

Article

Models of Air Pollution Propagation in the Selected Region of Katowice

Dariusz Foszcz^{1,*}, Tomasz Niedoba^{1,*}  and Jarosław Siewior²

¹ Environmental Engineering Department, Faculty of Mining and Geoengineering, AGH University of Science and Technology in Kraków, al. Mickiewicza 30, 30-059 Kraków, Poland; foszcz@agh.edu.pl

² Provincial Environmental Protection Inspectorate in Katowice, ul. Wita Stwosza 2, 40-032 Katowice, Poland; siewior@poczta.fm

* Correspondence: tniedoba@agh.edu.pl; Tel.: +48-12-617-20-56

Abstract: The paper deals with issues related to analyzing the spread of air pollution and pollutants in large urban agglomerations, specifically, the search for causality between meteorological conditions and the concentrations of particular substances. The pollutants SO₂ and PM₁₀ were selected for analysis, which, in addition to NO_x, CO, CO₂ and PM_{2.5}, contribute to smog, especially during the heating seasons. This analysis is particularly important because Polish environmental standards are more lenient than those in western EU states. Industrial activity, transport and heating systems based on coal-burning are still a big problem in Poland, and each year their gaseous and particulate emissions exceed air-quality limits. This paper presents a statistical analysis of data recorded at the air-quality monitoring station on Kossuth Street in Katowice concerning the heating seasons from 2013–2016. The verification of proposed parabolic models containing concentrations from previous time periods and statistically significant meteorological conditions was conducted for individual heating seasons as well for the whole set of data, which included the influence of wind speed and temperature. The models obtained proved that the selected form of a model is statistically significant, and its use may produce satisfactory forecast results and permit various environmental applications. The specified model might be used both for forecasting (verification and possibly updating coefficients to increase forecast accuracy) and analyzing the factors influencing pollution values. Such statistical analysis may be helpful in assessing the impact of measures adopted to reduce air pollution, particularly in large Polish cities.



Citation: Foszcz, D.; Niedoba, T.; Siewior, J. Models of Air Pollution Propagation in the Selected Region of Katowice. *Atmosphere* **2021**, *12*, 695. <https://doi.org/10.3390/atmos12060695>

Academic Editors: Jonathan E. Pleim and Alexandra Monteiro

Received: 21 April 2021

Accepted: 27 May 2021

Published: 29 May 2021

Keywords: air pollution; sulfur dioxide; PM₁₀; PM_{2.5}; regression models

1. Introduction

Air pollution is one of the world's most noticeable consequences of urbanization and transportation development. Poland, as one of the main coal producers because of the predominance of outdated home coal-burning furnaces, exceeds acceptable concentrations of particulate air pollutants [1,2]. In addition, the burning of waste in household furnaces, creates low-emissions of carcinogenic pollutants, i.e., benzo[a]pyrene and PM_{2.5} (particulate matter ≤2.5 μm in diameter). A major reason for this level of pollution is the level of energy poverty, which is unfortunately high compared to that of western EU countries. Exceedence occurs particularly during the heating season, i.e., between October and March when smog from increased emissions most often occurs despite Poland's huge expenditures on environmental protection over the last years, especially after it entered the EU. Levels of particulate pollutants still exceed the admissible values determined by the appropriate legal regulators, sometimes by several times: in this case, The Environmental Protection Act of 27 April 2001. In Poland, State Environmental Monitoring covers the whole country, with particular emphasis on places where the acceptable level of pollution is often exceeded. Levels of individual particulates in the air are assessed based on continuous automatic



Copyright: © 2021 by the authors. Licensee MDPI, Basel, Switzerland. This article is an open access article distributed under the terms and conditions of the Creative Commons Attribution (CC BY) license (<https://creativecommons.org/licenses/by/4.0/>).

or manual measurement carried out in accordance with accepted reference methods at fixed measurement points that are the most suitable for the selected area. The applicable regulation in this respect is the Regulation of the Minister of Environment of 13 September 2012 on the evaluation of levels of substances in ambient air.

Over the last years, one might have observed increased society activity concerning air pollution, especially in highly industrialized, densely populated regions. A good example of such social action is the establishment of Polish Smog Alert, an initiative associating movements concerned about poor air quality in Poland. High public pressure focused on environmental pollution has resulted in increased involvement of local authorities, especially in areas with high air pollution, but efforts to combat smog still have to increase.

In our country, smog during the heating season is quite frequent in areas such as Upper Silesia, the cities of Kraków or Skawina in Małopolskie Province, as well as in Warsaw and Łódź. The harmfulness of air pollutants to humans is well known and has been the subject of many scientific publications that describe the consequences of polluted inhaling such air over a long time.

According to polskialarmsmogowy.pl [3], a social movement in Rybnik that brings together activists fighting for the improvement of air quality, the number of days in 2017 when the limit values were exceeded was 53. For Kraków, it was 41. In 2019, the cities that had the most days exceeding air pollution limit were Pszczyna with 106 and Rybnik with 89. Krakow, as well as Rybnik, can be counted among the cities where local authorities have taken intensive action to reduce air pollution, an example of which might be a total ban on burning coal and wood in households in Kraków that has been in force since September 2019. The Polish Smog Alert activists emphasize that the list includes only cities where measurement stations of State Environmental Monitoring are located, so the absence of other cities from the list does not mean that smog is not a problem there. Some of the most polluted cities in 2020 include health and tourist resorts, such as Nowy Targ or Sucha Beskidzka [3], which have been the subject of scientific analysis [4]. The overall situation concerning smog was researched, among others, in the paper [5].

The proposed statistical analysis of the models and their coefficients and correlation to parameters such as temperature or wind speed permits the evaluation of changes concerning air pollution values in light of actions taken and independent natural factors.

Despite state initiatives to change the existing situation, there will probably still be periods when permissible levels are exceeded. Actions aimed at eliminating old coal-burning boilers from the market or limiting their marketing through legal regulations have so far failed to produce visible, measurable effects. In 2018, the government launched the “Clean Air” program, which, as the government announced, involves the destruction of three million old boilers, the so-called “black-smoke-belching stoves”, and spending 103 billion PLN on home insulation and boiler replacement over 10 years. However, improved air quality depends on its implementation and will be visible after several years.

Those most vulnerable to the risks associated with inhaling polluted air are the elderly, children and people with diagnosed respiratory, circulatory and nervous system disorders. Constant exposure to pollution might lead to hospitalization and in some cases death.

In the case of smog, preferably even before its appearance, it is reasonable to inform the public through the media about the hazards connected with it. Stochastic models estimating pollution concentrations in a selected area can also be used for this purpose. The authors of this work highlighted this aspect of the issue in previous publications [6–8], proving (with the assumed significance level $\alpha = 0.05$) the validity of applying a specific model for individual atmospheric conditions in the early warning system.

Therefore, from the prevention and warning point of view, it is important to develop the best possible statistical models to predict concentrations of gaseous pollutants (CO , CO_2 , SO_2 , NO_x) or particulate pollutants ($\text{PM}_{2.5}$ and PM_{10}). In statistical models, the variables are usually the concentrations of selected pollutants from the previous day (i.e., “background” concentrations) and meteorological conditions, mainly temperature, wind strength and direction, air humidity, and precipitation. It is worth noting that air

pollutant concentrations are usually stochastic. Therefore, in the authors' previous works, apparent variables were introduced in addition to the variables representing meteorological conditions and concentrations from previous days, [9,10]. However, these works were only concerned with SO₂ concentrations, whereas the present work takes into account concentrations of inhalable PM_{2.5} and PM₁₀ dust, which are dangerous to human health and monitored by Provincial Environmental Protection Inspectorate stations.

The growth of dust concentrations, in particular of PM_{2.5}, has also a big influence on the creation of smog. The elaborated models for dust permitted the verification and analysis of accepted model forms and compared their coefficients with heating seasons.

2. Models of Air Pollution Propagation

Generally, air pollution models can be divided according to the diagram in Figure 1.

