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Fixed-Time Pursuit–Evasion Differential Game with Grönwall Constraints in Hilbert Space l_2

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Abstract

This paper studies a pursuit–evasion differential game in the Hilbert space l_2 involving countably many pursuers and one evader. The players follow simple motion dynamics, while their control functions are subject to Grönwall-type constraints. The value of the game λ is defined as the infimum of the distances between the evader and the pursuers at a given terminal time θ. The pursuers aim to minimize this distance λ by the end of the game, while the evader aims to keep it as large as possible. Optimal strategies for the pursuers are constructed via the method of fictitious pursuers, and an explicit evasion strategy is derived. As a result, the optimal strategies of both sides and the value of the game are obtained.

Keywords: differential game; pursuer; evader; optimal strategies; attainability domain; game value; Grönwall-type constraints

1. Introduction

Differential game theory is an important branch of modern control theory and applied mathematics. The foundations of this theory were established in the pioneering monograph of Isaacs (1965). Later, substantial developments in the theory of differential games, including the theory of value functions and generalized Hamilton–Jacobi equations, were obtained by Subbotin (1984) and Petrosyan (1993). Further progress related to constructive methods in pursuit problems and the development of the Π-strategy was achieved in the works of Azamov and Samatov (2011) and Azamov et al. (2025).

There are a number of studies devoted to the examination of fixed-time-duration games. Among these, considerable attention has been devoted to multiplayer pursuit–evasion games involving one evader and multiple pursuers. In this direction, Ibragimov (1998) investigated a pursuit problem with several pursuers and established optimality conditions for the pursuers. Later, Ibragimov (2005) studied an optimal pursuit problem with countably many pursuers and one evader in a Hilbert space under geometric constraints on the controls. The fixed-time duration pursuit problem with integral constraints was subsequently considered by Ibragimov and Kuchkarov (2013). Infinite-dimensional pursuit

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problems and differential games described by systems of differential equations were also investigated in Ibragimov et al. (2019). Additional developments concerning many-pursuer pursuit–evasion games on manifolds were obtained in Kuchkarov et al. (2016).

Differential games with multiple pursuers have also been widely studied by other researchers. Ahmed et al. (2019) considered an optimal pursuit differential game involving many pursuers and one evader and constructed corresponding optimal strategies. Salimi and Ferrara (2019) analyzed pursuit problems with finitely and countably many pursuers and introduced the value of the game without imposing restrictive relations between the players' resources. The existence of evasion strategies under integral constraints was further studied in Pansera et al. (2025). Pursuit–evasion problems in Hilbert spaces with constrained energy resources were considered by Adamu et al. (2020) and by Badakaya et al. (2022b). Higher-order pursuit dynamics described by nth-order differential equations were investigated in Badakaya et al. (2022a). Differential games with state and integral constraints were analyzed by Sharifi et al. (2022). Fractional-order pursuit games involving groups of evaders were studied by Machtakova and Petrov (2024).

Alongside theoretical investigations, modern pursuit–evasion problems have attracted attention in engineering and computational sciences. Cooperative and distributed control methods for multi-agent pursuit systems were studied in Garcia et al. (2021); Lopez et al. (2020); Mozhegova and Petrov (2025); Wang et al. (2020); Zhang et al. (2024); P. Zhou and Chen (2024). Reach–avoid pursuit games with feedback strategies were investigated in Selvakumar and Bakolas (2021). Cheng and Yuan (2023) analyzed multiplayer pursuit–evasion games with obstacle avoidance and adaptive parameter estimation. Reach–avoid games in flow environments were considered in Deng et al. (2024). Containment and capture strategies for heterogeneous pursuers were investigated by S. Zhou et al. (2024). These works demonstrate the increasing relevance of pursuit–evasion models in distributed autonomous systems and intelligent control applications.

An important class of differential games involves constraints of the Grönwall type imposed on the players' control functions. Such constraints naturally arise in dynamical systems with exponentially growing resources and allow one to derive effective estimates by means of Grönwall's inequality (Gronwall, 1919). Differential games with Grönwall-type constraints were studied by Samatov et al. (2020), where a linear pursuit problem was investigated using the Π-strategy. Later, Rilwan et al. (2023) examined a differential game of one pursuer and one evader with Grönwall-type control constraints in $\mathbb{R}^n$ and established the value of the game together with guaranteed strategies of the players. Pursuit–evasion problems with inertial players under integral constraints were also investigated by Azamov and Turgunboeva (2025).

Motivated by the above-mentioned studies, in the present paper, we investigate a fixed-time pursuit–evasion differential game in the Hilbert space l_2 involving countably many pursuers and one evader under Grönwall-type constraints on the controls. The dynamics of the players are described by simple motion equations. The objective of the pursuers is to minimize the terminal distance to the evader, while the evader attempts to maximize this distance. We construct admissible strategies for the players and determine the value of the game. The obtained results extend earlier works devoted to pursuit–evasion games with geometric and integral constraints to the case of Grönwall-type constraints.

The analysis is based on the construction of attainability domains, the method of half spaces, the method of fictitious pursuers, and geometric properties of the Hilbert space l_2.

2. Statement of the Problem

In this section, we formulate the pursuit–evasion problem and introduce the key definitions used throughout the paper.

