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Research on the Characteristics of Agricultural Drought Disaster in China Based on Three-Dimensional Copula Function

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Abstract: Estimating the probability and consequences of drought disasters is an important task in drought risk assessment, which contributes to the development of mitigation strategies. Based on rainfall data from 2481 stations and the drought-affected arable land of each province from 1961 to 2021, a probabilistic analysis model of drought duration, drought severity and the proportion of affected farmland area (PAFA) was constructed by a three-dimensional copula function. The results show that the distribution functions of drought duration, drought severity and the PAFA are well given based on the principle of maximum entropy and can pass the Kolmogorov–Smirnov (K-S) distribution test with a significance level of 0.05. Among the three Archimedean copulas, Frank's method has a relatively better accuracy, suggesting that it captures the dependencies between drought characteristics better and is more suitable for constructing joint distribution functions. The value of PAFA in Gansu, Inner Mongolia, Shanxi and Liaoning is about 0.3, which is higher than other provinces. Drought duration and drought severity levels of 3 to 4 are the main causes of a PAFA greater than 0.3, which can be used as an early warning line for drought risk. At the same level of PAFA, the drought in the southern region lasted longer and was more intense.

Keywords: maximum entropy principle; three-dimensional copula; drought duration; drought severity; drought-affected area

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1. Introduction

Drought is a common natural phenomenon, mainly caused by a prolonged lack of water. Compared with disasters caused by heavy rainfall, floods and typhoons, drought is a relatively slow process. This relatively slow development does not make drought prevention and mitigation any easier. China is a drought-prone country, and drought-affected arable land accounts for nearly 50% of the area affected by all meteorological disasters each year, a value that far exceeds other natural disasters [1]. In 2022, the drought in the Yangtze River Basin was the most severe meteorological and hydrological drought since complete measured data became available. At the peak of the drought, a total of 52,452,000 people were affected, with 60,902,000 hectares of crops affected and 51.28 billion yuan in direct economic losses. Drought is a multidisciplinary cross-cutting issue, and although many scholars have carried out extensive research in the field of drought disaster risk and achieved remarkable results, they still cannot meet the current needs of society [2–5]. Strengthening drought-related research can help us better understand the processes and causes of drought occurrence, and this knowledge will contribute to the development of effective drought disaster prevention and mitigation measures [6–9].

Agricultural drought cannot be analyzed without the characterization of water sources, rainfall and runoff being the most important sources of water for agriculture. These

quantities have stochastic characteristics and are suitable for description by probabilistic models. For stochastic problems, finding the distribution function of a random variable is the basis of problem analysis. In the characterization of drought, drought duration, drought severity and drought-affected area are the variables of greater interest [10,11]. However, these variables are often dependent, and the actual calculations verify that there is a significant correlation between the variables [12,13]. For random variables that are not independent and have inconsistent types of distribution functions, there are difficulties in solving for the joint distribution function of these variables. The copula function is currently an effective method to solve this problem, and many researchers have used the copula function to conduct multivariate statistical analyses. For example, the copula function is applied in drought frequency characterization [14–16], drought risk analysis [12], drought indicator construction [17,18], in flood and streamflow forecasting [19,20], in wind energy forecasting [21,22] and heatwaves [23,24]. This is a good indication that the copula method has been favored by many scholars in the construction of distribution functions with multiple variables. In drought analysis, the modeling process usually starts with the isolation of drought characteristic variables (drought duration, drought intensity, drought kurtosis, etc.) from the time series data of drought indexes. Secondly, the parameters of the copula function are calculated to enable the frequency characterization of these variables. Most of the research is concerned with modeling two-dimensional copula analyses, and relatively little is done on copula analyses of three or more dimensions [12,25,26]. In terms of content, the characterization is mostly based on a single index, and little literature has established a link to the actual magnitude of the disaster. Hou et al. [12] attempted to establish a link between drought intensity and disaster variables based on the copula function. However, the reliability of the three-dimensional copula model in the paper was not fully verified due to the scarcity of disaster data. In fact, drought analyses using copula functions with dimensions greater than three are uncommon, mainly because they are also related to the sample size, while analyses involving drought disasters are rare. Due to the relative completeness of the disaster data in each province, an attempt was made to carry out drought disaster characterization based on a three-dimensional copula function on a provincial basis.

2. Data and Methodology

2.1. Data

The data consist of two parts: (1) the quality-controlled monthly precipitation data of 2481 stations in China from 1961–2021 provided by the China Meteorological Administration (CMA) and (2) the yearly data of total sown area of crops and drought-affected area in each province from 1961–2021 downloaded from the National Bureau of Statistics (NBS). Figure 1 shows the spatial distribution of the 2481 stations.

The filled colors in Figure 1 indicate the annual mean precipitation. Figure 1 shows that precipitation in Southeastern China can reach up to 1600 mm, while in parts of Northwestern China it is only 200 mm. The number of sites in the eastern region is much greater than the number of sites in the western region.

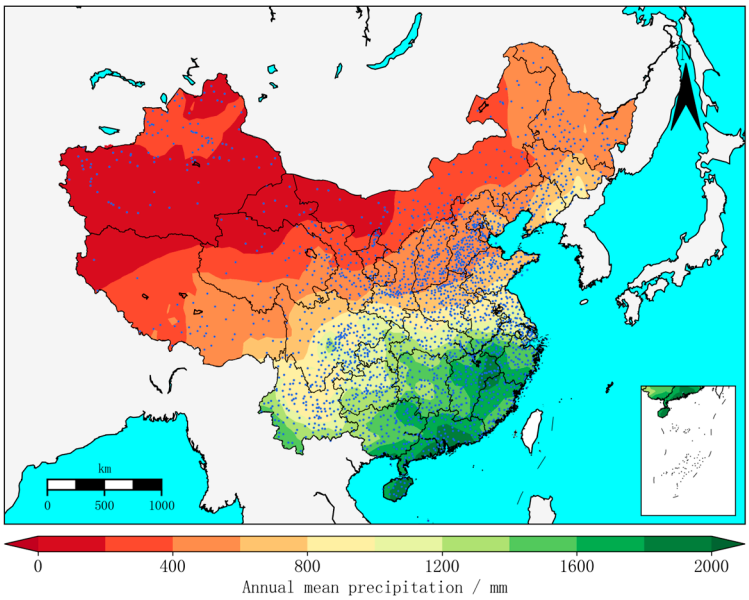


Figure 1. The spatial distribution of meteorological stations, where the blue dots in the figure indicate stations.

