves; short duration storms; Sherman equation; scale factor; Mexico

1. Introduction

Rain is defined by two variables: magnitude and frequency, which is known as the precipitation regime. The magnitude of rainfall is the total precipitation that occurs in millimeters, while the frequency is associated with the storm duration. The frequency in terms of probability is expressed by the return period. Therefore, the precipitation intensity is directly proportional to the return period and inversely proportional to the storm duration. The authors in [1] define intensity–duration–frequency curves (IDF) as the relationship between the mean precipitation intensity and the frequency of occurrence (inverse of the return period). Intensity–duration–frequency curves are applied worldwide in hydrological calculations to obtain the design precipitation depth, precipitation intensity, and return periods. These IDF curves are hydrologic risk analysis tools used in engineering for planning, design, and construction of hydraulic works, e.g., flood protection works [2]. Today, modern hydrology is looking for new and more precise mathematical formulations to fit these IDF curves. The first researcher to find the relationship between 24-h precipitation and short-term precipitation was [3], who found that



the rainfall depths of 5, 10, 15, and 30 min are inversely correlated with the rainfall depth of one hour, establishing empirical factors 0.29, 0.45, 0.57, and 0.79 respectively. Using data from nearly 200 rain gauges, Hershfield found values for non-dimensional factors that allowed the transformation of daily rainfall data to calculate IDF curves. In this research, frequency analysis was performed by fitting the Gumbel and Lognormal probability functions, using the maximum rainfall depth accumulated in 24 h to provide functional mathematical relationships for desegregation of rainfall in minutes. Even today, the same distributions are used for the analysis of the maximum annual precipitation associated with duration of 1, 2, 3, 6, 12, 18, 24, 48, and 72 h [4]. There are different methods or approaches to obtain IDF curves. A considerable literature on IDF curves has been published. The reference studies for IDF curves for Australia were conducted by [5–7]. In Belgium, the reference IDF curves were presented by [8,9]. In Canada, a number of investigations in the field of IDF curves were published by [10-16]. In Denmark [17,18] used Sherman's formulation to define IDF curves. In Sweden, detailed studies have also been formalized to define IDF curves [19-21]. To date, various methods have been developed and introduced in Spain to compute IDF curves [22-24]. In Taiwan, IDF curves were prepared according to the procedure used by [25–27]. Reference [28] have recently developed a methodology for select IDF curves in China. In Peninsular Malaysia, a software tool for estimated IDF curves has been developed as an Excel add-in by using Visual Basics for Applications [29]. A site in Java, Indonesia, was selected to demonstrate the application of IDF curves from 6-h rainfall to derive IDF curves for three sites where IDF curves exist [30]. In the United States, the National Oceanic and Atmospheric Administration (NOAA) Atlas #14 also provides IDF curves information and is usually used to construct design storms in the country [31]. Latin America is not an exception since the study of IDF curves is a frequent practice. The authors in [32] constructed IDF curves for the metropolitan region of Chile, for duration of 1, 2, 4, 6, 8, 12, and 24 h, and adjusted a probability function for extreme Gumbel-type events. They also built practical use tables that allow extrapolating curves to nearby areas with similar climate regimes. Also in Chile, [33] constructed IDF curves for the regions of Valparaiso and O'Higgins, from the analysis of which they observed that the maximum intensities do not follow a specific distribution pattern in these regions. One year later, this methodology was applied to develop IDF curves in the Bio-Bio and Araucania region, where it was determined that the maximum intensities increase as it progresses latitudinal from north to south [34]. In Argentina, [35] conducted a review of the parameters of the IDF curves for La Pampa region of for 30, 60, and 120 min duration. Recently, [36] determined the IDF regional curves for the Argentine Republic from pluviographic data for a discontinuous period of 25 years. In Uruguay, [37] updated the IDF curves for the Montevideo department. In Mexico, [38] used the formulations proposed by [39–42] to construct IDF curves from rainfall data near the Gulf of Mexico. In 1990 the authors of [43] were the first to analyze the relationship in Mexico between the precipitation depth in one hour (R_1^T) and the accumulated rainfall in 24 h (R_{24}^T) : $R = R_1^T / R_{24}^T$. In this research, it was found that at higher elevations above sea level, the value of R is the highest and vice versa. Similar work was updated for urban areas in 2010 [44], where an average ratio (R) of 0.479 with extreme values of 0.646 and 0.204 was obtained. In all the above-mentioned cases, the equations proposed by [39] are used [25,40,45,46]. Regarding storm duration, there is evidence of IDF curves constructed with storm data of less than 10 min, since most rain gauges measure at 10-min time intervals [26]. However, from the studies of [45], an empirical equation was proposed to estimate the rainfall depth for storm durations from 5 to 120 min. Reference [47] proposes a model that correlates intensity and storm duration, and this model is valid for any duration between 5 and 20 min and has been used to estimate the intensity of high-intensity storms [48]. It is to be mentioned that until now, no one has given physical sense to the parameters of the equations proposed to fit the IDF curves [49]. From the type of equation proposed by [39,40] it is demonstrated that for small values of storm duration, its intensity increases exponentially. This means that the intensity of precipitation is very sensitive to the storm duration. In cases where in the proposed equation for estimating IDF curves there is a parameter that is added to the storm duration (in the denominator), it would be appropriate for that parameter to have physical significance. In order to find the meaning of this parameter, it

would be necessary to have storm records for durations of less than 5 min. In fact, 5 min seems to be the critical duration that should be studied. Although the storm duration of 5 min does not make sense for the design of a hydraulic work, this duration allows IDF curves to be constructed and to be used in the correct estimation of storm intensities for greater storm durations [50]. Reference [51] tell us "The calculation of intensity-duration-frequency (IDF) relationships for rainfall requires records of rainfall accumulated over periods shorter than 1 day when the application is for small and/or urban catchments. For application to small and/or urban catchments, rainfall records of a sub-daily resolution are required. Whilst records of daily rainfall are common, records of rain falling over shorter periods (say, 5 min, from which records for longer duration can be derived) are much less common". Therefore, it is not clear what the critical storm duration will be. Usually, the IDF curve family is provided, where each curve corresponds to the duration-intensity relationship for a specific exceeding probability. Therefore, the main objective of this paper is to use Sherman's formulation [39] through storm data that has been measured in intervals of less than 5 min to obtain IDF curves. It is also intended to show that not all parameters in Sherman's formulation should be estimated numerically. Although the estimation of all these parameters comes from historical data, it is demonstrable that at least one of them must be inferred from the storm conditions observed in situ. In this regard, the importance of using physically meaningful parameters within our hydrological models is recognized.

