

## Article

# Multi-Wavelength Coherent-Perfect-Absorption—Laser Points and Half-Wave Phase Shift in Parity-Time-Reciprocal Scaling Symmetry Structure

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**Abstract:** Two slabs of dielectrics with different values of refractive index are synthesized to form a composite structure and the whole system satisfies parity-time-reciprocal scaling symmetry. Four coherent-perfect-absorber-laser points are induced by modulating the incident wavelength of light-waves and the loss/gain factor of materials. The locations of coherent-perfect-absorber-laser points in parameter space, which manifests the multi-wavelengths of coherent-perfect-absorber-lasers could be regulated by the incidence angle. Especially, a value of  $\pi$  phase shift in the transmitted/reflected light wave has been observed as well. This research has potential applications in multi-wavelength absorbers/lasers and half-wave phase shifters.



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**Keywords:** PTX symmetry; CPA-laser point; half-wave phase shifter; dielectric structure; laser

## 1. Introduction

The absorption and radiation of light waves by materials make the loss of the system inevitable through the interaction process of light and material [1]. By using the stimulated radiation of the gain medium or the parametric process of the nonlinear medium [2–4], the energy loss or gain of the system can be compensated. The introduction of loss and gain in non-Hermitian optical structures provides more freedom for photon manipulation.

The concept of parity-time (PT) symmetry has been transplanted from quantum systems to non-Hermitian optical systems. It has become a research hotspot in recent years and has a broad application prospect in laser technology [5,6], optical sensing [7], material engineering [8], and other fields. In non-Hermitian systems, the eigenvalues of Hamiltonians are usually complex numbers. However, when the Hamiltonian satisfies the PT symmetry, the eigenvalues of the Hamiltonian may also be real numbers. The complex function of the refractive index for the PT optical system could be written as  $n(x) = n * (-x)$ , viz. the real component of the refractive indice presents the symmetrical distribution, while the imaginary component is anti-symmetric. The asterisk represents the complex conjugate operation. Compared with the uniform distribution of complex permittivity, the complex permittivity modulated by space or time makes the non-Hermitian optical system show many unique optical properties, such as optical invisibility [9], one-way nonreflective transmission [10,11], electric field localization [12,13], and spectral singularity [14–16], which opens a new path for the search and exploration of new photonic devices [17–19]. The modulation of the complex permittivity includes the modulating of the real component and the imaginary component. It is found that spatial modulation of the real component of the permittivity could control the mode output, and the time modulation can be used

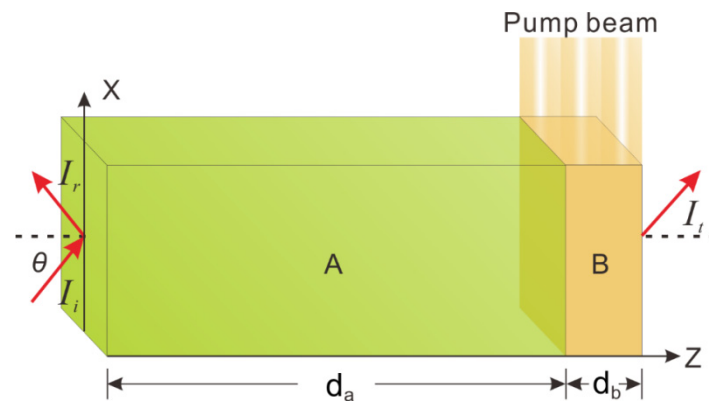
to construct nonreciprocal devices such as optical isolators [20]. The coherent-perfect-absorption–laser point (CPA–LP) [21–25] and nonreciprocal amplification [26] can be realized by modulating the imaginary part of the dielectric constant. CPA–LP is the reverse physics process of a laser, which could be utilized for an anti-laser, and it may completely absorb the electromagnetic wave with a specific frequency and waveform. Specifically, as two beams of coherent light waves with the same intensity incident on the upper and lower surfaces of the absorber, one can adjust the phase difference in the coherent lights to achieve the control of the absorption intensity from 0% to 100%. Instabilities of coherent perfect absorption in layered systems were analysed by alternating, amplifying, and absorbing media [27]. CPA–LP, as a special solution of the broken phase in PT-symmetric systems [28], may show more intuitive physical characteristics, which are of great significance to the study of non-Hermitian physics.

