

Article

Corresponding States for Volumes of Elemental Solids at Their Pressures of Polymorphic Transformations

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Abstract: Many non-molecular elemental solids exhibit common features in their structures over the range of 0 to 0.5 TPa that have been correlated with equivalent valence electron configurations. Here, it is shown that the pressures and volumes at polymorphic transitions obey corresponding states given by a single, empirical universal step-function $V_{tr}/L = -0.0208(3) \cdot P_{tr} + N_i$, where V_{tr} is the atomic volume in $\text{\AA}^3$ at a given transformation pressure P_{tr} in GPa, and L is the principal quantum number. N_i assumes discrete values of approximately 20, 30, 40, etc. times the cube of the Bohr radius, thus separating all 113 examined polymorphic elements into five discrete sets. The separation into these sets is not along L . Instead, strongly contractive polymorphic transformations of a given elemental solid involve changes to different sets. The rule of corresponding states allows for predicting atomic volumes of elemental polymorphs of hitherto unknown structures and the transitions from molecular into non-molecular phases such as for hydrogen. Though not an equation of state, this relation establishes a basic principle ruling over a vast range of simple and complex solid structures that confirms that effective single-electron-based calculations are good approximations for these materials and pressures. The relation between transformation pressures and volumes paves the way to a quantitative assessment of the state of very dense matter intermediate between the terrestrial pressure regime and stellar matter.

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1. Introduction

In difference to gases whose states can be described by the ideal gas- or the van der Waals equations of state, solids seem to defy the concept of a general equation of state. The range of structures and properties as well as their changes upon compression appear too large to be comprised within a single formula. Over a range of pressure of 0 to 500 GPa, many elements that are molecular solids at ambient conditions transform into atomic metals [1], while others that are normal metals at ambient conditions become semiconductors [2–4], and some monatomic elemental solids assume complex structures under compression that do not exist at ambient pressure [5,6].

These changes are not arbitrary but reflect general trends in the effect of pressure on the valence electron structure of solid matter over large pressure intervals [1–12]. The changes in the valence electron configuration reflect the different response of different orbital states to the increase in electron density upon compression [2,7,10–15]. The atomic volume often is markedly reduced upon such transitions. The transition from the bcc to the hcp structure in many transition metal elements is correlated with an increase in the hybridization of filled d-states with p-states of the next higher principal quantum number [16,17]. Similarly, the pressure-induced transformations of lanthanides from hcp to the Sm type to dhcp to fcc is explained through the increased occupancy of valence d-states [11–13], whereas the marked volume collapse and transitions to low symmetric

phases in lanthanides are assigned to delocalization of the 4f states [12,13,15]. A different type of delocalization occurs in the low-Z elemental alkaline metals, which at high pressure transform into electrides in which inner shell electron states overlap with valence s-states and non-bonding p-states are occupied on expenses of the bonding s- and p-hybrid states [2,4,14]. As a consequence, elements such as Li and Na, which are normal metals at ambient conditions, become semiconductors at pressures between 100 and 200 GPa [2–4]. Changes in the electronic valence states have been studied quantitatively for some alkaline metals, transition group elements, and lanthanides through X-ray spectroscopy [10,14–17]. These changes are correlated with structural transformations. It has been noted that these pressure-induced valence changes follow general correlations of the nuclear charge number with the pressure, volume, and structure type [5,8–10]. In addition, there are correlations between the atomic volumes of equivalent polymorphs of different elements [8,18].

Here, it is shown that all known non-molecular elemental solids follow a rule of corresponding states if only the volumes and pressures at the polymorphic transitions are considered. This observation extends to high-pressure atomic polymorphs of elements that are molecular at ambient conditions, such as O, N, S, P, Cl, Br, and I. Only experimental data are considered here (see Section 2), hence the relation presented is without any kind of simplifying assumptions or theory.

2. Materials and Methods

Only published volumes and transformation pressures of polymorphs of elemental solids are used in this paper. Recently published data and work conducted under hydrostatic or nearly hydrostatic conditions are given preference. This includes data obtained from samples embedded in He or Ne as pressure media or data from crystals grown from melt at a high pressure. Data from compression experiments without pressure media are only included if the average pressure has been below 10 GPa. However, the induced transformations in Ir, Pb, and Th under non-hydrostatic stresses are included for reference. Similarly, compression data of some lanthanides are included due to their principal interest, although most of these data were not and possibly could not be acquired with the use of hydrostatic media. The plotted volumes V_{tr} and pressures P_{tr} at the polymorphic transitions are generally those of the first observation of a high-pressure polymorph upon compression at 300 K. In a few cases in which the transformations exhibit hystereses larger than 5 GPa, the arithmetic middle pressure has been taken as P_{tr} . In all other cases, the hysteresis has been added to the uncertainty. For some elemental high-pressure polymorphs, the structures have been redetermined in more recent studies but without reassessment of the 300 K isotherms. In such cases, the atomic volume of the correct structure has been used to reassess the atomic volume at the pressure of the transformation that has been reported in the earlier studies. The complete data are given in Table 1 at the end of the paper.