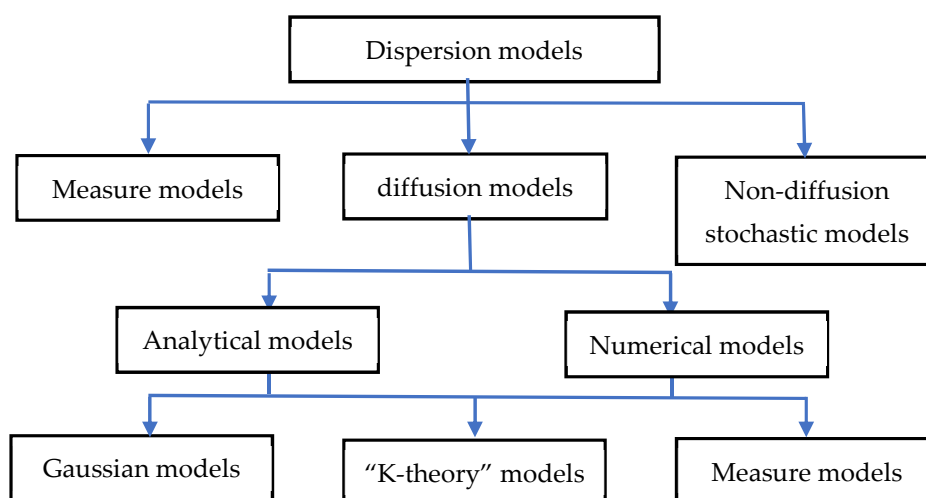


Figure 1. Division of dispersion models.

Air pollution concentration models have been the subject of much research, the results of which have been implemented in urban areas with satisfying results. Among these works, are the Stockholm model, presented in [11], and the Vienna model [12]. Among others, Hysenaj used dispersive models and their applications to predict air pollution in Tirana [13] to fulfill European emission norms. Markiewicz [14,15] discussed the methods used in air pollution dispersive models, among which the Eulerian, Lagrangian and Gaussian were distinguished. The division of these basic models is presented in Tables 1 and 2.

Table 1. Classification of air pollution dispersion models based on mathematical criteria [14].

Model Group (Basic Classes)	Air Pollution Dispersion Models
Eulerian models	Box models
	Analytical models
	Numerical, first-order closure models
	Numerical, higher-order closure models
	Large-scale eddy simulation models
Lagrangian models	Box models
	Particle models
Gaussian models	Traditional plume models
	New generation models
	Segmented plume or puff models

Table 2. Relationships between meteorological methods to determine meteorological data and air pollution dispersion models [14].

Meteorological Methods	Air Pollution Dispersion Models
Traditional methods	Traditional Gaussian plume models
	Segmented Gaussian plume of Gaussian puff
	Eulerian box models
	Eulerian analytical models
Meteorological pre-processors	New-generation Gaussian plume models
	Eulerian numerical models with the 1st order closure
	Eulerian box models
	Lagrangian box models
	Segmented Gaussian plume or puff models
Meteorological prognostic models	Eulerian numerical models with the 1st and higher order closure
	Eulerian large-scale eddy simulation models
	Lagrangian particle models

Furthermore, the methodology of applying dispersive models for the wet deposition of atmospheric pollutants was presented [16]. Such models include the Sulphur Transport and Emission Model (STEM II) [17], Regional Atmospheric Deposition Model RADM [18] and Atmospheric Deposition Oxidant Model ADOM [19]. Furthermore, the application of the Eulerian model for Nordic winter conditions was presented in [20]. Sulfur dioxide modeling of the industrial influence on air quality was also a main issue in [21,22]. Using Geographic Information System (GIS) to control the transportation air pollutants, including the application of previously developed models, was presented for Serbia in [23], for Bulgaria in [24] and for Egypt in [25]. Various improvements to the proposed dispersion models were discussed in many of the works. A review of them is presented in [26–29]. The introduction of a fuzzy synthetic evaluation model was done in [30]. Applications of the numerical simulation of pollution dispersion were conducted, for example, in [31,32], and various distribution functions were used for this in [33] and multiscale approaches were shown in [34,35]. Air pollution as a consequence of smog in big cities was discussed and modeled in [36–40]. The broad impact of different pollutants on the health of living organisms in different places of the world was discussed in [41–46]. The wide spectrum of modeling was discussed in [47], taking into consideration aspects of chemistry and meteorology as well different timescales with seasonal variation. The effect of the COVID-19 epidemic in the context of air pollution was analyzed in [48].

The station selected for analysis is on Kossuth Street in Katowice, located in the highly urbanized the Upper Silesian Industrial Region.

3. Correlation Analysis and Data Analysis

3.1. Information about Data

Data used to determine forecasting models for SO₂ and PM₁₀ concentrations originated from the automatic monitoring station located on Kossuth Street in Katowice. Measurements of pollutant concentrations and meteorological data are a result of increased interest in the environment and National Environment Monitoring (NEM), created at the beginning of the 1990s. NEM was founded to ensure reliable information about the environment following passage of the Inspection of Environmental Protection Act on 10 July 1991 [49]. One of the aspects of its activity was to create automatic monitoring stations for various sorts of pollutants. A system of such stations is found throughout the country, and 16 of them are located in the Silesian Voivodship (province). Furthermore, telemetric stations measure transport pollutants, and there is access to the mobile emission ambulance. Manual measurements cover SO₂ and NO₂ concentrations at 22 sites. The majority of the tasks of NEM after 2000 were related to integrating Poland into European Union structures and to adapt to European environmental management. The measuring station provides automatic hourly data measurements of pollutants: (PM₁₀, PM_{2.5}, SO₂) and meteorological features (wind direction, temperature, wind speed and humidity).

The map presented in Figure 2 shows the location of measuring stations in Southern Poland. Decisive for the choice of the Katowice station was its location and the fact that it is one of the few stations in Silesia that measure the level of $PM_{2.5}$. Both it and another station are characterized by continuity and recorded data availability.

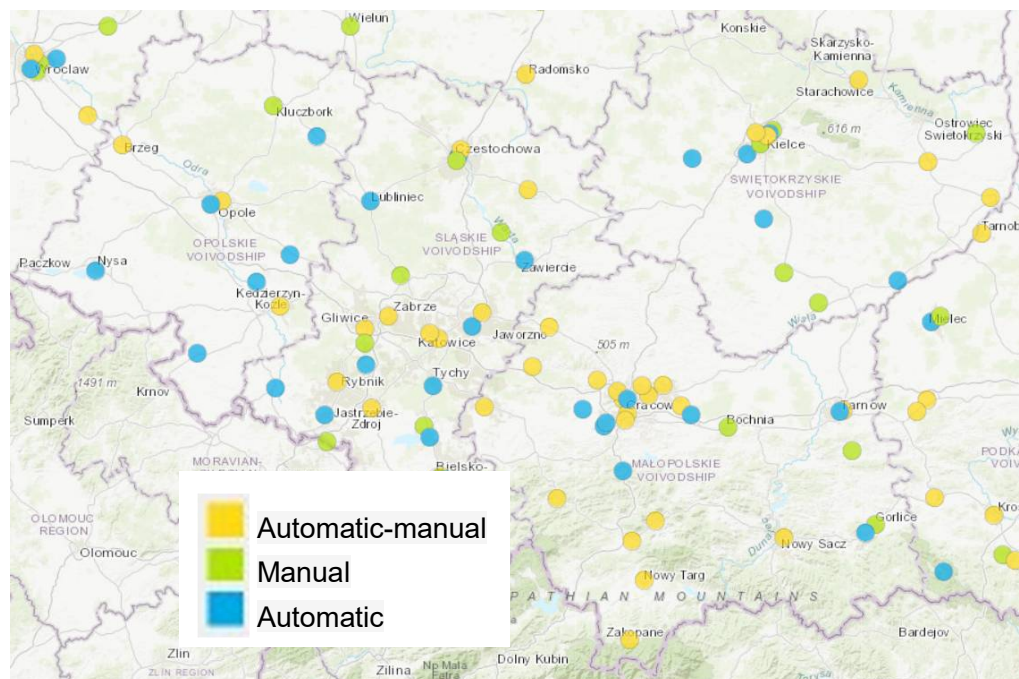


Figure 2. Location of air quality measuring stations in Southern Poland [50].

3.2. Correlation

Table 3 presents the correlation coefficients between individual values measured at the monitoring station in Katowice for the heating seasons 2013–2016. The maximum temperature and wind speed ($T_0 = 22.6\text{ }^{\circ}\text{C}$, $v_0 = 4.4\text{ m/s}$, respectively) were assumed as T_0 and v_0 , respectively.

One might observe a high correlation between dust concentration on day t and on day $(t - 1)$. It concerns both the smaller fraction, ($PM_{2.5}$) and the larger one (PM_{10}).

A similar trend may be observed in the case of sulfur dioxide. Correlation coefficients within the range of 0.9 indicated a high correlation between the current values and those measured on the previous day, which proved poor area ventilation caused by low wind speed and a high area roughness coefficient.