The PT-symmetric structures are special non-Hermitian systems. For quantum mechanics, if the Hamiltonian operator  $H$  and the parity time operator  $PT$  satisfy the reciprocity relation  $[H, PT] = 0$ , then the Hamiltonian is said to be PT-symmetric. The symbols  $T$  and  $P$  represent time and parity operators, respectively. In optical crystals, the condition of parity-time symmetry is equivalent to the existence of gain in one region of the system and the existence of equivalent loss in another region of symmetry. The scale of the loss and gain medium meeting should be the same size. The optical loss is easily doped and regulated within a large size range, but the optical gain can only be controlled within a small range. On the other hand, the loss/gain factor of the dielectric can be regulated by ion doping and two-wave nonlinear mixing. For example, the gain coefficient after beam coupling is significantly increased by two-wave mixing in  $Fe^{3+}$ -doped  $LiNbO_3$  [3] and Bi-doped  $Sn_2P_2S_6$  [29]. Inspired by the above studies, we here propose a PTX (parity-time-reciprocal scaling) symmetry dielectric structure. Based on PT symmetry, the structure increases the reciprocity symmetry of the space scale; that is, the optical gain size is reduced to  $1/G$  of the optical loss size ( $G > 1$ ), but at the same time, the optical gain coefficient is expanded to  $G$  times of the optical loss coefficient. Further, we explore CPA–LPs and their related characteristics in PTX-symmetric dielectric structures.

In this study, compound systems are synthesized, which contain two kinds of dielectric slabs A and B with different refractive indices. The loss/gain factor of dielectrics and the thickness of slabs A and B are modulated to satisfy parity-time-reciprocal scaling (PTX) symmetry. We first investigate multiple CPA–LPs arising from the parameter space consisting of the loss/gain factor and incident wavelength. Subsequently, regulation of the CPA–LPs is explored through the incident angle. Then, phase-shift characteristics of light waves at CPA–LPs are presented as well. Our research provides a theoretical or experimental project for designing multi-wavelength tunable lasers or half wave phase shifters.

## 2. Synthesis of PTX-Symmetric Dielectric Structure

The proposed geometrical structure is given in Figure 1. The symbols A and B denote two dielectric slabs with different refractive indices. Their geometrical thicknesses are denoted by  $d_a$  and  $d_b$ . The indices A and B are given by:  $n_a = n_{ar} + i^*n_{ai}$ ,  $n_b = n_{br} + i^*n_{bi}$ , where  $n_{ar}$  and  $n_{br}$ , respectively, represent the real part  $n_{ai}$  and the imaginary part  $n_{bi}$  and  $i$  is the imaginary unit. The incident, reflected, and transmitted light beams are denoted by the symbols  $I_i$ ,  $I_r$ , and  $I_t$ , respectively. The incident angle is denoted by  $\theta$  which represents the included angle between the incident beam and the horizontal direction (i.e., Z-axis). The host material of A can be set as  $LiNbO_3$ . Weak optical loss can be realized by doping iron ions ( $Fe^{2+}$ ) into the dielectric slab A and one can choose ion-doped  $LiNbO_3$  for the optical loss dielectric A [3]. The host material of slab B may be  $SiO_2$  and the optical gain coefficient is controlled by  $Er^{3+}$  doping in slab B along with the intensity regulation of the external pump beam. The refractive indices of materials and the thicknesses of slabs of the system are adjusted subtly, and finally, the entire dielectric structure meets the PTX symmetry.



**Figure 1.** Schematic of dielectrics structure with PTX symmetry.

The parameter  $z$  is the spatial position coordinate and the refractive index function of a composite system that satisfies the equation:  $n(z) = n^*(-z)$ ; then, the system follows PT-symmetry. The refractive index function is written in conjugate form, that is,  $n(z) = n_r(z) + i^*n_i(z)$ . In practice, the real component and imaginary component of the refractive index in materials need to be adjusted concurrently, so that the real refractive index meets even symmetry, viz.  $n_r(z) = n_r(-z)$ , while the imaginary refractive index satisfies odd symmetry, viz.  $n_i(z) = -n_i(-z)$ . The positive imaginary part of the index value represents the optics gain of the material, which can be realized by nonlinear two-wave mixing or ion doping. It shows that the optical gain in a broad-area semiconductor amplifier can be realized by the two-wave mixing, in which a pump beam and a signal beam with different frequencies, both a moving phase grating and a moving gain grating, are induced in the amplifier [30]. The optical gain is further provided through two-wave mixing in  $\text{Fe}^{3+}$ -doped  $\text{LiNbO}_3$  using the material's photorefractive nonlinearity, while optical loss is induced by  $\text{Fe}^{2+}$ -doping [3]. Otherwise, optical gain enhancements in Si nanocrystals via hydrogenation and  $\text{Ce}^{3+}$  ion doping and Bi-doped  $\text{Sn}_2\text{P}_2\text{S}_6$  crystals are through a two-beam coupling [29,31]. The coupling gain is induced by a higher effective electro-optic coefficient and an increase in the trap density compared with values in these pure nanocrystals or crystals. The negative imaginary refractive index component represents the optical loss in the system, and the optical loss can be realized by ion doping.