2.2. Methodology

2.2.1. Definition of Indicators

In this study, the widely used standardized precipitation index (SPI) [27] was chosen as an indicator for drought description [28,29]. The SPI is a multi-time scale drought index, with commonly used time scales of 1 month, 3 months, 6 months and 12 months. The SPI at different time scales was used to analyze different drought problems. For example, SPI-1 is commonly used for analyses of meteorological droughts, SPI-3 is commonly used for analyses of agricultural droughts, and SPI-6 or an SPI with a longer time scale is commonly used for analyses of hydrological droughts. The drought analysis in this paper is primarily intended to discuss agricultural drought, so SPI-3 was selected as a descriptor of drought magnitude. In order to maintain the same spatial scale as the PAFA, the SPIs were first calculated for each station, and the average SPI for each province was calculated from the stations included in each province. It is worth clarifying that the provincial average SPI is calculated by excluding missing data. The number of months in each year in which the SPI is lower than zero is used as the drought duration D, and the cumulative value of SPI lower than zero is used as the drought severity, which is calculated by the following equation:

$$S = -\sum_{i} SPI_i$$

The PAFA is calculated by reference to the National Standard of the People's Republic of China: Drought Hazard Levels, and is used to express the magnitude of drought impacts. The mathematical expression is as follows:

$$I = \frac{A_d}{A_p} \times 100\%$$

where A_d denotes the area of drought-affected area and A_p denotes the total area of crops.

When the distribution function of a variable is unknown, it is often necessary to compare several common distribution functions and choose the best-fitting function as the distribution function. In order to reduce this tedious comparative analysis, this paper adopts the maximum entropy theory to derive the distribution function, which has been proved to be effective in several articles [13,30].

2.2.2. Derive Distribution Function Based on Maximum Entropy Principle

Entropy is used to describe the uncertainty, chaos and dispersion of a given system. For a random variable X with a probability density function $f(x)$, the Shannon entropy [31] is defined as follows over the interval $[a, b]$:

$$H = -\int_a^b f(x) \ln f(x) dx$$

The univariate marginal distribution is obtained from the maximum entropy principle, a theorem formulated by Jaynes [32], which states that the probability density function is chosen such that the entropy achieves its maximum value under certain constraints. The constraints can be expressed as follows:

$$\int_a^b f(x) dx = 1$$

$$\int_a^b g_i(x) f(x) dx = \overline{g_i(x)}, i = 1, 2 \cdots, m$$

where the function $g_i(x)$ is often chosen as a power function.

Based on the maximum entropy principle, the probability density function can be derived, and the mathematical expression is as follows:

$$f(x) = \exp\left[-\sum_{i=0}^{m} \lambda_i g_i(x)\right]$$

The cumulative probability distribution can be expressed as follows:

$$F(x) = \int_a^x \exp\left[-\sum_{i=0}^{m} \lambda_i g_i(x)\right] dx$$

2.2.3. Copula Function

The copula function is an efficient way to be able to use the marginal distribution functions of different variables to construct their joint distribution functions. Following Sklar's theory [33], the joint distribution $F(x_1, x_1, \ldots, x_n)$ can be decomposed into a copula function and a marginal distribution function $F(x_1), F(x_2), \ldots, F(x_n)$ with the following mathematical expression:

$$F(x_1, x_1, \ldots, x_n) = C(F(x_1), F(x_2), \ldots, F(x_n)).$$

Currently there are three main categories of copula functions, elliptic, Archimedean and quadratic, while Archimedean copula functions are widely used due to their simplicity of computation [14,34,35]. Table 1 shows the commonly used three-dimensional Archimedean copula functions [36,37].

Table 1. Archimedes copula functions.

Name	Copula Function $C_\theta(\mu,\nu,\omega)$ [1]
Clayton	$(\mu^{-\theta} + \nu^{-\theta} + \omega^{-\theta} - 2)^{-1/\theta}$
Frank	$-\frac{1}{\theta}\ln[1 + \frac{(e^{-\theta\mu}-1)(e^{-\theta\nu}-1)(e^{-\theta\omega}-1)}{(e^{-\theta}-1)^2}]$
Gumbel–Hougaard	$\exp\left\{-[(-\ln\mu)^\theta + (-\ln\nu)^\theta + (-\ln\omega)^\theta]^{1/\theta}\right\}$

[1] μ, ν, ω denote the distribution functions of the variables; θ is the copula function parameter.

The parameters θ in the copula function can be solved by the maximum likelihood estimation method, and the copula function is selected by the root mean square error

(RMSE), i.e., finding the copula function type with the smallest RMSE, which is calculated as follows:

$$RMSE = \sqrt{\frac{1}{n}\sum_{i=1}^{n}(Pe_i - P_i)^2}$$

where Pe_i is the empirical probability derived directly from the samples and P_i is the theoretical probability calculated based on different copula functions in Table 1.

2.2.4. Joint Distribution

The joint distribution function is obtained from the marginal distribution functions $F_D(d)$, $F_S(s)$, $F_I(i)$ and Copula functions, which can be expressed as follows:

$$F(d,s,i) = P(D \le d, S \le s, I \le i) = C[F_D(d), F_S(s), F_I(i)]$$

which indicates the probability of a drought event where the drought duration, severity and PAFA are all less than or equal to a given value.

3. Results

3.1. The Drought Characteristics and Distribution Function Fitting Effect

Based on the SPI of each site, the average SPI of each provincial site is calculated, with an SPI equal to 0 as a threshold to identify the drought duration and drought severity of each year. The PAFA is calculated from agricultural statistics published by the National Bureau of Statistics. Taking Heilongjiang Province, the main grain producing area, as an example, the results are shown in Figure 2.

