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Understanding the Changes in Hydraulic Conductivity Values of Coarse- and Fine-Grained Porous Media Mixtures

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Abstract: Low permeability clay-sand mixtures are often used to construct hydraulic barriers to prevent contaminated water leaching from landfills and other waste disposal sites from polluting local groundwater aquifers. In order to engineer effective hydraulic barriers, a proper knowledge of the hydraulic conductivity of clay-sand mixtures is required. While there are several empirical models available in the literature that can be used to predict reductions in hydraulic conductivity values of coarse sand due to the presence of clay and other fine minerals, all these models require measurement of multiple physical properties of the porous media. The resulting empirical expressions have several parameters that need to be individually evaluated using multiple soil characterization tests. In this study, we propose a single parameter model that can be used to capture the variations in hydraulic conductivity value of different types of porous media mixtures using a scalable modeling framework. Several laboratory tests were conducted to measure the hydraulic conductivity values of a variety of coarse and fine glass bead mixtures. The coarse glass beads were used to simulate sand and small glass beads were used to simulate fine minerals such as silt and clay. The model results were further validated using the data derived from experiments conducted with natural sand and clay mixtures, and also using multiple literature-derived datasets. Our results show that the proposed model is a useful tool for describing the hydraulic conductivity values of various types of coarse- and fine-grained porous media mixtures.

Keywords: groundwater; hydraulic barrier; hydraulic conductivity; clay-sand mixtures; modeling

1. Introduction

Contamination of groundwater systems by landfill leachates is one of the common environmental problems. When an aquifer formation is highly permeable with no natural impervious soil layer, engineers use low permeability clay-sand mixture to construct hydraulic barriers that can prevent contaminated wastewater leaching from landfills and other waste disposal areas from polluting local groundwater aquifers [1–5]. The amount of clay required to achieve the desired level of low permeability mixture is evaluated by performing laboratory-scale conductivity tests, and these efforts can be time consuming and cost prohibitive [6–8].

Past studies have shown that the hydraulic conductivity of a clay-sand mixture will decrease with increase in clay percentage [3,8–12]. However, adding excess amount of clay can lead to swelling and shrinkage that can eventually result in cracking and increase the risk of leakage through preferential flow paths [2,13]. Also, at high clay levels, the mixture becomes more plastic and extremely difficult to compact [3]. Furthermore, the overall cost of the mixture will increase with increasing in clay content [2,4,8,14].

The most important parameter that controls the groundwater seepage processes through subsurface aquifers is the hydraulic conductivity of the system [9,15]. Without having a proper knowledge of the hydraulic conductivity value, it is impossible to design effective engineering barriers. Therefore, many investigators have conducted studies to develop mechanistic models that can predict the hydraulic conductivity value of clay-sand mixtures. Almost all currently available models are based on empirical formulations that use various physical properties of the materials used to develop the mixture and relate them to the effective hydraulic conductivity value of the mixture.

Chapuis [8] introduced an empirical equation to predict the hydraulic conductivity values of soil-bentonite mixtures. Several physical parameters were used to develop the model including porosity, bentonite content, the degree of saturation at the end of the test, grain-size distribution, and the compaction level estimated from the Proctor curve. Permeameter tests were performed to evaluate the conductivity values. Results of the study indicated that there is an inverse relation between hydraulic conductivity and amount of bentonite used in the mixture. No obvious correlation could be observed between the hydraulic conductivity and porosity values of the mixture. However, hydraulic conductivity correlated better with "efficient" porosity, which is related to the pore space available for fast-moving water. Note that the efficient porosity value is different from effective porosity, since it does not include the portion of immobile water that is retained at the surface of fine particles.

A five-variable regression model was developed by Benson et al. [16] to estimate the hydraulic conductivity of compacted soil liners. Results from the regression analysis indicated the five variables that were significantly correlated with the hydraulic conductivity value (analyzed using the natural logarithmic scale); the variables included: compactor weight, plasticity index, percent gravel, initial saturation, and percent clay. The coefficient of determination (R^2) of their regression model was 78%. The authors stated that the model can be used to understand the conditions needed to achieve required hydraulic conductivity values, however it should not be used to avoid performing hydraulic conductivity tests in the field or laboratory.

Another empirical model for predicting the hydraulic conductivity values was developed by Benson and Trast [17]. This model can determine the hydraulic conductivity of clay-sand mixture based on clay content, plasticity index, initial saturation, and compactive effort. The results from the falling-head hydraulic conductivity test indicated an inverse relationship between hydraulic conductivity and plasticity index, initial saturation, and compactive effort.

Mollins et al. [2] used the compaction permeameter falling head permeability test and an indirect test based on consolidation data to measure the hydraulic conductivity values of low and high clay content mixtures. Results showed that hydraulic conductivity values of the clay-sand mixture linearly correlated with the clay void ratio when plotted on a logarithmic scale. This study proposed a model that can predict the hydraulic conductivity of the mixture based on the clay content, its properties, sand porosity and tortuosity.

Sivapullaiah et al. [9] developed an equation aimed at predicting the hydraulic conductivity of bentonite-sand mixtures based on the void ratio and liquid limit of the mixture. The consolidation cell permeameter test was used to measure the hydraulic conductivity value. A linear relationship was established between the hydraulic conductivity of the mixture (used in the logarithmic scale) and the void ratio. Other studies have also explored the similar type of empirical relationship between hydraulic conductivity to the net void ratio of a mixture [11,18]. In contrast to these studies, Kenney et al. [5] and Castelbaum et al. [19] presented a relation between hydraulic conductivity and void ratio of the bentonite rather than the net void ratio for mixtures present with a sufficient bentonite content to be uniformly distributed to fill all the void spaces between sand particles.