Compared with PT symmetry, the PTX-symmetric condition of the dielectric structure demands the function of the refractive index to meet the equations:  $\sqrt{G}d_a = d_b/\sqrt{G}$ ,  $n_{ar} = n_b$  and  $n_{ai}/\sqrt{G} = \sqrt{G}n_{bi}$ . The parameter  $G$  is the scale transformation coefficient. For  $\text{LiNbO}_3$ , the real part of the refractive index is  $n_{ar} = n_{br} = 2.22$ , and the imaginary part is  $n_{ai} = 0.01^*q$  and  $n_{bi} = -0.1^*q$ , respectively. The mark  $q$  is called the loss/gain factor of optical media. The dielectric thicknesses are  $d_a = 5 \mu\text{m}$  and  $d_b = 0.5 \mu\text{m}$ . Here the scale transformation coefficient is set as  $G = 10$ .

For a transverse magnetic (TM) wave at the two ports of a layer, the electric and magnetic fields can be interconnected by the transfer matrix method (TMM) [32,33].

$$M_l = \begin{bmatrix} \cos \varphi_l & -\frac{i}{\eta_l} \sin \varphi_l \\ -i\eta_l \sin \varphi_l & \cos \varphi_l \end{bmatrix}, \quad (1)$$

The symbol is  $\varphi_j = 2\pi n_l d_l \cos \theta_l / \lambda$ . The parameter  $d_l$  is the thickness of dielectric and  $\theta_l$  is the angle of incidence. The symbol  $\eta_l = (\epsilon_0 \epsilon_r)^{1/2} / [(\mu_0 \mu_r)^{1/2} \cos \theta_l]$ . Then, the electric and magnetic fields at the first input port and last output port can be expressed as

$$\begin{pmatrix} E_0 \\ H_0 \end{pmatrix} = \prod_{l=1}^k M_l \begin{pmatrix} E_k \\ H_k \end{pmatrix} = \begin{bmatrix} m_{11} & m_{12} \\ m_{21} & m_{22} \end{bmatrix} \begin{pmatrix} E_k \\ H_k \end{pmatrix}. \quad (2)$$

Consequently, the transmission coefficient  $t$  and reflection coefficient  $r$  can be obtained as

$$t = \frac{2\eta_0}{(m_{11} + m_{12}\eta_k)\eta_0 + (m_{21} + m_{22}\eta_k)} \frac{\cos\theta_0}{\cos\theta_k}, \quad (3)$$

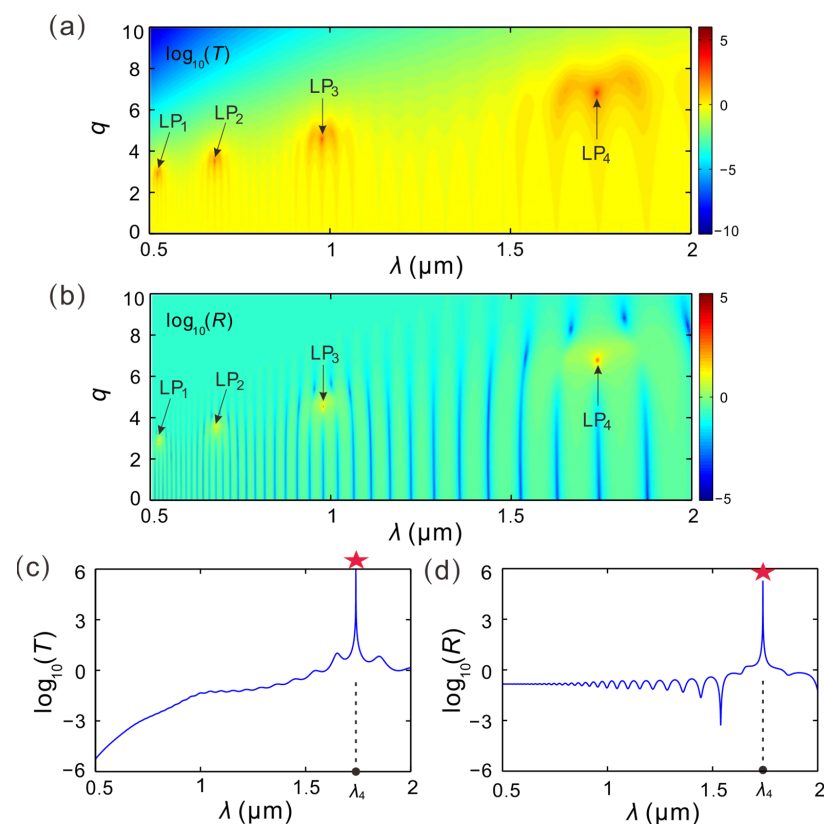
and

$$r = \frac{(m_{11} + m_{12}\eta_k)\eta_0 - (m_{21} + m_{22}\eta_k)}{(m_{11} + m_{12}\eta_k)\eta_0 + (m_{21} + m_{22}\eta_k)}, \quad (4)$$

where  $\eta_0 = \eta_k = (\varepsilon_0/\mu_0)^{1/2}/\cos\theta$ . The parameter  $\eta_0$  is the admittance in vacuum. The transmittance  $T$  of light beam and the reflectance  $R$  can be obtained by  $T = tt^*$  and  $R = rr^*$ , respectively.