The correlation coefficients between dust and sulfur dioxide were quite high, about 0.6, because they originated from the combustion of low-quality fuel and low emissions from household boilers that lack devices to reduce the emission of dust and gases. It is worth emphasizing that almost all the correlations were statistically significant, which resulted from a very large data sample. Therefore, it is an excellent basis for the development of statistical models for the reliable assessment of the propagation of selected air pollutants.

Table 3. Correlation coefficients for values measured at the station in Katowice in the years 2013–2016 (Kossuth street, N = 10,957).

Variable	Average	Standard Deviation	P (hPa)	Wind Direction (Degree)	V (m/s)	T (°C)	Humidity (%)	SO ₂	PM10	PM2.5	SO ₂ -1	PM10-1	PM2.5-1	(v-v ₀) ²	(T-T ₀) ²
P (hPa)	983.22	9.40	1.00	−0.03	−0.03	−0.08	−0.02	0.01	0.12	0.12	0.00	0.12	0.11	0.02	0.08
Wind_direction (degree)	178.35	90.19	−0.03	1.00	0.13	0.17	−0.01	−0.17	−0.23	−0.23	−0.16	−0.22	−0.22	−0.14	−0.20
v (m/s)	0.88	0.65	−0.03	0.13	1.00	0.15	−0.22	−0.16	−0.35	−0.36	−0.15	−0.32	−0.33	−0.99	−0.15
T (°C)	4.60	5.46	−0.08	0.17	0.15	1.00	−0.34	−0.33	−0.27	−0.34	−0.31	−0.26	−0.33	−0.16	−0.97
Humidity (%)	78.73	14.84	−0.02	−0.01	−0.22	−0.34	1.00	−0.02	0.10	0.17	−0.05	0.10	0.16	0.21	0.28
SO ₂	15.85	11.44	0.01	−0.17	−0.16	−0.33	−0.02	1.00	0.59	0.61	0.89	0.58	0.61	0.16	0.35
PM ₁₀	50.44	41.10	0.12	−0.23	−0.35	−0.27	0.10	0.59	1.00	0.98	0.54	0.94	0.92	0.36	0.27
PM _{2.5}	40.56	35.51	0.12	−0.23	−0.36	−0.34	0.17	0.61	0.98	1.00	0.57	0.93	0.94	0.37	0.34
SO ₂ -1	15.86	11.45	0.00	−0.16	−0.15	−0.31	−0.05	0.89	0.54	0.57	1.00	0.59	0.61	0.14	0.33
PM ₁₀ -1	50.47	41.14	0.12	−0.22	−0.32	−0.26	0.10	0.58	0.94	0.93	0.59	1.00	0.98	0.33	0.27
PM _{2.5} -1	40.58	35.56	0.11	−0.22	−0.33	−0.33	0.16	0.61	0.92	0.94	0.61	0.98	1.00	0.34	0.33
(v-v ₀) ²	12.79	4.21	0.02	−0.14	−0.99	−0.16	0.21	0.16	0.36	0.37	0.14	0.33	0.34	1.00	0.16
(T-T ₀) ²	353.78	203.71	0.08	−0.20	−0.15	−0.97	0.28	0.35	0.27	0.34	0.33	0.27	0.33	0.16	1.00

3.3. Wind Roses

Figure 3 presents the wind roses for the analyzed heating seasons. The analysis of wind direction in individual seasons showed that the dominant directions for this monitoring station were southwest and to a lesser extent southeast.

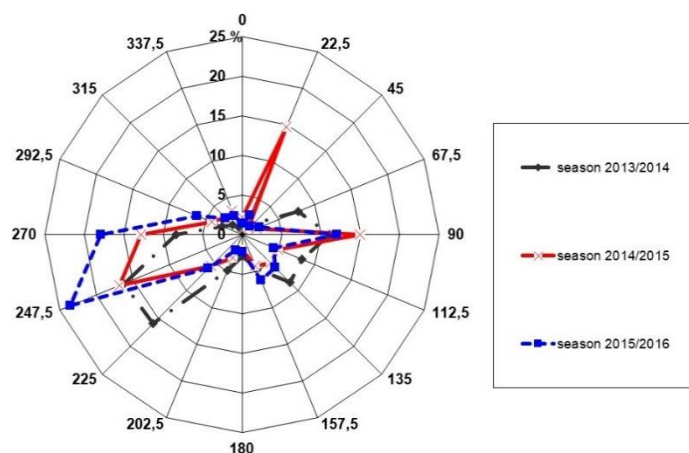


Figure 3. Wind rose for meteorological station, Katowice, seasons 2013/2014, 2014/2015, 2015/2016.

The wind directions for this meteorological station were comparable to those of the station in the Katowice agglomeration area and were cited in papers dedicated to the analysis of pollution in this area (Figure 3). The wind roses indicate that these wind directions were dominant for this region, which is meaningful in the context of pollution transport for the location of emission sources or how highly urbanized areas influence low emissions. This situation is well presented in Figure 4, which presents the location of industrial plants treated as sources of high air pollution. These were placed on the list of the plants having the largest negative influence on the environment (List “80”), which can be found in [51]. The figure shows wind roses presented in [7] in which the authors determined the models for three stations in Upper Silesia—Gliwice, Bytom and Piekary Śląskie for three variants:

- days in which air inflow at all stations came from quadrant (180°, 270°);
- days in which air inflow at certain station came from quadrant (180°, 270°);
- remaining days separately for heating seasons 1996–1997 and 1997–1998.

Such an approach to models made it possible to interpret coefficients and models for pollution transport or to search for rules (methods) of pollutant propagation in highly urbanized areas. Because of that, measuring stations are located small distances apart, so considering wind directions, one can expect that significant relocation of pollutants between stations may occur. It is important, then, that a station be located in an appropriate location, considering both the sources of the highest emissions and the direction of pollutant inflow from neighboring areas. Furthermore, the closest surrounding station is also important as it may influence changes in the parameters of pollution spreading. This was the basis for conducting analyses in this range.

3.4. Summary Statistics

In Table 2, the summary statistics for meteorological data and pollutant concentrations in the analyzed seasons were positioned. The data allowed us to analyze the obtained model forms in the context of coefficient values and their variation for individual seasons.

The analysis of averaged values in the heating seasons presented in Table 4 allowed initial comparisons of the levels of pollutant concentrations and meteorological conditions, which contribute in smog creation. Considering the temperature, the change between the lowest value in season 2014–2015 (4.31 °C) and highest (5.12 °C) is 0.81 °C. For wind speed, the season that had worst parameters for allowing an accumulation of pollutants because

of low speed was 2014–2015 (mean speed = 0.63 m/s) while the best was season 2015–2016 (mean speed = 1.08 m/s, almost two times higher). When comparing the heating seasons for SO₂ and PM air pollution values, it was worth noting that they were comparable in individual seasons for both mean and maximum values.

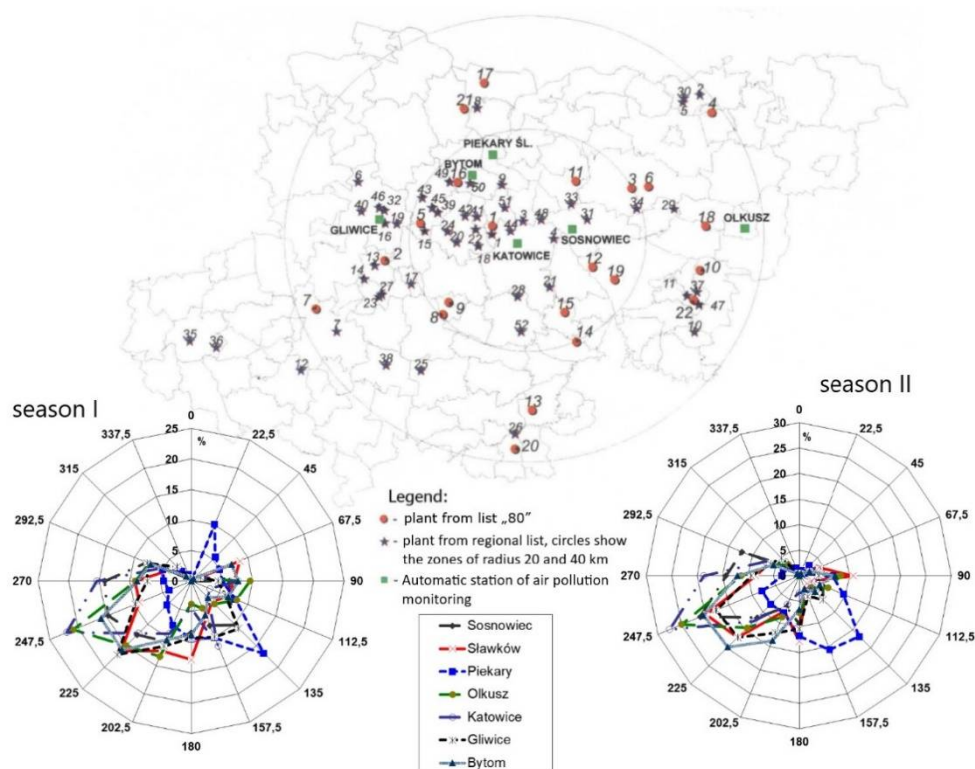


Figure 4. Location of industrial plants and measuring stations in the Upper Silesian Industrial Region and wind roses for analyzed heating seasons 1996–1997 and 1997–1998 with wind roses [7].