2. Materials and Methods

2.1. Sherman Parameterization

Reference [39] proposes the mathematical and graphic representation of the intensity (i)-duration (d)-frequency (T) curves as:

$$i_{d}^{T} = f(x) = \frac{P(T)}{Q(d)}; P(T), Q(d) \in \mathbb{R}[\text{time}]$$
(1)

In this expression, both the numerator and the denominator are a function of time. P(T) is a function of the mean number of years (T) and indicates the quartile for the cumulative frequency (1 - 1/T) of a probability distribution function of an aleatory variable $Y = \{I_j(d_j + C)^n; j = 1, \dots\}$. Which can be expressed as $P(T) = F_Y^{-1}(1 - \frac{1}{T})$ where $F^{-1}(\cdot)$ define the inverse of a probability distribution function [52]. The other term is also a function of time. If it is assumed that the denominator can be expressed as a polynomial that allows factoring, it can be written:

$$Q(d) = (d - C_1)^{n_1} (d - C_2)^{n_2} \cdots (d^2 + d + \dots + C_i)^{n_i} \cdots$$

This term is called the scale or duration factor. Substituted as the denominator in Equation (1) it can be written as:

$$i_{d}^{T} = \frac{kT^{m}}{\left(d^{\theta} + C\right)^{n}}$$
⁽²⁾

where

i rainfall intensity, in mm/h T return period, in years d storm duration, in minutes

k, m, n, C, and θ can be obtained from rainfall data by using optimization techniques and the least squares method.

A very important property of this expression is that the scale factor that characterizes the duration can be formulated independently of the distribution of annual maximum precipitation, defined by $F^{-1}(\cdot)$ [53]. Equation (2) is widely used, and several authors have proposed different values of the

parameters k, m, n, C, and θ . In all cases, these parameters are estimated by numerical, analytical, statistical and optimization procedures. Also, optimum values of θ can be calculated using a trial and error procedure [42,54]. In no case, however, is there any evidence that any of these parameters have physical significance. That is, the parameters in Equation (2) have not been proved to be related to any physiological, climatological or other environmental characteristics. Even today, in such varied latitudes and conditions, this practice is carried out by fitting the parameters of the duration factor, from the data sample [55]. Table 1 shows the values proposed by different authors in the parameterization of Equation (2). The most commonly used formulas are those known as Sherman and Bernard.

Formulation Known As	k	m	n	θ	С
Law of Montana [47]	-	-	0	-	-
Sherman [39]	-	-	-	1	-
Bernard [40]	-	0	1	1	0
Talbot/Linsley [56] ¹	-	0	1	1	-
Wenzel/Kimijima [57]	-	0	1	-	-
Chow [41]	-	-	1	1	-
Koutsoyiannis [42] ²	-	-	-	1	-
Seong [52]	-	1	$(n \cdot m)$	1	-

Table 1. Values of the coefficients of Equation (2) according to different authors.

¹ For duration (d) between 5 and 20 min and greater than 60 min. ² With $T^m = m - Ln[-Ln(1-1/T)]$.

To contrast the importance of parameter C, Figures 1 and 2 show a case where only the value of C changes. Equation (2) is used as an empirical formulation to construct IDF curves that tend to converge as duration (d) increases (see Figures 1 and 2), meaning that the longer the duration (d), the IDF curves will be seen as parallel lines. Thus, there is a value of C that causes the intensity of a 10-min rain to be almost the same as a 60-min storm. Equation (2) is a bivalent model since it is a function of the return period (T) and the storm duration (d); both in the time domain. For small duration rainfall, the IDF curves are asymptotic to the axis of the ordinates [2] (see Figure 2), and its exponential growth is conditioned precisely by the scale factor $(d^{\theta} + C)^{n}$. Reference [52] showed that the scale factor $(d^{\theta} + C)^n$ determines the statistical properties of historical data with which the IDF curves are constructed. Properties such as median, dispersion or asymmetry are reflected in the scale factor. This assumption becomes crucial, because from Equation (1) it is observed that parameter C is additive with the storm duration, and therefore it cannot be negative and must have time units. In this way, it is accepted that the statistics of the data sample can influence the differences in the intensity dispersion {Ij} when plotting the IDF curves. Certainly, in this process, the value of parameter C depends on several statistical conditions such as asymmetry or outliers of the sample [52]. However, the scientific literature published so far has not specified which duration or which statistics are related to the value of C. As already mentioned, research in this area has been limited to the estimation of the parameters k, m, n, C, and θ , using the technique of least squares or optimization.

"The Montana curve showed significant deviations at the lower time scales (for durations from 0 to 10 min), although giving a good representation of the decay of expected maximum rainfall intensities for larger durations (see Figure 1). The limitation of this formula is obvious, since it estimates rainfall intensities tending to infinity when time approaches to 0, and therefore a resulting overestimation of rainfall intensities for low t values" [22], p. 676.



Figure 1. Traditional representation of an intensity–duration–frequency (IDF) curve at station CH-03. k = 51.55; m = 0.53; n = 0.53; $\theta = 1$; C = 0.



Figure 2. Traditional representation of an IDF curve at station CH-03. k = 51.55; m = 0.53; n = 0.53; $\theta = 1$; C = 50.

2.2. Traditional Methodologies for Estimating IDF Curves

It is important to note that in 2001 the relevance of having precipitation measurements for short duration was already mentioned [22]. In that year it was specifically mentioned that there were no precipitation intensity measurements for less than 10 min rainfall. While different analytical IDF curves fit quite well quantiles for durations t > 10 min, becoming almost coincident, large deviations are found in the interval (0, 10) minutes. In the limiting case when $t \rightarrow 0$, the corresponding ordinate of the curve imax (t = 0) can be regarded as a maximum instantaneous rainfall intensity (never measured), which is strongly linked to the shape of the curve, in particular to its initial course (for t < 10 min) [22], p. 676. The traditional methodologies used for the estimation of IDF curves are detailed below.