3. Results

For all of the examined 113 elemental solids, the volume at the phase transition $V_{tr} = V(P_{tr})$ (P_{tr} = transition pressure) divided by the principal quantum number L exhibits a universal linear pressure dependence of $-0.0208(3) \cdot P_{tr}$ (P_{tr} in GPa; atomic volume in $\text{\AA}^3$; the adjusted R^2 of the linear fit was 0.9985; see Supplemental Figure S1). This is shown in Figure 1. Furthermore, within the uncertainties, the constant term of this equation provides a volume that for all of the examined elemental solids is nearly equal to $n \cdot 10 \cdot r_B^3$, where n is an integer number between 2 and 6, and r_B is the Bohr radius in $\text{\AA}$. This is shown in Figure 2. In other words:

$$V_{tr}/L + 0.0208(3) \cdot P_{tr} = N_i \approx n \cdot 10 \cdot r_B^3 \quad (1)$$

The actual values of the five distinct volumes for N_i are 2.88(8), 4.58(4), 5.51(5), 7.14(5), 8.53(6) $\text{\AA}^3/\text{at}$, which is 19.5(4), 30.9(4), 37.2(8), 48.2(1), and 57.6(5) times the cube of

the Bohr radius r_B^3 in $\text{\AA}^3$, respectively. Therefore, Figures 1 and 2 show that the volumes of elemental solids at their polymorphic transitions assume corresponding states. It is important to recall that this universal relation of corresponding states only holds for the pressures and volumes at the phase transitions: Within the stability field of each polymorph, the pressure–volume relation is controlled by its specific compressibility that itself depends on pressure and temperature and that, in general, is far from being linear, universal, and discrete. Equation (1) is therefore not an equation of state.

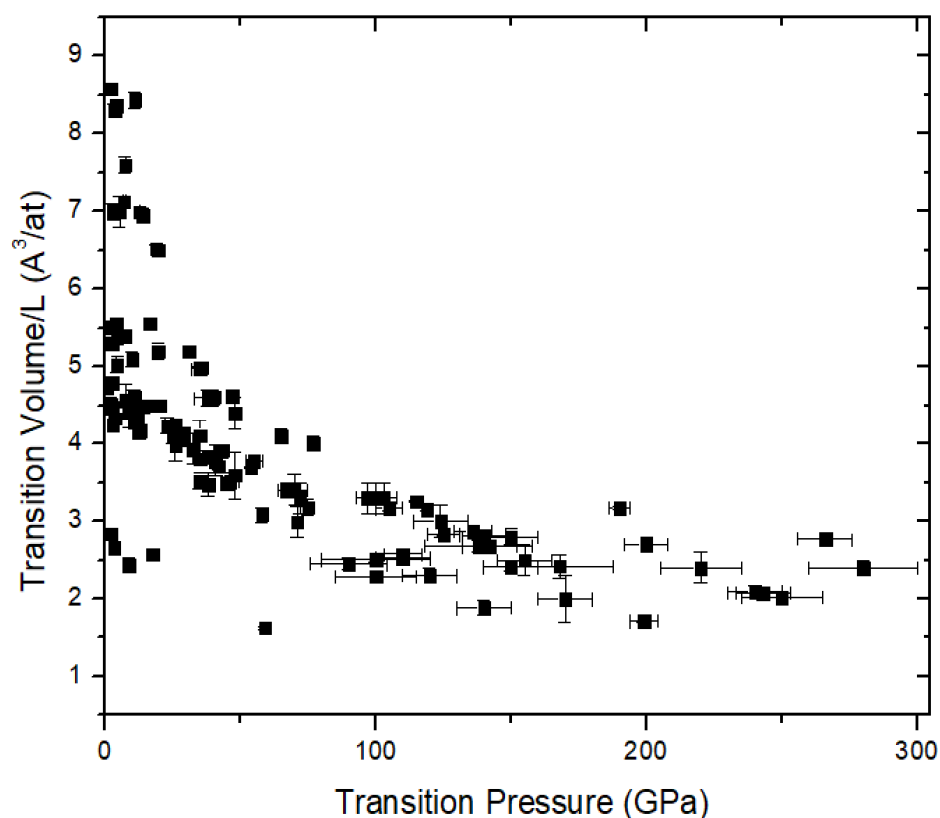


Figure 1. Volume of elemental polymorphs in $\text{\AA}^3/\text{at}$ at 300 K divided by the principal quantum number L as a function of pressure in GPa. The volumes are those that are observed at the pressure of their formation through direct phase transformation from lower pressure polymorphs; the plotted pressures are the pressures of the transformation. The linear relation between these volumes and pressures and the existence of five distinct sets of pressure–volume relations is clearly visible.