### 3. Multiple CPA–LPs and Half Wave Phase Shift

The transmittivity  $\log_{10}(T)$  in parameter space is described in Figure 2a. The parameter space is constituted by the loss/gain factor and the incident wavelength. The transverse magnetic wave is propagating along the Z-axis in the PTX-symmetric structure and the angle of incidence is fixed by  $\theta = 0^\circ$ . The logarithm  $\log_{10}(T)$  of the light transmittance  $T$  can be aimed at enhancing the contrast ratio. By modulating the values of  $\lambda$  in the range of  $[0.5 \mu\text{m}, 2 \mu\text{m}]$  and  $q$  in the range of  $[0, 10]$  synchronously, we could find the transmission extreme points in parameters space. The maximum transmitted point is commonly known as this CPA–LP, denoted by the symbol LP. Briefly, these four laser points are located at LP<sub>1</sub> [ $\lambda_1 = 0.5198 \mu\text{m}$ ,  $q_1 = 2.8529$ ], LP<sub>2</sub> [ $\lambda_2 = 0.6785 \mu\text{m}$ ,  $q_2 = 3.4835$ ], LP<sub>3</sub> [ $\lambda_3 = 0.9765 \mu\text{m}$ ,  $q_3 = 4.5646$ ], and LP<sub>4</sub> [ $\lambda_4 = 1.7393 \mu\text{m}$ ,  $q_4 = 6.8168$ ], and the corresponding transmittance values at these CPA–LPs are  $T_1 = 6.6405 \times 10^3$ ,  $T_2 = 2.7102 \times 10^4$ ,  $T_3 = 4.162 \times 10^4$ , and  $T_4 = 9.7791 \times 10^5$ , respectively.



**Figure 2.** (a,b) Transmittivity and reflectivity in parameter space formed by the loss/gain factor and incident wavelength, respectively. (c,d) Transmission and reflection spectra of light waves for  $q = 6.8168$ , respectively. The light wave is vertically incident. The red asterisk (☆) denotes the CPA-LP.

Figure 2b explores the reflection spectrum  $\log_{10}(R)$  in parameters space. We mark the four dark orange dots as  $LP_1$ ,  $LP_2$ ,  $LP_3$  and  $LP_4$ , representing four points with the maximum value of  $R$ . The positions of these four poles are exactly at  $[\lambda_1 = 0.5198 \mu\text{m}, q_1 = 2.8529]$ ,  $[\lambda_2 = 0.6785 \mu\text{m}, q_2 = 3.4835]$ ,  $[\lambda_3 = 0.9765 \mu\text{m}, q_3 = 4.5646]$ , and  $[\lambda_4 = 1.7393 \mu\text{m}, q_4 = 6.8168]$ , viz. the four CPA-LPs. The corresponding values of  $R$  for these four poles are  $R_1 = 1.0053 \times 10^3$ ,  $R_2 = 4.1495 \times 10^3$ ,  $R_3 = 6.8375 \times 10^3$ , and  $R_4 = 1.8268 \times 10^5$ , respectively.

To further prove the resonance characteristics of the CPA-LPs, we focus on one of the laser points to investigate its optical spectrum. Figure 2c describes the spectrum of transmission light waves for the parameter of  $q = 6.8168$ . The parameter  $q = 6.8168$  is exactly equal to the corresponding loss/gain factor of  $LP_4$ . The transmittivity of the light beams changes with the increase in the incident wavelength, and one can find that a sharp transmission peak appears in the spectrum of transmission beams, which is indicated by a red asterisk ( $\star$ ). The value of this transmission peak is  $\log_{10}(T) = 5.9903$ , and correspondingly, the wavelength of incidence is fixed by  $\lambda = 1.7393 \mu\text{m}$ , which also happens to be  $\lambda_4$ . The transmittance of this peak  $\log_{10}(T) = 5.9903 = \log_{10}(T_4)$  identifies with the transmittance at  $LP_4$ . The transmittivity roughly increases with the augment of the incidence wavelength as  $\lambda \leq 1.7393 \mu\text{m}$ . The transmitted intensity of light beams has been greatly improved as the incident wavelength tends to  $LP_4$  from the left side. As the value is  $\lambda > 1.7393 \mu\text{m}$ , the transmittivity decreases with the rise in the incident wavelength, while the transmitted intensity declines dramatically when the incident wavelength expands from the right of  $LP_4$ . Otherwise, the transmission spectrum curve is asymmetric to the CPA-LP of  $LP_4$ .