Table 4. Summary statistics for data in individual seasons.

Heating Season	Variable	Number of Observations	Summary Statistics (Katowice, Kossuth Street)			
			Mean	Minimum	Maximum	Standard Deviation
2013–2014	P (hPa)	4339	981.66	961.00	1002.00	8.41
	V (m/s)	4240	0.86	0.00	4.40	0.55
	T (st. C)	4342	4.63	−14.70	21.30	5.63
	Humidity (%)	4342	76.34	11.00	99.00	15.82
	SO ₂ (µg/m ³)	4140	13.90	1.00	115.00	9.85
	PM ₁₀ (µg/m ³)	4154	54.90	4.00	355.00	41.72
	PM _{2.5} (µg/m ³)	4226	43.63	1.00	319.00	34.95
2014–2015	P (hPa)	4142	983.05	941.00	1005.90	9.23
	V (m/s)	3506	0.63	0.00	3.50	0.62
	T (st. C)	4148	4.31	−11.70	22.60	5.73
	Humidity (%)	4146	79.07	27.00	99.30	14.53
	SO ₂ (µg/m ³)	4015	16.45	1.00	93.00	11.75
	PM ₁₀ (µg/m ³)	4313	51.07	2.97	356.96	39.14
	PM _{2.5} (µg/m ³)	4320	41.58	2.14	399.00	34.65
2015–2016	P (hPa)	4355	984.75	957.00	1007.10	9.63
	V (m/s)	4355	1.08	0.00	4.30	0.68
	T (st. C)	4355	5.12	−13.50	22.40	5.00
	Humidity (%)	4355	81.32	26.50	100.00	13.80
	SO ₂ (µg/m ³)	4341	16.92	1.95	107.44	12.11
	PM ₁₀ (µg/m ³)	4266	50.93	2.61	370.80	45.03
	PM _{2.5} (µg/m ³)	4293	40.98	1.80	320.20	39.38

Figures 5–8 show the comparison of the graphs of variation of SO_2 and PM_{10} concentrations based on temperature and wind speed for individual heating seasons.

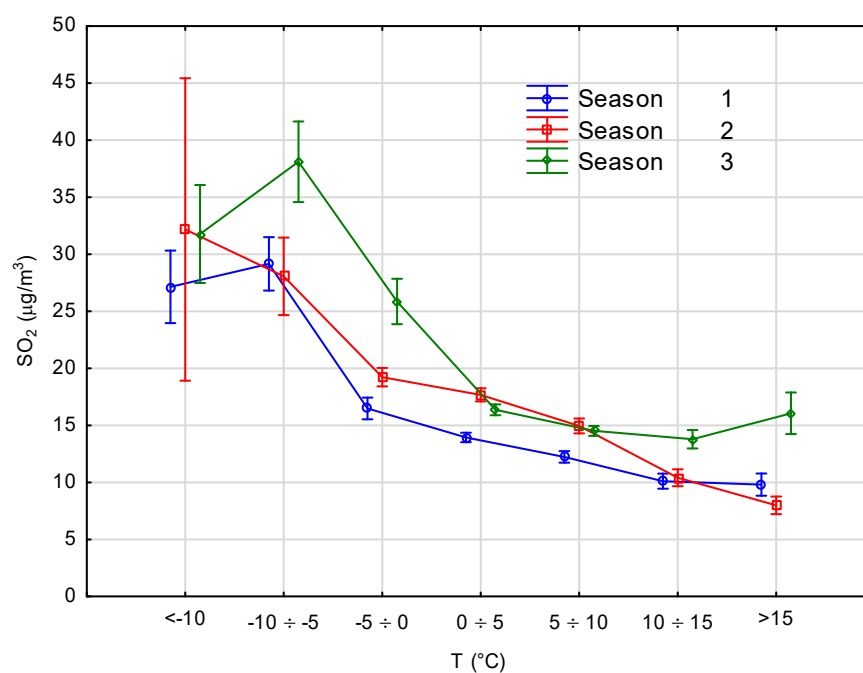


Figure 5. Relation between SO_2 concentration and temperature for the analyzed heating seasons.

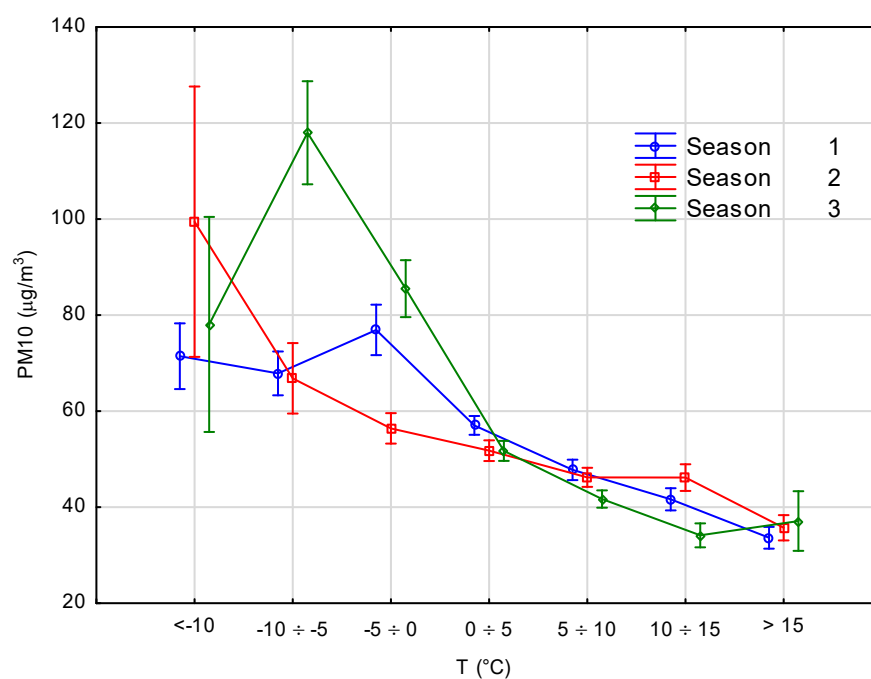


Figure 6. Relation between PM_{10} concentration and temperature for the analyzed heating seasons.

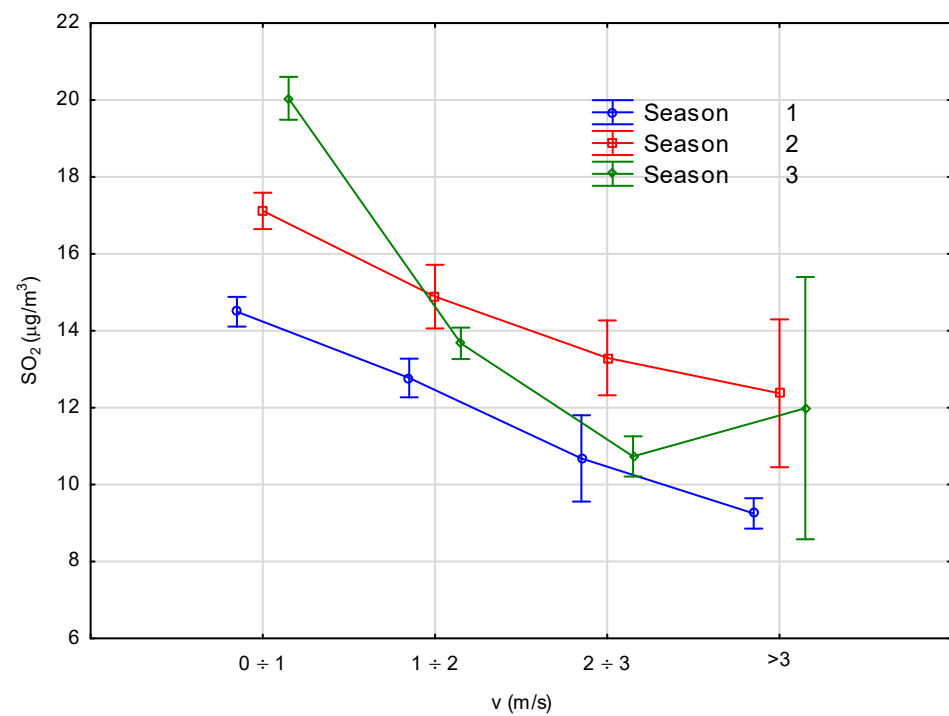


Figure 7. Relation between SO₂ concentration and wind speed for the analyzed heating seasons.

