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**Supplementary Materials for**  
**Interdecadal variations in the Walker circulation and its**  
**connection to inhomogeneous air temperature changes**  
**during 1961–2012**

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50 normalized PWC intensity index based on the MSF for the period 1961–2012. The  
51 regression coefficients statistically significant at the 5% confidence level are dotted in  
52 black.

53 **Introduction**

54 We provide figures in the Supplementary Materials to support the results discussed  
55 in the main text. Figures S1 and S2 serve as examples for the explanation of the  
56 “inhomogeneous” concept proposed in the main text. Figures S3–S7 display the results  
57 that are the interdecadal variations in the PWC and its connection to inhomogeneous  
58 air temperature changes by using the mass stream function (MSF) derived from the  
59 method proposed by Schwendike et al. [1,2]. Figures S3–S7 support the comparison of  
60 the method proposed by Schwendike et al. [1,2] and the three-pattern decomposition of  
61 global atmospheric circulation (3P-DGAC) method [3,4] in representing the PWC.

62 In addition, in the Supplementary Materials, we provide an explanation of the  
63 “inhomogeneous” concept proposed in the main text, a brief description of the method  
64 proposed by Schwendike et al. [1,2] (i.e., the local partitioning method of the  
65 overturning circulation in the tropics), and a brief introduction of the new dynamical  
66 equations derived from the 3P-DGAC method [3,4].

67

68 **Explanation of the “inhomogeneous” concept proposed in the main text**

69 In this section, we give an explanation of the “inhomogeneous” concept proposed  
70 in the main text. In this paper, the “homogeneous” part of one variable is defined as the  
71 global zonal mean of the variable, and the “inhomogeneous” part of the variable is  
72 defined as the difference between the variable and the “homogeneous” part of the  
73 variable (i.e., the variable minus the global zonal mean of the variable). Figures S1 and  
74 S2 are two examples that can be used to explain the “inhomogeneous” concept. Figures

75 S1a–c display the mean state of the annual mean meridionally averaged air temperature  
 76 between 5°S and 5°N, the “homogeneous” part of the air temperature, and the  
 77 “inhomogeneous” part of the air temperature, respectively. Figures S2a–c display the  
 78 mean state of the annual mean meridionally averaged zonal stream function (ZSF)  
 79 between 5°S and 5°N, the “homogeneous” part of the ZSF, and the “inhomogeneous”  
 80 part of the ZSF, respectively. Thus, the “inhomogeneous air temperature” in the main  
 81 text is the difference between the air temperature and the global zonal mean of the air  
 82 temperature.

83

84 **Local partitioning method of the overturning circulation in the tropics (see also**  
 85 **Schwendike et al. [1,2])**

86 According to Schwendike et al. [1,2], the horizontal vortex circulation

87  $\vec{V}_{rot} = u_{rot}\vec{i} + v_{rot}\vec{j}$  at low latitudes that satisfies

$$88 \quad \nabla_p \cdot \vec{V}_{rot} = \frac{1}{a \cos \varphi} \frac{\partial u_{rot}}{\partial \lambda} + \frac{1}{a \cos \varphi} \frac{\partial v_{rot} \cos \varphi}{\partial \varphi} = 0, \quad (S1)$$

89 is often neglected. Thus, the large-scale motion in the low latitudes is represented as  
 90 follows:

$$91 \quad \vec{V} = \vec{V}_{div} + \omega \vec{k}, \quad (S2)$$

92 which satisfies the continuity equation as follows:

$$93 \quad \nabla_p \cdot \vec{V}_{div} + \frac{\partial \omega}{\partial p} = \frac{1}{a \cos \varphi} \frac{\partial u_{div}}{\partial \lambda} + \frac{1}{a \cos \varphi} \frac{\partial v_{div} \cos \varphi}{\partial \varphi} + \frac{\partial \omega}{\partial p} = 0. \quad (S3)$$

94 By using Equations (S2) and (S3), Schwendike et al. [1,2] defined the zonal  
 95 overturning circulation  $\vec{V}_\lambda$  and the meridional overturning circulation  $\vec{V}_\varphi$  in the