We investigate an optimal pursuit problem in the Hilbert space l_2 involving countably many pursuers and a single evader. The Hilbert space l_2 consists of sequences $\delta = (\delta_1, \delta_2, \dots, \delta_j, \dots)$ such that

$$\sum_{j=1}^{\infty} \delta_j^2 < \infty,$$

where the inner product and norm are defined as

$$\langle \delta, \zeta \rangle = \sum_{j=1}^{\infty} \delta_j \zeta_j, \quad \|\delta\| = \sqrt{\langle \delta, \delta \rangle}.$$

Specifically, we consider a pursuit–evasion game in which the dynamics of countably many pursuers x_i and an evader y are described by the system of equations

$$\begin{aligned} &\dot{x}_i = u_i, \quad x_i(0) = x_{i0}, \quad i = 1, 2, \dots, m, \dots \\ &\dot{y} = v, \quad\ \ y(0) = y_0, \end{aligned} \tag{1}$$

where $x_i, x_{i0}, u_i, y, y_0, v \in l_2$, $u_i = (u_{i1}, u_{i2}, \dots, u_{in}, \dots)$ is the control parameter of the pursuer x_i and $v = (v_1, v_2, \dots, v_n, \dots)$ is the control parameter of the evader y. We denote by $\mathcal{I} = \{1, 2, \dots, m, \dots\}$ the countably infinite index set of all pursuers, and let $\theta > 0$ be a fixed terminal time.

We denote by $K(x_0, r) = \{x \in l_2 : \|x - x_0\| \le r\}$ the closed ball and by $L(x_0, r) = \{x \in l_2 : \|x - x_0\| = r\}$ the sphere in l_2 with center x_0 and radius r.

Definition 1. *For the i-th pursuer x_i ($i \in \mathcal{I}$), a function $u_i(t) = (u_{i1}(t), u_{i2}(t), \dots)$, $0 \le t \le \theta$, is called an admissible control if its coordinates $u_{ij} : [0, \theta] \to \mathbb{R}^1$ are Borel measurable and the constraint*

$$\|u_i(t)\|^2 \le \rho_i^2 + 2h \int_0^t \|u_i(s)\|^2 ds, \quad 0 \le t \le \theta, \tag{2}$$

is satisfied, where ρ_i and h are given positive constants.

It is assumed that the sequence $\{\rho_i\}_{i=1}^{\infty}$ is bounded.

Definition 2. *For the evader y, a function $v(t) = (v_1(t), v_2(t), \dots)$, $0 \le t \le \theta$, is called an admissible control if its coordinates $v_j : [0, \theta] \to \mathbb{R}^1$ are Borel measurable and the constraint*

$$\|v(t)\|^2 \le \sigma^2 + 2h \int_0^t \|v(s)\|^2 ds, \quad 0 \le t \le \theta, \tag{3}$$

is satisfied, where σ is a given positive constant.

Lemma 1 (Gronwall, 1919)**.** *If a measurable function $\eta(t)$, $t \ge 0$, satisfies for some $\rho > 0$ the condition*

$$\|\eta(t)\|^2 \le \rho^2 + 2h \int_0^t \|\eta(s)\|^2 ds, \quad t \ge 0, \tag{4}$$

then the inequality $\|\eta(t)\| \le \rho e^{ht}$ holds.

By Lemma 1, for the admissible controls $u_i(t)$ and $v(t)$, we have

$$\|u_i(t)\| \le \rho_i e^{ht}, \quad \|v(t)\| \le \sigma e^{ht}, \quad t \in [0, \theta]. \tag{5}$$

It is important to note that (5) does not imply (2) and (3). Therefore, the constraint (4) generalizes the geometric constraints of the form (5). One can easily verify that if

$$\|u_i(t)\| = \rho_i e^{ht}, \quad \|v(t)\| = \sigma e^{ht}, \quad t \geq 0,$$

then (2) and (3) hold, respectively.

Suppose the players have chosen admissible controls $u_i(\cdot)$ and $v(\cdot)$, respectively. Then their trajectories are given by

$$x_i(t) = (x_{i1}(t), x_{i2}(t), \ldots, x_{ij}(t), \ldots), \quad x_{ij}(t) = x_{ij}^0 + \int_0^t u_{ij}(s)ds,$$

$$y(t) = (y_1(t), y_2(t), \ldots, y_j(t), \ldots), \quad y_j(t) = y_j^0 + \int_0^t v_j(s)ds.$$

Consequently, $x_i(\cdot), y(\cdot) \in C(0,\theta;l_2)$, where $C(0,\theta;l_2)$ is the space of continuous functions on $[0,\theta]$

$$f(t) = (f_1(t), f_2(t), \ldots, f_j(t), \ldots) \in l_2, \quad t \geq 0,$$

with absolutely continuous components $f_j(t)$.

Definition 3. *If a function*

$$U_i(t, x_i, y, v) : [0,\theta] \times l_2 \times l_2 \times K(0, \sigma e^{ht}) \to K(0, \rho_i e^{ht})$$

is such that for any admissible control $v(t)$ of the evader y the system

$$\dot{x}_i = U_i(t, x_i, y, v), \quad x_i(0) = x_{i0}, \qquad \dot{y} = v, \quad y(0) = y_0,$$

admits a unique solution $(x_i(\cdot), y(\cdot))$ whose components are absolutely continuous and satisfy $x_i(\cdot), y(\cdot) \in C(0,\theta;l_2)$, then U_i is called a strategy of the pursuer x_i.

Definition 4. *The strategies $\hat{U}_i$ of the pursuers x_i, $i \in \mathcal{I}$, are called optimal if*

$$\inf_{U_1,\ldots,U_m,\ldots} \Lambda_1(U_1, \ldots, U_m, \ldots) = \Lambda_1(\hat{U}_1, \ldots, \hat{U}_m, \ldots),$$

where

$$\Lambda_1(U_1, \ldots, U_m, \ldots) = \sup_{v(\cdot)} \inf_{i \in \mathcal{I}} \|x_i(\theta) - y(\theta)\|,$$

and $v(\cdot)$ is any admissible control of the evader y.

