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Vertical Relationships within Platform Marketplaces

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Abstract: In two-sided markets a platform allows consumers and sellers to interact by creating sub-markets within the platform marketplace. For example, Amazon has sub-markets for all of the different product categories available on its site, and smartphones have sub-markets for different types of applications (gaming apps, weather apps, map apps, ridesharing apps, etc.). The network benefits between consumers and sellers depend on the mode of competition within the sub-markets: more competition between sellers lowers product prices, increases the surplus consumers receive from a sub-market, and makes platform membership more desirable for consumers. However, more competition also lowers profits for a seller which makes platform membership less desirable for a seller and reduces seller entry and the number of sub-markets available on the platform marketplace. This dynamic between seller competition within a sub-market and agents' network benefits leads to platform pricing strategies, participation decisions by consumers and sellers, and welfare results that depend on the mode of competition. Thus, the sub-market structure is important when investigating platform marketplaces.

1. Introduction

Over the last 10 years, smartphones have become ubiquitous. In 2013, the smartphone market reached 1 billion units sold to consumers worldwide.[1] In addition to being an important consumer good in its own right, the smartphone also provides consumers with the opportunity to purchase applications (apps) that can be used on the smartphone. For example, consumers can purchase gaming apps, weather apps, map apps, ridesharing apps, etc. and these apps make the platform more valuable to consumers. Apps are often provided by third party developers that do not produce the smartphone. Thus, the smartphone acts as a platform that creates a marketplace where consumers and app providers interact within sub-markets, the individual markets for different app types.

This structure exists in many economic markets: video game consoles provide consumers with sub-markets for many different genres of games that are developed by competing game developers; eReaders connect readers with book publishers creating sub-markets for different genres of books; and online marketplaces like Amazon connect consumers with product sub-markets ranging across many retail items. In these platform marketplaces, consumers care about the variety of content (the availability of many sub-markets) and the prices for content within these sub-markets. If there are many sub-markets on the platform and there is considerable competition within each sub-market so that the price of content is low, then the platform is very desirable for consumers. However,

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[1]   For more statistics on smartphones, see [1].

more competition within sub-markets also implies that joining the platform for a seller will be less profitable. Lower seller profits implies less entry by sellers onto the platform which results in fewer sub-markets on the platform marketplace. Thus, the competition structure affects network effects for both consumers and sellers that join the platform.

In this paper, I show how competition among sellers affects the platform's pricing strategies, the resulting levels of content and consumer participation on the platform, and the welfare that is generated on the platform. Participation by consumers and the availability of products on the platform are endogenously determined through the membership prices that the platform charges consumers and sellers. For a given set of platform prices, more competition reduces deadweight loss within each sub-market which increases total surplus. At the same time, more competition lowers profits which implies fewer sub-markets on the platform marketplace which decreases total surplus. Thus, there exists a tradeoff from changes in competition within a sub-market and the platform's pricing strategies, the amount of participation by consumers and sellers, and the welfare generated by the platform depend on the mode of competition that exists within each sub-market.

The tradeoff between deadweight loss and the number of sub-markets available to consumers leads to interesting pricing strategies by the platform. I find that when sellers have more market power within a sub-market, the platform lowers its prices to consumers and sellers so that there is more participation on each side of the platform. As participation is what generates surplus on the platform, this pricing strategy by the platform leads to several interesting results regarding welfare.

I find that less competition within each sub-market causes an increase in the number of sub-markets on the platform marketplace. The added surplus from additional sub-markets is enough to overcome the additional deadweight loss from increased seller market power. In other words, the total deadweight loss across all of the sub-markets from less competition at the seller level is overcome by the additional surplus generated from more sub-markets on the platform marketplace resulting in greater welfare. However, if the gains to consumer surplus from an increase in competition within a sub-market are sufficiently large then this argument fails and welfare improves. These findings show the importance of the sub-market structure when investigating pricing and welfare on a two-sided platform.