Method-a. By applying the logarithmic conversion, it is possible to convert the Equation (2) into a linear equation, and thus to calculate all the parameters related to the equation. $\log i = \log k + m \log T - n \log(d^{\theta} + C)$ [58]. It is the most used procedure in the literature and the most generalized [59]. Non-linear optimization procedures could also be applied [60].

Method-b. We proceeded by trial and error, adjusting the IDF curves by changing the value of C, until "visually" curves are transformed into straight lines when the axis of duration, in minutes, is represented in logarithmic scale [42].

2.3. Proposed Methodologies for Estimating IDF Curves

Method-c. Value of C weighted with the number of events (ST_i) for each analyzed storm duration (d_i) , in minutes, divided by the total number of recorded events (TST). An array similar to the one presented in Table 2 is used.

$$\overline{C} = \frac{1}{\text{TST}} \sum (\text{ST}_i \cdot d_i)$$
(3)

Method-d. Then C results from a change in intensity, directly proportional to the change in intensity with respect to duration (in logarithms). To complete the differential, a constant term is included that may or may not exist, depending on the statistics of the sample. This can be expressed as follows:

$$C f(i, \Delta t) = i \frac{\partial Ln i}{\partial Ln \Delta t} + t$$

Resolving the differential by the rule of the chain, we have:

$$y\frac{\partial w}{\partial z}$$
; $y\cdot\frac{\partial w}{\partial y}\cdot\frac{\partial y}{\partial x}\cdot\frac{\partial x}{\partial z}$; $y\cdot\frac{\partial y}{\partial x}\cdot\frac{1}{y}e^{z}$; $\frac{\partial y}{\partial x}e^{Lnx}$; $\frac{\partial y}{\partial x}x$

with y = i; $x = \Delta t$; $z = Ln(x) = Ln(\Delta t)$; w = Ln(x) = Ln(i) simplifying.

$$i \frac{\partial \operatorname{Ln} i}{\partial \operatorname{Ln} \Delta t} = i \frac{\partial \operatorname{Ln} i}{\partial i} \frac{\partial i}{\partial \Delta t} \frac{\partial \Delta t}{\partial \operatorname{Ln} \Delta t} = i \frac{\partial i}{\partial \Delta t} \frac{1}{i} e^{\operatorname{Ln}(\Delta t)} = \frac{\partial i}{\partial \Delta t} e^{\operatorname{Ln}(\Delta t)} = \left(\frac{\partial i}{\partial \Delta t} \Delta t\right) + t$$
(4)

This expression represents the change of C as a function of intensity. Therefore, the first term $\left[\frac{\partial i}{\partial \Delta t} \Delta t\right]$ corresponds to the change of intensity in a time interval during the entire time interval: This is the duration-weighted with the number of storms occurring between 5–20 min. The term that has a similar structure to the duration-factor of the formula of [39]. The second term (t) is the duration that characterizes that intensity and is the typical duration of a storm (d_t). The same can be written as the sum of the value of the weighted coefficient C plus the typical storm duration storm, in minutes, which is presented in situ. That is:

$$\hat{C} = \overline{C} + d_t \tag{5}$$

2.4. Queretaro Hydrometeorological Network

To date, in Mexico, we have access to precipitation data monitored every minute by a network of over 20 rain-gage stations in the center of the country (Queretaro State). This database includes records from June 2012 to October 2018, therefore, data from 523 storms monitored in said network are used every minute (Queretaro rainfall warning system, RedCIAQ). There are a few networks that monitor in real time every minute; but only with adjustment coefficients precipitation data for such short intervals can be estimated [58]. The typical storms that occur in each station were determined, that is, the temporal distribution or typical histogram for each station. In Table 2, the distribution of precipitation for the Queretaro city is observed. The purpose of using this database is to know the physical meaning of parameter C and its possible relationship with the statistics of the historical data sample. This study could stimulate debate, reveal, provide evidence, and contribute to the study of parameter C within the duration factor of the [39] equation. Studying storm intensities for a short

duration is not a new idea, and although these are not used for the design of hydraulic works, it is important to study intensities of up to 100 mm/h. These are documented and presented for duration less than 5 min [59,61]. The basin under study is located in the city of Queretaro, which has an area of 11,769 km². Likewise, the typical rainfall intensities (representative) for each station were studied. It is observed that in the entire raining season, the typical duration is 5 and 6 min, except for SJA-09 station (San Juan del Río city Queretaro), for which a more representative storm duration was 9 min. On the other hand, the station with the highest number of storms recorded was M-07 station, with a total of 82 storms for all months, and the one with the least storms recorded was SJA-09 station with 31 storms. Speaking about months, June is the month in which the highest number of storms was registered, with 108 storms analyzed, while in November only 30 storms were recorded. The analysis by duration showed that the duration of 5 min is the one that recorded the most events, with a total of 100 events for that duration, while the duration of 20 min was only recorded 11 times. It is important to note that as the duration increases, the number of events registered decreases. This proves the procedure

proposed by [39].

Table 2. Number of storms (ST_i) for some stations of Queretaro rainfall warning system (Reference)	edCIAQ).
---	----------

Duration of the Storm (minutes)										TST					
ID	5	6	7	8	9	10	11	12	13	14	15	16	18	20	101
C-01	9	5	1	5	8	4	2	3	0	1	3	4	1	2	48
B-02	7	14	2	3	4	10	7	5	6	4	3	1	4	3	73
CH-03	14	11	7	6	6	2	3	6	4	2	4	2	2	0	69
CC-04	10	1	0	6	3	5	0	3	1	0	3	0	1	0	33
C-05	7	10	7	8	7	6	2	5	2	0	4	7	3	3	71
ER-06	26	11	0	6	9	7	1	4	1	2	5	2	1	2	77
M-07	13	16	7	6	10	5	6	2	4	2	7	2	2	0	82
RP-08	10	2	7	2	1	3	4	0	2	3	1	1	2	1	39
SJA-09	4	4	4	1	6	3	2	2	2	2	0	1	0	0	31
-	100	74	35	43	54	45	27	30	22	16	30	20	16	11	523

Source [62].