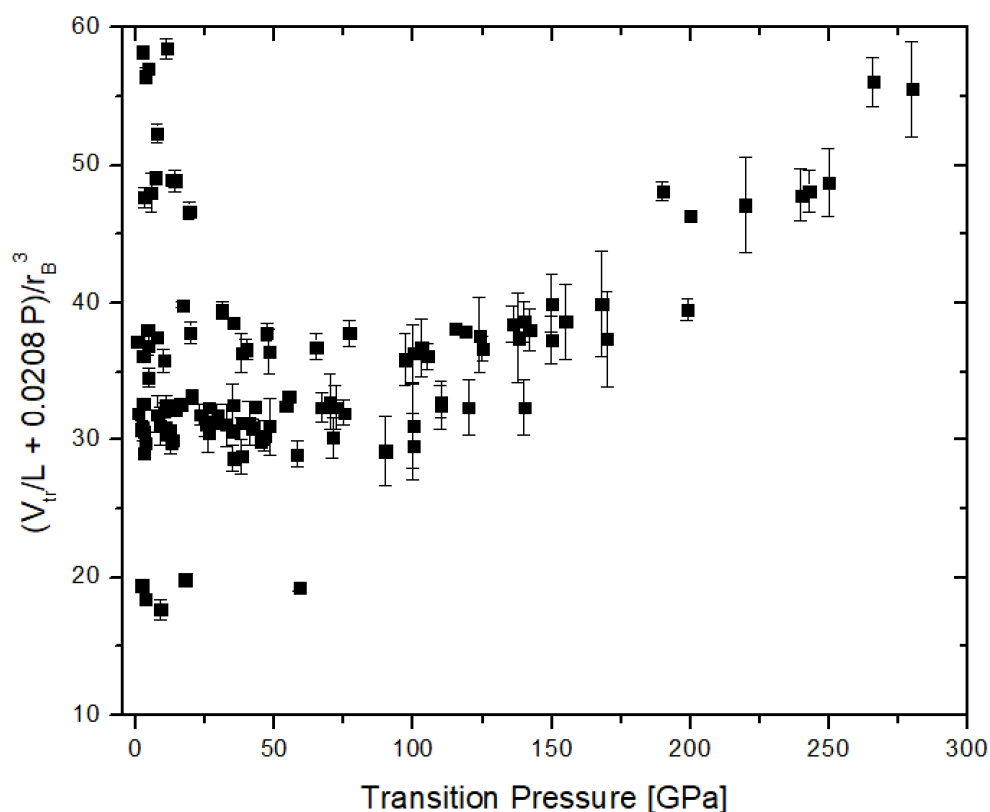


Figure 2. This figure shows the data from Figure 1 with the volume corrected for the linear pressure dependence from Equation (1). The remaining constant term of each set is approximately equal to integer multiples of 2, 3, 4, 5, and 6 of 10 times the cube of the Bohr radius.

4. Discussion

The ‘collective quantization’ shown in Figure 1 may, upon first glance, be assigned to the factorization by the principal quantum number L . However, the series of corresponding states N_i does not follow the rows of the periodic table. Rather, many elements with subsequent polymorphic transitions change between states N_i . For instance, the high-pressure polymorphs fcc, cl16, and oc120 of Na reside in $N_4 = 5.51(5) \text{ \AA}^3/\text{at} \approx 40 \cdot r_B^3$ along with K-IV, Rb-IV, -V, Cs-IV, and Ca-III to -IV; whereas the h4p-type Na resides in $N_5 = 7.14(5) \text{ \AA}^3/\text{at} \approx 50 \cdot r_B^3$ along with Rb-II and -III. In some cases, for instance Si, all polymorphs remain within one set N_i ; however, for most elements at least some polymorphs fall into different sets. For instance, Cs-II and Cs-III are in N_6 but Cs-IV and -V belong to N_4 and Cs-VI to N_3 . A sequence of stepwise transitions to sets with a lower state N_i along with an increasing transformation pressure is found for many elements: the high-pressure polymorphs of Rb fall into the sets N_5 (Rb-II and -III), N_4 (Rb-IV, -V, -VIII), and N_3 (Rb-VI, -VII); Li into N_6 (Li-II), N_4 (Li-III, -IV, -VI), and N_3 (Li-V); and the various polymorphs of the alkaline earths Ca, Sr, and Ba are found in the sets N_3 to N_6 (Figures 1 and 2), where the higher pressure polymorphs belong to sets with a smaller N_i . Lower sets N_i have markedly smaller volumes at a given pressure than the corresponding higher sets N_{i+1} (Figure 1), and the polymorphic transitions that involve changes $N_i \rightarrow N_{i-1}$ are accompanied by marked volume contractions $(V_{tr}^{LP} - V_{tr}^{HP})/V_0$ where V_{tr}^{LP} and V_{tr}^{HP} are the atomic volumes of the low- and the high-pressure polymorphs, respectively, at the pressure of the phase transitions; and V_0 is the volume of the ambient pressure phase at standard conditions. A smaller number of elements also exhibit transitions into higher sets N_{i+1} at higher pressure. In particular, this holds for the polymorphic transitions to Sc-V [19], bcc-Ti [20,21], hp4-Na [3], and Rb-VIII (Figure 1), but also for the elastic anomaly of Os around 290 GPa [22], which is taken here for an isostructural transition. There is a simple reason for these ‘upward transitions’ $N_i \rightarrow N_{i+1}$: at the given transformation

pressure, the corresponding volumes of the next lower set N_{i-1} would be negative. It is to be expected that for all elements at sufficiently high pressures, the cascade of successive, markedly contractive transitions $N_i \rightarrow N_{i-1}$ is followed by transitions $N_i \rightarrow N_{i+1}$ to polymorphs in higher sets with a less pronounced contraction once $N_i \rightarrow N_{i-1}$ reaches the limit of negative volumes. Transitions to higher sets come with smaller contractions $(V^{LP_{tr}} - V^{HP_{tr}})/V_0$ than those into lower sets.