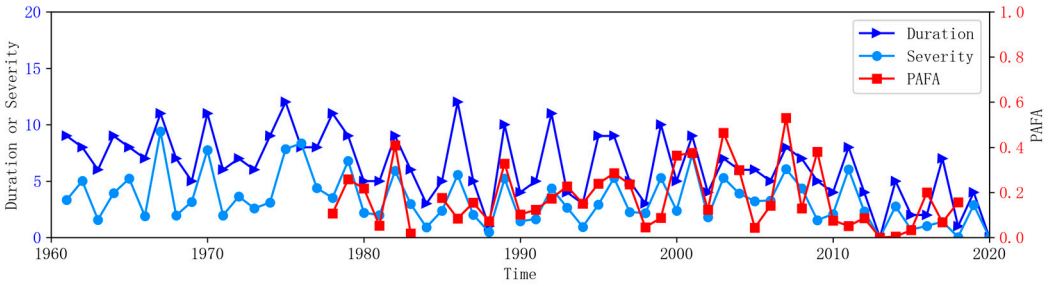


Figure 2. The time series of drought duration, drought severity and PAFA in Heilongjiang Province.

Due to missing disaster data, the PAFA from 1961 to 1977 is missing, and PAFA data are available for most provinces from 1978 to 2021. As shown in Figure 2, there is a significant correlation between variables. The calculations show that the correlation coefficient between drought duration and drought severity is 0.82, the correlation coefficient between drought duration and PAFA is 0.33 and the correlation coefficient between drought severity and PAFA is 0.49, and all of them have passed the significance test at a level of 0.05. Similar correlation coefficients were calculated for all provinces, and the mean values of the above correlation coefficients were, in order, 0.81, 0.47 and 0.51, which suggests that a correlation between variables is a common phenomenon. In terms of numerical magnitude, there is a strong linear relationship between drought duration and drought severity. However, in terms of the scatter of the two variables, the uncertainty of drought severity increases as the drought duration increases, and this phenomenon is common. In the case of Heilongjiang Province, for example, when the drought lasted nine months, the drought severity in the historical sample ranged from 3 to 8. This indicates that although the number of months of water shortage is the same, the magnitude of water shortage is significantly different. Smaller water deficits in agriculture can often be compensated for by irrigation, but for longer drought durations and larger water deficits, there may be no water that can be irrigated or insufficient irrigation capacity. This may be the reason

why drought duration and drought severity do not have large correlation coefficients with PAFA. Figure 3 gives the average of the historical data for each variable in each province.

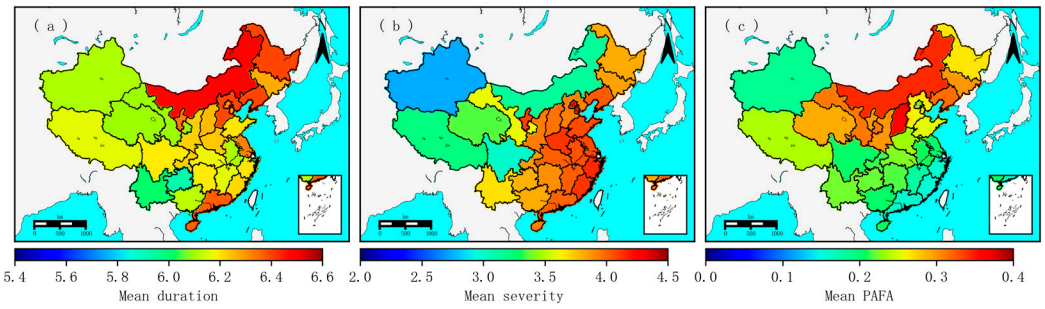


Figure 3. The annual average of the variables of each province: (**a**) drought duration; (**b**) drought severity; (**c**) PAFA.

From Figure 3a, it can be seen that the average drought lasts about 6 months per year in most parts of the country, which is consistent with the definition of the SPI, i.e., droughts and floods are 50/50. In addition, it can be seen from the figure that the average duration of drought in the northern regions of China, such as Inner Mongolia, Heilongjiang, Liaoning, and Hebei, is relatively slightly larger than 6.4 months, but the difference with 6 months is not significant. As can be seen from Figure 3b, the drought severity in the eastern and southern regions of China is significantly larger relative to the northwestern region, with a difference of up to two times in some areas. Figure 3c shows that the PAFA is significantly higher in the north than in the south, suggesting that the north is more prone to large-scale agricultural droughts, which may be related to the abundance of rainfall in the south and the development of irrigation technology. From the comparative analysis of the subplots in Figure 3, it can be seen that the long drought duration may be the main reason for the high PAFA in the northern region. Solving the distribution function of each variable is an important step in building a copula analytical model. Zuo et al. [13,30] showed that the distribution functions of drought duration and drought severity can be well derived based on the maximum entropy method, so the distribution functions of each variable in this paper will also be derived based on the maximum entropy method. The provincial PAFA data are all missing from 1961 to 1977, but this does not affect the calculation of the distribution function for this variable, which was calculated based on the 1978–2021 data. And the distribution function of drought duration and drought severity is calculated based on 1961–2021. According to the previous sensitivity analyses [38,39], when the sample size is greater than 30, the distribution functions obtained are almost identical. Still taking Heilongjiang Province as an example, Figure 4 gives a comparison between the empirical distribution and the distribution function derived from the maximum entropy principle.

As shown in Figure 4, the distribution function derived, based on the maximum entropy principle, is in good agreement with the empirical distribution, which indicates that the distribution function obtained from the principle of maximum entropy has a good fitting ability. The non-stationarity of the precipitation series is common, which makes the parameters of the distribution function not numerically constant. General distribution tests need to satisfy certain conditions, such as the assumption of normal distribution, large sample conditions, etc. The K-S test is a non-parametric test that does not have to assume the distribution of the data and is applicable to small samples, which makes the method suitable for the study in this paper. The results in Figure 4 passed the K-S test with a significant level of 0.05. Distribution tests and RMSE calculations were performed for all provinces, and the results are shown in Table 2.

