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Copyright Enforcement in Content-Sharing Platforms

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Abstract: This paper analyzes the choice between quality improvements and copyright litigation by a proprietary seller who faces a competitive threat from a content-sharing platform. The platform operates like a "public good" with contributors who share content and free-riders who only consume content while adding to congestion on the platform. When the proprietor can identify contributors in the platform with sufficient accuracy, a litigation strategy that targets contributors exacerbates free-riding behavior in the sharing platform and drives down platform quality. In contrast, investing in quality improvements for the copyrighted good does not affect contribution decisions on the platform, leading to a uniform decrease in the relative payoff for all users on the platform. The model presented in the paper shows that the proprietor finds litigation more profitable than quality improvements if she can target contributors accurately. Welfare analysis of the model shows that the proprietor has too high an incentive to invest in litigation and inefficiently low incentives for quality improvements of the copyrighted good.

Keywords: copyright enforcement; file-sharing; free-riding; public goods

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1. Introduction

Technological advances and the rapid digitization of content in the last two decades has led to the rise of several content-sharing platforms. This has posed a unique competitive challenge to traditional proprietary producers of entertainment products such as music and movies. Much of the challenge comes from the way in which the production of these platforms is organized. Sharing platforms often operate like a public good where every platform user's voluntary contribution of content is freely available. While some platforms such as YouTube may offer monetary incentives for users to contribute content, many platforms on social media and file-sharing networks such as BitTorrent are sustained by non-monetary incentives including altruism, reputational concerns, etc.

Proprietary entities, especially music distributors and movie rights holders, have responded to this growing threat by pursuing copyright infringement lawsuits against platform users. A well-known example of this is the slate of lawsuits initiated by the Record Industry Association of America (RIAA) and the Motion Pictures Association of America (MPAA), the trade groups that represents the music and movie industries respectively in the US, against users of file sharing networks such as Napster, Kazaa and BitTorrent in the early 2000s [1]. These lawsuits against individual infringers ended in 2008. However, the industry continues to monitor and penalize file-sharing activity with the help of Internet Service Providers where potential infringers are subject to slower internet connections or even suspension of users' internet access altogether.

These lawsuits, as well as on-going copyright enforcement by rights holders against individual platform users, have been controversial for several reasons. First, there are problems in accurately identifying infringers, and there have been several cases of "mistaken identity" where a non-violator has been sued, resulting in costly litigation [2,3]. Second, several industry analysts have argued that tough copyright enforcement has made proprietary entities slow to improve their quality by digitizing content and setting competitive prices that would benefit consumers of these entertainment products [4].

In this paper, I explore competitive strategies that copyright holders can use in the presence of sharing platforms. Specifically, I look at a copyright holder's choice between pursuing copyright enforcement through litigation against platform users and improving the quality of her own proprietary product. What makes this analysis interesting is the public good nature of the competitor, which, in this case, has unique implications for when and how copyright holders choose to litigate. As with any public good, user contributions to a sharing platform generate positive externalities whose value cannot be completely appropriated by the contributing user. This creates incentives for users to free-ride which, in the presence of congestion effects, impacts the quality of the shared platform. If the copyright holder can effectively target contributors in the platform through copyright litigation, she can drive down platform quality and hence improve her competitive position in the market.

I argue that copyright litigation and quality improvements affect demand for the copyrighted good in substantively different ways, and hence the choice between the two strategies depends on specific features of the sharing platform. Specifically, while an improvement in the copyrighted good's quality has a uniform impact on the relative payoff to both contributors and free-riders on the sharing platform, a well-targeted litigation strategy that places a higher burden of litigation on contributors can lower platform quality, and hence enable the copyright holder to set a high price while keeping her product's quality low.

To present this analysis, I look at a model of consumers who choose between joining the sharing platform and buying the proprietary good. All consumers receive altruistic utility from contributing content to the platform if they decide to join it; however, they have heterogeneous costs of contributing to the platform, so that some high-cost users in the platform free-ride. All users, contributors and free-riders also receive utility from consuming content on the platform. User utility depends on the quality of the platform which is subject to congestion effects. Congestion effects in turn depend on the proportion of contributors in the user population. The platform equilibrium is then defined by two margins—the joining margin and the contribution margin. I show that the pricing decision of the copyright holder affects both of these margins and influences congestion on the platform. Thus, the copyright holder controls platform quality through prices.

I then consider two competitive strategies that the copyright holder can employ to improve profits—quality improvements and copyright litigation. Improvements in quality enhance demand for the copyright holder by lowering the joining margin in the platform; however, quality improvements have no effect on the contribution margin and hence do not lower platform quality. On the other hand, copyright litigation, if targeted against content contributors, affects platform quality. The profitability of quality improvements in the copyrighted good relative to litigation depends on how well contributors can be targeted and how low the initial quality of the copyrighted good is.

Finally, I consider the welfare implications of each strategy to provide policy recommendations on copyright laws in online content. There are two kinds of inefficiencies that can potentially arise when the proprietary seller faces competition from a sharing group characterized by free-riding. On the one hand, the existence of free-riding reduces the quality of the shared good below the optimal level. This causes an underproduction of the shared good. Secondly, since the proprietor has market power, the equilibrium price chosen by the proprietor is too high, causing an inefficiently low proportion of users to buy the copyrighted good. This leads to an underproduction of the copyrighted good. I show that litigation by the proprietor exacerbates the first kind of inefficiency since it worsens free-riding. However, by inducing more consumers to buy the copyrighted good, it can reduce the inefficiency resulting from underproduction of the copyrighted good. I find that optimal investment then depends on the trade-off between these two inefficiencies.

The results of the welfare analysis have significant implications for public policy surrounding copyright enforcement. Many Western nations including the United States are making it easier to prosecute copyright infringers. I show through my model that copyright holders have higher-than-efficient incentives to invest in copyright litigation instead of quality improvements. Thus, I argue that current global trends towards strengthening copyright enforcement of online content are likely to have a detrimental impact on the quality of entertainment products.

My paper contributes to existing research on the effects of piracy and optimal copyright enforcement. Novos and Waldman (1984) [5] show that the existence of copying reduces the monopolist's incentive to increase the quality of her product, and hence they predict that an increase in copyright protection is likely to increase welfare. A more recent application has been to issues of software piracy and enforcement of intellectual property rights. Bae and Choi (2006) [6] claim that the short-run and long-run effects of intellectual property rights protection depend on whether it increases the reproduction costs or quality degradation costs of piracy. Sundararajan (2004) [7] analyzes Digital Rights Management (DRM) as a possible tool that sellers can use to discourage piracy. DRM often entails a cost in terms of reduced flexibility of the product, and hence the monopolist has to trade off deterring piracy with a value reduction in the legal good. Much of the research on piracy and copyright protection has assumed that the copied good is of inferior quality as compared with the original good. In contrast, I argue that, when the shared good exhibits public good characteristics, its quality depends on strategic decisions made by the copyright holder on prices and copyright enforcement.