Definition 5. *If a function*

$$V(x_1, \ldots, x_m, \ldots, y) : l_2 \times \cdots \times l_2 \times l_2 \to K(0, \sigma e^{ht})$$

is such that for any admissible controls $u_i(t)$ of the pursuers x_i, $i \in \mathcal{I}$, the system

$$\dot{x}_i = u_i, \quad x_i(0) = x_{i0},\ i \in \mathcal{I}, \qquad \dot{y} = V(x_1, \ldots, x_m, \ldots, y), \quad y(0) = y_0,$$

admits a unique solution $(x_1(\cdot), \ldots, x_m(\cdot), \ldots, y(\cdot)) \in C(0,\theta;l_2)$ whose components are absolutely continuous, then V is called a strategy of the evader y. The strategy is called admissible if it additionally satisfies the constraint

$$\|V(x_1(t),\dots,x_m(t),\dots,y(t))\|^2 \le \sigma^2 + 2h\int_0^t \|V(x_1(s),\dots,x_m(s),\dots,y(s))\|^2 ds, \quad t\in[0,\theta]. \tag{6}$$

Definition 6. *A strategy $\hat V$ of the evader y is called optimal if*

$$\sup_V \Lambda_2(V) = \Lambda_2(\hat V),$$

where

$$\Lambda_2(V) = \inf_{u_1(\cdot),\dots,u_m(\cdot)} \inf_{i\in\mathcal{I}} \|x_i(\theta) - y(\theta)\|,$$

$u_i(\cdot)$ are admissible controls of the pursuers x_i, $i\in\mathcal{I}$, and V is an admissible strategy of the evader y.

If

$$\Lambda_1(\hat U_1,\dots,\hat U_m,\dots) = \Lambda_2(\hat V) = \lambda,$$

then λ is called the *value of the game.*

Our aim is to find optimal strategies $\hat U_i$ for the pursuers ($i\in\mathcal{I}$), an optimal strategy $\hat V$ for the evader, and the value of the game.

3. Preliminary Results for a Single Pursuer

3.1. Attainability Sets

The *attainability set* of the i-th pursuer x_i at time θ is defined as the set of all points

$$x_i(\theta) = x_{i0} + \int_0^\theta u_i(s)ds,$$

where $u_i(\cdot)$ is an admissible control of x_i. This set coincides with the ball $K(x_{i0}, R_i(\theta))$, where

$$R_i(\theta) = \rho_i \int_0^\theta e^{hs}ds = \frac{\rho_i}{h}(e^{h\theta}-1).$$

Indeed, by Grönwall's lemma, we have

$$\|x_i(\theta) - x_{i0}\| = \left\| \int_0^\theta u_i(s)ds \right\| \le \int_0^\theta \|u_i(s)\|ds \le \int_0^\theta \rho_i e^{hs}ds = \frac{\rho_i}{h}(e^{h\theta}-1).$$

On the other hand, for any $\tilde x \in K(x_{i0}, R_i(\theta))$, using the control defined by

$$u_i(s) = \frac{he^{hs}}{e^{h\theta}-1}(\tilde x - x_{i0}), \quad 0\le s\le\theta, \tag{7}$$

we obtain

$$x_i(\theta) = x_{i0} + \int_0^\theta u_i(s)ds = x_{i0} + (\tilde x - x_{i0})\frac{h}{e^{h\theta}-1}\int_0^\theta e^{hs}ds = \tilde x,$$

that is, the pursuer x_i reaches the point $\tilde x$.

Next, we show that this control is admissible. By the definition of the set $K(x_{i0}, R_i(\theta))$, the inequality

$$\|\tilde x - x_{i0}\| \le R_i(\theta) = \frac{\rho_i}{h}(e^{h\theta}-1) \tag{8}$$

holds.

Substituting (7) into (2) yields

$$\left(\frac{h\|\tilde{x}-x_{i0}\|}{e^{h\theta}-1}\right)^2 e^{2ht} \le \rho_i^2 + \left(\frac{h\|\tilde{x}-x_{i0}\|}{e^{h\theta}-1}\right)^2 (e^{2ht}-1).$$

This inequality takes the form

$$\left(\frac{h\|\tilde{x}-x_{i0}\|}{e^{h\theta}-1}\right)^2 \le \rho_i^2,$$

which follows from (8). Consequently,

$$\|u_i(t)\|^2 \le \rho_i^2 + 2h\int_0^t \|u_i(s)\|^2 ds.$$

Therefore, the control $u_i(\cdot)$ is admissible.

Similarly, the attainability set of the evader y at time θ from the initial position y_0 is the ball $K(y_0, r(\theta))$, where

$$r(\theta) = \sigma \int_0^\theta e^{hs} ds = \frac{\sigma}{h}(e^{h\theta}-1),$$

and the proof proceeds analogously.

3.2. Auxiliary Game for a Single Pursuer

Now we consider an auxiliary game with one pursuer and one evader in a half-space X. We introduce the notations $x_i = x$, $\rho_i = \rho$, $x_{i0} = x_0$, and $R_i(\theta) = R(\theta)$. The half-space X is defined as follows: if $x_0 \ne y_0$, then

$$X = \left\{ z \in l_2 \mid 2\langle y_0 - x_0, z\rangle \le R^2(\theta) - r^2(\theta) + \|y_0\|^2 - \|x_0\|^2 \right\}, \quad \xi = \frac{y_0 - x_0}{\|y_0 - x_0\|}; \tag{9}$$

if $x_0 = y_0$, then

$$X = \{ z \in l_2 \mid \langle p, z - x_0\rangle \le R(\theta) \},$$

where p is an arbitrary unit vector.