4.1. Theoretical Explanation

‘Upward’ and ‘downward’ transitions between different states N_i can be correlated to changes in the valence electron configuration wherever such information has been obtained experimentally [10,15,17] or through calculation [14]. The fact that the volumes N_i are nearly equal to integer multiples of r_B^3 has a straightforward theoretical basis: It implies that the volumes that occur at polymorphic transitions of non-molecular elements are controlled by the number of valence electrons rather than by their interactions. Besides the limited experimental accuracy the electron exchange- and Coulomb interactions account for part of the deviations from Equation (1) and from integer N_i values; thus, the interaction terms are of the second order with respect to the volumes. In other words, Equation (1) is the equivalent to an ideal equation of state for elemental solids, only that it does not describe continuous pressure–volume changes but discrete changes that occur at the phase transitions. This finding can be further quantified by considering the effect of the different valence orbital states as based on the experimentally determined pressure-induced hybridization of d-states of the alkaline metals Rb, Cs, and K (the latter element exhibits no measurable d-state occupancy up to 40 GPa) [10]. Figure 1 may be re-parametrized in terms of pressure-induced changes in the d-state occupancy at the valence level. In particular, the d-state occupancy of the elemental polymorphs can be correlated with the five sets N_2 to N_6 shown in Figure 1 through their 0 GPa intercepts. This is shown in Figure 3a. For instance, the d-state occupancy of Cs increases for phase Cs-II to -VI from ~ 0.0 to ~ 0.6 , and for Rb from ~ 0.2 to 0.4 electrons between Rb-II and -VI [10]. The high-P polymorphs of Li and Na have little or no occupied d-states in this diagram and according to earlier computational work [2,4,14]. The correlation between $\Delta n(d)$ and N_i is linear, at least for the polymorphs K-II, Rb-II,-IV, Cs-II, and Cs-III (black squares in Figure 3a): Within uncertainty, the slopes are equal for all sets. In terms of multiples of the electron elemental charge, this linear correlation between d-state occupancy, V_{tr} , and L is:

$$N_i = N_i^0 - 6.11(14) \cdot 10^{-11} \cdot n(d) \cdot e \quad (2)$$

where $N_i^0 = N_i$ at $n(d) = 0$, the constant factor is given in $m^3/(C \cdot at)$, and e is the electron elemental charge. However, the correlation between N_i and $n(d)$ is not continuous: Equations (1) and (2) can be combined to give:

$$\frac{V_{tr}}{L} = -2.08(3) \cdot 10^{-32} \cdot P - 6.11(14) \cdot 10^{-11} \cdot \sum_{i=0}^{m-1} i \cdot n(d) \cdot e + N_m \quad (3)$$

where N_m is the value assumed at $n(d) = 0$, P is in GPa, V is in m^3/at , and e is the electron elemental charge. The effect of the $d \rightarrow p$ transfer on shifts into sets with a higher N_i has been discussed above. Overall, there are fewer data on pressure-induced changes in the valence p-state occupancy (summarized in Supplementary Figure S2, see refs. [10,16,17]), but within uncertainties, the relation between V_{tr}/L and $n(p)$ has the same slope as in (3) but with an opposed sign (consistent with the transfer into higher sets N_i , whereas changes in the d-state occupancy generally result in changes to a lower N_i). Implementing $n(p)$ into (3), dividing through r_B^3 , and using the approximation $N_i r_B^3 \sim 10 \cdot m$ gives:

$$\begin{aligned} \frac{V_{tr}}{L \cdot r_B^3} = & -0.1405(20) \cdot P + 10 \cdot m - 66.1(15) \cdot \sum_{i=0}^{m-1} i \cdot 0.15(4) + 66(2) \cdot \\ & \sum_{j=p}^m j \cdot 0.09(21) = -0.1405(20) \cdot P + 10 \cdot m - 10.0(6) \cdot \sum_{i=0}^{m-1} i + 6(1) \cdot \sum_{j=p}^m j \end{aligned} \quad (4)$$

where the pressure is given in GPa, the volumes N_i are taken as integer multiples $10 \cdot i \in \mathbb{N}$ (the set of natural numbers) of r_B^3 per atom, $p \leq n(d)$, and the increments $\Delta n(d)$ are taken as 0.151; hence: $66.1 \cdot 0.151 = 10$ d-electrons and accordingly $66(2) \cdot 0.09(1) = 6(1)$ for the p-electrons. Thus, within uncertainties, the factors of the sum terms equal the number of p- and d-states and an additional term for f-electrons with factor -14 may be added. The actual values N_i may be substituted for $10m$.

The volumes of elemental atomic solids at ambient conditions are not bound to Equation (1) because they do not represent volumes at phase transitions. However, the correlation $V = N_i \cdot L$ is within $\pm 10\%$ of the observed volumes at ambient conditions for 28 and within 20% for 38 out of the 42 non-molecular solid elements (Figure 3b). Considering that the atomic volumes of elemental metals are equal to the cube of the Wigner–Seitz radii, their approximate relation to $N_i \cdot L$ is equivalent to a relation recently found for the ionic radii [23]. However, in the present case of elemental solids the relation does not rely on any approximate model of bonding.