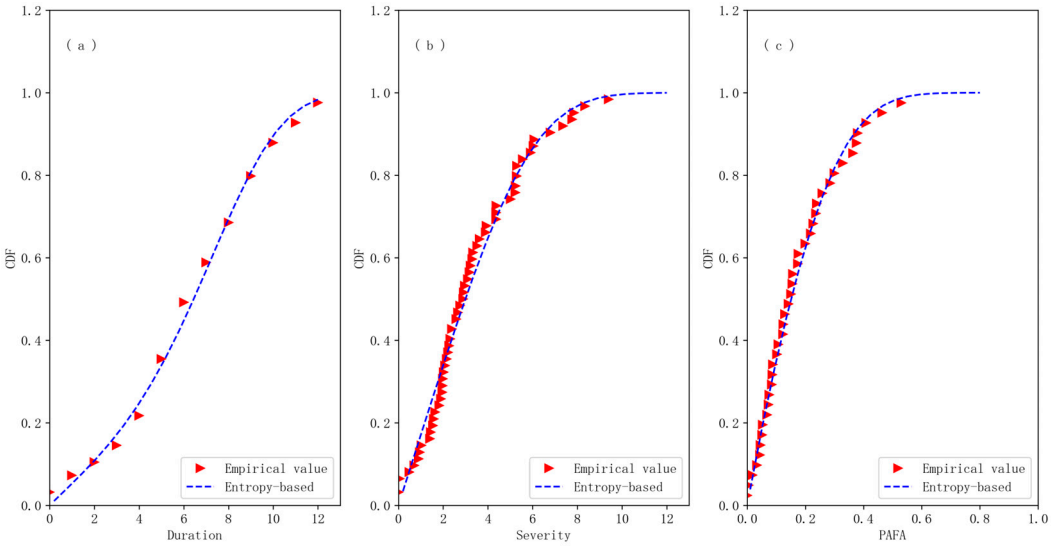


Figure 4. Comparative analysis of distribution function fitting in Heilongjiang Province: (**a**) drought duration; (**b**) drought severity; (**c**) PAFA.

Table 2. K-S distribution test and fitting error size.

	Drought Duration	Drought Severity	PAFA
K-S test pass rate	100%	100%	96.8%
RMSE	0.035	0.029	0.034

As can be seen in Table 2, drought duration and drought severity passed the distribution test in all provinces, but the PAFA did not reach 100%. The city of Shanghai, which did not pass the test, has seen a yearly decrease in the area under agricultural cultivation, from 790,000 hectares in 1961 to 264,000 hectares in 2021. At the same time, the drought-affected area was recorded only in 1994 with 114,000 hectares, while the other years mostly had no data and a few records of zero, so the distribution function test could not be performed. In almost all provinces, the deviation of the theoretical probability values of the variables from the empirical probability values is mostly less than 0.04, and this error is acceptable in practical problems, so the maximum entropy method is an effective method in deriving distribution functions. Based on the theoretical distribution function of each variable and data from 1978 to 2021, the maximum likelihood estimation method is used to solve the θ value of each copula function in Table 1.

The probabilities calculated based on the copula function are called theoretical probabilities, and the probabilities obtained from sample statistics are called empirical probabilities. It is necessary to verify whether the result calculated by the copula function is consistent with the empirical probability. The empirical and theoretical probabilities of all historical observation samples are calculated, and if they are equal, a scatter plot is drawn with the empirical probability as the horizontal coordinate and the theoretical probability as the vertical coordinate, which will lie on the diagonal line. Therefore, the closer the scatter point is to the diagonal line, the better the agreement between the two; otherwise, the worse the agreement. This plotting method is similar to Q-Q plotting, which can reflect the consistency of all sample probabilities. Meanwhile, based on the historical samples, the RMSE between the theoretical and empirical probabilities can be calculated, and the value reflects the average error size of the probability calculation. The results are shown in Figure 5.

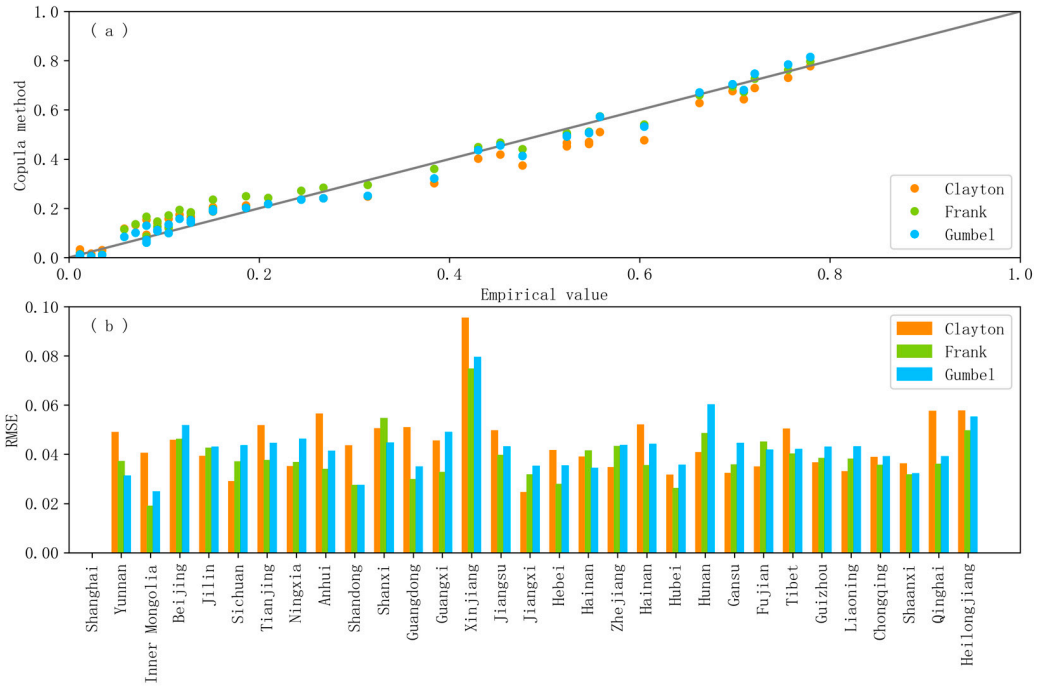


Figure 5. The fit effect of the copula function: (**a**) Heilongjiang Province; (**b**) RMSE.