Abichou et al. [6] conducted a study aimed at understanding the changes in microstructure and the hydraulic conductivity value of sand-bentonite mixtures at varying bentonite content. Simulated sand-bentonite mixtures were prepared using glass beads, to simulate sand particles, which were then mixed with powdered and granular bentonite. The use of glass beads helped to improve the visual properties of the mixtures. Results showed that pores available for water flow decreased as the bentonite content increased in the mixture, and this resulted in the reduction of hydraulic conductivity. In the case of mixtures prepared with powdered bentonite, the bentonite coated the glass bead particles, swelled, and later filled the pores. Little glass bead particles were coated with bentonite if mixtures were prepared using granular bentonite. In this case, granular bentonite particles occupied the pores between the glass bead particles and then absorbed the introduced water and swelled. In both cases, when sufficient bentonite was available to fill all the pores between glass bead particles, the hydraulic conductivity of the mixture was primarily controlled by the hydraulic conductivity of bentonite.

Dias et al. [20] investigated the effect of volume fraction and particle size ratio for binary mixtures of glass beads on the tortuosity coefficient. This was done because of the sensitivity of the tortuosity coefficient in estimating permeability values using the Kozeny–Carman equation that relates permeability with porosity, tortuosity, and grain size. Previous studies correlate tortuosity with porosity using a simplified formula, $T = 1/\varepsilon^n$, where T is tortuosity, ε is porosity, n is a power factor, which was proposed to be 0.04 by Mota et al. [21]. This simplified formula and the Kozeny–Carman equation were combined together and then a new formula for n factor was developed. Permeability tests were conducted and the data was used to measure the experimental values of n using the new formula. The measured n values were found to range between 0.4 and 0.5 and a model was used to correlate the changes of n with the changes in volume fraction and particle size ratio. The developed model helped to improve the accuracy of permeability values calculated using the Kozeny–Carman equation.

Our review indicates that while there are several types of models available for predicting the reduction in the hydraulic conductivity of a porous medium due to the presence of clay and other fine minerals, all these models are based on measuring multiple physical properties of the porous media used to develop the mixture. The resulting empirical expressions have several parameters that need to be individually evaluated by multiple soil characterization tests. It will be desirable if one can develop a model based on an effective scaling parameter that can capture the combined effects of multiple soil parameters. Therefore, the objective of this study is to develop an integrated scaling parameter that can be used to fully capture the variations in different types of physical properties into a unified framework. In this study, we have hypothesized that the changes in hydraulic conductivity values of fine and coarse grained mixtures can be corrected to the amount of fine material in the mixture. We collected several sets of laboratory data and also assembled multiple sets of literature-derived data to develop a scalable framework for modeling the changes in hydraulic conductivity values due to the presence of fine material.

2. Materials and Methods

2.1. Synthetic Coarse-Fine and Fine-Coarse Mixtures

Three sets of experiments with two using synthetic media and other using natural media were completed in this study. Materials used in the synthetic media experiments were different types of uniform coarse and fine glass beads. The coarse glass beads were used to simulate sand minerals and small glass beads were used to simulate fine minerals such as silt and clay. Our approach is similar to studies by Abichou et al. [6] and Dias et al. [20] that used glass beads to simulate clay-sand mixtures.

Figure 1 shows the two sets of synthetic media experiments completed in this study. First, a coarse porous medium was mixed with three different sized fine porous media to develop three different dry mixtures. The hydraulic conductivity (K_c) of the coarse porous medium is 920 m/d, and the hydraulic conductivity values (K_f) of the three fine porous media are: 228, 57, and 9 m/d. In the second set of experiments, a fine porous medium was mixed with three different coarse porous media. The hydraulic conductivity of the fine porous medium is 9 m/d, and the hydraulic conductivity values of the three coarse porous media are: 920, 228, and 57 m/d.

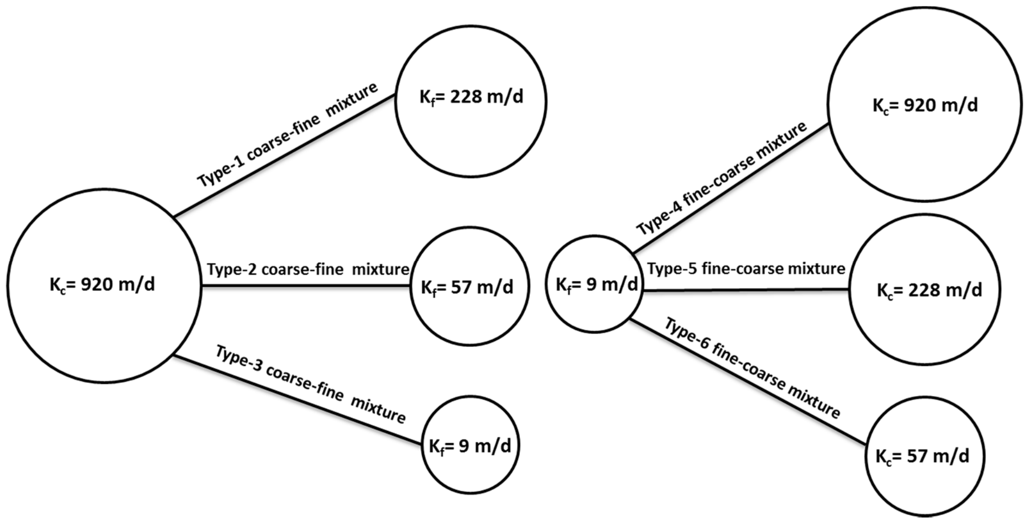

Figure 1. Schematic diagram illustrating the two sets of coarse-fine and fine-course porous media mixtures used in our laboratory experiments. K is hydraulic conductivity in m/day, the subscripts c and f stand for the coarse and fine particles. Each type of mixture was prepared with seven different fine percent levels: 0%, 5%, 10%, 15%, 20%, 25%, and 30%.