3. Results

Using the historical data of the 20 rain-gauge stations with complete data from June 2012 to October 2018, the calculation of their respective IDF curves was performed. At each station, only storms with duration of 5 to 20 min were selected. Following this, the traditional adjustment of IDF curves was carried out, estimating the parameters by multiple correlations. Table 3 shows the identification of some rain gauges stations, as well as the values of the typical duration of the storms, the number of storms analyzed for each station, and the values of the coefficient C calculated for the conditions described as follows. Method-a: Value obtained by the traditional multiple regression procedure. Method-b, "adjusting" the value of C to have the IDF curves transformed into a straight line, where the axis of the duration is represented in logarithmic scale [42]. Method-c, weighting C with the number of events for each storm duration analyzed, and method-d, formulation proposed with physical meaning of C.

Station ID	TST (d _t)	i _t (mm/h)	Method-a (min)	Method-b (min)	Method-c (min)	Method-d (min)
C-01	48(5)	18.0	13.55	9.85	10.0	14.85
B-02	73(6)	15.0	21.02	11.00	16.0	17.00
CH-03	69(5)	15.6	12.50	8.88	10.5	13.88
CC-04	33(5)	15.0	12.00	11.18	12.5	16.18
C-05	71(6)	15.0	13.03	10.98	8.0	16.98
ER-06	77(5)	15.6	12.07	7.70	13.0	12.70
M-07	82(6)	18.0	18.01	11.09	17.5	17.09
RP-08	39(5)	15.6	15.42	11.07	12.5	16.07
SIA-09	31(9)	15.0	13.51	11.29	12.5	20.29

Table 3. Values of coefficient C, for the proposed analysis conditions.

Adapted from [62]. i_t —is the typical (most frequent) intensity of total recorded storms (TST) for 5–20 min, in mm/h. d_t —is the typical storm duration in situ, in minutes.

It is verified that the correct value of C modifies the intensity of precipitation during the duration (d) of Sherman equation (1931) C, additionally to the duration of the storm. As an example, Table 4 shows the values of rainfall intensities (mm/h) calculated for different storm duration estimated at station CH-03 for different proposed values of parameter C. The proposed formulation to calculate C is in accordance to the values that are traditionally used to adjust the IDF curves, but in this case, if the conditions M-c and M-d are taken it can be said that the value of C has a physical meaning according to the duration and occurrence of storms.

Table 4. Rainfall intensities (mm/h) for different short duration storms, estimated at station CH-03 with different proposed values of parameter C (min).

d _i (min)	Typical Storm (In Situ)	C = 0	(M-a) C= 12.5	(M-b) C = 10.5	(M-c) 1 C = 8.88	(M-d) 2 C = 13.88
20	36.09	16.50	12.75	13.19	13.58	12.48
15	26.60	19.21	13.93	14.50	15.02	13.58
10	23.50	23.82	15.50	16.28	17.01	15.02
5	16.96	34.39	17.71	18.88	20.02	17.01
${}^1\overline{C} =$	$\frac{1}{69}[(14.5) + (11.6) +$	$-\cdots + (2.18) -$	+(0.20)] = 8.88 min	n; $\hat{C} = \overline{C} + d_t$	= 8.88 + 5 = 1	3.88 min (from

Equation (5)).

4. Discussion

At this point it is obvious that the value C has units of duration, being in this case minutes. However, it is interesting to compare the values obtained with this methodology for different return periods. Table 5 shows the precipitation intensities (mm/h) for different return periods, estimated at station CH-03 for 10 min duration and different proposed values for parameter C. As it can be seen in Table 4, the typical intensity for a storm in station CH-03 is 23.5 mm/h for 10-min duration. As already mentioned, there are storms records every minute for the last 6 years. It is, therefore, appropriate to compare this value with the values for Tr = 5 years in Table 5. It can then be concluded that any intensity obtained with equations with C \neq 0 is adequate. For validation of the results of Table 5, from database of storms recorded every minute, some values of intensity (mm/h) in the most severe storms at CH-03 station are shown next for the past 5 years: 12/06/2013 (21.8 mm/h); 06/07/2013 (15.27 mm/h); 15/09/2013 (17.3 mm/h); 19/11/2013 (25.3 mm/h); 20/06/2014 (26.7 mm/h). These results clearly show that there would be a serious error in trying to characterize an IDF curve considering C = 0. Therefore, the proposed methodology acquires an evident relevance.

Tr	C = 0	(M-a) = 12.5	(M-b) = 10.5	(M-c) = 8.88	(M-d) = 13.88
20	74.4	48.4	50.9	53.1	46.9
10	51.6	33.5	35.2	36.8	32.5
5	35.7	23.2	24.4	25.5	22.5

Table 5. Precipitation intensities (mm/h) for different return periods, estimated at station CH-03 for duration of 10 min and different proposed values of parameter C.

It was noted that for a short period (from 5 to 20 min) the initial part of the IDF curves becomes more sensitive to a given C value. The intensity of the rain shows an exponential decrease, with an increase in the storm duration. It seems obvious that for the same duration condition and return period there would be different intensity values depending on how C is calculated. As already mentioned, the parameter C must have time units and is a value associated with the duration of the analyzed storms. Thus, the correct value of C must be that of one of the cases analyzed (M-a, M-b, M-c, or M-d), but which of them? The values proposed for C (Table 3) are similar when looking at the columns M-a, M-b, and M-c, being virtually the same, for example for the CC-04 station. This indicates that the value of C, which normally adjusts with correlations, is a very close value to that obtained when weighting it with the duration and the number of storms. This combination of results supports the premise that C is a value that reaches a straight line when logarithms of duration are taken. In the literature, there are many examples where the authors have analytically calculated IDF curves including parameter C, but it is not mentioned that the calculated values of C are related to typical storm duration that could be inferred from the historical data. Some examples of this evidence are shown in Table 6.