The motion dynamics of the players in the auxiliary game are given by the following equations:

$$\begin{aligned} \dot{x} &= u, \quad x(0) = x_0, \\ \dot{y} &= v, \quad y(0) = y_0. \end{aligned} \tag{10}$$

We define the pursuer's strategy as follows: if $x_0 = y_0$, then

$$u(t) := v(t), \quad 0 \le t \le \theta; \tag{11}$$

if $x_0 \ne y_0$, then we set

$$u(t) = \begin{cases} v(t) - \langle v(t), \xi\rangle \xi + \xi\sqrt{e^{2ht}(\rho^2 - \sigma^2) + \langle v(t), \xi\rangle^2}, & 0 \le t \le \tau, \\ v(t), & \tau < t \le \theta, \end{cases} \tag{12}$$

where $\xi = \dfrac{y_0 - x_0}{\|y_0 - x_0\|}$ and τ, $0 \le \tau \le \theta$, is the first time at which the equality $x(\tau) = y(\tau)$ holds.

Lemma 2. *If $\sigma \le \rho$ and $y(\theta) \in X$, then the strategies (11), (12) in game (10) yield $x(\theta) = y(\theta)$.*

Proof. If $x_0 = y_0$, then by (11) we obtain $x(t) = y(t)$ for all $t \in [0, \theta]$; in particular, $x(\theta) = y(\theta)$.

Let $x_0 \neq y_0$. By (12), we have $y(t) - x(t) = \xi f(t)$, where

$$f(t) = \|y_0 - x_0\| + \int_0^t \langle v(s), \xi \rangle ds - \int_0^t \left(e^{2hs}(\rho^2 - \sigma^2) + \langle v(s), \xi \rangle^2 \right)^{1/2} ds. \tag{13}$$

Clearly, $f(0) = \|y_0 - x_0\| > 0$. We now prove that $f(\theta) \leq 0$, which guarantees the existence of some $\tau \in [0, \theta]$ with $f(\tau) = 0$.

To this end, define the two-dimensional vector function $g(t) = \left((e^{2ht}(\rho^2 - \sigma^2))^{1/2}, \langle v(t), \xi \rangle \right)$. We have

$$|g(s)| = \left(e^{2hs}(\rho^2 - \sigma^2) + \langle v(s), \xi \rangle^2 \right)^{1/2}.$$

Then,

$$\int_0^\theta |g(s)| ds \geq \left| \int_0^\theta g(s) ds \right| = \left(A^2 + \left(\int_0^\theta \langle v(s), \xi \rangle ds \right)^2 \right)^{1/2}, \tag{14}$$

where

$$A = \int_0^\theta (e^{2hs}(\rho^2 - \sigma^2))^{1/2} ds = \frac{\sqrt{\rho^2 - \sigma^2}}{h}(e^{h\theta} - 1).$$

Hence, by (13) and (14), we have

$$f(\theta) \leq \|y_0 - x_0\| + \int_0^\theta \langle v(s), \xi \rangle ds - \left(A^2 + \left(\int_0^\theta \langle v(s), \xi \rangle ds \right)^2 \right)^{1/2}. \tag{15}$$

According to the hypothesis of the lemma, $y(\theta) \in X$. Consequently, by (9), $\langle \xi, y(\theta) \rangle \leq q$, where

$$q = \frac{R^2(\theta) - r^2(\theta) + \|y_0\|^2 - \|x_0\|^2}{2\|y_0 - x_0\|}.$$

Since $y(\theta) = y_0 + \int_0^\theta v(s) ds$, we have

$$\langle \xi, y(\theta) \rangle = \langle \xi, y_0 \rangle + \int_0^\theta \langle v(s), \xi \rangle ds \leq q.$$

Therefore,

$$\int_0^\theta \langle v(s), \xi \rangle ds \leq q - \langle \xi, y_0 \rangle. \tag{16}$$

Now, to estimate the right-hand side of (15), we consider the function

$$\phi(t) = \|y_0 - x_0\| + t - \left(A^2 + t^2 \right)^{1/2}, \quad t \leq q - \langle \xi, y_0 \rangle.$$

The derivative of $\phi(t)$ is

$$\phi'(t) = 1 - \frac{t}{\sqrt{A^2 + t^2}} > 0.$$

Since $\phi'(t) > 0$, the function $\phi(t)$ is strictly increasing and has its maximum at $t = q - \langle \xi, y_0 \rangle$. Now we evaluate $\phi(q - \langle \xi, y_0 \rangle)$. Observe that

$$R^2(\theta) - r^2(\theta) = \frac{\rho^2 - \sigma^2}{h^2}(e^{h\theta} - 1)^2 = A^2.$$

Moreover,

$$\langle \xi, y_0 \rangle = \frac{\langle y_0 - x_0, y_0 \rangle}{\|y_0 - x_0\|} = \frac{\|y_0\|^2 - \langle x_0, y_0 \rangle}{\|y_0 - x_0\|}.$$

Consequently,

$$q - \langle \xi, y_0 \rangle = \frac{A^2 + \|y_0\|^2 - \|x_0\|^2 - 2(\|y_0\|^2 - \langle x_0, y_0 \rangle)}{2\|y_0 - x_0\|} = \frac{A^2 - \|y_0 - x_0\|^2}{2\|y_0 - x_0\|}.$$