Many of the original papers on platforms and two-sided markets [2–5], as well as subsequent work, assume that agents have homogeneous network effects. Furthermore, the magnitude of the network effect on each side of the market is independent of the network effect on the other side of the market. That is, the platform literature has abstracted from the sub-market structure that generates the network effects between consumers and sellers.

Solving for equilibria when agents have homogeneous network effects on each side of the market requires assumptions on participation decisions of agents. Another concern is that homogeneous network effects do not coincide with the empirical evidence of [6,7], who find that the network benefits that consumers receive from video games and apps vary across consumers. Thus, allowing for heterogeneity is important in modelling platforms. [8] develop a model with heterogeneous consumers and find equilibria that correspond to many platform markets, including smartphones and video game consoles. However, they do not consider the pricing relationship that exists between consumers and sellers.[2] This paper allows for heterogeneity and aims to illustrate the importance of the sub-market structure that exists within a platform in analyzing network effects, platform pricing strategies, and platform participation.

The relationship that exists between a platform and its sellers relates to the traditional models on vertical relationships which dates back to [10]. He shows that when a wholesaler and a retailer each have market power, double marginalization occurs, where each firm along the supply chain adds a market-power markup and final prices exceed the simple monopoly price. He finds that a

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[2]   [9] also show how heterogeneity on the consumer side plays a critical role in the platform profit maximization problem.

vertical merger between wholesalers and retailers improve efficiency by lowering the final price while increasing profits and consumer surplus.

The research on platforms relating to vertical relationships is very limited. One paper considering competition within one side of the market is [11] who consider a two-sided market model where one side of the market is has negative direct network externalities. That is, on the seller side of the market; more sellers makes the sellers worse off. In this paper, I consider a more general analysis on the seller side of the market where such negative direct network externalities are allowed. Furthermore, by analyzing this context more generally I am able to make interesting comparative statics regarding how the seller structure affects the platform. A similar model that considers direct network effects within one side of the market is [12]. However, they focus on the market two-sided market for health plans and make industry specific assumptions. This makes certain comparative statics that are of interest more difficult to consider.

The vertical relationship in a two-sided market for credit cards is analyzed in a different model by [13]. The focus of their work is on the fee structures for consumers and merchants used by credit card companies. They find that greater merchant competition leads to lower prices for consumers and increased welfare when there is a monopoly platform. This is consistent with the usual double marginalization result. However, Shy and Wang assume homogeneous agents on each side of the market and a network benefit structure that does not coincide with the empirical findings of [6,7]. Furthermore, participation on the consumer side of the market is exogenously given: consumers do not choose whether or not to join the platform. This paper is not limited by these assumptions.

The remainder of the paper is organized as follows. In Section 2, the general model of consumers, sellers within sub-markets, and the platform are introduced. In Section 3, the equilibrium for the entire two-sided platform and the effects of changes in competition within sub-markets on equilibrium participation and welfare are determined. In Section 4 a model of vertical integration between the platform and the seller side of the market is analyzed. Section 5 concludes, followed by an appendix that contains the proofs of all the formal findings.

2. The Model

There are three types of players: a platform, consumers who join the platform on one side of the market, and sellers who make up the other side. Consumers benefit by purchasing products that are available on the platform. Sellers must join the platform to make their products available to consumers. The platform earns profits by charging consumers and sellers to join the platform.[3] The sub-market structure that exists between the consumer and seller sides of the platform is developed first in the following subsection.