Table 6. Some evidence that authors calculate parameter C by some traditional procedure; but without recognizing that in their own work (the proof that C could be estimated with a physical meaning measured in situ).

Fact	Values	Reference
<i>"the rainfall time series that allows the correct transfer to the model is</i> 30 min <i>"</i>	The storm duration using in this study for IDF curves is 35 min .	[63], p.374
<i>"the storm index associated with the precipitation depth for design storms lasts </i> 60 min "	The IDF equation presented is of type [42] where C = 55	[64], p.16
"The parameter C varies between 0 and $C_{max} = 12d_{min}$ " their data vary between 0.083 h (5 min) and 0.167 h (10 min). The results of his work present two IDF equations.	C for the IDF curves were C = 0.189 (5 min) and C = 0.143 (10 min). The parameters of these equations are obtained by L-moments and an optimization routine.	[42], p.129
<i>"IDF curves for precipitation in the Monsoon area of Vietnam, identifies ratios 60-min rainfall intensity and duration for same return period"</i>	The ratios were fitted by Sherman's equation with C = 76.31	[65], p.100

The examples above are only some of them, where it can be observed that it is really important that the coefficient C has time units. Today, with climate change, storms of short duration and great intensity are a constant challenge. In addition, hydrological studies in a small urban basin require rainfall monitoring networks for short duration, e.g., 5 min [2].

In order to enrich the discussion, it is necessary to cite textually the work presented by [22]. This paper presented in 2001 shows evidence of the importance of fit IDF curves for durations less than 10 min. "No matter the kind of approach, it is clear from a practical point of view that the resulting IDF curves, developed under a more or less sophisticated methodology, should provide reliable predictions of extreme rainfall intensity for specified durations. These estimates become unreliable when very short durations are considered. This is due to non-existent data at those shorter time intervals and the inherent fluctuations and complexity that the rainfall process exhibits at those finer time scales. The problem becomes more apparent when extreme

hydrological regimes are under consideration, as it is the case of many Mediterranean regions. In these regions the highest rainfall intensities are generated by intense convective cells producing peaks of more than 300 mm/h. In those cases, the correct choice for the shape or temporal pattern to be used for the IDF curve in the interval (0, 10 min) is particularly critical" [22], p. 675. These considerations of intensities are very similar to those presented in the Mexican coasts, where hurricanes can generate intensities higher than 200 mm/h.

On the other hand, the development in hydrological modeling is towards ever more complex and physically realistic representations of the dynamic behavior of the earth system, driven by the need for better management of increasingly scarce resources. As we build more realistic and detailed models of environmental processes, we must also develop methods powerful enough to correct them. In particular, such models they must help explain to what degree a realistic representation of the real world has (or has not) been achieved and (more importantly) how the model should be improved [66]. This study confirms that the coefficient C of Sherman's formula is associated with real hydrological signatures. This means that curves IDF with the same parameter set, but a new meaning, implying a better representation of real rainfall intensity, duration and return period formulation. It is important to note that one of the main purposes of scientific hydrology is to develop better predictive models of rainfall-runoff processes where IDF curves are of very high significance [67]. In this way, models that incorporate relevant hydrological processes are able to simulate more realistically concerning hydrological signatures [68]. If we consider the factor C of Sherman's formula, as a parameter associated with the hydrological behavior of the site, then we can say that C is a hydrological signature. Taken together, these results suggest that realism is expressed as the performance of a model to simulate a variety of hydrologic signatures [68].

5. Conclusions

The results of this work have important implications for the future development of the IDF curves. First, C is independent of the return period (T). When the Sherman equation (1931) is used, the value of C must be obtained by weighting the duration of storms (5–20 min) and the number of registered events, plus the typical duration of the most frequent storm on site. Future and possible research questions should arise from these results. It is found that C should not be calculated by multiple methods of regression or optimization, but must be obtained from the statistics of the historical storm sample. These results help to understand that there is a physical meaning in the value of parameter C. Accepting that C is part of the duration factor of the Sherman equation, it is recommended that future studies, and even IDF curves already calculated, in very specific sites should be reviewed considering the physical meaning of parameter C. The results provide conclusive support to determine that the value of C is a physical condition of the site and the data sample. C is not a random variable, and it must be calculated in situ. The present conclusions seem to be consistent with those of other investigations that have already found values of C close to the duration and number of storms, but until now this has not been explicitly mentioned.

The results of this paper show that the C value affects IDF curves, especially in cases of short duration storms of less than 5 min. In these cases, a value of C = 0 causes an unrealistic value of rain intensity. *"Certain hydrological applications require estimates of maximum rainfall intensity values for short time durations, as it is the case of urban hydrology design among others. Short duration rainfall intensities are affected by a large uncertainty when durations under 10 or 5 min are considered, especially when they are produced during extreme convective rainfall events" [22], p. 675. The weighting between the duration of storms (5–20 min) and the number of events recorded proved to be equal to, or very close to, the value of C.*

Therefore, there is a physical interpretation of parameter C and it must have time units. It is recommended to make a traditional adjustment to the C value, and to compare it with the typical storm duration and the number of events measured in situ. The analysis of short-term storms is definitely important, and in this case, it was very favorable to have data measured every minute. The author of [69] was the one who proposed the Rainfall frequency atlas of the United States for duration from

30 min to 24 h and return periods from 1 to 100 years, and categorically declares that: "the question of whether a distribution of extreme rainfall is a function of storm type". He also wrote that "A study of more than 300 rainstorms, 15 major events in each of 23 densely gaged watersheds, has shown than gage density in defining the characteristic dimensions of the rainfall distribution" [70].

The presented paper uses data from 523 storms, measured every minute, and more than 30 storms in each rain gauge. It is considered adequate to demonstrate that short-duration storm data allow proper estimation of the C parameter.