There has been limited research looking at the issue of sharing groups where members contribute towards the production of the copied or pirated good. Bakos et. al. (1999) [8] consider sharing teams formed by pre-existing social relationships such as families. Recognizing that diversity in consumer valuations can harm the seller when she cannot price discriminate, they argue that the effect of sharing teams on seller profit depends on whether the variation in average valuation across teams is higher or lower than the variation in valuation across individuals. However, they assume that the group's willingness to pay for a good is simply the sum of the willingness to pay of individuals in that group. If the group members have differing valuations which is observable only to them, each member will understate their valuation in the hope of free-riding on other members' contributions [9]. The current analysis studies the effects of such free-riding behavior in sharing groups.

To my knowledge, there has been no research into the unique competitive dynamics that emerge when a proprietary copyright holder faces competition from a sharing platform with public good characteristics. I argue that the structure of production of the shared good has important implications for when and how copyright litigation can be profitable or welfare enhancing. Given that much of copyright infringement occurs in this format, my paper fills an important gap in literature.

An important insight from the analysis presented in this paper is that when the competing good has public good characteristics, the proprietor can exploit the free-riding behavior inherent in these sharing platforms to sustain a high price for her good even when a large proportion of the consumer population uses the shared good. Although there are many instances of this type of competition between a proprietary seller and a public good alternative formed by user-generated content, in order to make my analysis concrete, I focus on the case of file-sharing networks. File-sharing networks, the most prominent of which is BitTorrent, suffer from congestion effects when there are too many downloading users. When users log on to a file-sharing network, they can potentially take two actions: they can passively search and download music files provided by other "peers" on the network, and they can upload or share their own files for other users to download. While downloading music provides a direct benefit to the user, there is little or no direct benefit from uploading files. However, the quality of content on file-sharing networks depends on its users uploading a large variety of files with popular content onto the network. Free-riding affects the quality of the network in two ways. First, there is a direct effect in terms of reduced availability of content. The second is an indirect effect. As download requests from a large number of users are directed to a small number of uploading peers on the network, there is increased congestion leading to slower downloads speeds. Thus, these networks face a severe "public good" problem of securing enough contributions from their members to make them truly beneficial [10,11]. These characteristics of file-sharing networks make them unique targets of copyright litigation by copyright holders.

The paper is organized as follows. Section 2 outlines the model. In Section 3, I look at the relationship between the proprietor's price equilibrium and characteristics of the platform equilibrium in terms of congestion and free-riding. Section 4 describes the profitability of copyright litigation and its effect on the quality of the shared platform. Section 5 describes the proprietor's allocation of investment between litigation and quality improvements of the proprietary good. Section 6 concludes. All proofs are in the Appendix A.

2. Model

There is a unit measure of consumers in the population. Each consumer can decide whether to buy the copyrighted good from a proprietary seller or consume a shared good by joining a sharing platform that I denote by $\mathcal{N}$. The quality of the proprietary copyrighted good is determined by the proprietor, while the quality of the shared good is determined by the contributions made by the users on the platform. Before describing the value functions for each type of good, let us understand the context of the goods in question. The good being considered here can be interpreted in different ways. First, it can be thought of as a single piece of content such as a song or movie. Under this interpretation, the appropriate shared good to consider would be peer-to-peer file-sharing networks. In these networks every file is broken into packets and distributed across sharing peers who are users contributing a part of the file for other users to download. After a file has been downloaded by a user, her device may act as a "server" that the network can access to complete other users' download requests. Users can choose to supply bandwidth by making their downloaded file available for others to access or not. The quality of the downloads depends on how users contribute to the platform in this manner [12]. Since the good here is typically a direct copy of the copyrighted content, the quality of the original proprietary content can affect the quality of the file being shared.

Second, instead of a single piece of entertainment content, the good modeled here can also be thought of as a platform that provides a variety of entertainment content such as subscription services for music and entertainment. In many such sharing platforms, users share both copied content as well as some original content. Again, as with the case of the files-sharing networks, the sharing platform's quality is dependent on the number of users contributing content to it. However, the presence of original content or modifications of copied content reduces the sensitivity of platform quality on proprietary good quality relative to file-sharing networks. As described below, I account for varying levels of correlation between platform quality and proprietary quality in the model so as to provide a more robust analysis that could apply to both file-sharing networks as well as other sharing platforms.

Let us denote the quality of the proprietary good by Q. If the consumer purchases the proprietary good at price P, she gets a surplus of

$$U^o = Q - P.$$

Now, let us look at the payoffs to a user in the sharing platform. If an individual i decides to join the sharing platform, she has to decide how much to contribute to the group. Let c_i denote user i's contribution. I assume that $c_i \in \{0, 1\}$ so that the user faces a discrete choice of contributing 1 or 0 in $\mathcal{N}$. I refer to all users who join the sharing group but do not make a contribution as "free-riders." Users also face the possibility of being sued by the seller of the proprietary good. The probability of facing copyright litigation depends on their contribution to the platform. $\mathcal{L}(c_i)$ denotes a user's probability of litigation and D is the cost of litigation incurred by the infringing user.

The quality of the shared good to a user depends on the average contribution made by all other users in the group. Furthermore, the shared platform has both content that is directly pirated or copied from the proprietary good, as well as some original content. Given that some part of the shared good is typically copied from the proprietary good, its quality then also depends on the quality of the proprietary good itself. Thus, I model the quality of the shared good to user i as

$$\delta = (\alpha Q + 1) \sum_{j \in \mathcal{N}_{-i}} \frac{c_j}{n},$$

Here, $\alpha \in (0, 1)$ represents the dependence of shared good quality on the proprietary good. Low values of α then imply that improvements in the proprietary good do not have a significant effect on the quality of the shared product and much of the shared platform has original content. The magnitude of α can be thought to reflect the extent of copied content on the platform and it can also be an indicator of the quality of the copying technology. Before digital copying technology became widespread, it was often the case that copied content was not identical to the original. For example, at one time, pirating a movie literally meant carrying a video recorder to the cinema and recording the film playing on the cinema screen, often resulting in pirated copies with poor sound quality and shaky images with low resolution. This would have meant a low α in our model. In comparison, digital copies now are practically identical to the original, and hence would translate into higher values of α. $\mathcal{N}_{-i}$ is the set of all users in the sharing platform other than i, and n is the number of users in $\mathcal{N}_{-i}$. The payoff to the consumer from being in the sharing group is modeled as a function of the average contributions in order to capture congestion effects. Under this assumed functional form, quality decreases if a user joins the platform but does not contribute. Various studies have shown the presence of congestion in sharing platforms. For example, in the case of peer-to-peer file-sharing networks, a digital file such as a movie or song is typically broken up into small packets that are then distributed across participating users or "peers". When a file download is requested by a user, the system locates the component packets of the files across different peers. The more peers who contribute by sharing their files, the lower the congestion and quicker the download process for the file. On the other hand, as more peers download content without sharing files, the bandwidth available to other users decreases, and hence reduces the speed and quality of downloads [13–15].