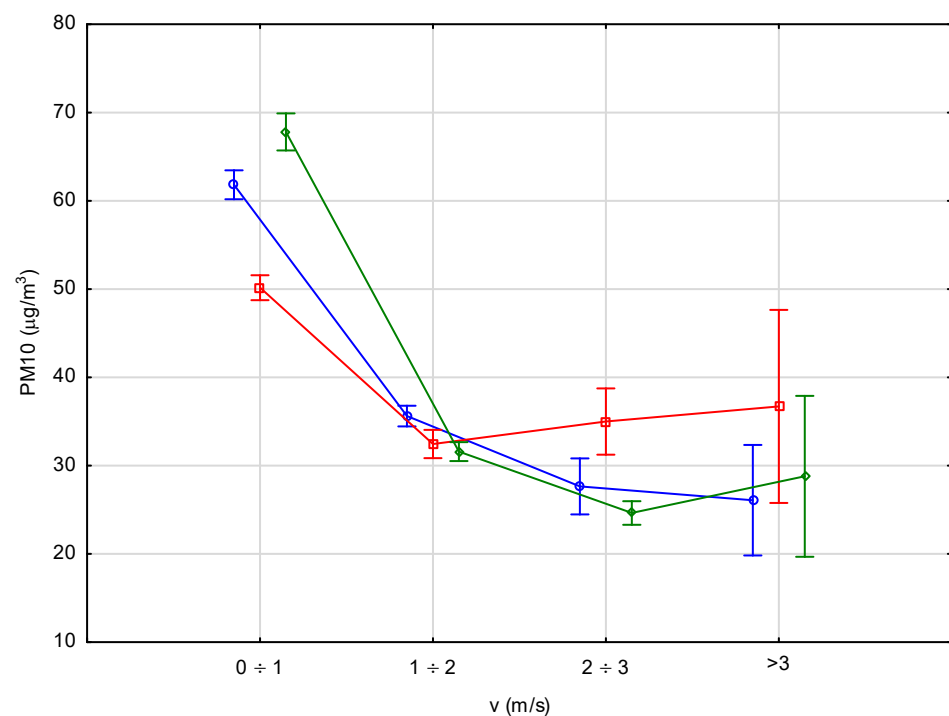


Figure 8. Relation between PM₁₀ concentration and wind speed for the analyzed heating seasons.

The graphs in Figures 5 and 6 clearly show that the concentrations of both SO₂ and PM₁₀ decreased with the rise in temperature, thus confirming assumptions about their parabolic character. However, this decrease was mostly noticeable for SO₂ in the range of negative temperatures and to a lesser extent for PM₁₀. The highest changeability of concentrations was observed at low temperatures—the highest concentrations for SO₂ were observed in 2015–2016—and the range of variation was very large below -5° , especially in 2014–2015. The variation of concentrations at higher temperatures was not high and related to a lower need for building heating.

Considering Figures 7 and 8 it can be seen that growth of wind speed significantly influenced a decrease in the concentration of both SO_2 and PM_{10} . If there had been no wind or a very weak on a certain day, then the phenomenon of pollutants bedding (accumulation) would have created smog. The greatest change in the concentration level was when the wind speed was between 1 and 2 m/s, causing a 50% decrease in pollutants. This was especially apparent for PM_{10} , which, together with $\text{PM}_{2.5}$, is the main cause of smog. The change of wind speed to higher values (up to 3 m/s) did not cause a further decrease in concentrations values, but the highest changeability was noticed for winds stronger than 3 m/s. However, days with such strong wind are rare in heating seasons. One of the reasons for the observed relation could have been the result of secondary dusting by strong wind. Considering the concentrations levels in relation to wind speed, all heating seasons can be treated as comparable.

Figures 9 and 10 show a deeper analysis of SO_2 and PM_{10} concentrations taking into consideration the influence of temperature and wind speed.

Figures 9 and 10 show that the data concerning the analyzed parameters were mostly similar. The biggest differences were observed for the lowest temperatures by low wind speed. In particular, season 2015–2016 stands out in this aspect. The largest variation of data also occurred because of a high wind speed value, but it concerned mainly the high value of standard deviation while the mean level was rather comparable. Furthermore, the features that noticeably influenced the analysis of the concentrations and their character were curvilinear and could be approximated by a parabolic function. That is why the parabolical model was accepted further in this paper.

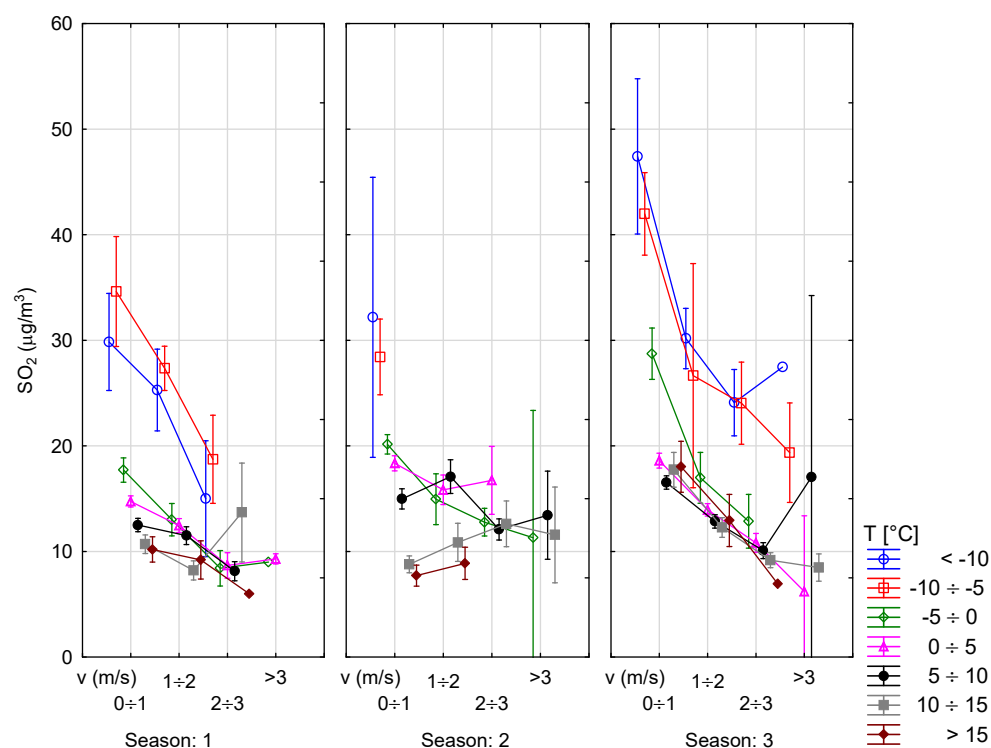


Figure 9. Relation between SO_2 concentration, temperature and wind speed.