Before mixing, the porous media were washed in tap water to remove any dust and then dried in an oven. Samples were prepared by mixing coarse porous media with the following amount (percent dry weights) of fine porous media (0%, 5%, 10%, 15%, 20%, 25%, and 30%). An innovative mixing procedure used by Dias et al. [22] was employed to thoroughly mix the coarse and fine glass beads. In this method, glycerol is used as a binder to fully mix the glass beads of different sizes. After packing the mixture, the glycerol was washed out by flushing water through the column. Use of glycerol allowed us to pack a uniform mixture within the column.

Hydraulic conductivity tests for glass bead mixtures were conducted by closely following the method described in ASTM-D2423-68 [23]. This method uses the constant head permeameter test to determine the hydraulic conductivity of materials with values greater than 1×10^{-5} m/s [24]. Two transparent plastic columns of diameter 1.9 cm, but different lengths were used to construct the permeameter. The length of the short column is 25 cm and the long column is 75 cm. A wire mesh was used at the bottom of the short column then the soil sample was added over this mesh and was compacted gradually. While packing, the column was kept under water to maintain fully saturated conditions. To avoid segregation, the column was gradually lowered into the water bucket as it was packed. After packing the short column, a long transparent column was connected to the short column using a rubber coupling.

Tap water was used as the permeant liquid, and prior to its use, the water was allowed to reach the lab temperature. This was done to avoid gas exchanges and air trapping while running the test. After the sample was compacted, a high hydraulic gradient was applied to wash out the glycerol before running the hydraulic conductivity tests. To ensure consistency, a pump was used to introduce the water flow into the sample. The flow rate, sample length, column cross-section area, and head loss were measured, and Darcy's law was applied to calculate the effective hydraulic conductivity value of the mixture.

2.2. Natural Clay-Sand Mixtures

Natural clay was sieved on mesh No. 60 and was used in this study. The hydraulic conductivity value of this clay is 1.12×10^{-3} m/d. Fine sand with a hydraulic conductivity value of 46.5 m/d was used. Aged deaired water equilibrated to laboratory conditions was used to avoid gas exchanges during the test. All the mixtures were prepared under saturated conditions.

Samples were prepared by mixing the sand with varying amount (percent weights) of clay (0%, 5%, 6%, 8%, 10%, 11%, 13%, 15%, 20%, 25%, and 30%). These mixtures were prepared based on the dry weight of the material. To obtain complete mixing, deaired water was added to the mixture and it was then physically stirred to prepare a well-mixed saturated slurry (glycerol was not added in this case). The clay-sand slurry was left in the mixing pan and covered for a period of about 48 h to let the clay fully saturate with water. During this period, water was added and the mixture was periodically stirred, whenever it is needed, to ensure complete mixing and full saturation.

A falling head permeameter was used; the test procedure closely followed the method described in ASTM-D5084-16a [24]. Within the permeameter, the clay-sand mixture was packed in between two sand layers. The bottom sand layer helped prevent the fine material washing away through the bottom screen. The top sand layer helped us to better compact the mixture. After the sample was packed, the falling head permeameter test was performed to determine the hydraulic conductivity value of the sample.

3. Results

3.1. Results of Coarse-Fine Porous Media Mixtures

Test results for the first set of synthetic coarse-fine porous media mixtures indicated that the hydraulic conductivity decreased as the fine percentage increased in the mixture (Figure 2). The reduction in hydraulic conductivity was significant when we started to add fines to the coarse material; however, the reductions became less significant when the percentage of the fine was greater than about 15%. Interestingly, the overall hydraulic conductivity of the mixture was almost close to the hydraulic conductivity of the fine particles when the percent of the fine was above 30%. This trend of decrease in hydraulic conductivity with increasing percent fines is similar to the observations made by others for different types of clay and sand mixtures [5,9,10,12,16].

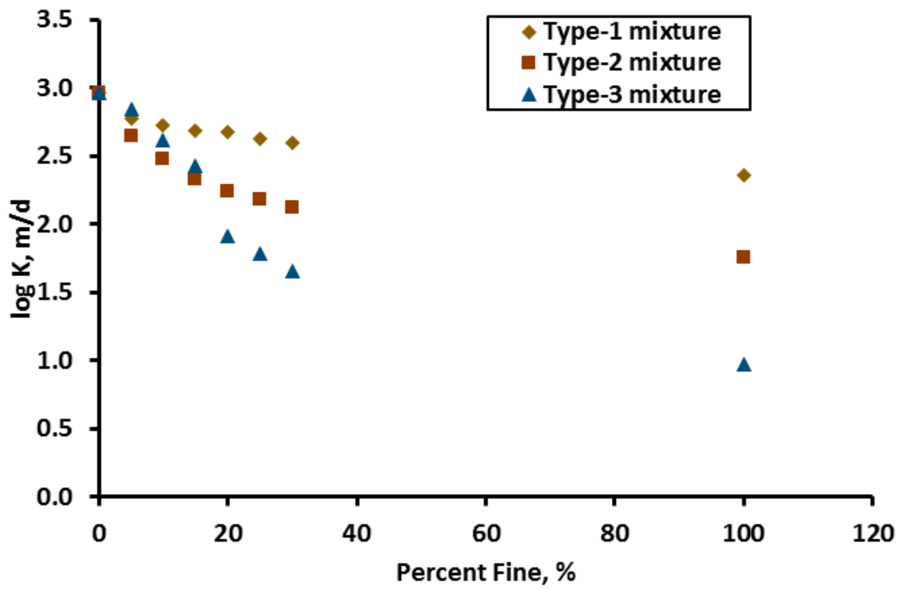

Figure 2. Decrease in the effective hydraulic conductivity values (K values plotted in log scale) of Type-1 (K_c = 920 m/day and K_f = 228 m/day), Type-2 (K_c = 920 m/day and K_f = 57 m/day), and Type-3 (K_c = 920 m/day and K_f = 9 m/day) coarse-fine mixtures. The seven mixtures for each type were prepared by varying the fine porous media content (percentage by weight) as: 0%, 5%, 10%, 15%, 20%, 25%, and 30%.