96 tropics as follows:

$$97 \quad \vec{V}_\lambda = u_{div} \vec{i} + \omega_\lambda \vec{k}, \quad \vec{V}_\varphi = v_{div} \vec{i} + \omega_\varphi \vec{k}, \quad (S4)$$

98 which satisfy the following continuity equations:

$$99 \quad \frac{1}{a \cos \varphi} \frac{\partial u_{div}}{\partial \lambda} + \frac{\partial \omega_\lambda}{\partial p} = 0, \quad (S5)$$

$$100 \quad \frac{1}{a \cos \varphi} \frac{\partial v_{div} \cos \varphi}{\partial \varphi} + \frac{\partial \omega_\varphi}{\partial p} = 0. \quad (S6)$$

101 Equations (S5) and (S6) ensure that the components  $u_{div}$ ,  $\omega_\lambda$ ,  $v_{div}$ , and  $\omega_\varphi$  can be  
 102 represented by the stream functions  $\psi_\lambda$  and  $\psi_\varphi$  as follows:

$$103 \quad u_{div} = -\frac{\partial \psi_\lambda}{\partial p}, \quad \omega_\lambda = \frac{1}{a \cos \varphi} \frac{\partial \psi_\lambda}{\partial \lambda}, \quad (S7)$$

$$104 \quad v_{div} = -\frac{\partial \psi_\varphi}{\partial p}, \quad \omega_\varphi = \frac{1}{a \cos \varphi} \frac{\partial \psi_\varphi \cos \varphi}{\partial \varphi}. \quad (S8)$$

105 Equations (S2), (S4), (S5), and (S6) can be combined to generate the following:

$$106 \quad \vec{V} = \vec{V}_{div} + \omega \vec{k} = \vec{V}_\lambda + \vec{V}_\varphi, \quad (S9)$$

107 which means that  $\vec{V}_\lambda$  and  $\vec{V}_\varphi$  represent a decomposition of the overturning  
 108 circulation at low latitudes. In addition, Equations (S5) and (S6) also represent a  
 109 decomposition of the continuity equation (S3). Schwendike et al. [1,2] referred to  
 110 Equation (S9) as the local partitioning of the overturning circulation in the tropics and  
 111  $\vec{V}_\lambda$  and  $\vec{V}_\varphi$  as the local Hadley and Walker circulations, respectively.

112 In the summary section of the main text, the zonal velocity  $u_{div}$  and mass stream  
 113 function (MSF)  $MSF = \frac{\pi a}{g} \int_p^0 u_{div} dp$  ( $p$  is pressure,  $a$  is the earth's radius, and  $g$   
 114 is the gravitational acceleration) of the local Walker circulation are used. Additional  
 115 details of the local partitioning method can be obtained from Schwendike et al. [1,2].

116 **New dynamical equations derived from the 3P-DGAC method (see also Hu et al.**  
 117 **[3,4])**

118 The new dynamical equations of the horizontal, meridional, and zonal circulations  
 119 can be obtained by combining the three-pattern decomposition of global atmospheric  
 120 circulation (3P-DGAC) method and the primary equations as follows:

$$121 \frac{\partial}{\partial t} \left( -\frac{\partial^2 H}{\partial \sigma^2} + \frac{1}{\sin \theta} \frac{\partial^2 R}{\partial \lambda \partial \sigma} \right) - \frac{\partial}{\partial \sigma} \pi' \frac{\partial H}{\partial \sigma} - \frac{\partial}{\partial \sigma} A' \frac{\partial W}{\partial \sigma} + \frac{\partial}{\partial \sigma} \left( A' \frac{\partial R}{\partial \theta} + \pi' \frac{1}{\sin \theta} \frac{\partial R}{\partial \lambda} \right)$$

$$122 + \frac{\partial}{a \partial \sigma} \tilde{L}'_1 \left( \frac{1}{\sin \theta} \frac{\partial R}{\partial \lambda} - \frac{\partial H}{\partial \sigma} \right) = \frac{R_0}{a^2 \sigma} \frac{\partial T}{\partial \theta}, \quad (S10)$$