It is easy to verify that

$$\begin{aligned} \phi(q - \langle \xi, y_0 \rangle) &= \|y_0 - x_0\| + \frac{A^2 - \|y_0 - x_0\|^2}{2\|y_0 - x_0\|} - \sqrt{A^2 + \Big(\frac{A^2 - \|y_0 - x_0\|^2}{2\|y_0 - x_0\|}\Big)^2} \\ &= \|y_0 - x_0\| + \frac{A^2 - \|y_0 - x_0\|^2}{2\|y_0 - x_0\|} - \frac{A^2 + \|y_0 - x_0\|^2}{2\|y_0 - x_0\|} = 0. \end{aligned}$$

Since ϕ increases strictly, inequality (16) implies

$$\phi\Big(\int_0^\theta \langle v(s), \xi \rangle ds\Big) \le \phi(q - \langle \xi, y_0 \rangle) = 0.$$

Observe that the right-hand side of (15) equals $\phi\big(\int_0^\theta \langle v(s), \xi \rangle ds\big)$. Hence, we obtain $f(\theta) \le 0$.

Thus $f(\tau) = 0$ for some $\tau \in [0, \theta]$, which gives $x(\tau) = y(\tau)$. For $t > \tau$, by (12), we have $u(t) = v(t)$, and therefore $x(t) = y(t)$ for all $t \in [\tau, \theta]$. In particular, $x(\theta) = y(\theta)$.

If $\rho = \sigma$, then (12) simplifies to

$$u(t) = v(t) - \langle v(t), \xi \rangle \xi + \xi\, |\langle v(t), \xi \rangle|,$$

the half-space X is defined by the following inequality:

$$X = \Big\{ z \in l_2 \mid 2\langle y_0 - x_0, z \rangle \le \|y_0\|^2 - \|x_0\|^2 \Big\}.$$

From (13), we have

$$\begin{aligned} f(t) &= \|y_0 - x_0\| + \int_0^t (\langle v(s), \xi \rangle - |\langle v(s), \xi \rangle|) ds \\ &\le \|y_0 - x_0\| + \int_0^t \langle v(s), \xi \rangle ds - \left| \int_0^t \langle v(s), \xi \rangle \right| ds. \end{aligned}$$

Since $y(\theta) \in X$ and $y(\theta) = y_0 + \int_0^\theta v(s)\, ds$, we obtain

$$2\langle y_0 - x_0, y_0 \rangle + 2\int_0^\theta \langle y_0 - x_0, v(s) \rangle ds \le \|y_0\|^2 - \|x_0\|^2.$$

Hence,

$$\int_0^\theta \langle v(s), \xi \rangle ds \leq -\frac{\|y_0 - x_0\|}{2}.$$

The function $\phi(t)$ takes the form $\phi(t) = \|y_0 - x_0\| + t - |t|$, $t \leq -\|y_0 - x_0\|/2$. We then have $\phi(t) = \|y_0 - x_0\| + 2t$, i.e., $\phi(t)$ is strictly increasing. Consequently,

$$f(\theta) \leq \phi\left(\int_0^\theta \langle v(s), \xi \rangle ds\right) \leq \phi\left(-\frac{\|y_0 - x_0\|}{2}\right) = 0.$$

Thus $f(t)$ is continuous, $f(0) > 0$ and $f(\theta) \leq 0$, so there exists some $\tau \in [0, \theta]$ with $f(\tau) = 0$. This completes the proof of Lemma 2. □

4. Main Result

We consider the game (1). Let

$$\lambda = \inf\left\{ l \geq 0 : K(y_0, r(\theta)) \subset \bigcup_{i=1}^{\infty} K(x_{i0}, R_i(\theta) + l) \right\}. \tag{17}$$

Theorem 1. *Assume there exists a nonzero $p_0 \in l_2$ such that, for all $i \in \mathcal{I}$, $\langle y_0 - x_{i0}, p_0 \rangle \geq 0$ and*

$$\sigma \leq \rho_i + \frac{\lambda h}{e^{h\theta} - 1}, \qquad i \in \mathcal{I}.$$

Then the number λ defined by (17) is the value of the game (1).

The proof of Theorem 1 relies on the following lemma, which are Assertions 4 and 5 in Ibragimov (2005).

Lemma 3 (Ibragimov, 2005)**.** *Suppose there is a nonzero vector $p_0 \in l_2$ satisfying $\langle y_0 - x_{i0}, p_0 \rangle \geq 0$ for all $i \in \mathcal{I}$.*

(a) If $K(y_0, r(\theta)) \subset \bigcup_{i=1}^{\infty} K(x_{i0}, R_i(\theta))$, then

$$K(y_0, r(\theta)) \subset \bigcup_{i=1}^{\infty} X_i,$$

where X_i is defined as follows:

$$X_i = \left\{ z \in l_2 \mid 2\langle y_0 - x_{i0}, z \rangle \leq R_i^2(\theta) - r^2(\theta) + \|y_0\|^2 - \|x_{i0}\|^2 \right\},\ x_{i0} \neq y_0;$$
$$X_i = \{ z \in l_2 \mid \langle p_0, z - y_0 \rangle \leq R_i(\theta) \},\ x_{i0} = y_0.$$

(b) If, for any $\varepsilon > 0$,

$$K(y_0, r(\theta)) \not\subset \bigcup_{i \in \mathcal{I}} K(x_{i0}, (R_i(\theta) - \varepsilon)_+) \quad (a_+ = \max\{0, a\}),$$

then there exists a point $\tilde{y} \in L(y_0, r(\theta))$ such that $\|\tilde{y} - x_{i0}\| \geq R_i(\theta)$ for all $i \in \mathcal{I}$.