Figure 2d studies the relationship between the incident wavelength and the reflectance of the light beam for the value of  $q = 6.8168$ . It can be found that the reflectance of the light waves changes with the wavelength of incidence. At  $\lambda = 1.7393 \mu\text{m}$ , there is a reflection peak, marked with a red asterisk ( $\star$ ) as well. The peak value of reflectance is taken as the logarithm  $\log_{10}(R) = 5.2617 = \log_{10}(R_4)$ , so the value is exactly equal to the reflectance at  $LP_4$ . Similarly, by adjusting the incident wavelength, the reflectivity of the light waves changes violently near the  $LP_4$ . The reflectance changes relatively gently as the incident wavelength moves away from  $LP_4$  on both sides.

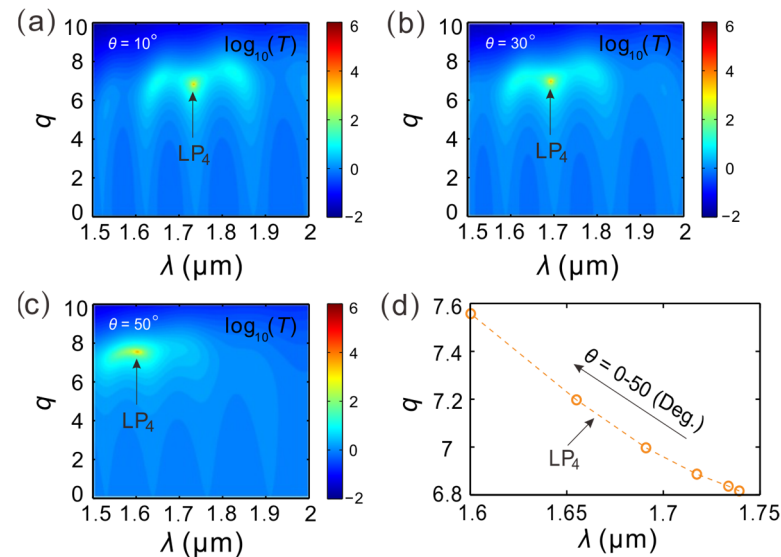
Furthermore, one can change the incident angle, and the other parameters remain unchanged. Along the Z-axis a transverse magnetic wave propagates in the PTX-symmetric dielectrics structure as the incidence angle is  $\theta = 0^\circ$ . Figure 3a provides the transmittivity of light waves around  $LP_4$  as  $\theta = 10^\circ$ . The CPA-LP ( $LP_4$ ) appears at  $[\lambda_4 = 1.7337 \mu\text{m}, q_4 = 6.8368]$  in parameter space. The transmittivity at this CPA-LP ( $LP_4$ ) is  $T = 2.0128 \times 10^5$ . For the value of incident angle  $\theta = 30^\circ$ , the CPA-LP ( $LP_4$ ) moves to  $[\lambda_4 = 1.691 \mu\text{m}, q_4 = 6.997]$  and its transmittance is  $T = 4.1908 \times 10^6$  as provided in Figure 3b. Increasing the angle of incidence to  $50^\circ$ , the position of the CPA-LP ( $LP_4$ ) is located at  $[\lambda_4 = 1.6003 \mu\text{m}, q_4 = 7.5576]$ , as shown in Figure 3c, and the corresponding transmittance is  $T = 1.0174 \times 10^6$ .

Changing the angle of incidence from the value of  $0^\circ$  to  $60^\circ$ , Figure 3d shows the relationship between the position of this CPA-LP ( $LP_4$ ) in parameter space and the incidence angle. One can see that with the augment of the angle value of incidence,  $LP_4$  moves to the upper left of the parameter space, viz. the corresponding wavelength of the laser decreases, and the loss/gain factor grows. In a similar way, the corresponding wavelengths of the other laser points of  $LP_1$ ,  $LP_2$ , and  $LP_3$  may change with the incidence angle as well. If the incident angle of the light wave changes, the position of the corresponding laser point in parameter space will change accordingly, that is, by changing the value of the incidence angle, the corresponding laser wavelength may be tuned.