2.1. Consumers and Sellers

On Side 1 there exists a mass of consumers, normalized to 1, with individual consumers indexed by $\tau \in [0,1]$ and consumer types are distributed uniformly. The mass consumers that decide to join the platform is denoted by $N_1 \in [0,1]$. Once a consumer joins the platform they can engage in transactions on the platform marketplace. Thus, a consumer benefits more from platform membership when the platform offers a greater variety of sub-markets. Furthermore, the competitive structure and the number of sellers within a sub-market also affects the amount of surplus consumers gain from a sub-market. Let $cs(n, \mathcal{C})$ be the consumer surplus that a consumer receives from a sub-market that a consumer is interested in being available on the platform marketplace, where n is the number of

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3   I will use the terminology "join the platform" in two ways. Depending on the type of platform market, consumers either join the platform (e.g., Amazon) or consumers purchase the platform (e.g., video game consoles, smartphones, digital devices, etc.).

sellers in the sub-market whose competition structure is defined by $\mathcal{C}$.[4] However, not all consumers are interested in all sub-markets. Some consumers are interested in many different sub-markets while other consumers are only interested in a few. More formally, suppose that a consumer of type τ is interested in a given sub-market with probability $(1 - \tau)$. That is, τ captures the probability that a consumer has any interest in participating in a given sub-market.[5]

When consumers join the platform they have access to all of the sub-markets that are available on the platform, N_2. Thus, by joining the platform consumer τ's expected utility is given by:

$$u_1(\tau, n, \mathcal{C}) = cs(n, \mathcal{C}) \cdot (1 - \tau) \cdot N_2 - P_1, \tag{1}$$

where P_1 is the price a consumer pays to join the platform. Every consumer's reservation utility is zero. Thus, a consumer τ joins the platform when $u_1(\tau) \geq 0$.

Side 2 contains sub-markets and their sellers. Sub-markets are indexed by $\theta \in [0, \infty)$ and the number of sub-markets that are available on the platform is denoted by N_2. I use θ to represent both a product and its sub-market.[6] Each seller of sub-market θ receives profits from a consumer that is interested in the sub-market given by $\pi(n, \mathcal{C})$. I assume profits are twice differentiable in n for a given competitive structure, $\mathcal{C}$.

For sellers, the fact that some consumers are uninterested in their sub-market implies that some consumers simply will not make a purchase. Thus, the expected profits for a seller from a consumer τ is given by $\pi(n, \mathcal{C}) \cdot (1 - \tau)$. Thus, when there are N_1 available consumers on the platform, a seller will have expected profits given by:[7]

$$\int_0^{N_1} \pi(n, \mathcal{C}) \cdot (1 - \tau) d\tau = \pi(n, \mathcal{C}) \cdot \left(1 - \frac{N_1}{2}\right) N_1. \tag{2}$$

Notice how seller profits change with the mass of consumers who join the platform. Sellers always prefer more consumers on the platform, $\frac{\partial \pi}{\partial N_1} > 0$, as this raises demand for their products. However, consumer heterogeneity implies that the marginal consumer who joins the platform is the least likely to purchase their products. Hence, sellers have decreasing marginal benefits from consumer participation on the platform, $\frac{\partial^2 \pi}{\partial N_1^2} < 0$. This differs from the previous literature, including [2,4,5] and subsequent work, including more recent papers by [8,13].

To allow for endogenous entry of sub-markets, sellers have different sunk costs. Let $c \cdot \theta$ be the sunk cost of developing a product in sub-market θ. That is, low θ-type sellers have lower sunk costs than high θ-sellers. Sellers earn profits by joining the platform and selling their products. The marginal cost of production is set to zero.

Thus, a seller of sub-market θ has expected utility from joining the platform which is given by:

$$u_2(\theta, n, \mathcal{C}) = \pi(n, \mathcal{C}) \left(1 - \frac{N_1}{2}\right) N_1 - c \cdot \theta - P_2, \tag{3}$$

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[4]　Note, consumer surplus as a function of n and $\mathcal{C}$ is more general than as a function of the set of prices set by the n sellers as the type of competitive structure can affect consumer surplus. One case occurs when products sold by the n sellers are differentiated. Hence, consumer surplus is simply defined by n and $\mathcal{C}$ and subsumes price changes.