4.1. Pursuers' Strategy Construction

Let z_i be fictitious pursuers defined by the following equations:

$$\dot{z}_i = \omega_i^{\epsilon}, \quad z_i(0) = x_{i0}, \quad \|\omega_i^{\epsilon}(t)\|^2 \le \tilde{\rho}_i^2(\epsilon) + 2h\int_0^t \|\omega_i^{\epsilon}(s)\|^2 ds, \tag{18}$$

where $\tilde{\rho}_i(\epsilon) = \rho_i + \frac{\lambda h}{e^{h\theta}-1} + \frac{\epsilon h}{e^{h\theta}-1}$ and ϵ, $0 < \epsilon < 1$, is an arbitrary number. Thus, for the fictitious pursuer z_i starting from x_{i0}, the attainability set at θ is the ball $K(x_{i0}, R_i^{\epsilon}(\theta))$, i.e., $K(x_{i0}, R_i(\theta) + \lambda + \epsilon)$.

Using Grönwall's lemma, it follows from (18) that

$$\|\omega_i^{\epsilon}(t)\| \le \tilde{\rho}_i(\epsilon)e^{ht}.$$

For the fictitious pursuers z_i, strategies are constructed as follows. If $x_{i0} = y_0$, then we set

$$\omega_i^{\epsilon}(t) = v(t), \quad 0 \le t \le \theta; \tag{19}$$

if $x_{i0} \ne y_0$, we set

$$\omega_i^{\epsilon}(t) = \begin{cases} v(t) - \langle v(t), \xi_i\rangle\xi_i + \xi_i\sqrt{e^{2ht}(\tilde{\rho}_i^2(\epsilon) - \sigma^2) + \langle v(t), \xi_i\rangle^2}, & 0 \le t \le \tau_i, \\ v(t), & \tau_i < t \le \theta, \end{cases} \tag{20}$$

where $\xi_i = \dfrac{y_0 - x_{i0}}{\|y_0 - x_{i0}\|}$ and τ_i, $0 \le \tau_i \le \theta$, is the first time when $z_i(\tau_i) = y(\tau_i)$. The strategies given in (19) and (20) are admissible. Indeed, if $x_{i0} = y_0$, by (19), we have $||\omega_i^{\epsilon}(t)|| = ||v(t)||$ on $0 \le t \le \theta$, which satisfies the inequality in (18). If $x_{i0} \ne y_0$, by (20), we have

$$||\omega_i^{\epsilon}(t)||^2 = ||v(t)||^2 + e^{2ht}(\tilde{\rho}_i^2(\epsilon) - \sigma^2), \;\; 0 \le t \le \tau_i,$$

and

$$||\omega_i^{\epsilon}(t)||^2 = ||v(t)||^2, \;\; \tau_i \le t \le \theta.$$

Therefore, in view of the admissibility of $v(t)$, the inequality in (18) is satisfied.

Remark 1. *All strategies defined by* (11), (12), (19), *and* (20) *are non-anticipative, the pursuers choose actions based on current and past information.*

We let the pursuers x_i use the following strategies:

$$u_i(t) = \frac{\rho_i}{\tilde{\rho}_i}\omega_i(t), \quad \tilde{\rho}_i = \rho_i + \frac{\lambda h}{e^{h\theta} - 1}, \quad 0 \le t \le \theta, \tag{21}$$

where

$$\omega_i(t) = v(t), \quad 0 \le t \le \theta,$$

if $x_{i0} = y_0$, and if $x_{i0} \ne y_0$, then

$$\omega_i(t) = \begin{cases} v(t) - \langle v(t), \xi_i\rangle\xi_i + \xi_i\sqrt{e^{2ht}(\tilde{\rho}_i^2 - \sigma^2) + \langle v(t), \xi_i\rangle^2}, & 0 \le t \le \tau_i, \\ v(t), & \tau_i < t \le \theta. \end{cases} \tag{22}$$

It should be noted that the time τ_i in (22) is taken to be exactly the capture time of the fictitious pursuer defined in (20). The real pursuer is not required to have captured the evader at this instant; it simply follows the given strategy up to τ_i and then proceeds with

$v(t)$. The admissibility of $\omega(t)$ can be proved in the same way as the admissibility of $\omega_i^\epsilon(t)$. Consequently, the strategy defined by (21) is admissible; that is, it satisfies condition (2).

4.2. Guaranteeing the Value λ for the Pursuers

We aim to show that the constructed strategies satisfy the following inequality:

$$\sup_{v(\cdot)} \inf_{i\in\mathcal{I}} \|y(\theta) - x_i(\theta)\| \leq \lambda. \tag{23}$$

By the definition of λ (17), the following holds for any $\epsilon > 0$:

$$K(y_0, r(\theta)) \subset \bigcup_{i=1}^{\infty} K(x_{i0}, R_i(\theta) + \lambda + \epsilon).$$

By the hypothesis of the theorem, $\langle y_0 - x_{i0}, p_0\rangle \geq 0$ for all $i \in \mathcal{I}$. Since Lemma 3(a) is true for arbitrary $R_i(\theta)$, it remains valid for $R_i(\theta) + \lambda + \epsilon$. Then, we have

$$K(y_0, r(\theta)) \subset \bigcup_{i=1}^{\infty} X_i^\epsilon$$

where for $x_{i0} \neq y_0$,

$$X_i^\epsilon = \left\{z \mid 2\langle y_0 - x_{i0}, z\rangle \leq (R_i(\theta) + \lambda + \epsilon)^2 - r^2(\theta) + \|y_0\|^2 - \|x_{i0}\|^2\right\},$$

and for $x_{i0} = y_0$,

$$X_i^\epsilon = \{z \mid \langle p_0, z - y_0\rangle \leq R_i(\theta) + \lambda + \epsilon\}.$$

Thus $y(\theta) \in X_s^\epsilon$ for some $s = s(\epsilon) \in \mathcal{I}$. Since $\tilde{\rho}_s(\epsilon) > \sigma$, it follows from Lemma 2 that $z_s(\theta) = y(\theta)$ when fictitious pursuers use strategies (19) and (20). Now we consider two possible cases for the initial position of the s-th pursuer.