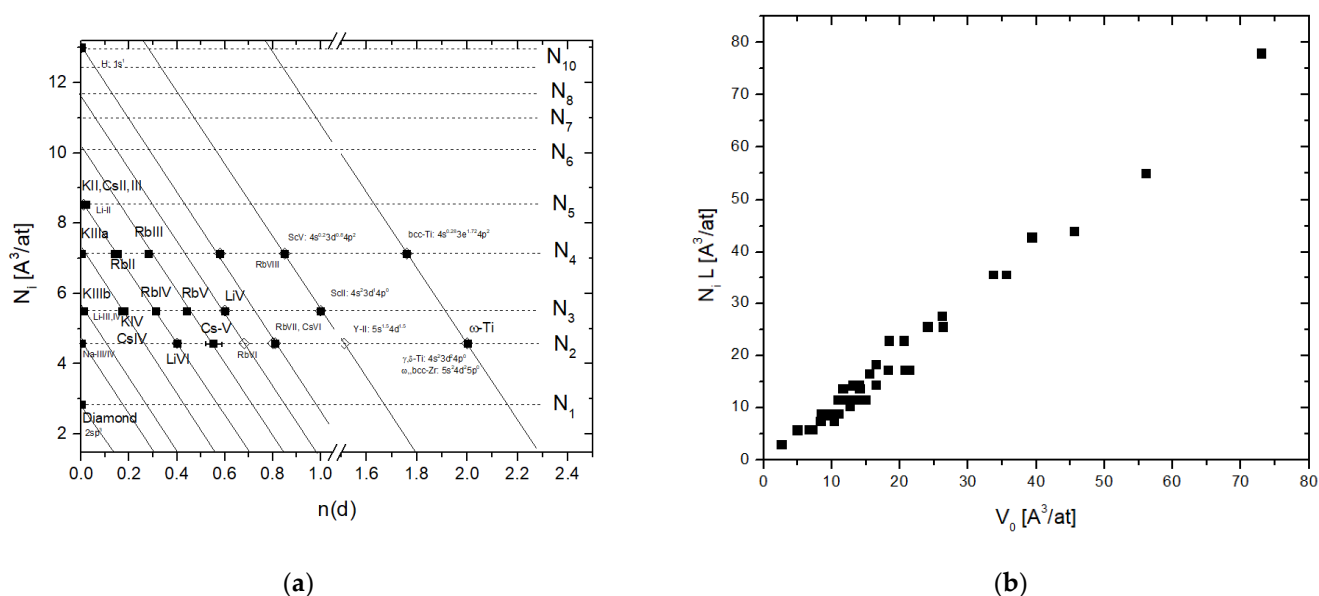


Figure 3. (a) Correlation between the normalized transformation volumes V_{tr} from Equation (1) and d-state occupancy. The volumes N_i from Equation (1) were used to define the V_{tr}/L sets of the alkaline metals [10] and of a selection of other elements as a function of d-state occupancy. Filled symbols: measured d-state occupancies; hollow symbols: estimated occupancies based on Equations (1) and (2). Solid lines: fitted correlation for KII, Rb-II and -IV, and Cs-II and -III. The same slope has then been applied to all data. Dashed lines give the values N_i and are guides for the eye. (b) Comparison of the atomic volumes of elemental non-molecular solids at ambient pressure and temperature with the calculated volumes $N_i \cdot L$. The correlation is linear with an adjusted $R^2 = 0.9928$ and a slope of 1.015(13), which reflects a general slight contraction of the elemental volumes relative to $N_i \cdot L$.

4.2. Molecular Gap and Forbidden Zone

There are two regions in Figure 1 that contain no values V_{tr}/L : (1) There is no observed V_{tr}/L equal to or smaller than $1.48 \text{ Å}^3/\text{at}$ at any pressure; $1.48 \text{ Å}^3/\text{at}$ equals 10 times the cube of the Bohr radius (per atom; this limiting value $10 r_B^3/\text{at}$ is defined here as N_1). Upon compression, elemental solids such as Au and Re reach this limit (see Figure 4) but without further known transformations [24,25]. There is no known elemental solid that undergoes a polymorphic transition between 0 and 300 GPa where the volume at the transition is equal to or smaller than $10 r_B^3$.

(2) The area between $P \sim 20$ GPa for $V_{tr}/L \sim 8 \text{ Å}^3/\text{at}$ and 200–300 GPa for $\sim 3 \text{ Å}^3/\text{at}$ contains no observations. This empty range has a nearly parabolic shape in Figure 1. It corresponds to volumes and pressures of molecular elemental solids (Figure 4).