The cost of making a contribution to the shared platform varies across users and is denoted by ν_i for user i. I assume a uniform distribution of consumers, i.e., $\nu_i \sim U[0, 1]$. Following the model of "impure altruism" by Andreoni (1990) [16], I assume that users also care altruistically about their own contribution. Every user obtains a utility of $\beta > 0$ from making a contribution to the shared good. Thus, the utility of users for contributing c_i in $\mathcal{N}$ is

$$U^N(c_i, \nu_i) = (\alpha Q + 1) \sum_{j \in \mathcal{N}_{-i}} \frac{c_j}{n} + \beta c_i - \nu_i c_i - D\mathcal{L}(c_i).$$

Finally, I assume that the proprietor can make an investment in two types of strategies when she faces a competitive threat from the sharing platform—(1) she can invest to increase the quality of the proprietary good, and (2) she can invest to pursue copyright litigation against users in the sharing platform.

The production function for investment in quality is $\phi(x)$. An investment of x in quality delivers quality $Q = Q_o + \phi(x)$ for the proprietary good, where Q_o is the initial quality of the proprietary good. I make standard assumptions on the quality production function—(1) quality is increasing in the level of investment or $\phi' > 0$, (2) the marginal return to investment approaches infinity at zero investment, $\lim_{x \to 0} \phi'(x) \to \infty$, (3) the rate of increase in quality weakly decreases as investment increases, i.e., $\phi'' < 0$ and (4) quality remains unchanged if there is no investment or $\phi(0) = 0$.

I assume a simple production function for investment in litigation where every dollar of investment leads to a unit increase in litigation costs for the user who faces the copyright infringement lawsuit. Thus, an investment of D leads to a litigation cost of D to every user who is sued by the proprietor.

In the following sections, I first model the investment choice as a binary decision between the two strategies, i.e., I assume that the proprietor either chooses to invest in improving the quality of the proprietary good or in copyright litigation. I then extend the analysis to consider a continuous choice of investment allocation between the two strategies in Section 5.

The timing of the game is as follows. The proprietor chooses a price P and investment allocation (x, D) where the first element in the allocation vector is the investment in quality and the second element is the investment in litigation. After observing, P, x and D, each consumer i decides whether to buy the copyrighted good or to join $\mathcal{N}$. If the consumer decides to join $\mathcal{N}$, she chooses her contribution level c_i. I restrict the analysis to symmetric equilibria. The solution concept used is Subgame Perfect Nash Equilibrium.

In the next two sections, I look at the effects of proprietary good quality and litigation separately. This is then followed by a section where I describe the proprietor's choice of investment in these two strategies.

3. Platform Equilibrium and Proprietary Good Quality

Let us start by looking at the case where there is no copyright litigation or quality improvements, i.e., $D = 0$ and $x = 0$, so that $Q = Q_o$. This benchmark analysis helps to highlight the relationship between the proprietary good quality and the incentives for free-riding in the sharing platform, which in turn also affects the quality of the shared good.

Consumer surplus from the proprietary good given price P is $U^o = Q_o - P$. Taking P as given, let us determine the platform equilibrium, i.e., user and contributor population. Every user has to make two decisions: the first is whether to buy the proprietary good at price P or join the shared platform. Second, if she decides to join the platform, she must decide whether or not to contribute towards the shared good. Let us take the contribution decision first. Since users have different contribution costs, the decision to contribute is determined by a threshold level of contribution cost ν^c, which I call the contribution margin. A user who joins $\mathcal{N}$ contributes if and only if $\nu_i \leq \beta$. This gives us a contribution margin of $\nu^c = \beta$. Potential free-riders are consumers with $\nu_i \geq \nu^c$, and potential contributors are consumers with $\nu_i < \nu^c$.

Given the contribution margin, we can determine the joining decision rule for users by looking at two cases. In the first case, all users contribute so that platform quality is $\delta = 1$. In this case, the marginal user who is indifferent between the shared good and the proprietary good is a contributor, and hence incurs a contribution cost. Let us denote the cost of the marginal user who joins by ν^J and call it the joining margin. The payoff to the marginal user-contributor with cost ν^J from joining $\mathcal{N}$ is then $U^N = \alpha Q + 1 + \beta - \nu^J$. Since this consumer is indifferent between joining and contributing in $\mathcal{N}$ and buying the proprietary good, ν^J must solve $\nu^J = \alpha Q + 1 + \beta - U^o$. In order for this to be an equilibrium with no free-riders it must be the case that $\nu^J \leq v^c$, i.e., $\alpha Q + 1 + \beta - U^o \leq \beta$, or $U^o \geq 1 + \alpha Q$.

If $U^o < 1 + \alpha Q$, then some of the users who join $\mathcal{N}$ are free-riders who do not contribute. Now, the marginal consumer who joins does not contribute, so that her payoff from the shared good is δ. Since every consumer in $\mathcal{N}$ has zero measure, no one consumer's decision to join the platform as a free-rider influences its quality. This means that if the marginal contributor who is indifferent between free-riding in the platform and contributing joins, then all other consumers also join $\mathcal{N}$. As a result, $\nu^J = 1$, $\delta = \beta$ and the platform is characterized by congestion and free-riding, which lowers its quality below 1.

Thus, the platform equilibrium ν_0^J is

$$\nu_0^J = \begin{cases} 1 \text{ if } U^o \leq 1 + \alpha Q, \\ 1 + \beta - U^o \text{ if } U^o \geq 1. \end{cases}$$

or

$$v_0^J = \begin{cases} 1 \text{ if } Q(1-\alpha) - P \leq 1, \\ 1 + \beta - U^o \text{ if } Q(1-\alpha) - P \geq 1. \end{cases}$$

The corresponding platform quality is

$$\delta_0 = \begin{cases} \beta \text{ if } Q(1-\alpha) - P \leq 1, \\ 1 \text{ if } Q(1-\alpha) - P \geq 1. \end{cases}$$

Observe that as the consumer surplus from the proprietary good increases, platform quality also increases. A high surplus from the proprietary good reduces the incentives to join $\mathcal{N}$ due to platform participation costs. However, as fewer users join, average contributions become larger and congestion is lower. U^o is high or low depending on the price charged by the proprietor. When the proprietor charges a high price, U^o is low, resulting in a large number of users, including free-riders, joining the shared platform. This leads to congestion and low platform quality. Conversely, if proprietary good price is low, fewer users join the shared platform, resulting in less congestion and higher platform quality.