[5]　This is common in platform marketplaces where consumers vary in the number of products that interest them.

[6]　Thus, more products on the platform coincide with more sub-markets on the platform.

[7]　Given that a consumer of type τ is interested in a sub-market with probability $(1 - \tau)$ and that consumers types are distributed uniformly, the consumers that join the platform will be the consumers with low τ-types and the total mass of consumers (N_1) will also identify the marginal consumer type that joins the platform. This gives the integral from 0 to N_1.

where P_2 is the price that a seller pays to join the platform. [8] Every seller's reservation utility is zero. Thus, a seller of type θ joins the platform when $u_2(\theta) \geq 0$.

2.2. The Platform and Timing of Play

The platform connects consumers with sellers. Every consumer pays a membership fee, P_1, to join the platform. For example, P_1 could be the monthly fee consumers pay to join Netflix or Hulu, or the retail price that consumers pay to purchase a smartphone or video game console. Similarly, the platform charges a membership fee, P_2, to sellers which gives sellers access to the consumers that join the platform.

The platform then maximizes profits with respect to prices P_1 and P_2. Platform profits are given by:

$$\Pi = N_1 \cdot P_1 + n \cdot N_2 \cdot P_2, \tag{4}$$

where N_1 is the number of consumers that join the platform, n is the number of sellers in each sub-market, and N_2 is the number of sub-markets so that $n \cdot N_2$ is the total number of sellers that join the platform. For simplicity, I will assume that the platform's marginal and fixed costs are zero.

The timing of play is as follows. First, the number of sellers in each sub-market, n, and the type of competition between sellers in a sub-market, $\mathcal{C}$, are given. The popularity of products is realized after sunk participation decisions are made. Thus, once consumers and sellers observe the sub-market structure they take expectations over the network gains from joining the platform. Given the expected gains to consumers and sellers from joining the platform, the platform sets prices, P_1 and P_2, which can be less than zero. Lastly, participation decisions are made and payoffs are realized.

3. Equilibrium

The aim of this paper is to determine how the amount of competition between sellers within sub-markets, characterized by the number of sellers and the competitive structure, affects the platform's pricing strategies and the welfare generated on the platform. In principle, consumer surplus and seller profits move in opposite directions as they are splitting the total surplus generated within a sub-market. However, the redistribution of surplus with changes in competition need not be one to one. For example, when sellers of homogeneous products compete *a la* Cournot then an increase in the number of sellers increases total surplus by eliminating deadweight loss. Similarly, if sellers have differentiated products within a sub-market then an increase in the number of sellers can increase consumer surplus more than it decreases sellers' profit. Thus, standard demand assumptions are used: $\frac{\partial cs(n,\mathcal{C})}{\partial n} \geq 0$ and $\frac{\partial \pi(n,\mathcal{C})}{\partial n} \leq 0$.[9] This gives a general characterization of sub-markets that allows for many types of demand and competitive structures to be analyzed.

The equilibrium concept is the standard Subgame Perfect Nash Equilibrium and so the problem can be solved using backward induction. Thus, for arbitrary prices set by the platform, the consumers and sellers play the participation subgame and make their participation decisions given the prices they observe. In light of the equilibria in the participation subgames for arbitrary prices, the platform sets its prices which completes the SPNE of the entire game.

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[8]   In reality, the number of sellers (n) depends on the price that the platform sets to sellers and so both the number of sellers and number of sub-markets that enter onto the platform depends on the platform's price to sellers (P_2). To maintain the tractability of a model with general sub-market competition ($\mathcal{C}$), the assumption of exogenously fixed n is made. One paper that considers n depending on P_2 is [14] but they must specify a very specific seller competition structure and this prevents a general analysis of platform outcomes across seller competition structures.

[9]   This implies that for any competition structure within sub-markets, when the number of sellers increases within a sub-market then consumer surplus for each consumer increases and each seller's profit decreases.