We used a following empirical equation to describe the reductions in the hydraulic conductivity values of all our coarse-fine porous media mixtures.

$$\log K(p) = (\log K_c - \log K_f) * \exp(-s * p) + \log K_f \quad (1)$$

where p is the percent of fine (e.g., 10 for 10 percent of fine), $K(p)$, K_c, and K_f are the hydraulic conductivity values of the mixture, coarse porous media, and fine porous media, respectively. The model uses a single fitting constant s, which is an empirical parameter employed to scale the results based on the percentage of fines in the mixture. The empirical model was fitted to the dataset shown in Figure 2 and the optimal values of the scaling parameter for Type 1, 2, and 3 mixtures that best fit all the experimental data are 0.04, 0.05, and 0.03 (see Figure 3), respectively. The nonlinear Solver Add-in available in Excel was used to evaluate these fitting parameters. Hydraulic conductivity values obtained by Equation (1) in comparison with experimental values for the first set of the three different types of mixtures are plotted in Figure 3 as a function of percent fine. The root mean square error (RMSE) values of log K calculated between the fitted model and experiment are summarized in Table 1. The coefficient of determination (R^2) values are 93.4%, 98.6%, and 95.4% for Type 1, 2, and 3 mixtures, respectively.

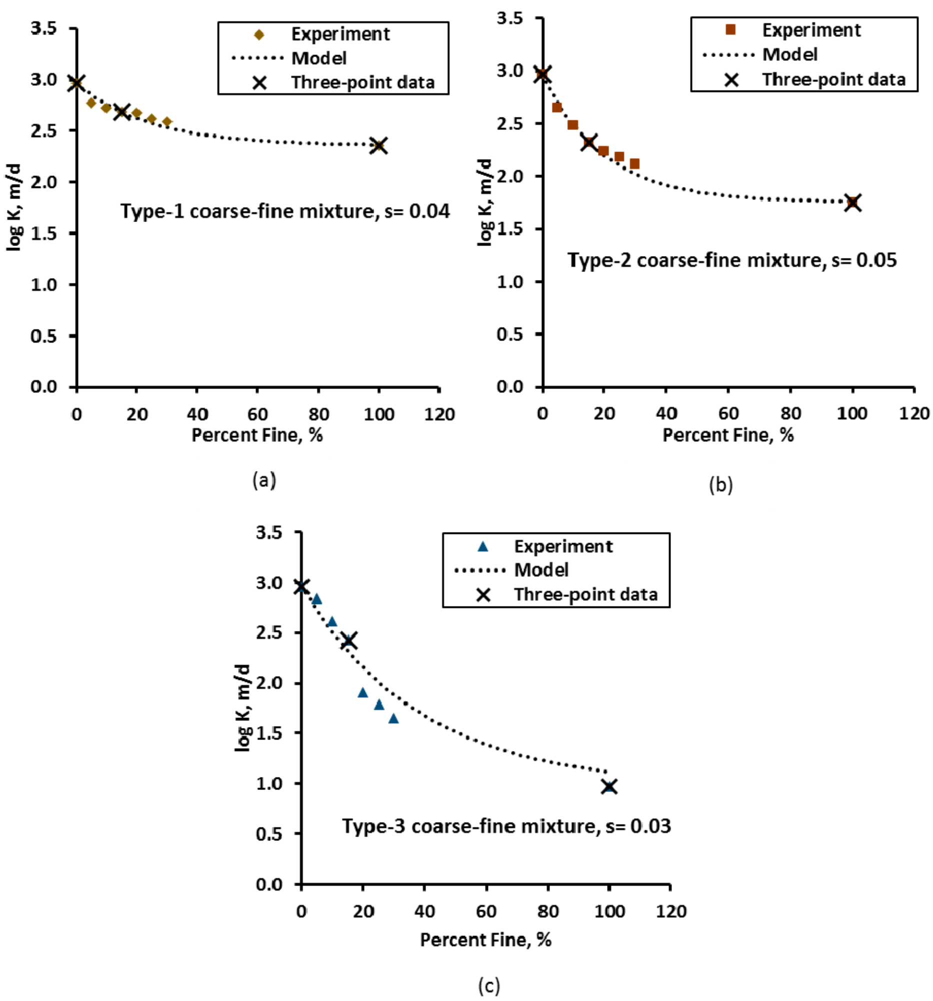

Figure 3. Comparison of fitted model results with experimental data for all three types of coarse-fine mixtures: (**a**) Type-1 (K_c = 920 m/day and K_f = 228 m/day), (**b**) Type-2 (K_c = 920 m/day and K_f = 57 m/day), and (**c**) Type-3 (K_c = 920 m/day and K_f = 9 m/day). The effective hydraulic conductivity K values are plotted on y axis in log scale. The cross symbol is used to identify the data points used in the three-point method discussed in Section 3.5.

Table 1. Summary of the scaling factor (*s*) values estimated for all the samples tested in this study and literature-derived experimental datasets.