$$123 \frac{\partial}{\partial t} \left( \frac{\partial^2 W}{\partial \sigma^2} - \frac{\partial^2 R}{\partial \theta \partial \sigma} \right) - \frac{\partial}{\partial \sigma} A' \frac{\partial H}{\partial \sigma} + \frac{\partial}{\partial \sigma} \pi' \frac{\partial W}{\partial \sigma} + \frac{\partial}{\partial \sigma} \left( A' \frac{1}{\sin \theta} \frac{\partial R}{\partial \lambda} - \pi' \frac{\partial R}{\partial \theta} \right)$$

$$124 + \frac{\partial}{a \partial \sigma} \tilde{L}'_1 \left( \frac{\partial W}{\partial \sigma} - \frac{\partial R}{\partial \theta} \right) = \frac{R_0}{a^2 \sigma} \frac{1}{\sin \theta} \frac{\partial T}{\partial \lambda}, \quad (S11)$$

$$125 \frac{\partial}{\partial t} (\Delta_3 R) + \frac{1}{\sin \theta} \left( \frac{\partial}{\partial \theta} \sin \theta A' \frac{\partial H}{\partial \sigma} - \frac{\partial}{\partial \lambda} \pi' \frac{\partial H}{\partial \sigma} \right) - \frac{1}{\sin \theta} \left( \frac{\partial}{\partial \theta} \sin \theta \pi' \frac{\partial W}{\partial \sigma} + \frac{\partial}{\partial \lambda} A' \frac{\partial W}{\partial \sigma} \right)$$

$$126 + \frac{1}{\sin \theta} \left( \frac{\partial}{\partial \theta} \sin \theta \pi' \frac{\partial R}{\partial \theta} + \frac{\partial}{\partial \lambda} \pi' \frac{1}{\sin \theta} \frac{\partial R}{\partial \lambda} - \frac{\partial}{\partial \theta} A' \frac{\partial R}{\partial \lambda} + \frac{\partial}{\partial \lambda} A' \frac{\partial R}{\partial \theta} \right)$$

$$127 + \frac{1}{a \sin \theta} \frac{\partial}{\partial \lambda} \tilde{L}'_1 \left( \frac{1}{\sin \theta} \frac{\partial R}{\partial \lambda} - \frac{\partial H}{\partial \sigma} \right) - \frac{1}{a \sin \theta} \frac{\partial}{\partial \theta} \sin \theta \tilde{L}'_1 \left( \frac{\partial W}{\partial \sigma} - \frac{\partial R}{\partial \theta} \right) = 0, \quad (S12)$$

$$128 \frac{R_0^2}{ac^2} \frac{\partial T}{\partial t} - \frac{R_0}{a\sigma} \left( \frac{1}{\sin \theta} \frac{\partial(\sin \theta H)}{\partial \theta} - \frac{1}{\sin \theta} \frac{\partial W}{\partial \lambda} \right) + \frac{R_0^2}{ac^2} \pi' T + \tilde{L}'_2 T = \frac{R_0^2}{ac^2} \frac{\varepsilon}{c_p}, \quad (S13)$$

129 where  $R$ ,  $H$ , and  $W$  represent the stream functions of the horizontal, meridional,

130 and zonal circulations, respectively.  $\pi' = \frac{u'}{\sin \theta} \frac{\partial}{\partial \lambda} + v' \frac{\partial}{\partial \theta} + \dot{\sigma} \frac{\partial}{\partial \sigma}$ ,

$$131 A' = 2\Omega \cos \theta + ctg \theta u' \quad , \quad c^2 = \frac{R_0^2 \bar{T}}{g} (\gamma_d - \bar{\gamma}) \quad , \quad \tilde{L}'_1 = -a \frac{\partial}{\partial \sigma} v_1 \left( \frac{g\sigma}{R_0 \bar{T}} \right)^2 \frac{\partial}{\partial \sigma} - a \mu_1 \nabla^2 \quad ,$$

$$132 \tilde{L}'_2 = -\frac{1}{a} \frac{\partial}{\partial \sigma} v_2 \left( \frac{g\sigma}{R_0 \bar{T}} \right)^2 \frac{\partial}{\partial \sigma} - \frac{1}{a} \mu_2 \nabla^2, \text{ and } \nabla^2 = \frac{1}{a^2 \sin^2 \theta} \frac{\partial^2}{\partial \lambda^2} + \frac{1}{a^2 \sin \theta} \frac{\partial}{\partial \theta} \sin \theta \frac{\partial}{\partial \theta}.$$