Theorem 1 (Unique Equilibrium). *There exists a unique equilibrium for all $\langle n, \mathcal{C} \rangle$. The unique equilibrium mass of consumers, number of sub-markets and platform prices are:*

$$N_1^* = \frac{n\pi(n) + cs(n)}{n\pi(n) + 2cs(n)}, \tag{5}$$

$$N_2^* = \frac{[n\pi(n) + cs(n)]^2}{4cn[n\pi(n) + 2cs(n)]}, \tag{6}$$

$$P_1^* = \frac{cs(n)^2 \cdot [n\pi(n) + cs(n)]^2}{8c[n\pi(n) + 2cs(n)]^2}, \tag{7}$$

$$P_2^* = \frac{n\pi(n) + cs(n)}{8[n\pi(n) + 2cs(n)]^2} \cdot [3n^2\pi(n)^2 + 9n\pi(n)cs(n) - 2cs(n)]. \tag{8}$$

All proofs are in the appendix. An examination of the equilibrium produces the following.

Proposition 1. *The equilibrium mass of consumers that the platform serves is decreasing in the number of sellers within sub-markets, $\frac{\partial N_1^*}{\partial n} < 0$.*

Proposition 1 states that an increase in the number of sellers within a sub-market always induces the platform to serve fewer consumers. An increase in the number of sellers creates two important incentives on the platform that drive this result. First, greater competition implies that the platform's existing consumers each receive additional surplus which raises the platform's marginal profit for an increase in the consumer price. Second, greater competition implies that an additional consumer generates less profit on the seller side which reduces in the marginal cost due to a decrease in the mass of consumers on the platform.[10] Thus, more competition implies that the marginal benefit from an increase in the consumer price increases while the marginal cost, in terms of fewer consumers for the seller side of the market, decreases. As a result, the platform raises its price so that fewer consumers join the platform.

Note that all else equal (i.e., the platform does not change its price with a change in the number of sellers), an increase in the number of sellers results in greater consumer surplus and less profit for sellers. More consumers join the platform but there are fewer sub-markets. Proposition 1 implies that the platform raises its consumer price so that the mass of consumers that join the platform actually decreases. This occurs since the gains to the platform from keeping its consumer price low and capturing additional marginal consumers are insufficient to overcome the two effects, greater marginal profits and lesser marginal costs, from a higher consumer price with fewer consumers.

An alternative perspective is to consider the price elasticity of demand for the platform. Consumer elasticity of demand for the platform is given by:[11]

$$\epsilon = \frac{\partial N_1}{\partial P_1} \cdot \frac{P_1}{N_1} = -\frac{1}{cs(n) \cdot N_2} \cdot \frac{P_1}{N_1}, \tag{9}$$

where the last equality holds since $P_1 = cs(n)(1 - N_1)N_2$. As consumer surplus increases, holding $\frac{P_1}{N_1 \cdot N_2}$ fixed, consumer elasticity of demand for platform membership becomes inelastic, or relatively unresponsive to changes in price. Thus, the platform's optimal response is to increase its consumer price resulting in fewer consumers or to decrease the number of sub-markets. The network effects that exist in this two-sided market imply that there also exist indirect effects, but the consumer elasticity

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10　Note, if there exists platform competition then this marginal cost is increasing and the extent depends on the platform's market power. In this case, the result is contingent on the platform having sufficient market power.

11　For ease of exposition, the $\mathcal{C}$ in $\pi(n, \mathcal{C})$ and $cs(n, \mathcal{C})$ will be suppressed so that $\pi(n)$ and $cs(n)$.

of demand for the platform suggests that the platform has an incentive to decrease the number of sub-markets. The following theorem shows that this is often the case.