Type of Mixture	*s* Estimated Using the Entire Dataset	*s* Evaluated Using the Three-Point Method [4]
Type-1 coarse-fine mixture	0.037 (RMSE [1] = 0.041, R^2 = 93.4%)	0.041 (RMSE = 0.045, R^2 = 92.3%)
Type-2 coarse-fine mixture	0.046 (RMSE = 0.041, R^2 = 98.6%)	0.050 (RMSE = 0.050, R^2 = 97.8%)
Type-3 coarse-fine mixture	0.031 (RMSE = 0.138, R^2 = 95.4%)	0.026 (RMSE = 0.169, R^2 = 93.0%)
Type-4 fine-coarse mixture	0.031 (RMSE = 0.138, R^2 = 95.4%)	0.026 (RMSE = 0.169, R^2 = 93.0%)
Type-5 fine-coarse mixture	0.040 (RMSE = 0.043, R^2 = 99.9%)	0.041 (RMSE = 0.044, R^2 = 98.7%)
Type-6 fine-coarse mixture	0.054 (RMSE = 0.046, R^2 = 95.6%)	0.056 (RMSE = 0.047, R^2 = 95.4%)
Clay-sand mixture	0.079 (RMSE = 0.188, R^2 = 97.3%)	0.077 (RMSE = 0.191, R^2 = 97.2%)
Montmorillonite-sand mixture [2]	0.074 (RMSE = 0.051, R^2 = 99.9%)	0.074 (RMSE = 0.051, R^2 = 99.9%)
Kaolinite-sand mixture [2]	0.034 (RMSE = 0.041, R^2 = 99.7%)	0.034 (RMSE = 0.041, R^2 = 99.7%)
Bentonite-silt mixture [3]	0.030 (RMSE = 0.134, R^2 = 99.4%)	0.032 (RMSE = 0.168, R^2 = 99.1%)

Note: [1] All root mean square errors (RMSEs) and coefficients of determination R^2 of log K were calculated using the entire dataset for each fitted model; [2] Data from Denson et al. [25]; [3] Data from Doley et al. [26]; [4] The details of the three-point method are described in Section 3.5.

3.2. Results of Fine-Coarse Porous Media Mixtures

The second set of experiments were completed using three different types of fine-coarse mixtures, where a uniform fine porous medium was mixed with three different types of coarse porous media at different dry weight percentages levels. Figure 4 shows the increase in hydraulic conductivity values as we increased the percentage of coarse material in the mixture. Similar to the coarse-fine mixtures, the amount of fines in the system controlled the effective hydraulic conductivity value of the mixture. Addition of coarse material into the fine porous media had little impact until about 85% of coarse material was added to the system, after which the system was highly influenced by the conductivity of the coarse material used to prepare the mixture.

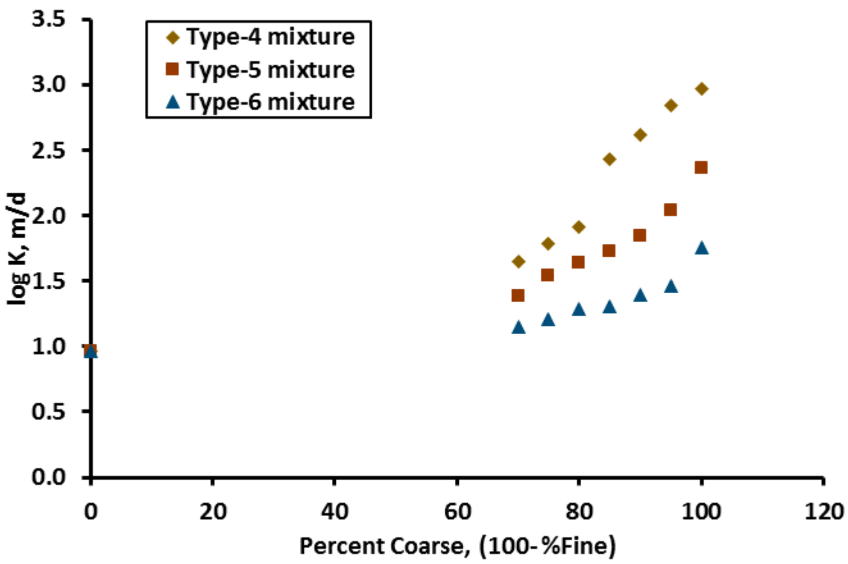

Figure 4. Increase in the effective hydraulic conductivity values (K values plotted in log scale) of Type-4 (K_c = 920 m/day and K_f = 9 m/day), Type-5 (K_c = 228 m/day and K_f = 9 m/day), and Type-6 (K_c = 57 m/day and K_f = 9 m/day) fine-coarse mixtures. The seven mixtures for each type were prepared by varying the coarse porous media content (percentage by weight) as: 0%, 70%, 75%, 80%, 85%, 90%, and 95%.

The empirical model used to describe the coarse-fine Type 1, 2, and 3 mixtures was also used to analyze the fine-coarse Type 4, 5, and 6 mixtures. The fitted values of the scaling parameter for Type 4, 5, and 6 mixtures are 0.03, 0.04, and 0.05, respectively. Figure 5 shows the experimental and modeled hydraulic conductivity values for the three types of fine-coarse mixtures characterized in this study. The coefficient of determination (R^2) values are 95.4%, 99.9%, and 95.6% for the three types of mixtures.