For instance, the volumes V_{tr} of the molecular polymorphs β , γ , δ , and ε of oxygen as well as the metallic molecular phase ζ -O₂ [26], are not compliant with Equation (1). The isotherm of ε -O₂ intersects $V_{tr}/L + N_6$ at 24 GPa without any observed transition ([26]; Figure 4). The isotherm of H₂ intersects the $V_{tr}/L + N_6$ at 26 GPa and $V_{tr}/L + N_5$ at 54 and 135 GPa and again $V_{tr}/L + N_6$ around 250 GPa, which is at the pressure of the transition to H₂-IV ([27], Figure 4), but neither of these intersections corresponds to the transition to an atomic metal [27]. However, the transition from molecular N₂ to the polymeric, semi-conducting cg-phase of nitrogen [28] falls onto the N_4 trajectory (Figure 1, Table 1). Cl₂ intersects $V_{tr}/L + N_6$ at 260 GPa, where it has been reported to become an atomic metal with an In-type structure [29] and thus follows the behavior predicted by Equation (1). Hence, the gap between 50–180 GPa for $V_{tr}/L + N_5$ and N_6 indirectly reflects the strength of the molecular bonds of these materials under compression and should be called a ‘molecular gap’. The intersection of the 300 K isotherm of the chlorine phase III (mc8; [29]) with $V_{tr}/L + N_5$ at 180 GPa may be coincidental because this phase is probably still molecular and the V_{tr} of the intermediate phase IV (i-oF3, [29]) falls in between $V_{tr}/L + N_5$ and $V_{tr}/L + N_6$. The isotherms of N, S, P, Br, and I all intersect the trajectories $V_{tr}/L + N_i$ at the pressures of the observed transitions to non-molecular phases [8,27,30,31] and fall mostly below the ‘molecular gap’ because their transitions to non-molecular solids occur at lower pressure.

Interestingly, the extrapolated pressure of the direct gap closure of H₂ that has been assessed through inelastic X-ray spectroscopy of H₂ [32] matches the intersection of the extrapolated isotherm of H₂ [27] and a trajectory $V_{tr}/L + N_9$ (of 13.0 to 13.3 Å³/at = 90 r_B³ by using the intersection of the trajectory N_9 in Figures 3a and 4) at around 0.5 TPa, which suggests that the transition from molecular to atomic hydrogen occurs at that pressure at 300 K.

Finally, it is noticed that Equation (1) could be used to assess the atomic volumes of polymorphs with extant structure analyses such as Ca-VII, Sc-III and -IV, Ga-IV, and metastable Ir-II to 8.0(5), 12.08(8), 10.3(11), 9.2(4), and 9.78(12) Å³/at at 170, 26, 35, 120, and 59 GPa, respectively (Table 1). The volume of hydrogen at the predicted transition at 490–530 GPa is estimated to 2.3(2) Å³/at. This and above values (marked with an asterisk) and the entire list of volumes V_{tr} and pressures P_{tr} are given in Table 1.

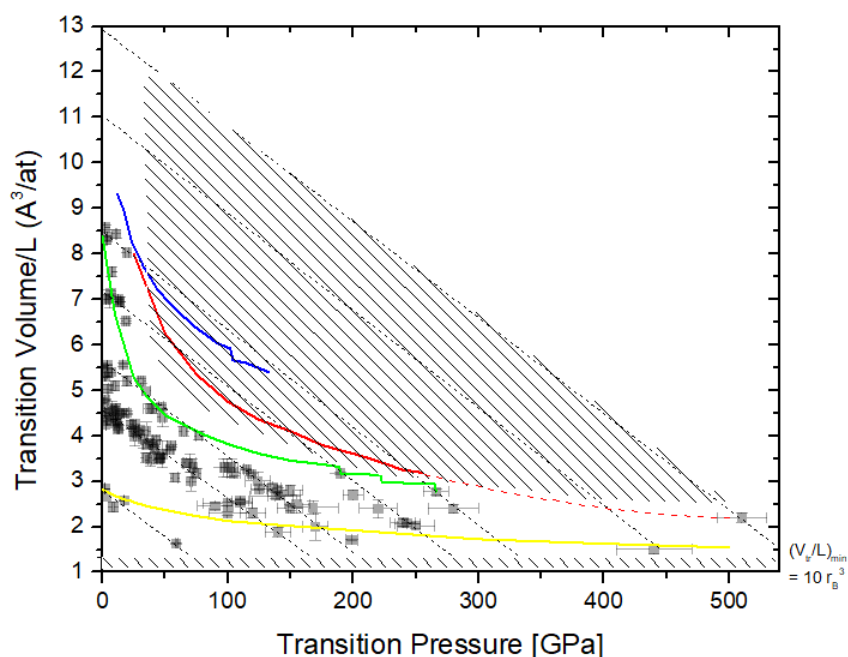


Figure 4. Data from Figure 1 with the equations of state for gold [24], chlorine [29], hydrogen [27], and oxygen [26] shown as yellow, green, red, and blue lines, respectively. Dashed lines indicate the

distinct sets that defined V_{tr}/L versus P as defined by Equation (1). The additional data points for hydrogen [32] and the second elastic anomaly of osmium [22] have been added. Hachured areas mark the molecular gap and the forbidden zone below $V_{tr}/L = 10 \cdot r_B^3$.

Table 1. Columns from left to right: Name of polymorph, pressure of phase transition at 300 K, volume of phase transition divided by the principal quantum number L , volume of phase transition at 300 K, and reference(s).