The scatter in Figure 5a indicates the comparison between the theoretical probability and the empirical probability obtained by using different copula functions. As can be seen from the scatter positions, there is little difference in the results of the three types of copula calculations. In Figure 5a, the scatter closer to the upper right corner is basically a drought event with a long drought duration, a large drought severity and a large PAFA. Scatters closer to the lower left corner indicate drought events with a small drought duration, small drought severity and small PAFA. The distribution of the two-sided scatter suggests that the probabilities of extreme drought events are more accurate. That is, the consistency of the right-tailed probabilities is better than the consistency of the left-tailed probabilities. Figure 5b shows the results of the RMSE for each province, from which it can be seen that the probability calculation error is the largest in Xinjiang, and other provinces are close. Most of the probability deviations calculated by the Clayton, Frank, and Gumbel methods were below 0.05, and the RMSE averages for multiple provinces were 0.044, 0.038, and 0.042, respectively. It has been shown that Gumbel applies to the characterization of variables with upper-tailed correlations, Clayton applies to the characterization of variables with lower-tailed correlations, and Frank applies to the characterization of variables with symmetric correlations [36]. The development process of drought indicates that drought duration, drought severity and PAFA achieve maximum and minimum at the same time, and this conclusion can be verified from Figure 2. In addition, this paper is biased towards the analysis of average features, not tail features, so the Frank type is more appropriate. The RMSE calculations also show that the Frank method is better. On balance, the Frank method will be used for all subsequent calculations.

3.2. Joint Probability Analysis

Figure 6 gives the distribution function under three different PAFAs using Heilongjiang as an example.

In the national standard of drought hazard, PAFAs of 10%, 30% and 50% are the thresholds for levels 1, 2 and 3, respectively. The surfaces from high to low in Figure 6 correspond to joint distribution functions with PAFAs of 10%, 30% and 50%, respectively. As shown in Figure 6, the probability of a PAFA less than 30% in Heilongjiang Province is close to 1, while the probability of a PAFA less than 10% is less than 0.4. The results for the other provinces are similar to the above results. In order to highlight how much

drought duration and drought severity are mainly responsible for different PAFAs, drought duration of 5, 7 and 9 months were used as the boundaries for the drought duration level, and the level classification of drought severity was obtained with the same probability. The level classification is shown in Table 3.

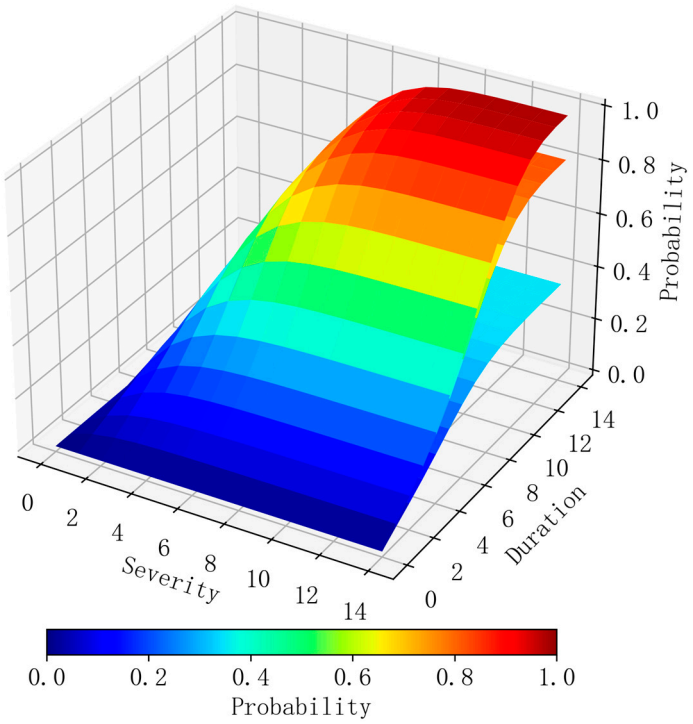


Figure 6. Joint distribution.

Table 3. Level classification.

Level	Drought Duration	Drought Severity	PAFA
1	$D \leq 5$	$S \leq 2.41$	$I \leq 0.1$
2	$5 < D \leq 7$	$2.41 < S \leq 4.05$	$0.1 < I \leq 0.3$
3	$7 < D \leq 9$	$4.05 < S \leq 6.01$	$0.3 < I \leq 0.5$
4	$9 < D$	$6.01 < S$	$0.5 < I$

In Table 3, the proportion of drought events corresponding to level 1 for drought duration and severity is 0.41, level 2 is about 0.24, level 3 is about 0.2 and level 4 is about 0.15. When the PAFA is known, the size of the probability under different drought duration and drought severity levels can be calculated. The result of probability $P(d_1 < D \leq d_2,\ s_1 < S \leq s_2 | i_1 < I \leq i_2)$ is shown in Figure 7.

The results in Figure 7 are averaged across provinces, using Figure 7a as an example, which indicates the probability $P(d_1 < D \leq d_2,\ s_1 < S \leq s_2 | 0 < I \leq 0.1)$. The range of intervals in this equation take the values in Table 3. From Figure 7a, it can be seen that the main cause of the emergence of a PAFA less than 0.1 is a drought with a drought duration and drought severity of level 1. From Figure 7b, the main cause of the emergence of a PAFA between 0.1 and 0.3 is droughts of a drought duration and drought severity on a level of 1 to 3. From Figure 7c,d, the main cause of the emergence of a PAFA greater than 0.3 is a drought duration and drought severity level of 3 to 4. Therefore, historical data show that drought durations more than 7 months are highly susceptible to the occurrence of droughts with a large PAFA and that preparedness for such drought events should be strengthened. Since the values given in Figure 7 are an average of the probabilities across provinces, there are differences in the probabilities across provinces. The results for Heilongjiang are similar to those in Figure 7, while Jilin Province, which is adjacent to Heilongjiang, exhibits

different characteristics. The main reason for a PAFA less than 0.3 in Jilin Province is a drought with a drought duration of level 1 and a drought severity of level 1–2. The result for a PAFA less than 0.1 is similar to Figure 7a, except that the probability is greater, with a value of 0.62. When the PAFA is between 0.3 and 0.5, the results in Jilin Province are similar to Figure 7b, while the results in Heilongjiang Province are similar to Figure 7c. This is a significant difference between the two provinces. The main causes for a PAFA greater than 0.5 were similar in both provinces, with results similar to Figure 7d. For a more detailed comparison of the differences between provinces, taking the drought event with the highest probability, i.e., a drought duration less than 5 months, drought severity less than 2.41 and PAFA less than 0.1, as an example, the probability magnitude of the event in each province is given in Figure 8.