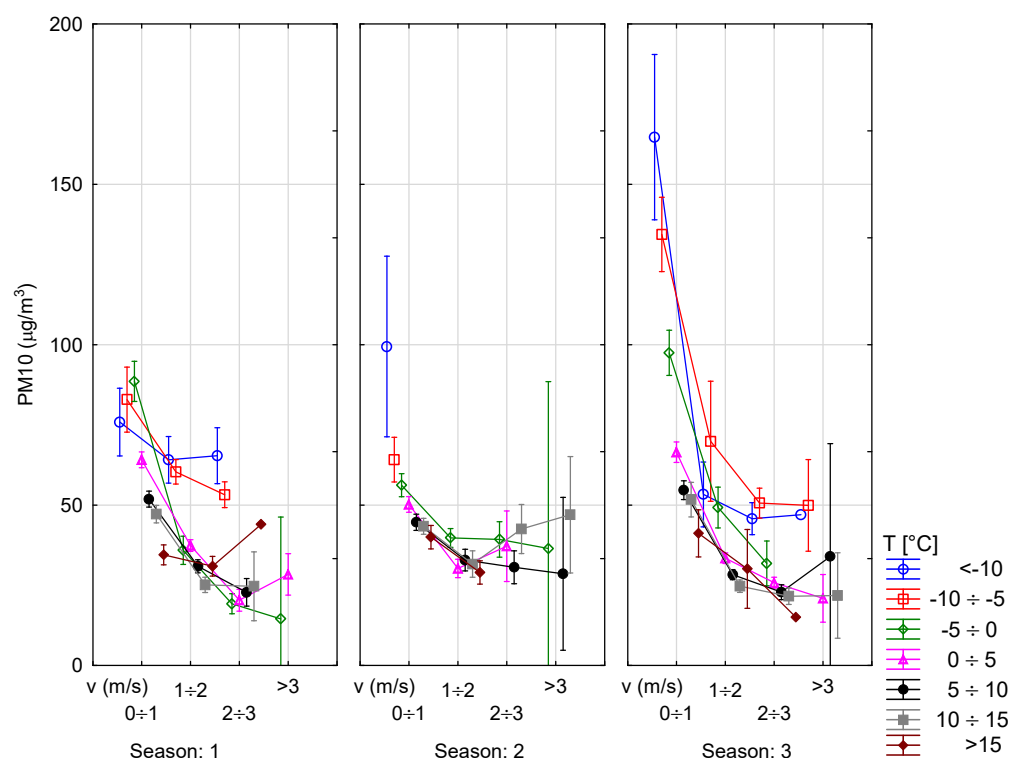


Figure 10. Relation between PM₁₀ concentration, temperature and wind speed.

4. Modelling Results and Discussion

A good mathematical model to describe the daily average SO₂ concentrations in cities (measurement stations) of the Upper Silesian Industrial Region is

$$\text{SO}_2(t) = a + b\text{SO}_2(t-1) + c(v - v_0)^2 + d(T - T_0)^2 \quad (1)$$

where T_0 is the temperature (around 20°) determined from the peak of the parabolic function $\text{SO}_2(T)$, and v_0 is the wind speed (around 6 m/s) determined from the peak of parabolic function $\text{SO}_2(v)$ [52].

Such a model was used between 2000 and 2013 to study the propagation of SO₂ concentrations in the cities in Upper Silesia and was based on a similar model that was used in Vienna. The models obtained at that time were characterized by the level of the coefficient of determination R^2 at 50–70% [9,10]. It was decided to verify the adopted form of the model by adapting it to the values from the 2013–2016 heating seasons. The data, collected by the air quality monitoring station in Katowice on Kossuth Street, were statistically processed and, based on them, regression equations were derived. For the collected data on both weather conditions and recorded concentration levels, a linear regression analysis was performed for the three heating seasons. The analysis gave us models which were used to obtain forecasts close to real values. A proper fit of the model was confirmed by high coefficients of determination R^2 . Comparing the concentrations obtained from the prediction to the concentrations measured at the stations, we also estimated errors using the mean square error MSE according to Formula (2), which were also at a satisfactory level.

$$\text{MSE} = \sqrt{\frac{\sum_{i=1}^n (S_i - \check{S}_i)^2}{n - k}} \quad (2)$$

where S_i is the empirical concentration (measured at the station); $\check{S}_i$ is the theoretical concentration (obtained from the model); n is the number of measurements taken; and k is the number of variables in the model.

The obtained models were presented using formulas according to Equation (1), where: b is a parameter value; R is the correlation index; $\text{Cor. } R^2$ is the corrected coefficient of distribution; F is the value of the F-Snedecor test; and p is a significance level for the F-Snedecor test. In the brackets below the model, the values of the coefficient errors were placed; we accept as a rule that the coefficient is statistically important if its absolute value is at least two times larger than the value of the error. The statistically important coefficients values at significance level 0.05 are marked by red color.

Therefore, the mathematical formulas for this model are:

- Regression model for SO_2 concentrations in the heating season 2013–2014, Katowice, Kossuth Street.

$$\text{SO}_2 = 0.8297 \cdot \text{SO}_2(t-1) + 0.0909 \cdot (v - v_0)^2 + 0.0040 \cdot (T - T_0)^2 - 0.2294 \quad (3)$$

[0.0086] [0.0219] [0.0004] [0.3370]

$N = 3938$; $R = 0.8626$; $\text{Cor. } R^2 = 0.7440$ $F(3, 3934) = 3814.3$ $p < 0.0000$ $\text{MSE} = 5.0115$

- Regression model for SO_2 concentrations in the heating season 2014–2015, Katowice, Kossuth street.

$$\text{SO}_2 = 0.8720 \cdot \text{SO}_2(t-1) + 0.0434 \cdot (v - v_0)^2 + 0.0028 \cdot (T - T_0)^2 + 0.4015 \quad (4)$$

[0.0078] [0.0213] [0.0005] [0.3401]

$N = 3776$; $R = 0.9092$ $\text{Cor. } R^2 = 0.8264$ $F(3, 4324) = 6869.0$ $p < 0$, $\text{MSE} = 5.3768$

- Regression model for SO_2 concentrations in the heating season 2015–2016, Katowice, Kossuth street.

$$\text{SO}_2 = 0.8785 \cdot \text{SO}_2(t-1) + 0.1187 \cdot (v - v_0)^2 + 0.0033 \cdot (T - T_0)^2 - 0.3915 \quad (5)$$

[0.0069] [0.0189] [0.0004] [0.2494]

$N = 4328$; $R = 0.9092$ $\text{Cor. } R^2 = 0.8264$ $F(3, 4324) = 6869.0$ $p < 0$, $\text{MSE} = 5.0449$

For the assumed significance level = 0.05, one can see that all coefficients of the regression equation concerning the heating seasons are statistically significant (except for the value of the absolute term). Furthermore, the value of the R^2 coefficient exceeded 70%, which meant that these models were statistically better than models developed in previous years. It has to be stated that for all three heating seasons the models had a comparable form, i.e., the values of model parameters were similar and the level of R^2 was comparable. This meant that the seasons were similar and could easily be adapted to subsequent seasons. This might be due to the fact that concerning winters in recent years, one did not see large fluctuations regarding meteorological conditions. In addition, the model obtained for all three periods combined was also presented.

- Regression model for SO_2 concentrations in three heating seasons 2013–2016 total, Katowice, Kossuth Street.

$$\text{SO}_2 = 0.8691 \cdot \text{SO}_2(t-1) + 0.0681 \cdot (v - v_0)^2 + 0.0031 \cdot (T - T_0)^2 + 0.0696 \quad (6)$$

[0.0044] [0.0109] [0.0002] [0.1649]

$N = 12042$; $R = 0.8924$; $\text{Cor. } R^2 = 0.7963$; $F(3, 12038) = 15689$ $p < 0.0000$ $\text{MSE} = 5.1530$

What is more, due to the smog in many Polish cities during winter, models for PM_{10} concentrations, taking into account analogous initial equation models, were developed.

- Regression model for PM_{10} concentration for heating season 2013–2014, Katowice, Kossuth Street.

$$\text{PM}_{10} = 0.8802 \cdot \text{PM}_{10}(t-1) + 0.9822 \cdot (v - v_0)^2 + 0.0090 \cdot (T - T_0)^2 - 9.3522 \quad (7)$$

[0.0064] [0.0708] [0.0011] [0.9871]

$N = 4111$; $R = 0.9327$; $\text{Cor. } R^2 = 0.8699$ $F(3, 4107) = 9158.0$; $p < 0.0000$ $\text{MSE} = 14.972$

- Regression model for PM10 concentration for heating season 2014–2015, Katowice, Kossuth Street.

$$\text{PM}_{10} = \underset{[0.0057]}{0.9150} \cdot \text{PM}_{10}(t-1) + \underset{[0.0543]}{0.4211} \cdot (v - v_0)^2 + \underset{[0.0011]}{0.0015} \cdot (T - T_0)^2 - 2.8190 \quad (8)$$

$N = 4295$; $R = 0.9321$ Cor. $R^2 = 0.8687$ $F(3, 4291) = 9474.2$; $p < 0.0000$ $MSE = 14.073$

- Regression model for PM10 concentration for heating season 2015–2016, Katowice, Kossuth Street.