Proposition 2 (Sub-Markets). *The equilibrium number of sub-markets on the platform is decreasing in the number of sellers within sub-markets unless the increase in consumer surplus that results from additional sellers within sub-markets is relatively large. That is* $\frac{\partial N_2^*}{\partial n} < 0$, *unless* $\frac{\partial cs(n)}{\partial n} > \frac{cs(n)}{2} + \left(-\frac{\partial \pi(n)}{\partial n} \right) \cdot \left(\frac{n^2 \pi(n)}{2cs(n)} + \frac{3n}{2} \right)$.

On the seller side of the market, more competition has two first order effects. First, having lower seller profit implies that the platform must reduce its price to sellers to attract or maintain the number of sub-markets which reduces the platforms incentive to provide sub-markets. Second, an increase in consumer surplus within each sub-market implies that the platform has an incentive to provide more sub-markets. Thus, the platform only provides more sub-markets when the increase to consumer surplus is the larger of the two effects.

Given this equilibrium, welfare and the effects on welfare from changes in the sub-markets are investigated. Total welfare generated by the platform is given by:

$$W^* = \frac{[n\pi(n) + cs(n)]^4}{32cn[n\pi(n) + 2cs(n)]^3} \cdot [3n\pi(n) + 10cs(n)]. \tag{10}$$

In platform markets, surplus is generated by the interaction between the two sides of the market. Thus, holding network benefits fixed, a reduction in the number of consumers or in the mass of sub-markets on the platform results in lower surplus (welfare). With this in mind, consider how an increase in the number of sellers in a sub-market affects total welfare generated by the platform. From Proposition 1 the mass of consumers decreases, which lowers welfare. Furthermore, the number of sub-markets decreases unless the increase in consumer surplus is relatively large. Thus, when the number of sellers in sub-markets, n, increases then total welfare generated by the platform, W^*, will decrease unless the increase in consumer surplus, $cs'(n)$, is relatively large.

Proposition 3 (Welfare). *The total welfare generated by the platform is decreasing in the number of sellers in a sub-market,* $\frac{\partial W^*}{\partial n} < 0$, *unless the increase in consumer surplus from a greater number of sellers in a sub-market,* $\frac{\partial cs(n)}{\partial n}$, *is sufficiently large.*

Proposition 3 implies that one can expect the platform to generate less welfare unless there is a significant increase to consumer surplus that results from additional sellers (e.g., if sellers have differentiated products and the sub-markets are not saturated).

4. Content Provided by the Platform

Now suppose that the platform provides the content on the platform. That is, given the competitive structure that exists within each sub-market, $\mathcal{C}$, the platform chooses the number of products that are available to its consumers in that sub-market, $n(\theta)$, and the price that consumers pay to join the platform, P_1. In this case, the platform chooses the number of sellers in each sub-market θ to maximize total surplus generated in that sub-market minus the total cost. This implies that the platform sets each product's price equal to marginal cost so that there is no deadweight loss within a sub-market and sets $n(\theta)$ to maximize $cs(n, \mathcal{C}) - c\theta n$. Thus, the platform solves the following problem:

$$\max_{N_1, N_2} \Pi = N_1 \cdot P_1 - \int_0^{N_2} c \cdot \theta \cdot n(\theta) \, d\theta \tag{11}$$

$$\text{s.t.} \quad P_1 = cs(n(\theta)) \cdot (1 - N_1)N_2. \tag{12}$$

When the platform provides content it is able to reduce two forms of deadweight loss. First, there no longer exists any deadweight loss within a sub-market as there are no longer product price

markups by sellers that have market power. The platform sets product prices equal to marginal cost which eliminates deadweight loss. Second, the platform provides the optimal number of products within each sub-market by accounting for the cost of the sub-market, $c\theta n$. Thus, the platform reduces the total costs of providing content on the platform. Consequently, that when the platform provides content, welfare is improved.[12] Thus, the following corollary follows:

Theorem 2 (Welfare with Integration). *When the platform and the product side of the market are fully integrated, welfare increases.*