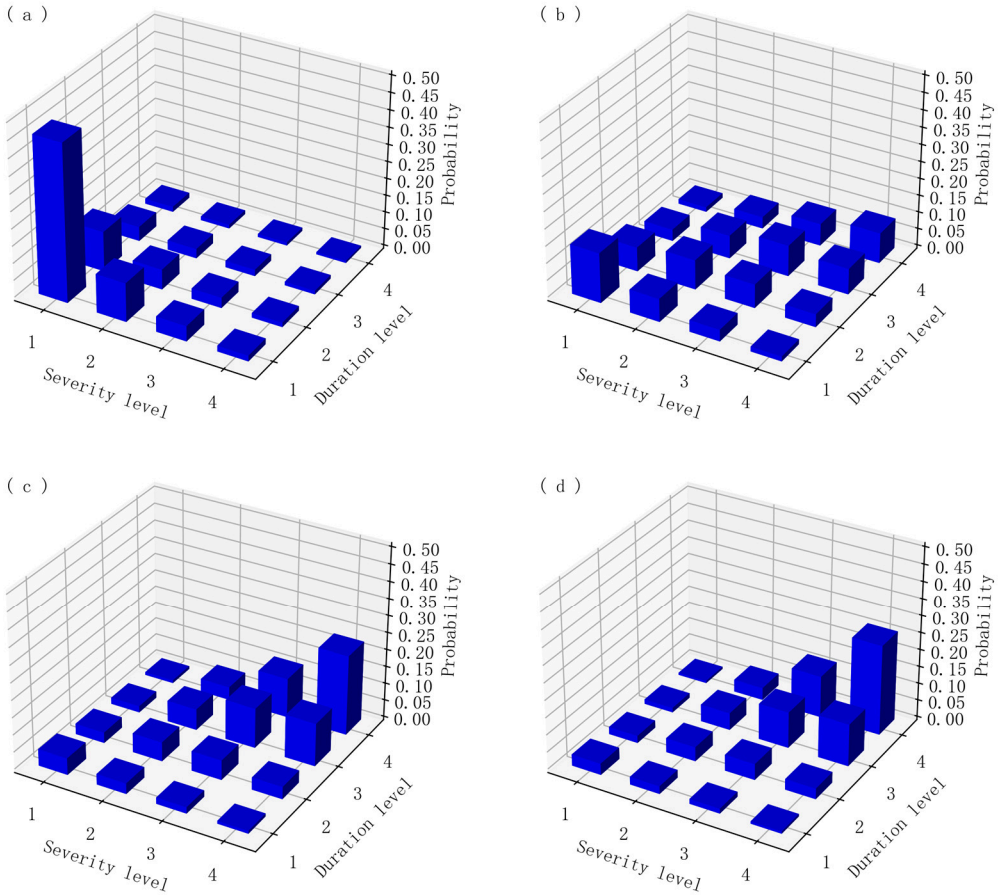


Figure 7. The probability of different drought types: (**a**) $I \leq 0.1$; (**b**) $0.1 < I \leq 0.3$; (**c**) $0.3 < I \leq 0.5$; (**d**) $0.5 < I$.

As can be seen from Figure 8, although it is the same type of drought event, the north and south of China show a large difference. The probability of the occurrence of a drought event in the northern region is about 0.7, while the probability of the occurrence of a drought event in the southern region is about 0.4 to 0.6. Probabilistic analyses of other types of drought events show that, for the same PAFA, it can occur in the northern region with a smaller drought duration and drought severity, while the southern region requires a larger drought duration and drought severity. For example, at a PAFA of 10% to 30%, Jiangsu, Anhui, Zhejiang, Jiangxi, Fujian and Guangdong need a drought duration greater than 9 months and a drought severity greater than 6, whereas most of the northern regions can experience this damage rate when the drought duration is less than 5 months and the drought severity is less than 2.4. This indicates that longer drought durations and greater drought severity are needed in the south to achieve the same rate of damage as in the

north, which may be related to the relative development of water facilities and irrigation technology in the south.

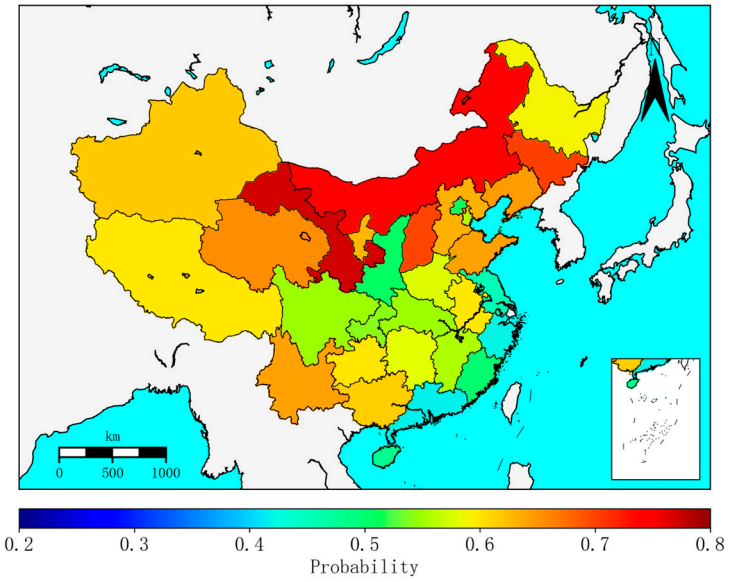


Figure 8. The distributed feature of $P(D \leq 5,\ S \leq 2.41,\ I \leq 0.1)$.