Proposition 1 describes the pricing and platform equilibrium for different ranges of Q and β. In order to restrict the number of cases to consider, I assume that fewer than half of the users contribute if they join the platform, i.e., $\beta \leq \frac{1}{2}$. I also restrict $Q_o \geq \frac{\beta}{1-\alpha}$, so that even in the absence of investments in quality, the proprietary good is a superior substitute for the shared good if it is characterized by congestion. At the same time, it is possible for the shared platform to have higher quality than the proprietary good if more users who join contribute.

I make the following indifference assumptions. If a consumer is indifferent between buying from the proprietor and using the shared good, she chooses to buy the proprietary good. This ensures that the proprietor's profit function is continuous in prices. If a user is indifferent between contributing in the sharing platform and free-riding, she free-rides.

Proposition 1. *There exists $\widetilde{Q}_o > \frac{\beta}{1-\alpha}$, such that the following describes the equilibrium outcome when there is no investment in litigation.*

(a) *If $0 < Q_o \leq \frac{\beta}{1-\alpha}$, then everyone joins $\mathcal{N}$ even if the price of the copyrighted good is zero, i.e., $v_0^{J*} = 1$ and $P_0^* = 0$.*

(b) *If $\frac{\beta}{1-\alpha} < Q_o < \widetilde{Q}_o$, then a positive measure of users buy the copyrighted good, but the proprietor does not capture the entire market, i.e., $v_0^{J*} = \beta$ and $P_0^* = (1-\alpha)Q_o - \beta$.*

(c) *If $Q_o \geq \widetilde{Q}_o$, then the proprietor captures the entire market, so that at $P_0^* = (1-\alpha)Q_o - 1 - \beta$.*

When the initial quality of the proprietor's good is lower than the minimum possible quality of the shared good, then no one buys the copyrighted good, even at zero price. At the other extreme, if initial quality of the copyrighted good is high enough, then even the user with the lowest marginal cost buys the copyrighted good. If Q_o is in an intermediate range, there is a positive measure of consumers to buy the copyrighted good as well as a positive measure of users in the platform. We can see that when quality of the proprietary good is low, the platform equilibrium has free-riders. In contrast, at higher quality levels, the platform equilibrium is comprised only of contributors. Thus, the presence of congestion and the quality of the platform is affected by the quality of the proprietary substitute.

Given the effect that Q_o has on platform quality, let us understand when and how quality improvements increase proprietor profits. Proposition 2 summarizes the increase in profits from an investment in quality of x.

Proposition 2. *The increase in proprietor profits following an investment in quality of x, given initial quality Q_o, is as follows:*

(a) For $Q_o \in \left[0, \frac{\beta}{1-\alpha}\right]$, if $\phi(x) \leq \frac{\beta}{1-\alpha} - Q_o$, profits do not change. If $\frac{\beta}{1-\alpha} - Q_o \leq \phi(x) \leq \widetilde{Q}_o - Q_o$, increase in profit is $[(1-\alpha)(Q_o + \phi(x)) - \beta](1-\beta)$ and if $\phi(x) \geq \widetilde{Q}_o - Q_o$, profit increases by $(1-\alpha)(Q_o + \phi(x)) - \beta - 1$.

(b) For $Q_o \in \left[\frac{\beta}{1-\alpha}, \widetilde{Q}_o\right]$, if $\phi(x) \leq \widetilde{Q}_o - Q_o$, increase in profit is $(1-\alpha)\phi(x)$. If $\phi(x) \geq \widetilde{Q}_o - Q_o$, increase in profit is $(1-\alpha)\phi(x) - 1 + \beta((1-\alpha)Q_o - \beta)$.

(c) For $Q_o \geq \widetilde{Q}_o$, increase in profit is $(1-\alpha)\phi(x)$.

An improvement in quality allows the proprietor to affect the joining margin; however, the contribution margin in the platform, $\nu^c = \beta$, remains unaffected, as does platform quality. Then, the increase in the marginal user's surplus from the proprietary good is exactly $\phi(x)$. As we see in the next section, copyright litigation targeted against contributors affects the marginal user's relative payoff from the shared good in two ways—first is a direct effect of the increase in cost of using the platform and second is an indirect effect of a lower contribution margin, resulting in lowering platform quality. The direct effect is similar in effect to an increase in the quality of the proprietary good.

4. Copyright Litigation against Contributors

I now look at how copyright litigation affects the platform equilibrium and the profits to the proprietor. To focus on how litigation affects proprietary prices and the shared platform, I begin by fixing $x = 0$ so that quality of the proprietary good is the initial quality, $Q = Q_o$. In order to distinguish the equilibrium here from the equilibrium without litigation derived in the previous section, I use the subscript L for all endogenous variables—ν_L^J, ν_L^c, δ_L and P_L.

To maximize the impact of litigation, the proprietor would ideally like to inflict a loss of D only on contributors in the platform, as this ensures maximal costs on users of the platform. However, I assume that the proprietor cannot observe contributors on the platform, and hence cannot target them perfectly. The proprietor, instead, receives a signal about user contributions, $s \in \left\{s^H, s^L\right\}$ which has the following conditional density.

$$
\begin{aligned}
\Pr\left(s^H | c_i = 1\right) &= \rho, \\
\Pr\left(s^H | c_i = 0\right) &= 1 - \rho,
\end{aligned}
$$

where $\rho \in \left[\frac{1}{2}, 1\right]$ denotes the informativeness of the signal received by the proprietor. So, if $\rho = \frac{1}{2}$, the signal is completely uninformative. If $\rho = 1$, the proprietor can perfectly distinguish contributors from free-riders. Based on her signal, suppose the proprietor chooses a litigation strategy where she sues with probability $\lambda\left(s^k\right) = \lambda^k \in [0,1]$, if she observes signal s^k, $k \in \{H, L\}$, then the litigation threat faced by a user in $\mathcal{N}$ is given by

$$
\mathcal{L}(c_i) = \begin{cases} \rho\lambda^H + (1-\rho)\lambda^L \text{ if } c_i = 1, \\ (1-\rho)\lambda^H + \rho\lambda^L \text{ if } c_i = 0. \end{cases}
$$

Given the proprietor's litigation strategy, and signal generating function, the expected litigation cost for a free-rider is $D\left[(1-\rho)\lambda^H + \rho\lambda^L\right]$, while the expected litigation cost faced by a contributor is $D\left[\rho\lambda^H + (1-\rho)\lambda^L\right]$. As before, a user in the sharing group contributes only if her marginal cost of contributing is low enough. Comparing the payoff to the user from contributing with the payoff from free-riding gives us a cut-off marginal cost of contribution, ν_L^c, which is

$$
\nu_L^c = \beta - D(2\rho - 1)\left(\lambda^H - \lambda^L\right).
$$

A user in the platform contributes only if $\nu_i \leq \nu_L^c$. Note that if $\lambda^H > \lambda^L$, so that contributors are targeted in litigation, $\nu_L^c < \beta$, i.e., the incentive to contribute in the platform is lower than without litigation. Moreover, the impact on contributions is increasing in the informativeness of the signal. If the signal does not contain any information about contributors, i.e., if $\rho = \frac{1}{2}$, then the difference in incentives falls to zero, and the proprietor can do no better in reducing contributions with litigation than without.