$$\text{PM}_{10} = \underset{[0.0059]}{0.9008} \cdot \text{PM}_{10}(t-1) + \underset{[0.0595]}{0.7926} \cdot (v - v_0)^2 + \underset{[0.0013]}{0.0067} \cdot (T - T_0)^2 - 6.3078 \quad (9)$$

$N = 4249$; $R = 0.9473$ Cor. $R^2 = 0.8973$ $F(3, 4245) = 12367$; $p < 0$, $MSE = 14.446$

- Regression model for PM10 concentrations for all three heating seasons 2013–2016, Katowice, Kossuth Street.

$$\text{PM}_{10} = \underset{[0.0034]}{0.9099} \cdot \text{PM}_{10}(t-1) + \underset{[0.0212]}{0.3571} \cdot (v - v_0)^2 + \underset{[0.0008]}{0.0052} \cdot (T - T_0)^2 - 6.4848 \quad (10)$$

$N = 12655$; $R = 0.9375$ Cor. $R^2 = 0.8789$ $F(3, 12651) = 30619$; $p < 0.0000$ $MSE = 14.583$

Analyzing the coefficients of the models developed for PM_{10} concentrations, one could observe that the proposed model form is also effective for this type of air pollution. The R^2 values concerning this model were close to 90%, which meant that statistically, these models give a fair amount of potential estimate accuracy. However, it should be noted that concerning heating season 2014–2015, the parameter of the regression equation concerning the influence of temperature was not found to be significant. Most likely, this was because of the autocorrelation, which caused the obtained coefficient to be subject to additional error. However, this did not change the fact that in the other two seasons, the model proved to be significant. Concerning the 2014–2015 season, the significance level for the obtained parameter related to temperature change was only slightly higher than the assumed level of 5%.

This meant that the proposed regression model can be successfully applied for the purpose of warning and preventing practices concerning concentrations of selected air pollutants. There is no other work concerning this aspect of forecasting pollutant concentrations (SO_2 and PM_{10}), which makes it difficult to compare this with other works. These problems were the subject of older works [11,12]. Currently, thanks to the environmental protection activities, this topic is no longer such an interesting area of research.

5. Conclusions

The statistical analysis of concentration and meteorological data presented in the paper, obtained at the monitoring station in Katowice in the heating seasons 2013–2016, allows the following conclusions to be drawn:

- There is a high correlation between SO_2 and dust concentrations at moment t and previous concentrations. The correlation coefficients between individual pollutants measured at the same time are at a satisfactory level.
- The proposed form of the model can generate good starting material when attempting to create a stochastic model that, depending on the direction of research, might provide better results for predicting concentrations of the studied type of air pollution. The analyses confirmed the parabolic character of the relationship between SO_2 and PM_{10} concentrations and the meteorological parameters of wind speed and temperature. This was especially visible for low temperature values. For SO_2 , the mean concentration for low temperatures was around 30–40 $\mu\text{g}/\text{m}^3$ and for higher values around 15. In case of PM_{10} the same relations were 70–120 $\mu\text{g}/\text{m}^3$ and for positive values, 30–50 $\mu\text{g}/\text{m}^3$.

- The results obtained from individual heating seasons were similar to the results obtained from analyzing the years 2013–2016, as a whole.
- The application of the above-mentioned statistical methods describing the measurement data obtained from the station allows for the analysis of their quantitative and qualitative dependencies, which is not always possible when using only numerical models or estimation methods (e.g., neural networks). The obtained errors were quite comparable for all seasons. In case of SO₂ it was equal to about 5 µg/m³ and in case of PM₁₀ around 15 µg/m³.
- The proposed model can be treated as a unified form. Even if the model from a previous year is used for a new one, the fit is quite good. The very important parameter in the model is the accumulation of a pollutant from a previous time. That means that the model coefficients are quite stable for a certain area. They may change if area's urbanization or location of industrial plants changes.
- The observation of the model coefficient values and variations, as well as the influence of meteorological conditions (wind speed and temperature) allows an eventual evaluation of the effects of activities to change heating systems in buildings as described at the beginning of the paper.

Author Contributions: Conceptualization; formal analysis; methodology; validation; writing—original draft preparation; investigation, D.F. and T.N.; data curation; formal analysis; resources; J.S.; writing—review and editing, T.N. and J.S. All authors have read and agreed to the published version of the manuscript.

Funding: This research received no external funding.

Institutional Review Board Statement: Not applicable.

Informed Consent Statement: Not applicable.

Data Availability Statement: Not applicable.

Acknowledgments: The authors want to express gratitude towards the late prof. dr hab. Tadeusz Tumidajski for his invaluable contribution to research on monitoring and modelling of air pollutions propagation. The article is developed as a part of the project No. 16.16.100.215.

Conflicts of Interest: The authors declare no conflict of interest.

References

1. Kleczkowski, P. *Smog w Polsce*; Wydawnictwo Naukowe PWN: Warszawa, Poland, 2019.
2. Moussiopoulos, N. (Ed.) *Air Quality in Cities*; SATURN—EUROTRAC 2 Subproject Report; Springer: Berlin/Heidelberg, Germany, 2003.
3. Polski Alarm Smogowy. Available online: polskialarmsmogowy.pl (accessed on 30 November 2020).
4. Kobus, D.; Merenda, B.; Sówka, I.; Chlebowska-Styś, A.; Wroniszewska, A. Ambient air quality as a condition of effective healthcare therapy on the example of selected Polish health resorts. *Atmosphere* **2020**, *11*, 882. [CrossRef]
5. Wielgosiński, G.; Czerwińska, J. Smog Episodes in Poland. *Atmosphere* **2020**, *11*, 277. [CrossRef]
6. Foszcz, D.; Niedoba, T.; Saramak, D.; Gawenda, T. The Methodology of the SO₂ Concentration Forecasting in the Cities for the Preventive Purposes in the Example of the City of Kraków. In Proceedings of the 3rd International Conference of PhD Students, Engineering Sciences, Miskolc, Hungary, 13–19 August 2001; Volume 2, pp. 569–576.
7. Foszcz, D.; Niedoba, T.; Siewior, J. The Methods of Forecasting of SO₂ and Suspended Dust Concentrations for Warning Purposes in the Example of Selected Polluted Regions in Poland. In *Ecosystems and Sustainable Development*; Tiezzi, E., Brebbia, C.A., Jorgensen, S., Almorza Gomar, D., Eds.; WIT Press: Southampton/Boston, UK, 2005; pp. 477–491.
8. Kunysz, J.; Foszcz, D.; Gawenda, T.; Tumidajski, T. Modele adaptacyjne jako metoda prognozowania średniodobowych stężeń SO₂. *Ochr. Powietrza Probl. Odpadów* **2001**, *35*, 138–143.
9. Siewior, J.; Tumidajski, T.; Foszcz, D.; Niedoba, T. Prognozowanie stężeń zanieczyszczeń powietrza w GOP-ie modelami statystycznymi. *Rocz. Ochr. Sr.* **2011**, *13*, 1261–1274.
10. Tumidajski, T.; Siewior, J.; Foszcz, D.; Niedoba, T. Ocena wpływu stężeń zanieczyszczeń powietrza w GOP-ie na jakość powietrza w rejonie Opola i Kędzierzyna-Koźla. *Rocz. Ochr. Sr.* **2014**, *16*, 519–533.
11. Bringfelt, B. Important factors for the sulphur dioxide concentration in central Stockholm. *Atmos. Environ.* **1971**, *5*, 949–972. [CrossRef]