In the parameter space constituted by the incident light wavelength and the loss/gain factor, the PTX-symmetric dielectric structure supports multi-wavelength laser points. Multi-wavelength laser points correspond to the maximum points of transmittance and reflectance, which could be utilized for obtaining ideal multi-wavelength lasers. The wavelengths of the laser correspond to the wavelength values at these multi-wavelength laser points. The multi-wavelengths of the laser can be flexibly adjusted by the value of the incidence angle. Compared with PT-symmetric aperiodic photonic crystals, which

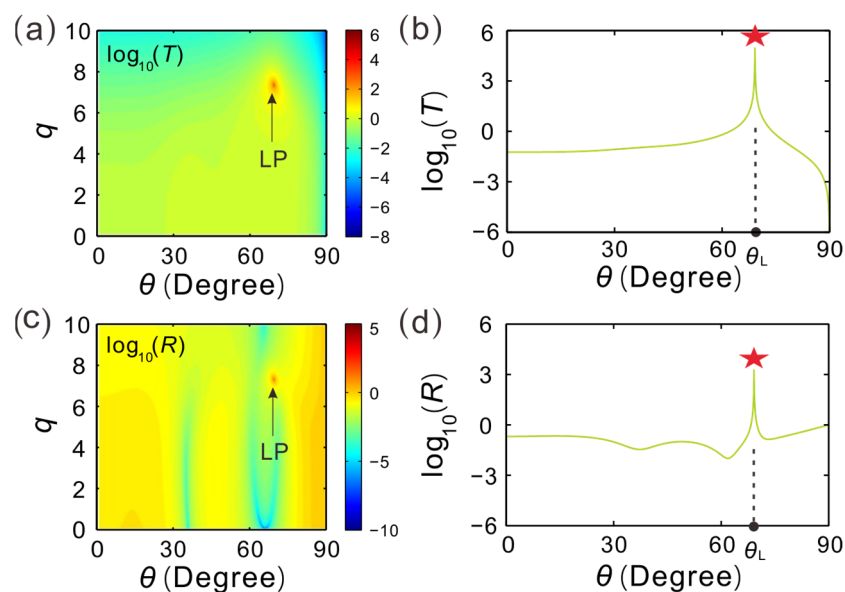


also are used for realizing multi-wavelength laser, the PTX-symmetric dielectric proposed here has a simple structure. However, the reduction in the size of the gain material in multi-wavelength lasers is at the cost of increasing the gain coefficient of the material.



**Figure 3.** (a–c) Transmittivity in the parameter space constituted by loss/gain factor and incident wavelength for  $\theta = 10^\circ$ ,  $30^\circ$  and  $50^\circ$ , respectively. (d) CPA–LP<sub>4</sub> varying with incident wavelength in parameter space.

For a fixed incident wavelength, the dependence of the CPA–LP on the direction of the reflected light wave in the PTX-symmetric dielectric structure is discussed here. Figure 4a demonstrates the intensity transmitted light versus the loss/gain factor and the incident angle. One could find that there is a CAP–LP in the parameters space. The according parameters of this CAP–LP are given by the incident angle of  $\theta = 69.175^\circ$ , the loss/gain factor  $q = 7.3417$ , and the maximum transmittivity of  $T = 8.9702 \times 10^4$ .



**Figure 4.** (a,c) Transmittance and reflectance in parameters space formed by loss/gain factor and incident angle. (b,d) Transmission spectrum and reflection spectrum for  $q = 7.3373$ . Incidence wavelength is  $\lambda = 1.31 \mu\text{m}$  for (a–d).

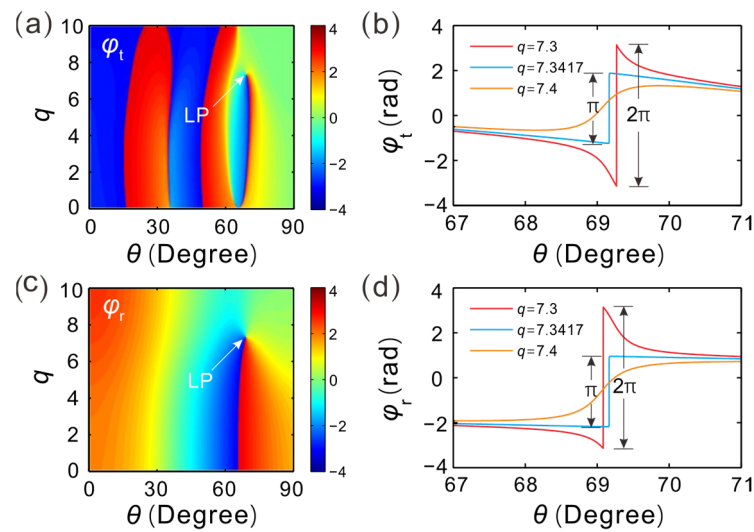
For the loss/gain factor value  $q = 7.3417$ , Figure 4b describes the corresponding transmission spectrum changing with the incidence angle. The transmittance changes with the incidence angle. At  $\theta = 69.175^\circ$ , there is a sharp transmission peak marked with a red asterisk ( $\star$ ). At the labeled asterisk, the maximum value is  $\log_{10}(T) = 4.9528 = \log_{10}(T_L)$ , which exactly equates to the relevant value of transmittance for the laser point LP, as shown in Figure 4a. The transmission intensity increases slowly with the increase in the incident angle for  $\theta \leq 69.175^\circ$ , while for  $\theta > 69.175^\circ$ , the transmission intensity decreases rapidly with the rise in the incidence angle. As the incident angle reaches the peak from the right or the left sides of this laser point LP, the transmission intensity could be greatly improved. Therefore, one can modulate the parameters of the laser point LP through the loss/gain factor and the incidence angle.