Similarly, if $\lambda^H = \lambda^L$, $\nu_L^c = \beta$, so that there is no difference in litigation risk between free-riders and contributors and, again, contribution incentives remain unaffected by litigation.

Since contributors can also generate a low signal with a positive probability, it may be optimal for the proprietor to assign a positive litigation probability to users who generate a low signal, i.e., $\lambda^L > 0$ is possible in a profit-maximizing equilibrium. The optimal λ^L thus needs to be solved as part of the proprietor's profit maximization problem along with price. However, it is always profit maximizing to litigate all users with a high signal. This is stated in the lemma below.

Lemma 1. *The proprietor always chooses $\lambda^H = 1$ if she invests in litigation.*

Given D, the expected threat of litigation is $\rho\lambda^H + (1-\rho)\lambda^L$ for a contributor and $\rho\lambda^L + (1-\rho)\lambda^H$ for a free-rider. Both are increasing in λ^H. Moreover, as λ^H increases, the average contributions in the sharing group decrease, since the relative threat of litigation for contributors is now higher, and hence more users contribute to the platform. Hence, increasing λ^H reduces the payoff to consumers from joining the sharing group both by increasing the threat of litigation and by lowering average contributions in the sharing group.

Given that $\lambda^H = 1$, as long as $\lambda^L < 1$, the probability of litigation faced by a contributor exceeds the threat imposed on a free-rider. Furthermore, the difference in the threat faced by contributors relative to free-riders increases as λ^L is lowered.

Let us now consider the implications of choosing $\lambda^L = 1$. Here, the proprietor initiates litigation against all users in the platform to inflict damages of D without distinguishing between users and contributors. When $\lambda^H = \lambda^L = 1$, we know that contribution incentives are unaffected by litigation, i.e., $\nu_L^c = \beta$, and hence platform quality with litigation is the same as without litigation, i.e., $\delta = \delta_L$. Since all users face litigation costs D, the user payoff from the platform is either $\delta + \beta - \nu_i - D$ if the user is a contributor or $\delta - D$ if the user is a free-rider. It is easy to see that, given the initial quality of the copyrighted good Q_o and price P, the demand faced by the proprietor when $\lambda^L = 1$ is identical to the demand generated by a quality enhancement of $\phi(x) = \frac{D}{1-\alpha}$. Thus, the optimal λ^L chosen by the proprietor simultaneously allows us to compare the relative profitability of quality improvements and litigation when the two are equivalent in magnitude. This is summarized in the following lemma.

Lemma 2. *If $\lambda^L = 1$, then, for any given price, P of the copyrighted good and investment x in quality such that $\phi(x) = \frac{D}{1-\alpha}$, proprietor profit from copyright litigation is equal to proprietor profit with quality improvement, i.e., $\pi_q^* = \pi_L^*$.*

This result illustrates an important distinction in the effects of investment in quality and litigation targeting contributors on demand for the copyrighted good. Looking at the difference in user payoff between the copyrighted good and the platform, targeting contributors through litigation disproportionately affects contributors, while quality improvements uniformly affect the relative payoff to all users, both contributors and free-riders. In terms of the average difference in user payoff across both types of consumers, however, quality improvements created a larger wedge than litigation with $\lambda^L < 1$.

In the next subsection, taking the price of the proprietary good as given, I first describe how consumers choose between the proprietary good and the shared good and derive the proprietor's optimal litigation strategy in terms of λ^L. I then consider the implications of free-riding for the proprietor's pricing decision in the subsequent subsection. Following the result presented in Lemma 2, for the remainder of this section, I describe a discrete investment choice for the proprietor between investing D in copyright litigation and an equivalent investment in quality of x, where $\phi(x) = \frac{D}{1-\alpha}$. This simplification helps in highlighting the different mechanisms by which each type of investment affects prices, demand and proprietor profit. In Section 5, I generalize the results to a continuous investment allocation between the two strategies.

4.1. Platform Equilibrium

In order to describe free-riding in the sharing platform, let us set aside the pricing decision and take the consumer surplus from the proprietary good, $U^o = Q_o - P$, as given. As before the consumers' decision rule to join the sharing group takes the form of a cut-off level of marginal cost, ν_L^J, and her contribution decision is another cost threshold denoted by ν_L^c.

Lemma 3.

(a) *For every λ^L there exists $\nu_L^J \in [0,1]$, such that a consumer joins the platform if and only if $\nu_i \leq \nu_L^J$.*

(b) *For $U^o + D \leq 1 + \alpha Q_o + \beta$, there exists $\widehat{\lambda}^L$, such that every user in the platform strictly prefers to contribute if and only if $\lambda^L > \widehat{\lambda}^L$.*

Lemma 3 describes the relationship between the joining margin and λ^L. Part (b) of the lemma says that the platform equilibrium is characterized by the presence of free-riding if and only if the proprietor targets contributors well enough in her litigation. In order to see why this is the case, let us look at the effect on the two margins that characterize the sharing group.

As explained in the previous section, the presence of free-riding in the platform depends on the size of the joining margin, ν_L^J, relative to the contribution margin, ν_L^c. The contribution margin increases as λ^L increases because the differential litigation risk faced by a contributor relative to a free-rider falls. Thus, platform quality increases as λ^L increases. On the other hand, the joining margin increases as λ^L increases, since the overall threat of litigation for a user in the platform, given by $\rho + (1-\rho)\lambda^L$, becomes larger. When the proprietor cannot observe contributions perfectly, targeting contributors in litigation has two effects on the incentives to join the platform. On the one hand, lowering λ^L reduces incentives to contribute among users who are already using the platform. This drives down platform quality. However, reductions in λ^L also lead to a decrease in the overall litigation threat to users joining the platform since the proprietor misses some contributors who generate a low signal. The relative strengths of the two effects depends on how well the proprietor can distinguish contributors from free-riders in the platform and also on the consumer payoff from the copyrighted good. Proposition 3 describes the conditions under which litigation with $\lambda^L < 1$ is profitable to the proprietor when she faces a binary investment choice between investing D in copyright litigation and investing x in quality improvement of an equivalent magnitude, i.e., $\phi(x) = \frac{D}{1-\alpha}$.