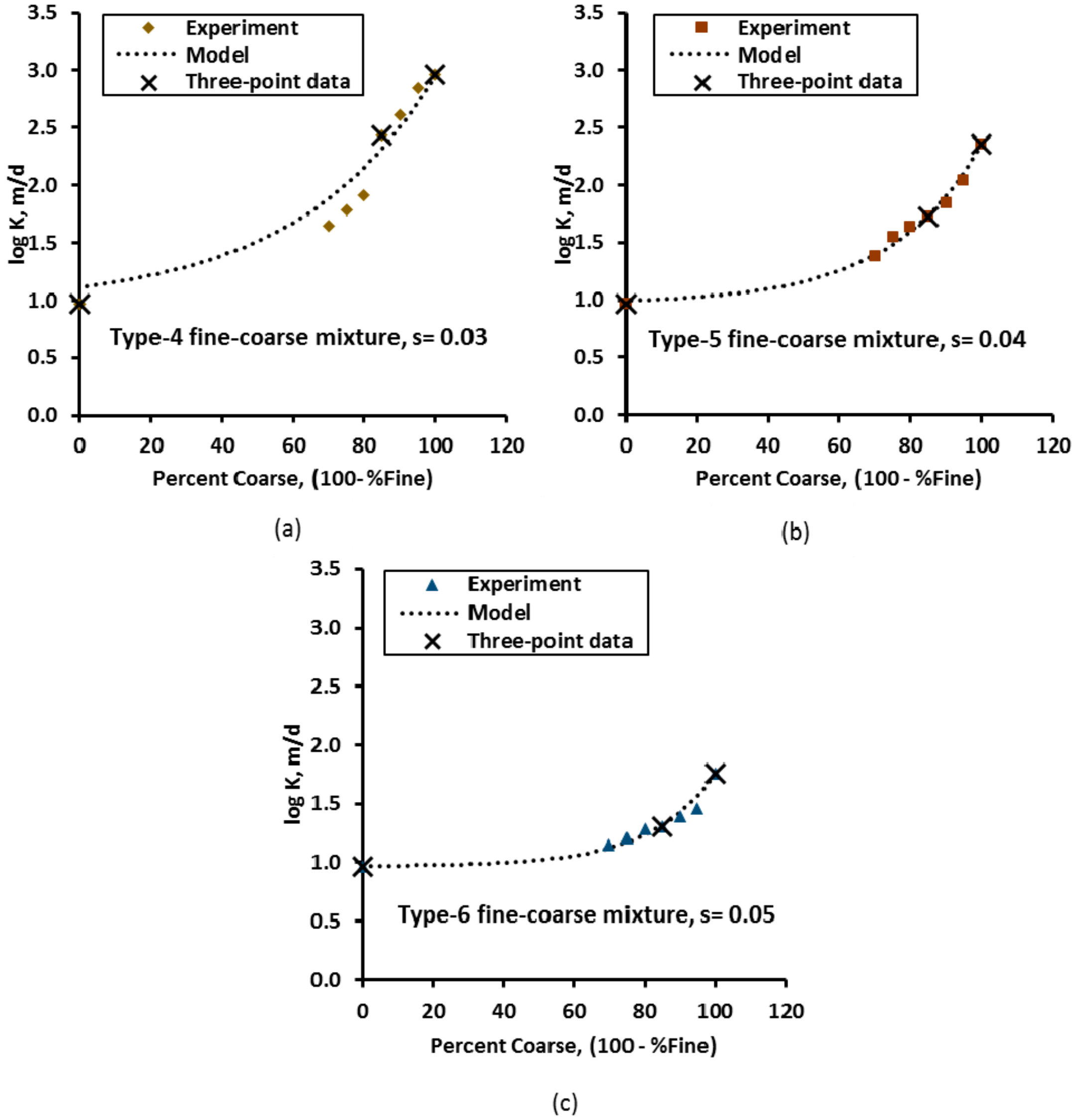

Figure 5. Comparison of fitted model results with experimental data for all three types of fine-coarse mixtures: (**a**) Type-4 (K_c = 920 m/day and K_f = 9 m/day), (**b**) Type-5 (K_c = 228 m/day and K_f = 9 m/day), and (**c**) Type-6 (K_c = 57 m/day and K_f = 9 m/day). The cross symbol is used to identify the data points used in the three-point method discussed in Section 3.5.

3.3. Results of Clay-Sand Mixtures

The performance of the model was tested using an exploratory dataset collected using natural clay and sand. Figure 6 shows both experimental data and model results for a natural sand, with saturated hydraulic conductivity value of 46.5 m/day, mixed with a natural clay with saturated hydraulic conductivity value of 1.1×10^{-3} m/day. The behavior of the mixture at varying clay content was similar with the behavior of the different types of synthetic mixtures discussed in the previous section. The optimal fitted value of the scaling parameter *s* for the clay-sand system was estimated to be 0.08. This result gave a value of 97.3% for the coefficient of determination R^2.

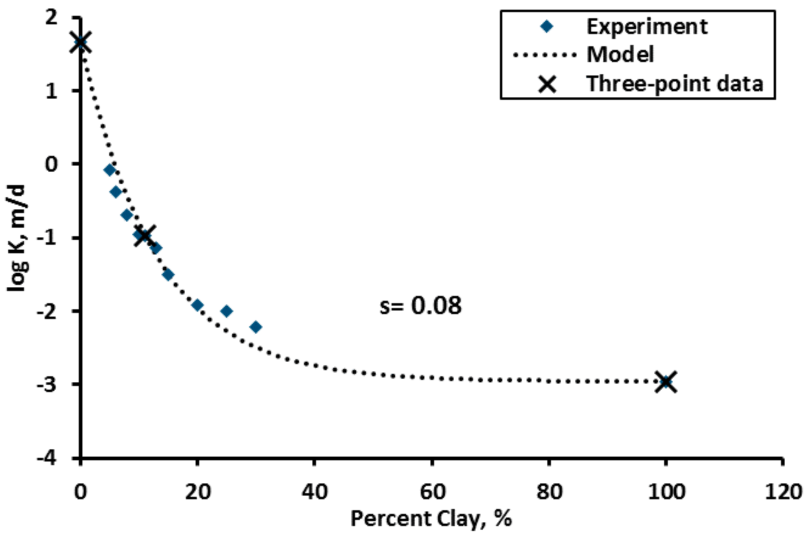

Figure 6. Comparison of fitted model results with experimental data for the natural clay and sand mixture. The cross symbol is used to identify the data points used in the three-point method discussed in Section 3.5.