Figure 4c provides the reflectance of light waves varying with the incidence angle in parameters space. In order to increase the contrast, the logarithm  $\log_{10}(R)$  of the light wave reflectance  $R$  is also taken in simulations. There is one LP appearing in the quantitative value range of parameters space, and its reflectance is extremely high, with a value of  $R_L = 1.8214 \times 10^3$ . For a fixed quantity of the loss/gain factor  $q = 7.3417$ , the corresponding reflection spectrum for light waves is described in Figure 4d. Obviously, the reflectance increases as the incident angle changes around  $\theta = 69.175^\circ = \theta_L$ , and the reflectance reaches the maximum value at  $\theta_L$  marked with a red asterisk ( $\star$ ). The reflectance at this point is  $\log_{10}(R) = 3.2604 = \log_{10}(R_L)$ . In the same way, the reflectivity of light waves changes dramatically near this CPA-LP. When the incident angle of the light waves is far from the peak on both sides, the reflectance change is relatively flat. In conclusion, for the PTX-symmetric dielectric structure proposed here, the CPA-LP is precisely located at the pole of the transmitted light beam or the reflected light beam.

Generally, the transmission coefficient can be expressed as  $t = E_o/E_i$ , where  $E_i$  denotes the input electric field strength, while  $E_o$  represents the transmission electric field strength, both of which are complex. Further, the transmission coefficient is written in the form of the complex index,  $t = |t| \exp(i\varphi_t)$ ,  $E_o = |E_o| \exp(i\varphi_o)$ , and  $E_i = |E_i| \exp(i\varphi_i)$ . From this, we can obtain  $E_o = t^* E_i = |t| |E_i| \exp[i(\varphi_t + \varphi_i)]$ , viz. the phase  $\varphi_o = \varphi_t + \varphi_i$  of the transmitted electric field. Similarly, the phase  $\varphi_o = \varphi_r + \varphi_i$  of the reflected electric field can be obtained. That is to say, the phase of the electric field is closely related to the reflection coefficient phase and the transmission coefficient phase. Finally, the phase of the electric field can be adjusted by modulating the reflection coefficient phase or the transmission coefficient phase.

For a fixed incident wavelength  $\lambda = 1.31 \mu\text{m}$ , the transmission coefficient phase of the light waves in parameters space is formed by the loss/gain coefficient and the incident angle is demonstrated in Figure 5a. One could find that the CPA-LP is located at  $[\theta_L = 69.175^\circ, q_L = 7.3417]$  and the transmission coefficient phase  $\varphi_t$  is singular at the CPA-LP. This means that near the CPA-LP, the phase of transmission coefficient is very sensitive to the incident angle and the loss/gain factor. If the incident angle undergoes a minor change, the transmission coefficient phase will vary dramatically.

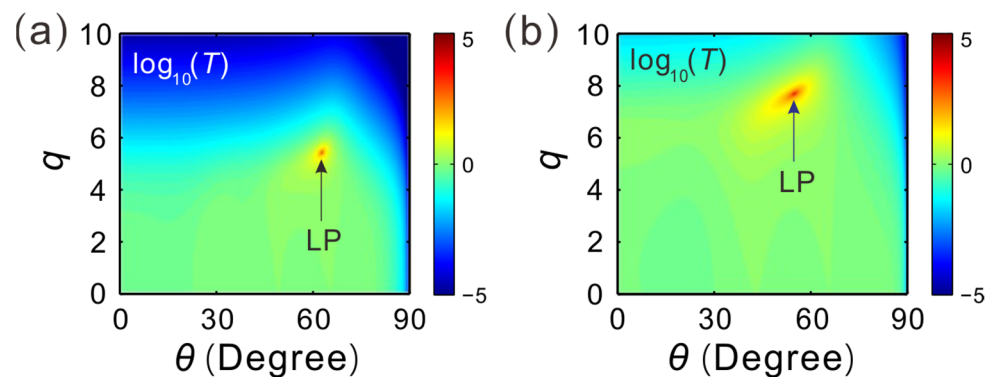
Figure 5b provides the transmission coefficient phase varying with the incident angle. For demonstrating the change in the transmission coefficient phase, we here take three specific values of the loss/gain factor to demonstrate. The incident wavelength is provided by  $\lambda = 1.31 \mu\text{m}$ . The parameter value  $q = 7.3417$  is just equal to the loss/gain factor of the CPA-LP. One of the other two parameters is greater than  $q = 7.3417$  and the other is less than  $q = 7.3417$ . One can see that at  $\theta = 69.175^\circ = \theta_L$ , there is a  $\pi$  phase jump in the transmission coefficient phase  $\varphi_t$ . For the loss/gain factor  $q = 7.3$ , it has a jump of  $2\pi$  in phase in the transmission coefficient phase curve as the incident angle increases. The  $2\pi$  phase change is meaningless and can be ignored. For the loss/gain factor  $q = 7.4$ , the transmission coefficient phase changes slowly and continuously with the varying of the incident angle. In short, the variation in the transmission coefficient phase is exactly equal to  $\pi$  at the CPA-LP, and this phase jump effect can be used in half-wave phase shifters for transmitted light waves. The phase shift is controlled by changing the incident angle.