12. Bolzern, P.; Fronza, G.; Runze, E.; Uberhuber, C. Statistical analysis of winter sulphur dioxide concentration data in Vienna. *Atmos. Environ.* **1982**, *16*, 1899–1906. [\[CrossRef\]](#)
13. Hysenaj, M. Dispersion model prospective of air pollution in Tirana. *Geogr. Tech.* **2019**, *14*, 10–19. [\[CrossRef\]](#)
14. Markiewicz, M.T. Methods of determining meteorological data used in air pollution dispersion models. *Environ. Prot. Eng.* **2007**, *33*, 75–86.
15. Markiewicz, M.T. *Podstawy Modelowania Rozprzestrzeniania Się Zanieczyszczeń w Powietrzu Atmosferycznym*; Oficyna Wydawnicza Politechniki Warszawskiej: Warsaw, Poland, 2004.
16. Markiewicz, M.T. Methods of the wet deposition description in air pollution dispersion models. *Environ. Prot. Eng.* **2007**, *33*, 113–120.
17. Carmichael, C.R.; Peters, L.K.; Saylor, R.D. The STEM II regional scale acid deposition model: Physical concept and formulation. Part I. *Atmos. Environ.* **1991**, *25*, 2077–2090. [\[CrossRef\]](#)
18. Chang, J.S.; Brost, R.A.; Isaksen, P.S.A.; Madronich, S.; Middleton, P.; Stockwell, W.R.; Walcek, C.J. A three-dimensional Eulerian acid desposition model. Physical concepts and formulation. *J. Geophys. Res.* **1987**, *92*, 14681–14700. [\[CrossRef\]](#)
19. Venkatram, A.; Karamandani, P.K. Testing a comprehensive acid deposition model. *Atmos. Environ.* **1989**, *22*, 737–747. [\[CrossRef\]](#)
20. Hamer, P.D.; Walker, S.-E.; Sousa-Santos, C.; Vogt, M.; Vo-Thanh, D.; Lopez-Aparicio, S.; Schneider, P.; Ramacher, M.O.P.; Karl, M. The urban dispersion model EPISODE v. 10.0—Part I: An Eulerian and sub-grid scale air quality model and its application in Nordic winter conditions. *Geosci. Model. Dev.* **2020**, *13*, 4323–4353. [\[CrossRef\]](#)
21. Cirtina, D.; Pecingina, J.; Cirtina, L. Study on Mathematical Modeling of the Dispersion Sulfur Dioxide by Burning Fossil Fuels. In Proceedings of the International Multidisciplinary Scientific GeoConference and EXPO SGEM, Albena, Bulgaria, 17–26 June 2014; pp. 523–531.
22. Morawska-Horawska, M. Stochastyczne modele prognozy średniego dobowego stężenia SO₂ dla Krakowa. *Wiadomości IMGW* **1988**, *9*, 3.
23. Ilić, I.; Vuković, M.; Štrbac, N.; Urošević, S. Applying GIS to control transportation air pollutants. *Pol. J. Environ. Stud.* **2014**, *23*, 1849–1860.
24. Vlaknenski, T.; Stoychev, P.; Chuturkova, R. Dispersion modeling of atmospheric emissions of particulate matter (PM₁₀) and evaluation of the contribution of different sources of air pollution in the town of Svishtov, Bulgaria. *J. Sci. Appl. Res.* **2014**, *5*, 202–212.
25. Zahron, E.M.M. A novel approach to improve the air quality predictions of air pollution dispersion modelling systems. *Int. J. Environ. Res.* **2013**, *7*, 205–218.
26. El-Harabawi, M. Air quality model ling, simulation and computational methods: A review. *Environ. Rev.* **2013**, *21*, 149–179. [\[CrossRef\]](#)
27. Juda-Rezler, K. *Oddziaływanie Zanieczyszczeń Powietrza na Środowisko*; Wyd. OWPW: Warsaw, Poland, 2006.
28. Holnicki-Szulc, P. *Modele Propagacji Zanieczyszczeń Atmosferycznych w Zastosowaniu do Kontroli i Sterowania Jakością Środowiska*; Akademicka Oficyna Wydawnicza EXIT: Warsaw, Poland, 2006.
29. Khoo, H.H.; Spedding, T.A.; Houston, D.; Taplin, D. Application of modeling and simulation tools in costs and pollution monitoring. *Environmentalist* **2001**, *21*, 161–168. [\[CrossRef\]](#)
30. Gorai, A.K.; Upadhyay, A.K.; Goyal, P. Design of fuzzy synthetic evaluation model for air quality assessment. *Environ. Syst. Decis.* **2014**, *34*, 456–469. [\[CrossRef\]](#)
31. Billiaiev, M.M.; Slavinska, O.S.; Kyrychenko, R.V. Numerical simulation of pollution dispersion in urban street. *Ekol. Transporti* **2017**, *70*, 23–28. [\[CrossRef\]](#)
32. Jury, M.R.; Wilczak, J.M. Mesoscale numerical models: Cost-effective input to dispersion prediction schemes. *Clean Air* **1992**, *8*, 12–14. [\[CrossRef\]](#)
33. Preto, W.H.; Cremasco, M.A. Application of probability density functions in modelling annual data of atmospheric NO_x temporal concentration. *Chem. Eng. Trans.* **2017**, *57*, 487–492.
34. Hammond, J.K.; Chen, R.; Mallet, V. Meta-modeling of a simulation chain for urban air quality. *Adv. Mod. Simul. Eng. Sci.* **2020**, *7*, 1–22.
35. Kim, Y.; Wu, Y.; Seigneur, C.; Roustan, Y. Multi-scale modeling of urban air pollution: Development and application of a Street-in-Grid model (v1.0) by coupling MUNICH (v1.0) and Polair3D (v1.8.1). *Geosci. Model Dev.* **2018**, *11*, 611–629. [\[CrossRef\]](#)
36. Chen, H.; Xue, B. Research on the factors affecting regional smog in China—Based on spatial panel model. *Mod. Econ.* **2019**, *10*, 1292–1309. [\[CrossRef\]](#)
37. Deng, L.; Yu, M.; Zhang, Z. Statistical learning of the worst regional smog extremes with dynamic conditional modeling. *Atmosphere* **2020**, *11*, 665. [\[CrossRef\]](#)
38. Hu, D.; Jiang, J. A study of smog issues and PM_{2.5} pollutant control strategies in China. *J. Environ. Prot.* **2013**, *4*, 746–752. [\[CrossRef\]](#)
39. Yu, X.; Shen, M.; Shen, W.; Zhang, X. Effects of land urbanization on smog pollution in China: Estimation of spatial autoregressive panel data models. *Land* **2020**, *9*, 337. [\[CrossRef\]](#)
40. Zwoździak, J. *Prognozy i Analizy Stężeń Zanieczyszczeń w Powietrzu w Regionie Czarnego Trójkąta*; Oficyna Wydawnicza Politechniki Wrocławskiej: Wrocław, Poland, 1998.

41. Finzi, G.; Tebaldi, G. A mathematical model for air pollution forecast and alarm in an urban area. *Atmos. Environ.* **1982**, *16*, 2055–2090. [[CrossRef](#)]
42. Rani, N.L.A.; Azid, A.; Khalit, S.I.; Juahi, H.; Samsudin, M.S. Air pollution index trend analysis in Malaysia, 2010–2015. *Pol. J. Environ. Stud.* **2018**, *27*, 801–807. [[CrossRef](#)]
43. Hamid, A.; Atique, S.A.; Huma, Z.; Uddin, S.G.M.; Asghar, S. Ambient air quality & noise level monitoring of different areas of Lahore (Pakistan) and its health impacts. *Pol. J. Environ. Stud.* **2019**, *28*, 623–629.
44. Huete-Morales, M.D.; Quesada-Rubio, J.-M.; Navarette Álvarez, E.; Rosales-Moreno, M.J.; Del-Moral-Ávila, M.J. Air quality analysis in the European union. *Pol. J. Environ. Stud.* **2017**, *26*, 1113–1119. [[CrossRef](#)]
45. Saramak, A. Comparative analysis of indoor and outdoor concentration of PM10 particulate matter on example of Cracow City center. *Int. J. Environ. Sci. Technol.* **2019**, *16*, 6609–6616. [[CrossRef](#)]
46. Spychała, M.; Domagajska, J.; Ćwieląg-Drabek, M.; Marchwińska-Wyrwał, E. Correlation between length of life and exposure to air pollution. *Pol. J. Environ. Stud.* **2020**, *29*, 1361–1368. [[CrossRef](#)]
47. Baklanov, A.; Zhang, Y. Advances in air quality modeling and forecasting. *Glob. Transit.* **2020**, *2*, 261–270. [[CrossRef](#)]
48. Nichol, J.E.; Bilal, M.; Qiu, Z. Air pollution scenario over China during COVID-19. *Remote Sens.* **2020**, *12*, 2100. [[CrossRef](#)]
49. Dziennik Ustaw. *J. Laws* **2019**, pos. 1355, as amended.
50. Inspekcja Ochrony Środowiska. Available online: <http://powietrze.gios.gov.pl/pjp/maps/measuringstation> (accessed on 20 January 2021).
51. Jarzębski, L. *Raport o Stanie Środowiska w Województwie Katowickim w Latach 1995–1996*; Biblioteka Monitoringu Środowiska: Katowice, Poland, 1997.
52. Tumidajski, T.; Foszcz, D.; Gawenda, T. Wybrane aspekty zmian zanieczyszczeń powietrza miast Górnośląskiego Okręgu Przemysłowego. *Zesz. Nauk. Wydziału Budownictwa Śynierii Sr.* **2001**, *20*, 573–580.