The term, fully integrated means that the platform provides all the content. In digital marketplaces, fully integrated platforms do not exist. However, Costco and Sam's Club resemble this case. These marketplaces sell products to their members with prices equal to marginal costs and then charge members a membership fee to shop in the marketplace. Thus, the platform implements the same two-tier pricing scheme that is used by Costco and Sam's Club when it acts as the marketplace product provider.[13]

5. Conclusions

In this paper, the relationship between a platform and its sub-markets is considered. The mode of competition that exists within sub-markets affects the network effects between consumers and sellers, which in turn affect agents' participation decisions and platform pricing strategies. The network benefits that consumers and sellers receive from joining the platform are determined by consumer demand and the competitive structure that exists among sellers for these products. If there is less competition within a sub-market, then the price of the product will be higher, resulting in greater network gains for sellers but lower network gains for consumers. However, the size of the network also matters. More consumers on the platform increases demand for a product, and more products available on the platform makes participation on the platform more desirable for consumers.

When the number of sellers within a sub-market increases, competition within a sub-market increases, each seller receives less expected profit from a given consumer; thus, the platform has less of an incentive to provide sellers with additional consumers resulting in the platform serving fewer consumers. I find that the platform reduces consumer participation when the number of sellers increases, and this result is robust to many types of competitive structures within sub-markets.

In many platform markets, mergers between the platform and the product or seller side of the market are common. For example, in its online marketplace, Amazon connects consumers with sellers but it is often a seller itself. Similarly, many video games are developed by the console developers. I find that efficiency increases when the platform integrates with the seller side of the market. With integration two forms of surplus destruction are mitigated. First, the platform maximizes the surplus generated within each sub-market with respect to the number of product sellers within that sub-market; hence, surplus destruction through redundant sunk costs is minimized. Second, the platform sets the price of each product equal to marginal cost so that there is no distortion in product provision. The distortion that remains is the platform's market power on the consumer side of the market. I find that integration leads to greater platform profits, total consumer surplus, and total welfare generated on the platform. Even though full integration is unlikely to occur in many platform markets, this serves as a base case for policymakers where such integration is a concern.

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[12] For more on the platform's decision to act as a marketplace or as a retailer see [15,16].
[13] See [17] for a complete analysis on the extent to which a platform would tend to integrate with the seller side of the market.

Appendix

Proof of Theorem 1: within each sub-market, $\langle n, \mathcal{C} \rangle$, the platform sets prices P_1 and P_2 to maximize profit given by Equation (4). The game is solved backwards. For observed platform prices, the marginal agents that join the platform on each side of the market, τ^c for consumers and θ^c for sub-markets, identify the total mass of consumers and sub-markets on the platform. Given both consumer and seller types are distributed uniformly we have that the marginal type directly corresponds to the number of agents on each side: $N_1 = \tau^c$ and $N_2 = \theta^c$. Thus, Equations (1) and (3) imply $u_1(\tau = N_1, n, \mathcal{C}) \equiv 0$ and $u_2(\theta = N_2, n, \mathcal{C}) \equiv 0$. Thus, the platform's problem is:

$$\max_{N_1, N_2} N_1 \cdot P_1 + n \cdot N_2 \cdot P_2$$

$$\text{given } P_1 = cs(n) \cdot (1 - N_1) \cdot N_2,$$

$$\text{and } P_2 = \pi(n) \left(1 - \frac{N_1}{2} \right) N_1 - c \cdot N_2.$$

For ease of exposition, the $\mathcal{C}$ in $\pi(n, \mathcal{C})$ and $cs(n, \mathcal{C})$ will be suppressed so that we have $\pi(n)$ and $cs(n)$.