133 A simplified model of the new dynamical equations can be obtained as follows:

$$134 \frac{\partial}{\partial t} \left( -\frac{\partial^2 H}{\partial \sigma^2} + \frac{1}{\sin \theta} \frac{\partial^2 R}{\partial \lambda \partial \sigma} \right) = \frac{R_0}{a^2 \sigma} \frac{\partial T}{\partial \theta}, \quad (S14)$$

135 
$$\frac{\partial}{\partial t} \left( \frac{\partial^2 W}{\partial \sigma^2} - \frac{\partial^2 R}{\partial \theta \partial \sigma} \right) = \frac{R_0}{a^2 \sigma} \frac{1}{\sin \theta} \frac{\partial T}{\partial \lambda}, \quad (\text{S15})$$

136 
$$\frac{\partial}{\partial t} (\Delta_3 R) = -\pi'_R (\Delta_2 R - f), \quad (\text{S16})$$

137 
$$\frac{\partial}{\partial t} \left( \frac{R_0^2}{a^2 c^2} T \right) = \frac{R_0}{a^2 \sigma} \frac{1}{\sin \theta} \left( \frac{\partial}{\partial \theta} \sin \theta H - \frac{\partial W}{\partial \lambda} \right) - \frac{R_0^2}{a^2 c^2} \pi'_R T, \quad (\text{S17})$$

138 where  $\pi'_R = \frac{u'_R}{\sin \theta} \frac{\partial}{\partial \lambda} + v'_R \frac{\partial}{\partial \theta}$  and  $\Delta_2 = \frac{1}{\sin^2 \theta} \frac{\partial^2}{\partial \lambda^2} + \frac{1}{\sin \theta} \frac{\partial}{\partial \theta} \left( \sin \theta \frac{\partial}{\partial \theta} \right)$ .

139 More details on the new dynamical equations derived from the 3P-DGAC method  
 140 and its simplified model can be obtained in Hu et al. [3,4].

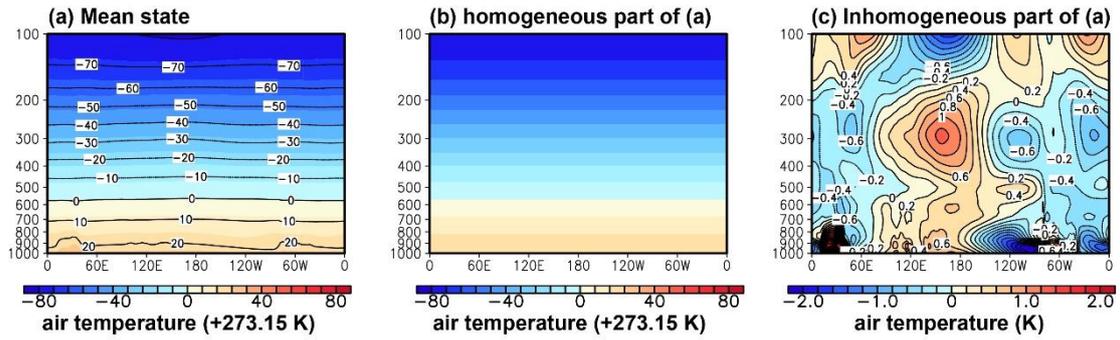
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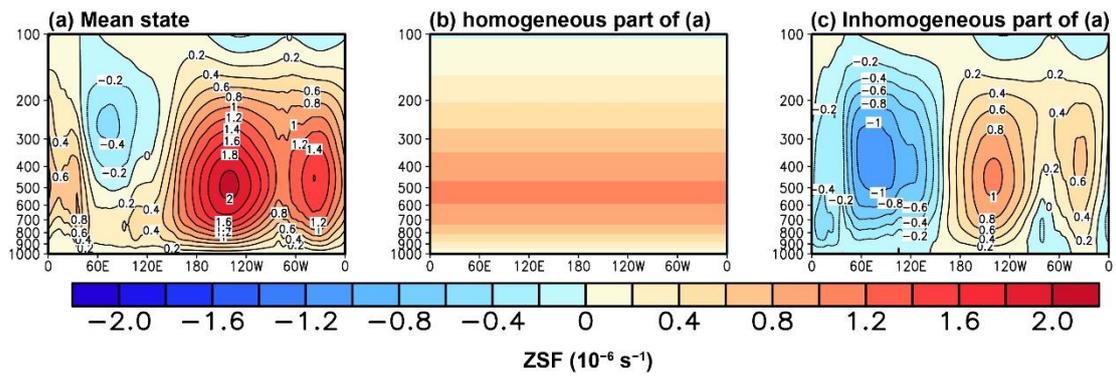
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156 <https://doi.org/10.1007/s00382-017-3763-1>