3.4. Comparison of Model Results with Literature-Derived Experimental Datasets

The validity of the model in representing the changes in hydraulic conductivity of the clay-sand mixtures at varying clay content levels was further tested using data from previous studies. Denson et al. [25] reported the hydraulic conductivity of clay-sand mixtures at varying clay content for both montmorillonite and kaolinite clay. The purpose of their study was to investigate the reduction in hydraulic conductivity of a sand by introducing different amount of clay. The hydraulic conductivity of the clay-sand mixture reduced to practically zero when montmorillonite and kaolinite content were about 3 and 16 percent, respectively. Figure 7 shows both model results and experimental data from Denson et al. [25] for montmorillonite and kaolinite mixed with sand. These results indicate that experimentally measured hydraulic conductivity values at different clay contents are well described by our model—Equation (1).

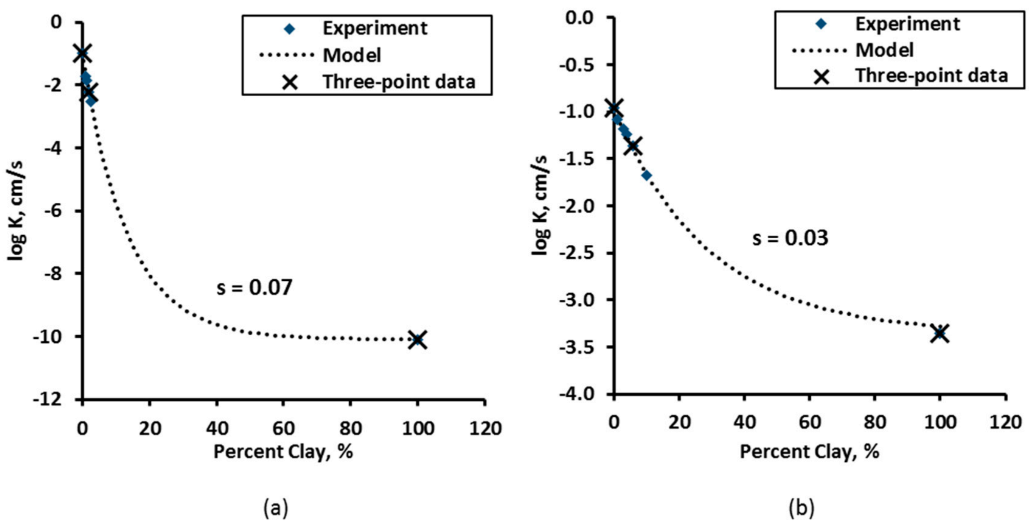

Figure 7. Comparison of our fitted model results with experimental data from Denson et al. [25] for clay-sand mixtures: (**a**) montmorillonite clay, and (**b**) kaolinite clay. The cross symbol is used to identify the data points used in the three-point method discussed in Section 3.5.

Also, the dataset from a study carried out by Doley et al. [26] was used to test the validity of the proposed model. Figure 8 shows both experimental data and model results for bentonite mixed with locally available silty soil in Brahmaputra, India. Several hydraulic conductivity tests were performed to investigate the effect of bentonite content at various percentages on the locally available soil in order to find the optimum percentage that can be used to get the required low hydraulic conductivity value. The study recommended mixing the available soil with 20 to 30 percent dry weight of bentonite to modify and use the available soil as low conductivity liner material.

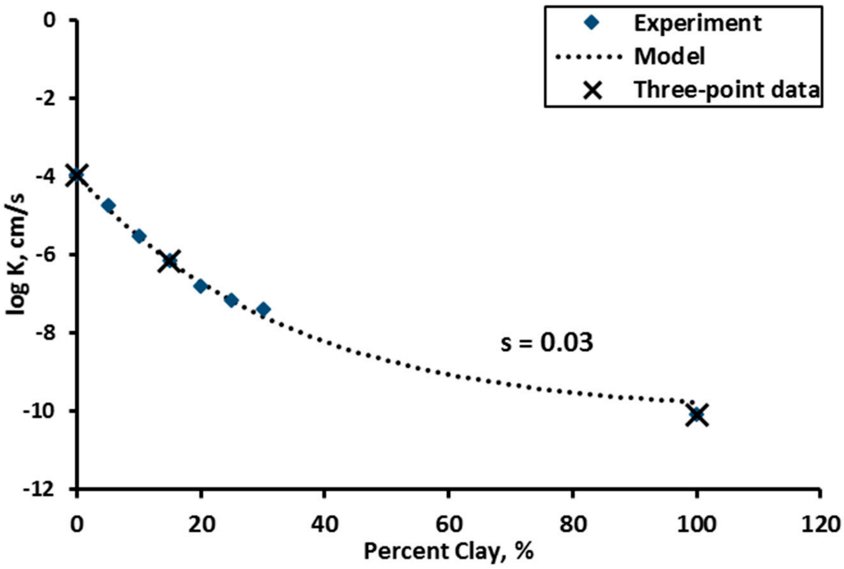

Figure 8. Experimental hydraulic conductivity values for bentonite-silt mixtures [26] at different bentonite content in comparison with fitted model results using the proposed model—Equation (1). The cross symbol is used to identify the data points used in the three-point method discussed in Section 3.5.