It is not intended to find a "universal" relationship. The relationship proposed by [39] is adequate. However, the adjustments of the parameters of this equation are always made in the numerical form, without recognizing that parameter C is the typical duration of storms in situ. It should also be considered a measurable physical value (with ability to predict hydrological signatures), and not a calculated one. This work is intended to allow modifying the way in which the parameter C is calculated in the future. Certainly, a more precise and realistic formula of IDF will provide the estimation of extreme events with less uncertainty. Finally, for future research, the development of intensity–duration–frequency (IDF) curves under the uncertainty of climate change can be carried out, for example, by relating the value of parameter C to precipitation anomalies.

Author Contributions: A.G.-L. and C.E.S. developed the initial and final versions of this manuscript and analyzed the data together. S.B.J.H. investigation and resources.

Funding: This research received funding of Secretaria de Educacion Publica (SEP-Mexico). Programa para el Desarrollo Profesional Docente (PRODEP) and was supported by the Fund for the Reinforcement of Research UAQ-2018 Research and Postgraduate Division (FOFIUAQ/FIN201911).

Acknowledgments: The authors are grateful to the Risk Management Unit of the UNESCO Regional Office of Science for Latin America and the Caribbean. We express our sincere gratitude to Zelmira May, Ivonne Cruz, Diletta Assorbi, Paula Santos, Will Logan, Raisa Barragan and Vincent Taks for discussions on the topic of this work. We acknowledge the constructive comments of the two anonymous reviewers.

Conflicts of Interest: The authors declare no conflict of interest.

References

- 1. Willems, P. Compound intensity/duration/frequency-relationships of extreme precipitation for two seasons and two storm types. *J. Hydrol.* **2000**, 233, 189–205. [CrossRef]
- 2. Shrestha, A.; Babel, M.; Weesakul, S.; Vojinovic, Z. Developing Intensity–Duration–Frequency (IDF) Curves under Climate Change Uncertainty: The Case of Bangkok, Thailand. *Water* **2017**, *9*, 145. [CrossRef]
- 3. Hershfield, D. Extreme rainfall relationships. *ASCE J. Hydraul. Div.* **1962**, *88*, 73–92.
- 4. Vivekanandan, N. Analysis of hourly rainfall data for the development of IDF relationships using the order statistics approach of probability distributions. *Int. J. Manag. Sci. Eng. Manag.* **2013**, *8*, 283–291. [CrossRef]
- 5. Dunkerley, D. Identifying individual rain events from pluviograph records: A review with analysis of data from an Australian dryland site. *Hydrol. Process.* **2008**, *22*, 5024–5036. [CrossRef]
- Dunkerley, D. Rain event properties in nature and in rainfall simulation experiments: A comparative review with recommendations for increasingly systematic study and reporting. *Hydrol. Process.* 2008, 22, 4415–4435. [CrossRef]
- 7. Dunkerley, D. How do the rain rates of sub-event intervals such as the maximum 5- and 15-min rates (I5orI30) relate to the properties of the enclosing rainfall event? *Hydrol. Process.* **2010**, *24*, 2425–2439. [CrossRef]
- 8. Van de Vyver, H. A multiscaling-based intensity–duration–frequency model for extreme precipitation. *Hydrol. Process.* **2018**, *32*, 1635–1647. [CrossRef]
- 9. Hosseinzadehtalaei, P.; Tabari, H.; Willems, P. Precipitation intensity–duration–frequency curves for central Belgium with an ensemble of EURO-CORDEX simulations, and associated uncertainties. *Atmos. Res.* **2018**, 200, 1–12. [CrossRef]
- 10. Adamowski, K.; Bougadis, J. Detection of trends in annual extreme rainfall. *Hydrol. Process.* **2003**, *17*, 3547–3560. [CrossRef]
- 11. Bougadis, J.; Adamowski, K. Scaling model of a rainfall intensity–duration–frequency relationship. *Hydrol. Process.* **2006**, *20*, 3747–3757. [CrossRef]