157

158 **Figure S1 a** Mean state (1961–2012) of the annual mean meridionally averaged air  
 159 temperature (K) between 5°S and 5°N using the JRA-55 reanalysis dataset. **b**  
 160 homogeneous part of **a** (global zonal mean of **a**). **c** inhomogeneous part of **a** (**a** minus  
 161 **b**).

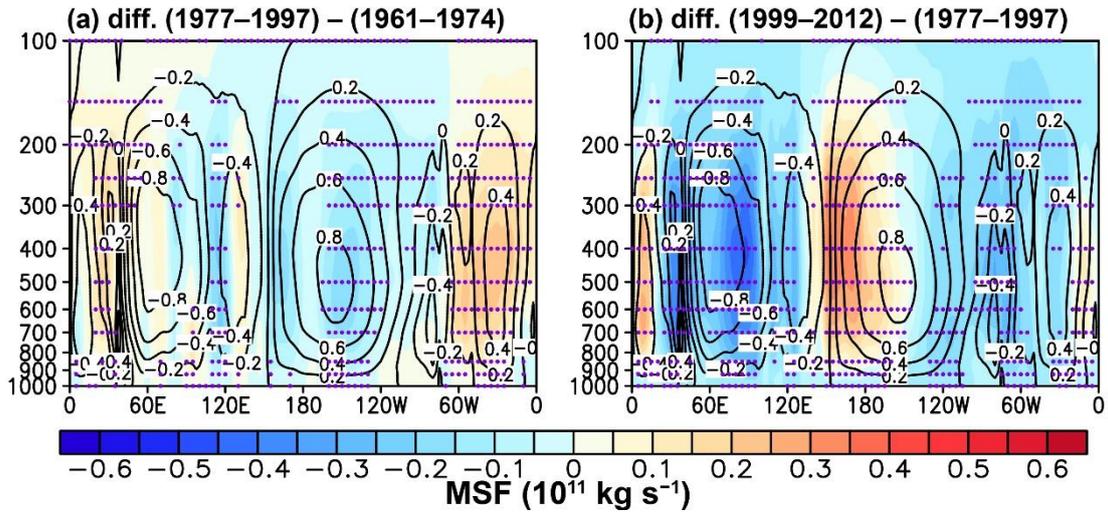


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163 **Figure S2 a** Mean state (1961–2012) of the annual mean meridionally averaged ZSF