This is a standard maximization problem with two control variables. Solving the platform's problem gives the unique equilibrium of the entire game. The first order conditions are given by:

$$\frac{\partial \Pi}{\partial N_1} = cs(n) N_2 (1 - 2N_1) + n \cdot N_2 \pi(n)(1 - N_1),$$

$$\frac{\partial \Pi}{\partial N_2} = N_1 cs(n)(1 - N_1) + n \cdot \pi(n) \left(1 - \frac{N_1}{2} \right) N_1 - 2cnN_2.$$

Setting the $\frac{\partial \Pi}{\partial N_1} = 0$ gives Equation (5) and setting $\frac{\partial \Pi}{\partial N_2} = 0$ gives while using the N_1^* given by Equation (5) provides Equation (6). Substituting N_1^* and N_2^* into the platforms pricing constraints, Condition (A), gives Equations (7) and (8).

To prove uniqueness, the second order conditions must satisfy the standard conditions of a unique maximum with two variables: $\Pi_{N_1, N_1} < 0$, $\Pi_{N_2, N_2} < 0$, and $\Pi_{N_1, N_1} \cdot \Pi_{N_2, N_2} - \Pi_{N_1, N_2} \cdot \Pi_{N_1, N_2} > 0$. The second order conditions are given by:

$$\Pi_{N_1, N_1} = -2cs(n) N_2 - n \cdot N_2 \pi(n) < 0,$$

$$\Pi_{N_2, N_2} = -2c < 0,$$

$$\Pi_{N_1, N_2} = cs(n)(1 - 2N_1) + n \cdot \pi(n)(1 - N_1).$$

To show $\Pi_{N_1, N_1} \cdot \Pi_{N_2, N_2} - \Pi_{N_1, N_2} \cdot \Pi_{N_1, N_2} > 0$, note that we have $\Pi_{N_1, N_2} = 0$ give N_1^*. This implies the conditions for a global maximum are satisfied and we have the unique equilibrium. $\square$

Proof of Proposition 1: Taking the derivative of Equation (5) with respect to n implies that $\frac{\partial N_1^*}{\partial n} < 0$ if and only if $\frac{\pi'(n)}{\pi(n)} < \frac{cs'(n)}{cs(n)}$ which holds since $cs'(n) \geq 0 \geq \pi'(n)$ and $cs(n), \pi(n), \geq 0$. $\square$

Proof of Proposition 2: Taking the derivative of Equation (6) with respect to n implies that $\frac{\partial N_2^*}{\partial n} > 0$ if and only if $\frac{\partial cs(n)}{\partial n} > \frac{cs(n)}{2} + \left(-\frac{\partial \pi(n)}{\partial n} \right) \cdot \left(\frac{n^2 \pi(n)}{2cs(n)} + \frac{3n}{2} \right)$. $\square$

Proof of Proposition 3: Taking the derivative of Equation (10) with respect to n implies that $\frac{\partial W^*}{\partial n} > 0$ if and only if

$$4n \cdot cs'(n)[n^2\pi(n)^2 + 4n\pi(n)cs(n) + 10cs(n)^2] + n\pi(n)[3n^2\pi(n)^2 + 34n\pi(n)cs(n) + 10cs(n)^2]$$
$$> n^2\pi(n)^2[19cs(n) - \pi'(n)6n^2] - \pi'(n)34n^3\pi(n)cs(n)$$
$$+ cs(n)^2[36n\pi(n) - \pi'(n)56n + 10cs(n)].$$

Essentially this requires that $cs'(n)$ be large relative to $n\pi(n)$ and $cs(n)$ and that $n\pi'(n)$ is non-negative. $\square$

Proof of Theorem 2: When the platform provides content it is able to reduce two forms of deadweight loss. First, there no longer exists any deadweight loss within a sub-market as there are no longer product price markups by sellers that have market power. The platform sets product prices equal to marginal cost which eliminates deadweight loss. Second, the platform provides the optimal number of products within each sub-market by accounting for the cost of the sub-market, $c\theta n$. Thus, the platform reduces the total costs of providing content on the platform. Consequently, that when the platform provides content, welfare is improved. $\square$

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