3.5. A Simple Three-Point Method to Estimate the Scaling Parameter

In all of the above discussions, the model fitting exercise was completed using all available data points. Based on further analysis we have found that in order to approximately estimate the value of the model parameter (the scaling factor *s*), one would at least need three data points. Fortunately, two of these points are end members (fine and coarse material) and therefore all we need is the value of one coarse-fine mixture. As an example, if our model is used to describe the Doley et al. [26] dataset, there is no need to measure the hydraulic conductivity values for many bentonite-silt mixtures in order to find the bentonite percentage that would yield the required low conductivity value. All one needs to measure is the hydraulic conductivity value of just one mixture of sand and bentonite. In order to save the time required to perform the hydraulic conductivity tests, we recommend using an optimal mixture with the lowest possible bentonite content required to achieve evenly distributed bentonite throughout the mixture. The hydraulic conductivity value of this mixture coupled with the hydraulic conductivity of the end members (sand and bentonite) are sufficient to adequately fit and determine the scaling factor *s*. The fitted model can then be used to estimate the hydraulic conductivity values for any bentonite content ranging from 0 to 100 percent. We applied this approach to estimate the scaling factor *s* for all our laboratory datasets and for all literature-derived datasets. The estimated values of scaling parameter and the corresponding RMSE values are summarized in Table 1. The three experimental data points used for this approach are shown in Figures 3 and 5–8 for each dataset; note these three data points are marked with a cross symbol. All root mean square errors (RMSEs) and coefficients of determination R^2 of log K in Table 1 were calculated using the entire dataset for fitted models developed using the three-point method; therefore, these RMSEs and R^2 values can be compared with the values of corresponding fitted model developed using the entire dataset. For all different types of mixtures, very little difference was observed between RMSE and R^2 values for fitted models developed using the entire dataset and using the three-point method. Therefore, the three-point method is an acceptable approach for estimating the value of the model parameter *s*.

4. Discussion and Conclusions

In typical engineering projects, a soil mixture with a hydraulic conductivity value of at least 1×10^{-7} cm/s is required to construct hydraulic liners [16,27,28]. The percentage of clay and sand to be used vary based on the properties of the materials used to develop the mixture and compaction conditions during the construction of the liner. A laboratory test conducted by Garlanger et al. [12] recommended a minimum bentonite content of 6% to be mixed with locally available material, for a landfill site in central Florida, to achieve a hydraulic conductivity value of 1×10^{-8} cm/s or less. In another study, Gleason et al. [27] stated that a bentonite content of $\leq$15% is able to achieve a bentonite-sand mixture with a hydraulic conductivity value of less than 1×10^{-7} cm/s. The required bentonite percentage was varied based on the type of bentonite, sodium or calcium bentonite, and sand type. Usually, the bentonite content ranged between 5 and 15 percent [8]. In all these studies, multiple laboratory experiments were conducted to determine the bentonite content required to achieve the target hydraulic conductivity value. The model proposed in this study can estimate the hydraulic conductivity values of coarse-fine mixtures with fine content varying between 0 and 100 percent without using multiple experimental data points. In practical applications, in order to estimate the value of the model parameter (the scaling factor *s*), one would need at least three data points. In the proposed three-point method these values can include the hydraulic conductivity values of the two end members (pure coarse material and pure fine material), and at least one critical mixture. We recommend using a mixture with the lowest possible fine (or clay material) percentage required to achieve a uniform, well-mixed mixture. A good rule of thumb is to use about 10 to 20 percent of fine material. These three data can be used to fit the model and estimate the value of the scaling parameter *s*, and then the fitted model can be used to calculate the hydraulic conductivity of mixtures with various amounts of fines. Table 1 compares the values of *s* evaluated using the three-point method against the values estimated using the entire dataset. The refitted *s* values for all synthetic coarse-fine and fine-coarse mixtures, clay-sand mixtures, and literature-derived experimental datasets using this three-point approach were close to the values estimated using the full set of data.

To summarize, in this study, we have investigated the performance of a scalable model that used for predicting the changes in the hydraulic conductivity value of coarse and fine porous media mixtures due to the presence of different amounts of fines. Several laboratory experiments that represented the percentage of fines ranging from 0 to 30 were conducted using simulated coarse-fine and fine-coarse synthetic porous media mixtures. The value of the hydraulic conductivity of the coarse porous media decreased as the percent fine increased in the mixture. The reduction in hydraulic conductivity was significant when we started to add fine material to the coarse material; however, the reductions became less significant when the percentage of fine exceeded about 15%. Typically, the overall hydraulic conductivity value of the mixture was almost close to the hydraulic conductivity of the fine particles when the percent of the fine was above 30%.

In the past, others have attempted to predict reductions in the hydraulic conductivity of coarse sand due to the presence of clay material and other fine minerals. However, all available models require measurement of multiple physical properties of the porous media mixture. The resulting empirical expressions have several model parameters that need to be individually calibrated by conducting multiple soil characterization tests. In this study, we have proposed a simpler model that uses a single model parameter to estimate the hydraulic conductivity values of different types of mixtures. The proposed model successfully correlated the changes in hydraulic conductivity values of a variety of coarse-fine mixtures with the amount of fine material in the mixture. We have tested the model performance using several new laboratory datasets, and also using multiple literature-derived datasets.

It is important to note that this method is suitable only for modeling artificial mixtures and should not be used to predict the hydraulic conductivity value of undisturbed, heterogeneous natural porous media. The proposed framework is a useful tool for modeling the hydraulic properties of various types of engineered mixtures. The model can help design optimal mixtures without conducting multiple experiments that could be time consuming and cost prohibitive.

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