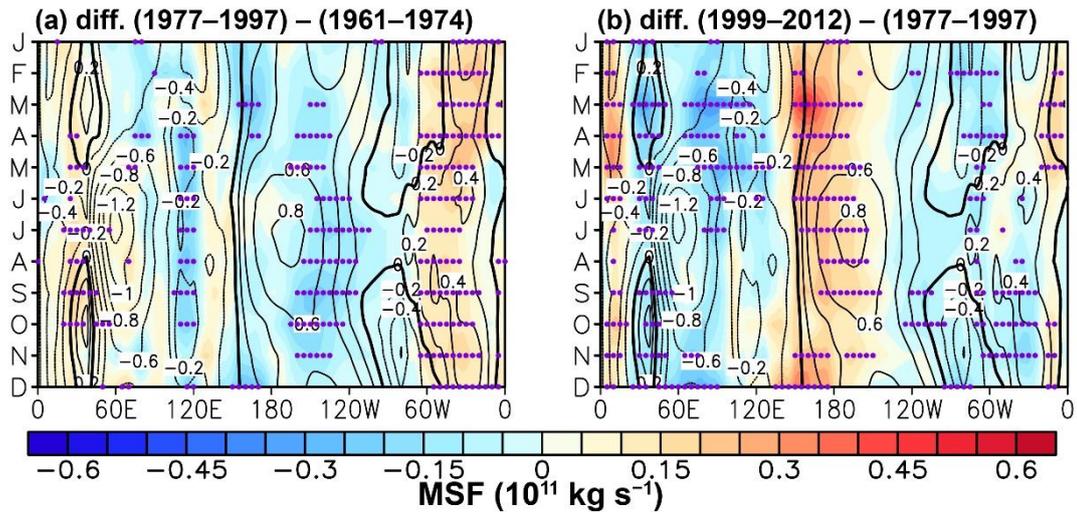
164 ( $10^{-6} \text{ s}^{-1}$ ) between  $5^{\circ}\text{S}$  and  $5^{\circ}\text{N}$  using the JRA-55 reanalysis dataset. **b** homogeneous

165 part of **a** (global zonal mean of **a**). **c** inhomogeneous part of **a** (**a** minus **b**).



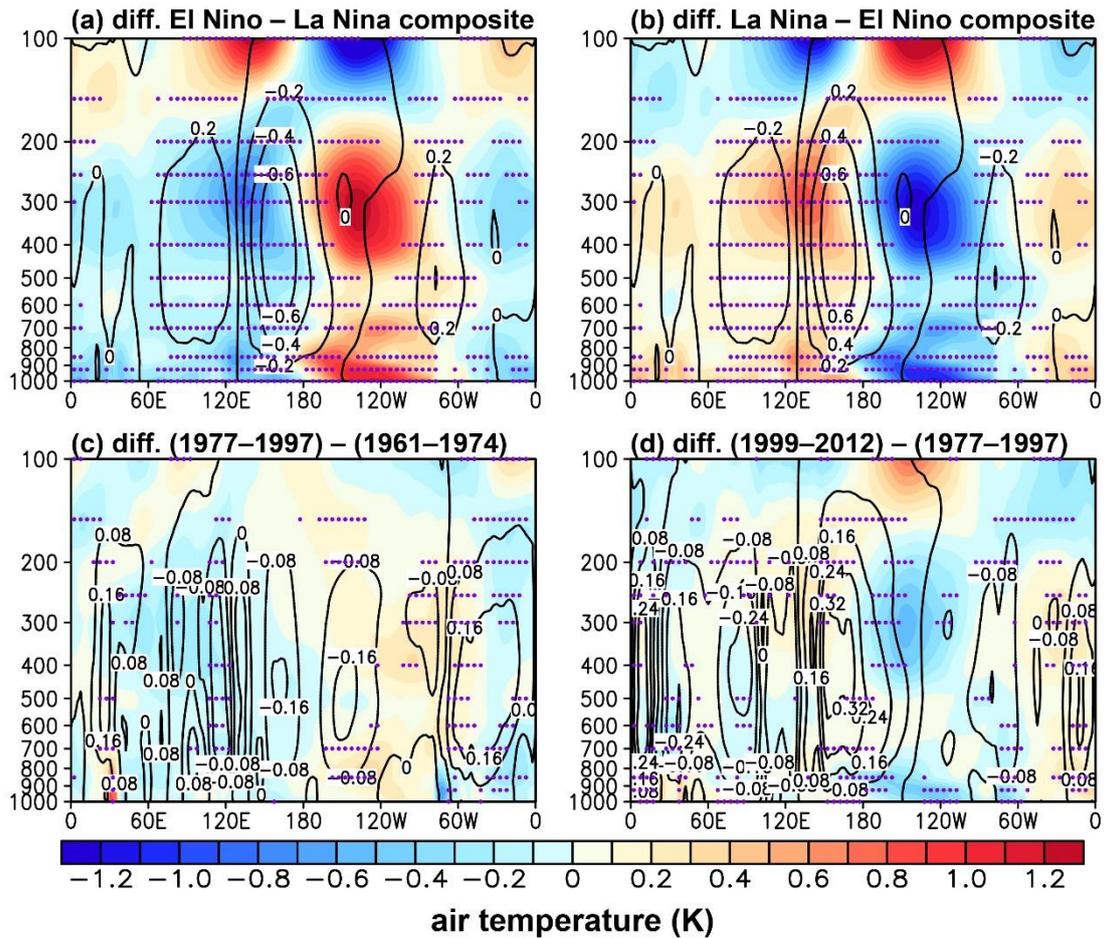
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167 **Figure S3** Contours in **a** and **b**: mean state (1961–2012) of the annual mean zonal  
 168 circulations along the equator ( $5^{\circ}\text{S}$ – $5^{\circ}\text{N}$ ); shading: differences in the annual mean zonal  
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 171 are dotted in purple. The zonal circulations are represented by the MSF ( $10^{11} \text{ kg s}^{-1}$ )  
 172 derived from the method developed by Schwendike et al. [1,2].