4. Conclusions and Discussion

In this paper, the SPI on a 3-month time scale was chosen as an indicator to describe agricultural drought. Drought duration and severity were isolated from the SPI-3 series. Meanwhile, the PAFA was obtained for each year based on the area sown and the area affected by drought published by the National Statistical Office. A probabilistic analytical model incorporating drought duration, drought severity and PAFA was developed using a three-dimensional copula function, which was used to analyze the characteristics of drought in each province of China. The results show that the average drought duration in the north of China is slightly greater than that in the south, but both are near six months, and the average results of drought severity show that the value in the southern region is higher than that in the north, and that the average PAFA in the northern region of China can reach more than 30%, while that in the southern region mostly ranges from 10% to 20%. The distribution functions derived from the maximum entropy principle fit the empirical distributions of the three variables well, and all pass the K-S distribution test with a significance level of 0.05. The errors in calculating the joint probability are similar for the three copula functions, and the analysis of the RMSE shows that the Frank type is better. Nearly 65% of the droughts were events with a PAFA of less than 20%, and drought duration and drought severity at a level of 3 to 4 were the main causes of a PAFA above 30%. At the same level of PAFA, droughts in the southern region last longer and are more intense than those in the northern region. The reason may be related to the relative abundance of water resources in southern China and better economic conditions for upgrading irrigation capacity.

From the above analyses, it can be seen that the probabilistic analysis model of drought duration, drought severity and PAFA can be well constructed by using the copula function. In terms of the practical application of the model, the following need to be noted: (1) Although there is a sample of nearly 40 years, the sample size is still small, and the stability of the model lacks assessment. Since precipitation has an interdecadal character and the 40-year PAFA happens to be on a similar time scale to the interdecadal, the distribution function of the PAFA may be unstable. With the development of the economy, there has been a significant increase in irrigation capacity. In order for the copula model to better characterize the joint probability of the current drought duration, drought severity and PAFA, it becomes necessary to reduce the influence of irrigation factors on the copula

model. Selecting samples from recent years is more practical, but the treatment is prone to the problem of an insufficient sample size, for example by selecting data from the last 10 years. The aforementioned increase in irrigation capacity can be seen in Figure 2, where the post-2010 PAFA data are significantly lower than the pre-2010 values, a phenomenon that is reflected in the data for multiple provinces. A small sample size affects the computational stability of the distribution function, as well as the stability of parameter θ of the copula function, both of which will affect the accuracy of the joint probability. (2) Due to the different growing periods of crops in the north and south, the drought duration, drought severity and PAFA in this paper are not consistent in the starting and ending time and can be adjusted according to the local planting characteristics in the specific application. (3) Since the spatial extent of drought impacts is generally large, it can exceed the area of one province or several provinces. Therefore, it is reasonable to use provinces as the unit in the spatial scale. Of course, some provinces with large spatial scales do need to be cautious in conducting analyses based on the 3-D copula model.

Based on the copula model, a link can be established between the hazard factors and the damage rate. Early warning lines can be derived from frequency and return period analyses, e.g., when the drought duration and drought severity reach a certain value, the irrigation of crops needs to be intensified. A stability analysis of the model, analysis of the effect of irrigation on PAFA and joint probabilistic characterization of drought duration, drought severity and soil moisture during the crop growth period are all issues worth exploring in the future.

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References

1. Zhang, X.; Duan, Y.; Duan, J.; Jian, D.; Ma, Z. A daily drought index based on evapotranspiration and its application in regional drought analyses. *Sci. China Earth Sci.* **2022**, *65*, 317–336. [CrossRef]
2. Zhang, Q.; Yao, Y.; Li, Y.; Huang, J.; Ma, Z.; Wang, Z.; Wang, S.; Wang, Y.; Zhang, Y. Causes and Changes of Drought in China: Research Progress and Prospects. *J. Meteorol. Res.* **2020**, *34*, 460–481. [CrossRef]
3. Zhang, J.; Chen, H.; Zhang, Q. Extreme drought in the recent two decades in northern China resulting from Eurasian warming. *Clim. Dyn.* **2019**, *52*, 2885–2902. [CrossRef]
4. Naumann, G.; Cammalleri, C.; Mentaschi, L.; Feyen, L. Increased economic drought impacts in Europe with anthropogenic warming. *Nat. Clim. Chang.* **2021**, *11*, 485–491. [CrossRef]
5. Gong, Z.; Feng, G.; Dogar, M.M.; Huang, G. The possible physical mechanism for the EAP–SR co-action. *Clim. Dyn.* **2018**, *51*, 1499–1516. [CrossRef]
6. Huang, J.; Yi, Y.; Wang, S.; Chou, J. An analogue-dynamical long-range numerical weather prediction system incorporating historical evolution. *Q. J. R. Meteorol. Soc.* **1993**, *119*, 547–565. [CrossRef]
7. Chamani, R.; Monkam, D.; Djomou, Z.Y. Analysis of return periods and return levels of Yearly July-September extreme droughts in the West African Sahel. *Clim. Dyn.* **2019**, *52*, 3421–3433. [CrossRef]
8. Jenkins, K.; Warren, R. Quantifying the impact of climate change on drought regimes using the Standardised Precipitation Index. *Theor. Appl. Clim.* **2015**, *120*, 41–54. [CrossRef]