Phase	P_{tr} [GPa]	V_{tr}/L [$\text{\AA}^3/\text{at}$]	V_{tr} [$\text{\AA}^3/\text{at}$]	Reference(s)
Li-fcc	7.5(1)	7.6(1)	15.2(2)	[33–35]
Li-hR1	39.7(1.0)	4.6(1)	9.2(2)	[34,36]
Li-c16	48(2)	4.4	8.8(4)	[34,35]
Li-oC40	67(2)	3.4(1)	6.8(2)	[35]
Li-oC24	97(3)	3.3(2)	6.6(4)	[35]
Na-fcc	67(1)	4.10(5)	12.3(30)	[36]
Na-cl16	115(2)	3.26(1)	9.77(3)	[3]
Na-oP8	119(2)	3.15(1)	9.45(6)	[3]
Na-h4p	200(8)	2.7(1)	8.1(3)	[2]
K-fcc	11.0(5)	8.4(1)	33.74(4)	[37]
K-III	22(43)	5.64(5)	22.57(3)	[5]
K-IIIb	31(2)	5.2(2)	20.8(4)	[38]
K-IV (oP8)	54(2)	3.69(1)	14.78(4)	[39]
Rb-II	7.0(5)	7.13(6)	35.65(30)	[37]
Rb-III	14.3(2)	6.95(1)	34.77(5)	[5]
Rb-IV	16.7(1)	5.56(1)	27.8(15)	[5,40]
Rb-V	19.6(5)	5.2(1)	26.0(5)	[41]
Rb-VI	48.1(5)	3.6(3)	18.0(1.5)	[41]
Rb-VII	>70	3.4(1)	17(1)	[41]
Rb-VIII	220(15)	2.4(1)	12(1)	[41]
Cs-II	2.37(1)	8.58(1)	51.47(6)	[42]
Cs-III	4.20(2)	8.37(1)	50.22(6)	[5]
Cs-IV	4.30(2)	5.83(1)	32.28(6)	[43]
Cs-V	10(1)	5.1(2)	30.6(1.2)	[44]
Cs-VI	72(2)	2.6(2)	15.6(1.2)	[44,45]
Mg-bcc	47(2)	4.63(3)	13.9(2)	[46]
Ca-II	19.5(15)	8.02(2)	32.08(8)	[47,48]
Ca-III	35(3)	5.0(1)	19.9(4)	[49,50]
Ca-IV	124(10)	3.0(2)	12.0(8)	[50]
Ca-V/VI	155(15)	2.5(2)	10.0(8)	[50]
Ca-VII	170 *	2.0 *	8	this work
Sr-II	3.5(2)	8.30(8)	41.5(4)	[48]
Sr-III	26.0(6)	3.98(19)	19.9(9)	[5,49]
Sr-IV	35(1)	4.1(2)	20.5(10)	[5,49,51]
Sr-V	46(3)	3.5(1)	17.5(5)	[5,49,51]
Ba-II	5.5(1)	7.14(4)	42.84(24)	[48]
Ba-IV	12.6(1)	7.0(1)	42.0(6)	[52]
Ba-V	45(2)	3.5(1)	21.0(6)	[53]
Sc-II	20(1)	4.5(1)	18.0(4)	[54]
Sc-III	104(5)	3.02(2) *	12.08(8)	[54]
Sc-IV	140(10)	2.58(27) *	10.3(1.1)	[54]
Sc-V	240(10)	2.09(7)	3.36(28)	[19]