Case 1: $x_{s0} = y_0$. In this case, by (19), we have $\omega_s^\epsilon(t) = \omega_s(t) = v(t)$. Consequently, the strategy for the pursuer x_s given by (21) takes the form $u_s(t) = \frac{\rho_s}{\tilde{\rho}_s} v(t)$. Using the fact that $z_s(t) = y(t)$ for all $t \in [0, \theta]$, we obtain

$$\|y(\theta) - x_s(\theta)\| = \|z_s(\theta) - x_s(\theta)\| = \left\|\int_0^\theta \left(v(t) - \frac{\rho_s}{\tilde{\rho}_s} v(t)\right) dt\right\| = \left(1 - \frac{\rho_s}{\tilde{\rho}_s}\right) \left\|\int_0^\theta v(t) dt\right\|.$$

Applying Lemma 1 to the evader's control, we have $\|v(t)\| \leq \sigma e^{ht}$. Therefore,

$$\left\|\int_0^\theta v(t) dt\right\| \leq \int_0^\theta \|v(t)\| dt \leq \sigma \int_0^\theta e^{ht} dt = \frac{\sigma}{h}(e^{h\theta} - 1).$$

Substituting the expression for $\tilde{\rho}_s = \rho_s + \frac{\lambda h}{e^{h\theta}-1}$, we compute

$$1 - \frac{\rho_s}{\tilde{\rho}_s} = \frac{\frac{\lambda h}{e^{h\theta}-1}}{\rho_s + \frac{\lambda h}{e^{h\theta}-1}} = \frac{\lambda h}{\tilde{\rho}_s(e^{h\theta} - 1)}.$$

Combining these estimates yields

$$\|y(\theta) - x_s(\theta)\| \leq \frac{\lambda h}{\tilde{\rho}_s(e^{h\theta} - 1)} \cdot \frac{\sigma}{h}(e^{h\theta} - 1) = \frac{\lambda \sigma}{\tilde{\rho}_s}.$$

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By the hypothesis of Theorem 1, we have $\sigma \leq \tilde{\rho}_s$. Consequently,

$$\|y(\theta) - x_s(\theta)\| \leq \lambda.$$

Case 2: $x_{s0} \neq y_0$. In this case, using (21), we obtain

$$\begin{aligned} \|y(\theta) - x_s(\theta)\| = \|z_s(\theta) - x_s(\theta)\| &= \left\| \int_0^\theta \left(\omega_s^\epsilon(t) - \frac{\rho_s}{\tilde{\rho}_s}\omega_s(t) \right) dt \right\| \\ &\leq \int_0^\theta \|\omega_s^\epsilon(t) - \omega_s(t)\| dt + \int_0^\theta \left\| \left(1 - \frac{\rho_s}{\tilde{\rho}_s} \right) \omega_s(t) \right\| dt. \end{aligned} \tag{24}$$

Next, we estimate the right-hand side of the last inequality. We first establish that

$$\lim_{\epsilon \to 0} \sup_{i \in \mathcal{I}} \int_0^\theta \|\omega_i^\epsilon(t) - \omega_i(t)\| dt = 0. \tag{25}$$

Indeed, if $x_{i0} = y_0$, then by (19), $\omega_i^\epsilon(t) = \omega_i(t) = v(t)$, and (25) is obvious. Consider the case $x_{i0} \neq y_0$. Then, by (20), we have

$$\begin{aligned} \int_0^\theta \|\omega_i^\epsilon(t) - \omega_i(t)\| dt = \int_0^{\tau_i} \Big[&\left(e^{2ht}(\tilde{\rho}_i^2(\epsilon) - \sigma^2) + \langle v(t), \xi_i \rangle^2 \right)^{1/2} \\ &- \left(e^{2ht}(\tilde{\rho}_i^2 - \sigma^2) + \langle v(t), \xi_i \rangle^2 \right)^{1/2} \Big] dt. \end{aligned}$$

Since

$$\begin{aligned} &\left(e^{2ht}(\tilde{\rho}_i^2(\epsilon) - \sigma^2) + \langle v(t), \xi_i \rangle^2 \right)^{1/2} - \left(e^{2ht}(\tilde{\rho}_i^2 - \sigma^2) + \langle v(t), \xi_i \rangle^2 \right)^{1/2} \\ &= \frac{|\tilde{\rho}_i^2(\epsilon) - \tilde{\rho}_i^2| e^{2ht}}{\left(e^{2ht}(\tilde{\rho}_i^2(\epsilon) - \sigma^2) + \langle v(t), \xi_i \rangle^2 \right)^{1/2} + \left(e^{2ht}(\tilde{\rho}_i^2 - \sigma^2) + \langle v(t), \xi_i \rangle^2 \right)^{1/2}} \\ &\leq \frac{|\tilde{\rho}_i^2(\epsilon) - \tilde{\rho}_i^2| e^{2ht}}{\left(e^{2ht}(\tilde{\rho}_i^2(\epsilon) - \sigma^2) \right)^{1/2} + \left(e^{2ht}(\tilde{\rho}_i^2 - \sigma^2) \right)^{1/2}} \\ &\leq \frac{|\tilde{\rho}_i^2(\epsilon) - \tilde{\rho}_i^2| e^{2ht}}{\left(e^{2ht}(\tilde{\rho}_i^2(\epsilon) - \tilde{\rho}_i^2) \right)^{1/2}} = (\tilde{\rho}_i^2(\epsilon) - \tilde{\rho}_i^2)^{1/2} e^{ht} \end{aligned}$$