Proposition 3. *For any given price of the copyrighted good P, the following describes the proprietor's choice of investment when $\phi(x) = \frac{D}{1-\alpha}$.*

(a) *If $U^o \geq 1 + \alpha Q_o + \beta - D$, then the proprietor invests in improvement in quality.*

(b) *If $U^o \leq 1 + \alpha Q_o + \beta - D$, then there exists $\overline{\rho} \in \left[\frac{1}{2}, 1\right]$, such that the proprietor targets contributors in litigation by setting $\lambda^L < 1$ if and only if $\rho \geq \overline{\rho}$.*

Given that lowering λ^L has two opposing effects on the incentives to use the shared good, the profitability of choosing litigation with $\lambda^L < 1$ depends on the relative strengths of the two effects. The impact of targeting contributors on platform quality, in turn, depends on ρ as well as U^o. With a high ρ, the proprietor can detect contributors more accurately. In this case, she can lower the probability of litigation on free-riders relative to contributors without significantly reducing the overall threat of litigation from joining in the sharing group. At the same time the wedge that targeting creates between the incentive to contribute versus free-ride is large when contributors can be identified accurately. Targeting contributors through litigation is also more effective when the surplus to consumers from the proprietary good, U^o, is low. A high U^o induces a relatively small measure of low marginal cost users to join the platform, even without litigation or quality improvements. Since the joining margin is low, free-riding incentives are likely weak, so that targeting contributors through litigation is less effective in increasing demand for the proprietary good relative to an improvement in quality of the copyrighted good.

4.2. The Pricing Equilibrium

As the previous subsection showed, the proprietor can capture a larger share of the market through litigation targeting contributors when the surplus to consumers from the proprietor's good is relatively low and the proprietor's signal is accurate enough. For any given initial quality Q_o of the proprietary good, the payoff from buying the copyrighted good is low if the proprietor sets a high price. On the other hand, setting a low price allows her to capture a larger share of the market. Given that a smaller measure of low-cost consumers joins the platform when price is low, free-riding incentives are weak, so that litigation targeting contributors is not as effective and the proprietor is better-off investing in quality.

The two investment strategies yield different price equilibriums. The proprietor can thus choose between two outcomes—(1) she can target platform contributors in litigation and set a high price for the copyrighted good, or (2) she can invest in quality to ensure a high demand for the proprietary good at a low price. In the first case, the high price lowers U^o and thus ensures a joining margin with a low contribution margin. With strong free-riding incentives in $\mathcal{N}$, litigation is effective in driving down contributions allowing the proprietor to maintain a high price. In the second case, the low price makes the copyrighted good attractive to a large fraction of the consumers. As only a small proportion of low marginal cost users join the platform, incentives to contribute in $\mathcal{N}$ are high and free-riding is low. Here, the proprietor prefers to invest in quality that lowers the relative payoff from the platform for all potential users.

The choice between the high-price litigation strategy and the low-price quality investment strategy ultimately depends on ρ, i.e., the ability to identify contributors accurately in the platform. When ρ is high, litigation is effective in targeting contributors. Hence, it has a strong effect on platform quality, and the price advantage from litigation is more attractive than the market-share advantage from quality improvements.

Proposition 4. *Let $\left(P_L^*, \nu_L^J\right)$ and $\left(P_q^*, \nu_q^J\right)$ represent the optimal price and measure of users joining the platform if the proprietor adopts litigation investment D and a quality investment of $\phi(x) = \frac{D}{1-\alpha}$, respectively. Then,*

(a) $P_L^* \geq P_q^* \geq P_0^*$ *and* $\nu_0^J \geq \nu_L^J \geq \nu_q^J$.

(b) *For every Q_o, there exists a $\widetilde{\rho} \in \left(\frac{1}{2}, 1\right)$, such that if $\rho < \widetilde{\rho}$, the proprietor chooses quality investment with $P^* = P_q^*$ and $\nu^J = \nu_q^J$, and if $\rho \geq \widetilde{\rho}$, then she chooses litigation with $P^* = P_L^*$ and $\nu^J = \nu_L^J$.*

(c) *$\widetilde{\rho}$ decreases as Q_o increases.*

Part (a) of Proposition 4 states that litigation against contributors leads to a high price but low market share for the proprietary good compared with quality investment. Part (b) states that litigation is profitable when the proprietor's signal is highly informative, and part (c) implies that the proprietor's incentive to invest in litigation increases as the initial quality of the proprietary good increases. Thus, the model predicts that we are likely to see more copyright lawsuits when the proprietary good is of poor quality. At the same time, the results also suggest more optimistically that investment in quality leads to a virtuous cycle as quality improvements are likely to incentivize further investments in quality and lower levels of litigation.

5. Quality Investment and Copyright Litigation

In this section, I generalize the investment decision by allowing the proprietor to simultaneously invest in both quality improvement and copyright litigation. I model this decision now as an allocation of I dollars between the two strategies. Thus, the proprietor chooses x and D to maximize profits, such that $x + D = I$. The proprietor's constrained maximization problem can be written as

$$\max_{x \in [0,I],\, D \in [0,I]} \pi_L(D)$$

$$s.t.\ x + D = I.$$

Given the constraint, the proprietor's allocation can be described by her choice of D, since $x = I - D$.

As before, characteristics of the platform equilibrium depend on the price of the proprietary good. When proprietary price is low, only a small user population joins the platform, so that there is no congestion or free-riding in the shared platform. Conversely, when proprietary price is high, many high-cost users are also induced to join the shared platform as free-riders, and this leads to congestion and lower platform quality. The proposition below describes the price equilibrium. P_L^{nf*} represents an equilibrium price where there is no free-riding in the platform, and P_L^{f*} denotes an equilibrium price where the platform exhibits congestion.

Proposition 5. *There exists $\widehat{D}$, such that if $D \leq \widehat{D}$, the $P^* = P_L^{nf*}$, and the platform equilibrium is characterized by no free-riding. If $D \geq \widehat{D}$, $P^* = P_L^{f*}$, and the platform equilibrium is characterized by congestion and free-riding.*

Proposition 5 states that the proprietary price depends on the allocation of investment towards litigation and quality. When a large proportion of the investment budget is used to improve proprietary quality, the platform equilibrium is characterized by high platform quality and no congestion. Two factors drive this result. First, high investment in quality leads to high quality for the proprietary good. As described in previous sections, a high Q leads to higher U^o, which then attracts more users towards the proprietary good, and only a few low-cost users who contribute content to the platform are induced to join the platform. Second, when D is low, the expected litigation cost from contributing content is also lower. This also encourages more users who join the platform to contribute. The reverse is true when the proprietor invests a larger proportion in raising litigation costs for users.

Given the price equilibrium, Proposition 6 describes the profit-maximizing allocation of investment between quality and litigation.

Proposition 6.

(a) *There exists $\widehat{Q}_o$, and for every $Q_0 > \widehat{Q}_o$, there exists $\widehat{\rho} \in \left[\frac{1}{2}, 1\right]$, such that the equilibrium investment allocation $D^* \leq \widehat{D}$, and the platform equilibrium is characterized by no free-riding if $Q_o \geq \widehat{Q}_o$ and $\rho \leq \widehat{\rho}$. In all other cases, $D^* > \widehat{D}$, and the platform equilibrium is characterized by congestion and free-riding.*

(b) *D^* is increasing in α, i.e., $\frac{\partial D^*}{\partial \alpha} \geq 0$.*

Part (a) of the proposition describes the factors that influence the equilibrium investment allocation chosen by the proprietor. There are two main factors that affect the proprietor's investment allocation. The first is the initial quality of the proprietary good.