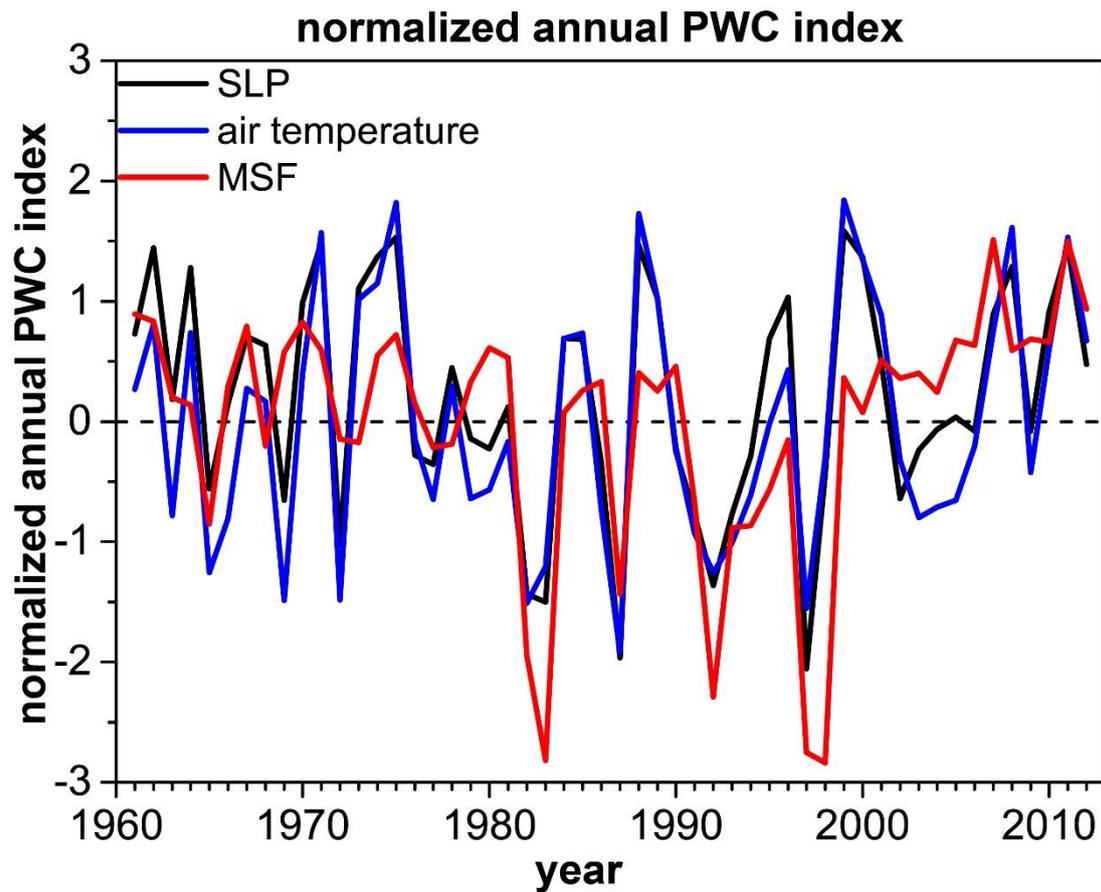
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 175 (shading) in the monthly mean of vertically averaged zonal circulations. The zonal  
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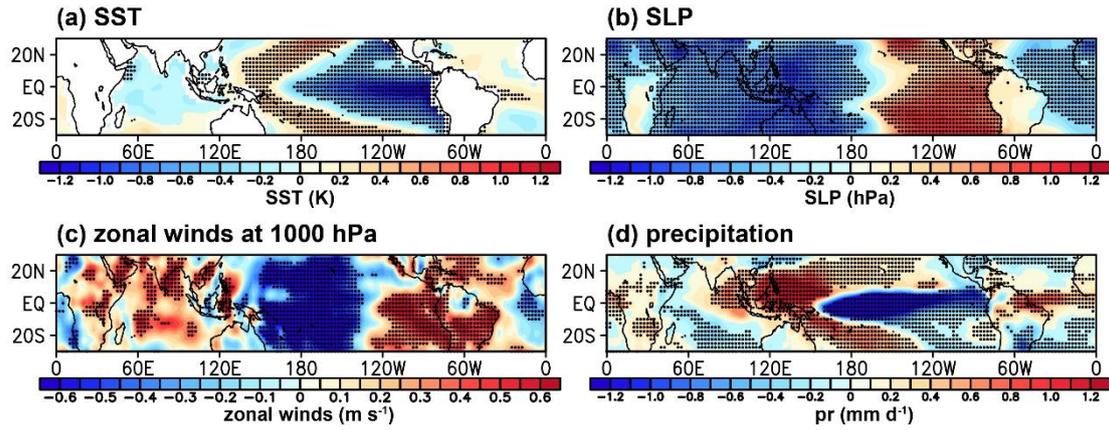
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179 **Figure S5** Differences in the zonal circulation (contours;  $10^{11} \text{ kg s}^{-1}$ ) and  
 180 inhomogeneous air temperature (shading; K) along the equator ( $5^{\circ}\text{S}$ – $5^{\circ}\text{N}$ ) between **a**  
 181 El Niño and La Niña, **b** La Niña and El Niño, **c** the periods 1961–1974 and 1977–1997,  
 182 and **d** the periods 1977–1997 and 1999–2012. The zonal circulations are represented by  
 183 the MSF derived from the method developed by Schwendike et al. [1,2].



184

185 **Figure S6** Time series (1961–2012) of the normalized annual mean tropical PWC  
 186 intensity indices based on the SLP (defined as the mean SLP over the region (5°S–5°N,  
 187 80°W–160°W) minus the mean SLP over the region (5°S–5°N, 80°E–160°E); black  
 188 line), air temperature (defined as in Equation (13) in the main text; blue line), and MSF  
 189 (defined as the mean value of the MSF over the region (5°S–5°N, 140°W–180°, 300–  
 190 600 hPa); red line)



191

192 **Figure S7** Spatial pattern of the regressed **a** SST (K), **b** SLP (hPa), **c** zonal winds of the  
 193 zonal circulation at 1000 hPa ( $\text{m s}^{-1}$ ), and **d** precipitation ( $\text{mm d}^{-1}$ ) against the  
 194 normalized PWC intensity index based on the MSF for the period 1961–2012. The  
 195 regression coefficients statistically significant at the 5% confidence level are dotted in  
 196 black.