- 12. Cunderlik, J.; Ouarda, T. Regional flood–rainfall duration-frequency modeling at small ungaged sites. *J. Hydrol.* **2007**, 345, 61–69. [CrossRef]
- 13. Adamowski, J.; Adamowski, K.; Bougadis, J. Influence of trend on short duration design storms. *Water Resour. Manag.* **2009**, *24*, 401–413. [CrossRef]
- 14. Burn, D.; Taleghani, A. Estimates of changes in design rainfall values for Canada. *Hydrol. Process.* **2013**, 27, 1590–1599. [CrossRef]
- Burn, D. A framework for regional estimation of intensity-duration-frequency (IDF) curves. *Hydrol. Process.* 2014, 28, 4209–4218. [CrossRef]
- Soulis, E.; Sarhadi, A.; Tinel, M.; Suthar, M. Extreme precipitation time trends in Ontario, 1960–2010. *Hydrol. Process.* 2016, 30, 4090–4100. [CrossRef]
- 17. Harremoës, P.; Mikkelsen, P. Properties of extreme point rainfall I: Results from a rain gauge system in Denmark. *Atmos. Res.* **1995**, *37*, 277–286. [CrossRef]
- Madsen, H.; Arnbjerg-Nielsen, K.; Mikkelsen, P. Update of regional intensity-duration-frequency curves in Denmark: Tendency towards increased storm intensities. *Atmos. Res.* 2009, 92, 343–349. [CrossRef]
- 19. Olsson, J.; Niemczynowicz, J.; Berndtsson, R.; Larson, M. An analysis of the rainfall time structure by box counting—Some practical implications. *J. Hydrol. (Amst.)* **1992**, *137*, 261–277. [CrossRef]
- 20. Bengtsson, L.; Milloti, S. Extreme storms in Malmö, Sweden. Hydrol. Process. 2010, 24, 3462–3475. [CrossRef]
- 21. Olsson, J.; Willen, U.; Kawamura, A. Downscaling extreme short-term regional climate model precipitation for urban hydrological applications. *Hydrol. Res.* **2012**, *43*, 341–351. [CrossRef]
- 22. Garcia-Bartual, R.; Schneider, M. Estimating Maximum Expected Short-Duration Rainfall Intensities from Extreme Convective Storms. *Phys. Chem. Earth Part B* **2001**, *26*, 675–681. [CrossRef]
- 23. Salas, L.; Fernandez, J. "In-site" regionalization to estimate an intensity—Duration—Frequency law: A solution to scarce spatial data in Spain. *Hydrol. Process.* **2007**, *21*, 3507–3513. [CrossRef]
- 24. Garcia-Marin, A.; Ayuso-Muñoz, J.; Jimenez-Hornero, F.; Estevez, J. Selecting the best IDF model by using the multifractal approach. *Hydrol. Process.* **2012**, *27*, 433–443. [CrossRef]
- 25. Yu, P.; Chen, C. Potential of extending the rainfall intensity–duration–frequency relationship to non-recording rain gauges. *Hydrol. Process.* **1997**, *11*, 377–390. [CrossRef]
- Yu, P.; Cheng, C. Incorporating uncertainty analysis into a regional IDF formula. *Hydrol. Process.* 1998, 12, 713–726. [CrossRef]
- 27. Lee, K.; Ho, J. Design Hyetograph for Typhoon Rainstorms in Taiwan. *J. Hydrol. Eng.* **2008**, *13*, 647–651. [CrossRef]
- 28. Wang, S.; Wang, H. Extending the Rational Method for assessing and developing sustainable urban drainage systems. *Water Res.* **2018**, *144*, 112–125. [CrossRef]
- 29. Chang, K.; Lai, S.; Faridah, O. RainIDF: Automated derivation of rainfall intensity–duration–frequency relationship from annual maxima and partial duration series. *J. Hydroinform.* **2013**, *15*, 1224–1233. [CrossRef]
- 30. Liew, S.; Raghavan, S.; Liong, S. How to construct future IDF curves, under changing climate, for sites with scarce rainfall records? *Hydrol. Process.* **2013**, *28*, 3276–3287. [CrossRef]
- Knighton, J.; Walter, M. Critical rainfall statistics for predicting watershed flood responses: Rethinking the design storm concept. *Hydrol. Process.* 2016, *30*, 3788–3803. [CrossRef]
- 32. Pizarro, R.; Garcia-Chevesich, P.; Valdes, R.; Dominguez, F.; Hossain, F.; Folliott, P.; Olivares, C.; Morales, C.; Balocchi, F.; Bro, P. Inland water bodies in Chile can locally increase rainfall intensity. *J. Hydrol.* **2013**, *481*, 56–63. [CrossRef]
- 33. Pizarro, R.; Aravena, D.; Macaya, K.; Abarza, A.; Cornejo, M.; Labra, M.; Pavez, M.; Roman, L. *Curvas Intensidad-Duracion-Frecuencia Para la Zona Centro sur de Chile. Talca, Chile*; Programa Hidrologico Internacional de UNESCO (PHI) para America Latina y el Caribe. PHI-VI/Documento Tecnico N° 7. Montevideo, Uruguay, 2007. Available online: https://unesdoc.unesco.org/ark:/48223/pf0000228195 (accessed on 1 April 2019).
- 34. Pizarro, R.; Valdes, R.; Garcia-Chevesich, P.; Vallejos, C.; Sangüesa, C.; Morales, C.; Balocchi, F.; Abarza, A.; Fuentes, R. Latitudinal Analysis of Rainfall Intensity and Mean Annual Precipitation in Chile. *Chil. J. Agric. Res.* **2012**, *72*, 252–261. [CrossRef]
- 35. Puricelli, M. Rainfall extremes modeling under shortage of data and uncertainty in the Pampean region (Argentina). *Cuad. Investig. Geogr.* **2018**, *44*, 719. [CrossRef]