The lower the quality of the proprietary good, the more likely it is that the proprietor spends a large proportion of her investment budget on litigation. Thus, just as we saw from the results in Proposition 4, investment in quality is likely to be self-reinforcing. The second factor that determines the magnitude of D is the informativeness of the proprietor's signal about contributors in the platform. The more easily she is able to target contributors, the greater the marginal benefit from investment in litigation, and a larger proportion of investment goes towards litigation rather than quality improvements.

Part (b) of the proposition states that as the sensitivity of shared good quality on proprietary good quality increases, the proprietor's investment in litigation increases, while her investment in quality improvements correspondingly decreases. This is an intuitive result. α indicates the spillover of quality improvements from the proprietary good to the shared good. As α becomes higher, every unit increase in Q also increases δ, and hence makes competition more intense for the proprietor. This limits the profitability of quality improvements for the proprietor. If copying technology is very good, we would expect α to be considerably high, and hence the proprietor is more likely to favor a competitive strategy based on copyright enforcement.

Finally, let us understand the policy implications of strengthening copyright laws by looking at the efficiency of the proprietor's investment allocation.

Welfare Implications

In the current analysis, since the variation in population arises from differences in the marginal cost of contribution, the monopoly outcome without competition from the platform is first-best. If the platform did not exist, the proprietor would choose monopoly price Q and all consumers would buy the proprietary good. The proprietor would thus extract the entire surplus. Here, litigation is clearly inefficient, and hence all investment should be made in quality for maximizing total surplus form a first-best perspective.

However, we can look at the second-best efficient outcome when the proprietor faces competition from the platform. The question asked here is: Given the nature of the platform and price equilibrium that would result for any given allocation of investment, what is the welfare-maximizing investment allocation? There are two kinds of inefficiencies that can potentially arise in the market for any given D. First, due to the "public good" nature of the platform, the presence of free-riding causes an underproduction of the shared good for any given price, and litigation strategy chosen by the proprietor. Second, when free-riding on the platform is high, so that platform quality is low, there is also an inefficiency arising from the fact that too many consumers use the inferior shared good, as market power allows the proprietor to price the proprietary good too high to exclude some consumers who should be using the high-quality proprietary good. In this case, by allowing the proprietor to serve a larger proportion of the market, litigation can reduce underproduction of the proprietary good. At the same time, litigation also exacerbates underproduction of the shared good. Thus, the optimal investment allocation D^o then trades-off these two effects. Hence, we see that it is possible for $D^o > 0$, meaning that allowing some level of copyright enforcement through litigation is optimal in the second-best outcome.

However, the proprietor over-invests in litigation relative to the second-best level. This is because investment in D increases the likelihood of free-riding and congestion in the platform, and hence exacerbates the underprovision of the shared good. This increases the proprietor's incentive to increase D, while worsening the underprovision inefficiency of the shared platform. Furthermore, the inefficiency becomes larger as α increases. This happens due to two reasons. First, as explained in Proposition 6, the proprietor's investment in quality decreases as α increases by making competition from the shared good more intense. Second, when quality improvements by the proprietor spillover into higher quality for the shared good, it generates a positive externality. Higher α means a stronger effect of this positive externality, and hence a higher efficient level of investment in quality of the proprietary good. Thus, I find that D^o decreases with α. A higher equilibrium D^* combined with a lower efficient D^o means that the wedge between them, $D^* - D^o$, increases with α. The proposition below states this result.

Proposition 7. *Let D^o denote the second-best efficient level of investment in litigation, then,*

(a) $0 < D^o < D^*$.

(b) $D^* - D^o$ *increases as* α *increases, i.e.,* $\frac{\partial}{\partial \alpha}[D^* - D^o] > 0$.

Proposition 7 provides important implications for public policy surrounding copyright enforcement. Copyright infringement of digital goods including online content in the US is regulated under the Digital Millennium Copyright Act (DMCA) passed in 1998. The DMCA requires internet service providers (ISPs) to issue "notice and takedown" procedures so that copyright holders have an easy way to disable infringing content. ISPs are also required to have a "repeat infringer policy" to terminate the account of users with multiple infringement violations. A recent legislation proposed in the US Senate in 2020 strengthens these procedures by essentially mandating ISPs to install filters that monitor user uploads [17,18]. Other countries, such as New Zealand, France, the United Kingdom and South Korea, have also pursued enforcement and potential litigation against individual file-sharers by setting up public monitoring bodies. Proposition 7 provides a case against current global trends towards strengthening copyright enforcement of online content and predicts that it is likely to have a detrimental impact on the quality of entertainment products. This is especially true if α is high in the presence of digital copying technology that makes copies of the proprietary good almost close to the original.

6. Conclusions and Discussion

This paper analyzed the incentives for copyright litigation and quality improvements by the seller of a proprietor good facing competition from a shared content platform. I used a model of public good contributions with congestion effects to describe production of the shared good on the platform. In this set-up, platform quality is sensitive to the proportion of contributors relative to free-riders. I showed that litigation by the proprietor that targets contributors on the platform may increase congestion by encouraging free-riding, and hence bring down platform quality. Such an effect is found to be strong when the proprietor is able to identify contributors in the platform with greater accuracy and when the initial quality of the proprietary good is low, so that a large proportion of the consumer population joins the platform as free-riders. On the other hand, investments in quality improvements on the proprietary good are found to be more profitable compared with copyright litigation when initial quality of the copyrighted good is high. When this is the case, only a small proportion of free-riders relative to contributors join the platform, making congestion on the platform low. Under these circumstances, litigation is unlikely to have a large impact on platform quality, while an increase in proprietary goods' quality can draw many of the users away from the platform and towards the proprietor's good. Finally, I used the model to compare welfare effects of investment in litigation and quality improvements in the copyrighted good, and I found that the proprietor's incentive to invest in litigation relative to quality is greater than the efficient incentive. The conclusion drawn from this result is that public policy aimed towards strengthening copyright protection is likely to have an adverse effect on the quality of copyrighted goods.

While the model and analysis presented in this paper is fairly robust to changes in functional specification, I would like to conclude by discussing a few aspects of copyright and piracy that the model does not directly address. First, the model does not explicitly describe the costs of making copies of original works that are then uploaded and shared on the platform. One may argue that the price paid for the original copyrighted good is also part of the cost of contributing. If this were the case, the joining decision for a contributor becomes independent of proprietary good price since she has to buy the proprietary good in order to contribute on the platform. While this modification will alter the price equilibrium, the correlation between proprietary good quality and platform congestion highlighted in Proposition 1 continues to hold. The relative incentive for quality improvement and copyright litigation also remains qualitatively unchanged as it continues to be the case that an increase in quality affects both free-riders and contributors in a uniform way, while copyright litigation asymmetrically affects contributors. However, in my opinion, the price of the original is unlikely to be an important factor influencing piracy. Digital goods are practically costless to reproduce, and once the original material has been obtained, many copies can be made at very low or zero marginal cost. Hence, the price of the original good is unlikely to be a significant component of contributing costs on the platform. This is even more true in the case of file-sharing networks, where the act of contributing does not require purchase of proprietary content but rather supplying an already downloaded file to other users on the platform.