and $\tau_i \leq \theta$, we obtain

$$\begin{aligned} \int_0^\theta \|\omega_i^\epsilon(t) - \omega_i(t)\| dt &\leq (\tilde{\rho}_i^2(\epsilon) - \tilde{\rho}_i^2)^{1/2} \int_0^\theta e^{ht} dt \\ &= \sqrt{\frac{2\tilde{\rho}_i h}{e^{h\theta} - 1}\epsilon + \frac{h^2}{(e^{h\theta} - 1)^2}\epsilon^2} \cdot \frac{e^{h\theta} - 1}{h} \\ &= \sqrt{\epsilon} \sqrt{\frac{2\tilde{\rho}_i(e^{h\theta} - 1)}{h} + \epsilon} \leq H\sqrt{\epsilon}, \end{aligned}$$

where H is some positive number that does not depend on i. The proof of relation (25) is complete.

For the second integral in (24),

$$\int_0^\theta \left\| \left(1 - \frac{\rho_s}{\tilde{\rho}_s}\right) \omega_s(t) \right\| dt \le \left| 1 - \frac{\rho_s}{\tilde{\rho}_s} \right| \tilde{\rho}_s \int_0^\theta e^{ht} dt = \frac{\lambda h}{e^{h\theta} - 1} \cdot \frac{e^{h\theta} - 1}{h} = \lambda.$$

Thus, in Case 2, $\|y(\theta) - x_s(\theta)\| \le \lambda + H\sqrt{\epsilon}$.

Combining Cases 1 and 2, we have shown that for the pursuer x_s, the distance at the terminal time satisfies $\|y(\theta) - x_s(\theta)\| \le \lambda + H\sqrt{\epsilon}$. Since $0 < \epsilon < 1$ is arbitrary, this proves inequality (23). Hence, when the pursuers use the strategies given in (21), inequality (23) is satisfied.

4.3. Guaranteeing λ for the Evader

We construct a strategy for the evader, which guarantees that the distance at the terminal time is at least λ. In other words, we establish the inequality

$$\inf_{u_1(\cdot),\ldots,u_m(\cdot),\ldots} \inf_{i \in \mathcal{I}} \|y(\theta) - x_i(\theta)\| \ge \lambda \tag{26}$$

holds, where $u_1(\cdot), \ldots, u_m(\cdot), \ldots$ are arbitrary admissible controls of the pursuers.

If $\lambda = 0$, inequality (26) is trivial for any admissible evader control. For $\lambda > 0$, by the definition of λ, for every $\varepsilon > 0$ we have

$$K(y_0, r(\theta)) \not\subset \bigcup_{i=1}^{\infty} K(x_{i0}, R_i(\theta) + \lambda - \varepsilon).$$

From Lemma 3(b), it follows that there exists a point $\tilde{y} \in L(y_0, r(\theta))$ such that

$$\|\tilde{y} - x_{i0}\| \ge R_i(\theta) + \lambda.$$

Hence

$$\|\tilde{y} - x_i(\theta)\| \ge \|\tilde{y} - x_{i0}\| - \|x_i(\theta) - x_{i0}\| \ge R_i(\theta) + \lambda - R_i(\theta) = \lambda. \tag{27}$$

Let the evader use the control

$$v(t) = \frac{h(\tilde{y} - y_0)}{e^{h\theta} - 1} e^{ht}.$$

This control is admissible. Indeed, since $\tilde{y} \in L(y_0, r(\theta))$, we have

$$\|\tilde{y} - y_0\| = r(\theta) = \frac{\sigma}{h}(e^{h\theta} - 1).$$

It follows that

$$\|v(t)\| = \frac{h}{e^{h\theta} - 1} \|\tilde{y} - y_0\| e^{ht} = \sigma e^{ht}.$$

Substituting this into (3) yields

$$\|v(t)\|^2 = \sigma^2 e^{2ht} = \sigma^2 + \sigma^2(e^{2ht} - 1) = \sigma^2 + 2h \int_0^t \sigma^2 e^{2hs} ds = \sigma^2 + 2h \int_0^t \|v(s)\|^2 ds.$$

Thus,

$$\|v(t)\|^2 = \sigma^2 + 2h \int_0^t \|v(s)\|^2 ds,$$

which shows that $v(t)$ satisfies (3) with equality. Therefore, the evader's control is admissible.

For this control, by (27), inequality (26) holds. In fact,

$$y(\theta) = y_0 + \int_0^\theta v(s)ds = y_0 + \frac{h(\tilde{y} - y_0)}{e^{h\theta} - 1} \int_0^\theta e^{hs} ds = y_0 + \frac{h(\tilde{y} - y_0)}{e^{h\theta} - 1} \cdot \frac{e^{h\theta} - 1}{h}$$
$$= y_0 + (\tilde{y} - y_0) = \tilde{y}.$$

Thus, the theorem is proved.

5. Conclusions

We have studied a pursuit–evasion differential game in the Hilbert space l_2 involving countably many pursuers and a single evader. The control functions of the players are subject to Grönwall-type constraints. We used some ideas from the work of Ibragimov (2005), where a simple motion pursuit–evasion differential game with geometric constraints was studied. For the case of Grönwall-type constraints, the value of the game λ has been found. Optimal strategies for the pursuers and the evader have been constructed. To construct the optimal strategies of the pursuers, the method of fictitious pursuers has been used.

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