**Figure 5.** (a,c) Transmission and reflection coefficient phase versus loss/gain factor and incident angle. (b,d) Transmission and reflection coefficient phases versus incidence angle for loss/gain factor  $q = 7.3, 7.3417$  and  $7.4$ , respectively.

Figure 5c gives the relationship between the incident angle and the phase of reflection coefficient. The phase  $\phi_r$  at the CPA–LP experiences a dislocation. Figure 5d depicts the phase of the reflection coefficient changing with the incident angle for different loss/gain factors. In comparison with the characteristics as shown in Figure 5b, it can be found that the changing trend in the phase  $\phi_r$  is the same as that in phase  $\phi_t$  for the three given loss/gain factors. For  $q = 7.3417$ , the phase of  $\phi_r$  changes suddenly at the CPA–LP, and the corresponding reflection coefficient phase change is exactly equal to  $\pi$  as well. This effect can also be used in half-wave phase shifters for reflected light waves, and the shift in phase can be controlled by varying the value of the incident angle.

For different incident wavelengths, Figure 6 gives the corresponding transmittance of light waves. There is one CPA–LP in Figure 6a and another CPA–LP in Figure 6b as well. The LP appears at  $[\theta_L = 62.85^\circ, q_L = 5.4454]$  and the peak value of the transmittance is  $\log_{10}(T) = 2.8508$  for  $\lambda = 0.85 \mu\text{m}$ , whereas the other LP is situated at  $[\theta_L = 54.8^\circ, q_L = 7.6977]$  and the maximum transmittance is  $\log_{10}(T) = 6.5361$  for  $\lambda = 1.55 \mu\text{m}$ . It demonstrates that the laser resonance may be realized by adjusting the loss/gain coefficient and the incident angle for different incident light wavelengths. At the laser point, the phase jump of  $\pi$  occurs in both the reflection coefficient phase and the transmission coefficient phase, thus realizing the half-wave shift in the phase of the transmitted light wave and the reflected light wave.



**Figure 6.** (a,b) Transmittance of light wave in parameter space for  $\lambda = 0.85 \mu\text{m}$  and  $1.55 \mu\text{m}$ , respectively. Parameter space is constituted by loss/gain factor and incident angle.



With a fixed incident wavelength, the PTX-symmetric dielectric structure supports a resonant laser point, that is, a CPA–LP. This point corresponds to the maximum transmittance and reflectance of light waves simultaneously. The reflection coefficient phase and the transmission coefficient phase have a phase jump of  $\pi$  at the laser point as well. This phase jump effect could be utilized to realize the half-wave phase shift in the transmission light wave and the reflection light wave. Furthermore, for different incident wavelengths, the positions of the CPA–LP in the parameter space are different, viz., the corresponding value of the incident angle and the loss/gain coefficient at the CPA–LP changes.

#### 4. Conclusions

In summary, multiple CPA–LPs are researched in the dielectric structures with PTX symmetry. Along the Z-axis, two kinds of dielectric sheets are arrayed to form one-dimensional dielectric composite structures. The thickness of the dielectric and optical loss/gain of the dielectric system are meticulously modulated to satisfy PTX symmetry. PTX symmetry is based on PT symmetry; meanwhile, the spatial scale reciprocity symmetry is added to reduce the optical gain factor to  $1/G$  ( $G > 1$ ) of the optical loss factor, while the optical gain coefficient is  $G$  times that of the optical loss coefficient. The parameter  $G$  refers to the scale transformation coefficient. Four CPA–LPs appear in the parameter space and the corresponding wavelengths for these CPA–LPs decrease with the increase in the incident angle. In particular, for a specific incident wavelength, the PTX-symmetric dielectric structure supports CPA–LPs as well, and the phases of reflection and transmission coefficients simultaneously undergo a  $\pi$  phase jump at the CPA–LPs, which can be utilized to achieve an accurate half-wave phase shift. Otherwise, the position of the CPA–LP varies with the light wavelength. This research can be applied to multi-wavelength absorbers/lasers and half-wave phase shifters.

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