9. Hao, Z.C.; Hao, F.H.; Singh, V.P.; Ouyang, W. Quantitative risk assessment of the effects of drought on extreme temperature in eastern China. *J. Geophys. Res.-Atmos.* **2017**, *122*, 9050–9059. [CrossRef]
10. Cancelliere, A.; Salas, J.D. Drought length properties for periodic-stochastic hydrologic data. *Water Resour. Res.* **2004**, *40*, 389–391. [CrossRef]
11. Huang, S.; Li, P.; Huang, Q.; Leng, G.; Hou, B.; Ma, L. The propagation from meteorological to hydrological drought and its potential influence factors. *J. Hydrol.* **2017**, *547*, 184–195. [CrossRef]
12. Hou, W.; Yan, P.C.; Feng, G.L.; Zuo, D.D. A 3D Copula Method for the Impact and Risk Assessment of Drought Disaster and an Example Application. *Front. Phys.* **2021**, *9*, 656253. [CrossRef]
13. Zuo, D.D.; Hou, W.; Hu, J.G. An Entropy-Based Investigation into Bivariate Drought Analysis in China. *Water* **2017**, *9*, 632. [CrossRef]
14. Mirabbasi, R.; Fakheri-Fard, A.; Dinpashoh, Y. Bivariate drought frequency analysis using the copula method. *Theor. Appl. Climatol.* **2012**, *108*, 191–206. [CrossRef]
15. Naderi, K.; Moghaddasi, M.; Shokri, A. Drought occurrence probability analysis using multivariate standardized drought index and copula function under climate change. *Water Resour. Manag.* **2022**, *36*, 2865–2888. [CrossRef]
16. Shiau, J.T. Fitting Drought Duration and Severity with Two-Dimensional Copulas. *Water Resour. Manag.* **2006**, *20*, 795–815. [CrossRef]
17. Feng, K.; Su, X.; Zhang, G.; Javed, T.; Zhang, Z. Development of a new integrated hydrological drought index (SRGI) and its application in the Heihe River Basin, China. *Theor. Appl. Clim.* **2020**, *141*, 43–59. [CrossRef]
18. Van de Vyver, H.; Van den Bergh, J. The Gaussian copula model for the joint deficit index for droughts. *J. Hydrol.* **2018**, *561*, 987–999. [CrossRef]
19. Naseri, K.; Hummel, M.A. A Bayesian copula-based nonstationary framework for compound flood risk assessment along US coastlines. *J. Hydrol.* **2022**, *610*, 128005. [CrossRef]
20. Yang, X.; Chen, Z.; Qin, M. Joint probability analysis of streamflow and sediment load based on hybrid copula. *Environ. Sci. Pollut. Res.* **2023**, *30*, 46489–46502. [CrossRef]
21. Moradian, S.; Olbert, A.I.; Gharbia, S.; Iglesias, G. Copula-based projections of wind power: Ireland as a case study. *Renew. Sustain. Energy Rev.* **2023**, *175*, 113147. [CrossRef]
22. Huang, Y.; Zhang, B.; Pang, H.; Wang, B.; Lee, K.Y.; Xie, J.; Jin, Y. Spatio-temporal wind speed prediction based on Clayton Copula function with deep learning fusion. *Renew. Energy* **2022**, *192*, 526–536. [CrossRef]
23. Li, X.; Ren, G.; Wang, S.; You, Q.; Sun, Y.; Ma, Y.; Wang, D.; Zhang, W. Change in the heatwave statistical characteristics over China during the climate warming slowdown. *Atmos. Res.* **2021**, *247*, 105152. [CrossRef]
24. Chen, S.; Zhao, J.; Dou, H.; Yang, Z.; Li, F.; Byun, J.; Kim, S.W. A study on the monitoring of heatwaves and bivariate frequency analysis based on mortality risk assessment in Wuhan, China. *Front. Public Health* **2024**, *12*, 1409563. [CrossRef]
25. Latif, S.; Simonovic, S.P. Compounding joint impact of rainfall, storm surge and river discharge on coastal flood risk: An approach based on 3D Fully Nested Archimedean Copulas. *Environ. Earth Sci.* **2023**, *82*, 63. [CrossRef]
26. Latif, S.; Simonovic, S.P. Trivariate Joint Distribution Modelling of Compound Events Using the Nonparametric D-Vine Copula Developed Based on a Bernstein and Beta Kernel Copula Density Framework. *Hydrology* **2022**, *9*, 221. [CrossRef]
27. McKee, T.B.; Doesken, N.J.; Kleist, J. The relationship of drought frequency and duration to time scales Preprints. In Proceedings of the 8th Conference on Applied Climatology, Anaheim, CA, USA, 17–22 January 1993; pp. 179–183.
28. Hayes, M.; Svoboda, M.; Wall, N.; Widhalm, M. The Lincoln declaration on drought indices: Universal meteorological drought index recommended. *Bull. Am. Meteorol. Soc.* **2011**, *92*, 485–488. [CrossRef]
29. Hao, Z.; Yuan, X.; Xia, Y.; Hao, F.; Singh, V.P. An overview of drought monitoring and prediction systems at regional and global scales. *Bull. Am. Meteorol. Soc.* **2017**, *98*, 1879–1896. [CrossRef]
30. Hong, X.; Guo, S.; Xiong, L.; Liu, Z. Spatial and temporal analysis of drought using entropy-based standardized precipitation index: A case study in Poyang Lake basin, China. *Theor. Appl. Clim.* **2015**, *122*, 543–556. [CrossRef]
31. Shannon, C.E. A Mathematical Theory of Communication. *Bell Syst. Tech. J.* **1948**, *27*, 379–423. [CrossRef]
32. Jaynes, E.T. Information theory and statistical mechanics, I. *Phys. Rev.* **1957**, *106*, 620–630. [CrossRef]
33. Sklar, A. *Fonctions de Répartition à N Dimensions et Leurs Marges*; Publications de l'Institut de Statistique de l'Université de Paris: Paris, France, 1959; pp. 229–231.
34. AghaKouchak, A.; Bárdossy, A.; Habib, E. Conditional simulation of remotely sensed rainfall data using a non-Gaussian v-transformed copula. *Adv. Water Resour.* **2010**, *33*, 624–634. [CrossRef]
35. Serinaldi, F.; Bonaccorso, B.; Cancelliere, A.; Grimaldi, S. Probabilistic characterization of drought properties through copulas. *Phys. Chem. Earth Parts A/B/C* **2009**, *34*, 596–605. [CrossRef]
36. Nelsen, R.B. *An Introduction to Copulas*; Springer: New York, NY, USA, 2006.
37. Bezak, N.; Mikoš, M.; Šraj, M. Trivariate Frequency Analyses of Peak Discharge, Hydrograph Volume and Suspended Sediment Concentration Data Using Copulas. *Water Resour. Manag.* **2014**, *28*, 2195–2212. [CrossRef]

38. Zuo, D.D.; Hou, W.; Wu, H.; Yan, P.C.; Zhang, Q. Feasibility of Calculating Standardized Precipitation Index with Short-Term Precipitation Data in China. *Atmosphere* **2021**, *12*, 603. [CrossRef]
39. Zuo, D.D.; Hou, W.; Wu, H.; Zhang, Q.; Yan, P.C. Sensitivity analysis of standardized precipitation index to climate state selection in China. *Adv. Clim. Chang. Res.* **2022**, *13*, 42–50. [CrossRef]

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