- 36. Pizarro, R.; Sangüesa, C.; Bro, P.; Ingram, B.; Vera, M.; Vallejos, C.; Morales, C.; Olivares, C.; Balocchi, F.; Fuentes, R.; et al. *Curvas Intensidad Duracion Frecuencia Para las Regiones Metropolitana, Maule y Biobio. Intensidades Desde 15 Minutos a 24 Horas*; Programa Hidrologico Internacional de UNESCO (PHI) para America Latina y el Caribe. PHI-VII/Documento Tecnico N° 29; Universidad de Talca: Montevideo, Uruguay, 2013; 129p.
- 37. Silveira, L.; Usera, G.; Alonso, J.; Scavone, M.; Chreties, C.; Perera, G.; Gonzalez, M. Nuevas curvas intensidad-duracion-frecuencia de precipitacion para el departamento de Montevideo, Uruguay. *Agrociencia Urug.* **2014**, *18*, 113–125.
- 38. Pereyra-Diaz, D.; Perez-Sesma, J.; Gomez-Romero, L. Ecuaciones que estiman las curvas intensidad-duracion-periodo de retorno de la lluvia. *GEOS* **2004**, *24*, 46–56.
- 39. Sherman, C.W. Frequency and Intensity of Excessive Rainfall at Boston, Mass. *Trans. Am. Soc. Civil Eng.* **1931**, *95*, 951–960.
- 40. Bernard, M.M. Formulas for rainfall intensities of long durations. *Trans. Am. Soc. Civil Eng.* **1932**, *96*, 592–606.
- 41. Chow, V.; Maidment, D.; Mays, L. *Hidrologia Aplicada*; Mc Graw-Hill: Santa Fe de Bogota, CO, USA, 1994; 584p.
- 42. Koutsoyiannis, D.; Kozonis, D.; Manetas, A. A mathematical framework for studying rainfall intensity–duration–frequency relationships. *J. Hydrol.* **1998**, 206, 118–135. [CrossRef]
- 43. Campos, D.; Gomez, R. Procedimiento para obtener curvas IDT a partir de registros pluviometricos. *Ing. Hidraul. Mex.* **1990**, *5*, 39–52.
- 44. Campos, D. Intensidades maximas de lluvia para diseño hidrologico urbano en la republica mexicana. *Ing. Investig. Technol.* **2010**, *11*, 179–188.
- 45. Bell, F. Generalized rainfall-duration-frequency relationships. ASCE J. Hydraul. Div. 1969, 95, 311–327.
- 46. Chen, C. Rainfall intensity-duration-frequency formulas. J. Hydraul. Div. Am. Soc. Civ. Eng. 1983, 109, 1603–1621. [CrossRef]
- 47. Chocat, B. *Encyclopedie de L'hydrologie Urbaine et de L'assainissement;* Collection: Eaux Pluviales Lavoisier: Paris, France, 1997; 1136p.
- 48. Notaro, V.; Liuzzo, L.; Freni, G.; La Loggia, G. Uncertainty Analysis in the Evaluation of Extreme Rainfall Trends and Its Implications on Urban Drainage System Design. *Water* **2015**, *7*, 6931–6945. [CrossRef]
- 49. Da Silva, C.; Schardong, A.; Garcia, J.; Oliveira, C. Climate Change Impacts and Flood Control Measures for Highly Developed Urban Watersheds. *Water* **2018**, *10*, 829. [CrossRef]
- 50. Sanches Fernandes, L.; Pereira, M.; Morgado, S.; Macario, E. Influence of Climate Change on the Design of Retention Basins in Northeastern Portugal. *Water* **2018**, *10*, 743. [CrossRef]
- 51. Svensson, C.; Clarke, R.; Jones, D. An experimental comparison of methods for estimating rainfall intensity–duration–frequency relations from fragmentary records. *J. Hydrol.* **2007**, *341*, 79–89. [CrossRef]
- 52. Seong, K. Deriving a practical form of IDF formula using transformed rainfall intensities. *Hydrol. Process.* **2014**, *28*, 2881–2896. [CrossRef]
- 53. Asikoglu, O.; Benzeden, E. Simple generalization approach for intensity–duration–frequency relationships. *Hydrol. Process.* **2012**, *28*, 1114–1123. [CrossRef]
- 54. Lopcu, Y. Modeling the Intensity–Duration–Frequency Relationships of Annual Maximum Storms. Master's Thesis, Civil Engineering, Dokuz Eylul University, Izmir, Turkey, 2007.
- 55. Sane, Y.; Panthou, G.; Bodian, A.; Vischel, T.; Lebel, T.; Dacosta, H.; Quantin, G.; Wilcox, C.; Ndiaye, O.; Diongue-Niang, A.; et al. Intensity–duration–frequency (IDF) rainfall curves in Senegal. *Nat. Hazard. Earth Syst.* **2018**, *18*, 1849–1866. [CrossRef]
- 56. Linsley, R.; Kohler, M.; Paulhus, J. Applied Hydrology; Mc Graw-Hill: New York, NY, USA, 1949; 689p.
- 57. Wenzel, H. 1982. Rainfall for urban stormwater design. In *Urban Storm Water Hydrology;* Water Resources Monograph 7; American Geophysical Union: Washington, DC, USA, 1982; pp. 35–67.
- 58. Willems, P.; Arnbjerg-Nielsen, K.; Olsson, J.; Nguyen, V. Climate change impact assessment on urban rainfall extremes and urban drainage: Methods and shortcomings. *Atmos. Res.* **2012**, *103*, 106–118. [CrossRef]
- 59. Elsebaie, I. Developing rainfall intensity–duration–frequency relationship for two regions in Saudi Arabia. *J. King Saud Univ. Eng. Sci.* **2012**, 24, 131–140. [CrossRef]
- 60. Mendoza, C.; Trasviña, J.; Gutierrez-Lopez, A. Empleo del algoritmo GRG Nonlinear en el calculo de intensidades de lluvia. *Revista NTHE* **2018**, *24*, 11–15.

- 61. Sumner, G. The prediction of short-duration storm rainfall intensity maxima. *J. Hydrol.* **1978**, *37*, 91–100. [CrossRef]
- 62. Gutierrez-Lopez, M.; Barragan, R. Ajuste de curvas IDF a partir de tormentas de corta duración. *Tecnol. Cienc. Agua* **2019**, *10*. [CrossRef]
- 63. Olsson, J.; Berggren, K.; Olofsson, M.; Viklander, M. Applying climate model precipitation scenarios for urban hydrological assessment: A case study in Kalmar City, Sweden. *Atmos. Res.* **2009**, *92*, 364–375. [CrossRef]
- 64. Cheng, K.; Wei, C.; Cheng, Y.; Yeh, H. Effect of spatial variation characteristics on contouring of design storm depth. *Hydrol. Process.* **2003**, *17*, 1755–1769. [CrossRef]
- 65. Minh Nhat, L.; Tachikawa, Y.; Takara, K. Establishment of Intensity–Duration–Frequency Curves for Precipitation in the Monsoon Area of Vietnam. *Annu. Disas. Prev. Res. Inst. Kyoto Univ.* **2006**, *49*, 93–103.
- 66. Gupta, H.; Wagener, T.; Liu, Y. Reconciling theory with observations: Elements of a diagnostic approach to model evaluation. *Hydrol. Process.* **2008**, *22*, 3802–3813. [CrossRef]
- 67. Euser, T.; Winsemius, H.; Hrachowitz, M.; Fenicia, F.; Uhlenbrook, S.; Savenije, H. A framework to assess the realism of model structures using hydrological signatures. *Hydrol. Earth Syst. Sci.* **2013**, *17*, 1893–1912. [CrossRef]
- 68. Jehn, F.; Chamorro, A.; Houska, T.; Breuer, L. Trade-offs between parameter constraints and model realism: A case study. *Sci. Rep.* **2019**, *9*, 10729. [CrossRef] [PubMed]
- 69. Hershfield, D. Rainfall Frequency Atlas of the United States for Durations from 30 Min to 24 H and Return Periods from 1 to 100 Years; Pap. No. 40; U.S. Dep. Commerce, Weather Bur. Tech.: Washington, DC, USA, 1961; 115p.
- Hershfield, D. Rainfall Input for Hydrologie Models. In *Geochemistry, Precipitation, Evaporation, Soil-Moisture, Hydrometry; Reports and Discussions. Publication (78);* General Assembly of Bern; International Association of Scientific Hydrology (IASH): Gentbrugge, Belgium, 1968; pp. 177–188.



© 2019 by the authors. Licensee MDPI, Basel, Switzerland. This article is an open access article distributed under the terms and conditions of the Creative Commons Attribution (CC BY) license (http://creativecommons.org/licenses/by/4.0/).