ω -Zr	12.5(10)	4.15(5)	20.75(25)	[55]
Zr-bcc	35(2)	3.5(10)	17.5(50)	[55,56]
Zr-bccII	58–60	3.08(5)	15.4(3)	[55]
ω -Ti	0(5)	4.375(15)	17.5(6)	[20]
γ -Ti	110(10)	2.55(5)	10.2(2)	[20,21]
δ -Ti	150(10)	2.41(5)	9.64(20)	[20,21]
Ti-bcc	243(10)	2.07(3)	8.28(12)	[21]
V rhombohedral	250(20)	2.02(5)	8.08(20)	[57]
Fe-hcp	17.6(1)	2.57(1)	10.28(4)	[58]
Os-hcpII	140(10)	1.88(10)	11.28(60)	[22]
Os-hcpIII	440(20)	1.5(1)	9.0(6)	[22]
Sn-bct	10.8(1)	4.60(1)	23.00(5)	[59]
Sn-bco	32.5(10)	3.93(1)	19.65(5)	[59]
Sn-bcc	40.8(15)	3.78(1)	18.9(5)	[59]
Pb-hcp	14(1)	6.95(10)	41.7(6)	[60]
Pb-bcc	142(10)	2.68(2)	16.08(12)	[5,61]
Ga-II	2.0(2)	4.52(8)	18.08(32)	[5]
Ga-III	2.8(1)	4.24(5)	16.96(24)	[62]
Ga-V	10.5(2)	4.358(5)	17.432(24)	[5]
Ga-IV	120(10)	2.3(1)*	9.2(4)	[62,63]
Tl-fcc	3.5(5)	4.3(2)	25.8(1.2)	[64]
Ge-II	10.6	4.28(8)	17.12(32)	[65]
Ge-hp	75(3)	3.18(10)	17.72(40)	[18,66]
Ge-Cmca	140(120–160) *	2.68(7)	10.72(28)	[66]
Ge-hcp	>170	2.4(1)	9.6(4)	[18]
Diamond	2.0(1)	2.837(1)	5.674(2)	[67]
Si-II	12(1)	4.3(1)	12.9(3)	[68]
Si-V	16(2)	4.43(2)	13.29(6)	[69]
Si-VI	38(2)	3.83(1)	11.49(3)	[69]
Si-VII	42(2)	3.71(2)	11.13(6)	[69]
N, cg-phase	110(7)	2.58(3)	5.16(6)	[28]
P-A7	4.5(1)	5.03(5)	15.09(15)	[70]
P-sc	10(1)	4.54(2)	13.62(6)	[70]
P-ph	140(8)	2.8(1)	8.4(3)	[70]
P-bcc	280(20)	2.4(1)	7.2(3)	[70]
As-sc	25(1)	4.1(1)	16.4(4)	[18]
As-III	46(3)	3.5(1)	14.0(4)	[18]
As-IV	125(10)	2.84(2)	11.36(8)	[18]
Sb-II	8.6(2)	4.42(20)	22.1(1)	[71]
Sb-V	28(3)	4.05(6)	20.25(30)	[71]
Bi-II	2.55(1)	5.30(2)	31.80(12)	[8]
Bi-III	2.70(1)	4.48(9)	26.88(54)	[8]
Bi-V	7.7(1)	4.57(10)	27.42(60)	[8]
S-II	3.0(2)	7.0(1)	21.0(3)	[72]
S-III	38(2)	4.6(1)	13.8(3)	[72]
S-IV	103(6)	3.3(1)	9.9(3)	[72]
S-V	150(10)	2.8(1)	8.4(3)	[72]
Se-VII	14(1)	4.5(1)	18.0(4)	[72]
Se-III	26(1)	4.25(7)	17.0(3)	[72]
Se-IV	35(1)	3.8(1)	15.2(4)	[72]
Se-V	77(2)	4.0(1)	16.0(4)	[72]

Se-VI	136(7)	2.87(7)	11.48(28)	[18]
Te-II	4.0(1)	5.56(1)	27.80(5)	[5,73]
Te-III	7.4(1)	5.40(1)	27.00(5)	[5,73]
Te-V	29.2(7)	4.25(2)	21.25(10)	[5,73]
Te-VI	102(5)	3.3(2)	16.5(1)	[74]
I-bct	43(3)	3.9(1)	19.6(1)	[75,76]
I-fcc	55(4)	3.78(2)	18.9(1)	[75,76]
Th-bct	100(30)	2.51(1)	17.57(7)	[77]
γ -Ce	0.5(1)	4.70(1)	28.20(6)	[78]
PrtI2	180(10)		23.90(14)	[79]
Sm-dhcp	0.4(1)	5.51(1)	33.06(6)	[80]
Sm-hR9	13(1)	4.2(1)	25.2(6)	[81]
Sm-oF8	90(10)	2.45(8)	14.70(48)	[81]
Nd-oF8	100(10)	2.30(4)	13.80(24)	[82]
Eu_IV	31.5(20)	3.32(2)	19.9(1)	[83]
Gd-dhcp	9(3)	4.3(1)	26	[84]
Gd-fcc	26(2)	3.7(1)	22.2(6)	[85]
Gd-dfcc	33(2)	3.43(10)	20.5(6)	[85]
Gd-VIII	60.5(3)	2.86(10)	17.15(61)	[85]
Dy-hR9	2.5(2)	4.79(1)	28.74(6)	[86]
Cl-IV	266(10)	2.77(6)	8.31(18)	[29]
Ir-II	59	1.63(2)*	9.78(12)	[5,87]

* Parameters calculated using Equation (1).

5. Conclusions

The volume and pressure of phase transformations of non-molecular elemental solids obeyed a universal relation of discrete corresponding states. These states, as defined in Equation (1), were close to integer multiples of the cube of the Bohr radius. Therefore, the volumes and pressures of polymorphic transitions of these solids were controlled by the number of valence electrons as specified in Equation (4). This finding was in accordance with the commonly used effective single-electron-based computational approaches for modeling elemental solids within the range of 0 to 0.5 TPa. However, Equation (1) can be used for constraining elemental metal volumes for first-principal- and empirical-potential-based calculations, thus removing the weakest point in these approaches (the proper assessment of volumes). The volumes of polymorphs at the phase transitions exhibited a general linear dependence on the pressure of the transition (see Equation (1)). Though not an equation of state, this relation establishes a very simple principle that rules over a vast range of simple and complex solid structures and a range of pressure of 0.5 TPa.

Supplementary Materials: The following supporting information can be downloaded at: <https://www.mdpi.com/article/10.3390/cryst12121698/s1>, Figure S1: Volume of elemental polymorphs in $\text{\AA}^3/\text{at}$ divided by the principal quantum number L as function of pressure in GPa and mapped onto the set N_4 , Figure S2: Correlation between the normalized transformation volumes from Equation (1) and p -state occupancy.

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