Second, it is worth pointing out that although this paper is modeled around investment in copyright litigation against individual users, the results predicted by this model can be applied to many other strategies that copyright owners use to limit piracy by making it harder for contributors to share content beyond copyright litigation. Any copyright protection strategy that increases the cost of copying and uploading content on shared platforms is applicable to the analysis described in the paper.

Original content producers have used a number of different strategies to combat and piracy and compete with shared alternatives. In many cases, copyright holders have used secondary liability to pursue litigation against the platform itself rather than individual users. Examples of this type of litigation include the lawsuit against Napster and Grokster in the early 2000s. The DMCA Safe Harbor Act limits this type of litigation as long as the platform acts expeditiously to remove infringing content on its website once it has been notified by the copyright holder. Both secondary liability litigation and the use of Safe Harbor provisions incentivize the platform to monitor and track down users who upload content, thus making it harder for users to contribute content to the platform. Investment in such lawsuits and monitoring platform content by copyright owners has the same effect qualitatively on the shared good value as direct infringement lawsuits against users.

Apart from legal and regulatory protections, copyright holders also use technological protections for content, prominently Digital Rights Management (DRM). DRM is the use of technology to restricts the unauthorized distribution and modification of copyrighted content. It involves the use of codes that prohibit the copying of content or limits the number of devices that can access purchased copyrighted material, limiting the number of digital copies that can made. While DRM has not completely eliminated piracy, it has also made it more difficult to share copyrighted content, and hence would have a similar effect on the quality of the shared platform as greater investment in copyright litigation does [19].

The analysis in the current paper, thus, applies to a broad range of interventions that a proprietor may use to influence the quality of the competing shared alternative. My analysis suggests that in all cases where investment in copyright protection lowers the value of the competing shared alternative, the proprietor underinvests in the quality of her original good relative to what is efficient. As a result, regulatory policies that allow greater monitoring of user data on sharing platforms and easier litigation against users and platform owners will likely disincentivize quality improvements by proprietary owners of entertainment content, and consequently have an adverse effect on consumer welfare.

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Appendix A

Proof of Proposition A1. Suppose there is no investment in litigation, i.e., $D = 0$, then the profit to the proprietor is given as follows:

$$\pi_0 = \begin{cases} P \text{ if } P \leq (1-\alpha)Q_o - 1 - \beta \\ \left(1 - \nu_0^J(P)\right)P \text{ if } (1-\alpha)Q_o - 1 - \beta \leq P \leq (1-\alpha)Q_o - 1 \\ (1-\beta)P \text{ if } (1-\alpha)Q_o - 1 \leq P \leq (1-\alpha)Q_o - \beta \\ 0 \text{ otherwise.} \end{cases}$$

$$\nu_0^J(P) = 1 + \beta - (1-\alpha)Q_o + P.$$

In all cases, $\frac{d^2}{dP^2}\pi_o(P) \leq 0$, so that the second order condition is satisfied.

If $(1-\alpha)Q_o \leq \beta$, then $P_0^* = 0$ and $\nu_0^* = 1$. If $\beta \leq (1-\alpha)Q_o \leq 1$, then $P_0^* = (1-\alpha)Q_o - \beta$ and $\nu_0^{J*} = \beta$. If $1 \leq (1-\alpha)Q_o \leq 1+\beta$, then it can be shown that $\frac{\partial \pi_0}{\partial P}\Big\rfloor_{(1-\alpha)Q_o - 1} \geq 0$. This means that $P_0^* = (1-\alpha)Q_o - \beta$ and $\nu_0^{J*} = \beta$. If $(1-\alpha)Q_o \geq 1+\beta$, then we have to consider three different cases. If $1+\beta \leq (1-\alpha)Q_o \leq 2-\beta$, then $P_0^* = (1-\alpha)Q_o - \beta$ and $\nu_0^{J*} = \beta$ since $\frac{\partial \pi_0^u}{\partial P}\Big\rfloor_{(1-\alpha)Q_o - 1} \geq 0$. If $2-\beta \leq (1-\alpha)Q_o \leq 2+\beta$, then we have to compare $\pi_0^{nf} = \frac{((1-\alpha)Q_o - \beta)^2}{4}$ and $\pi_0^f = (1-\beta)((1-\alpha)Q_o - \beta)$. It can be shown that $\pi_0^{nf} - \pi_0^f$ is increasing in Q_o for $2-\beta \leq (1-\alpha)Q_o \leq 2+\beta$ and it is negative at $(1-\alpha)Q_o = 2+\beta$. This is because when $(1-\alpha)Q_o = 2+\beta$, $\pi_0^{nf} - \pi_0^f$ is increasing in β and negative at $\beta = \frac{1}{2}$. Thus, $\pi_0^{nf} - \pi_0^f \leq 0$ for Q_o in this range, which means that again $P_0^* = (1-\alpha)Q_o - \beta$ and $\nu_0^{J*} = \beta$. For $(1-\alpha)Q_o \geq 2+\beta$, we have to compare $\pi_0^{nf} = ((1-\alpha)Q_o - 1 - \beta)$ with $\pi_0^f = (1-\beta)((1-\alpha)Q_o - \beta)$. Again $\pi_0^{nf} - \pi_0^f$ is increasing in $(1-\alpha)Q_o$, and it is negative at $(1-\alpha)Q_o = 2+\beta$. Define

$$\widetilde{Q}_o = \frac{(1+\beta)}{\beta(1-\alpha)} - \frac{(1-\beta)}{(1-\alpha)} = \frac{1+\beta^2}{\beta(1-\alpha)}.$$

If $Q_o \geq \widetilde{Q}_o$ then $\pi_0^f \leq \pi_0^{nf}$.

$$P_0^* = \begin{cases} 0 \text{ if } (1-\alpha)Q_o \leq \beta \\ (1-\alpha)Q_o - \beta \text{ if } \beta < (1-\alpha)Q_o \leq (1-\alpha)\widetilde{Q}_o \\ (1-\alpha)Q_o - 1 - \beta \text{ if } (1-\alpha)Q_o > (1-\alpha)\widetilde{Q}_o. \end{cases}$$

and

$$\nu_0^{J*} = \begin{cases} 1 \text{ if } (1-\alpha)Q_o \leq \beta \\ \beta \text{ if } \beta < (1-\alpha)Q_o \leq (1-\alpha)\widetilde{Q}_o \\ 0 \text{ if } (1-\alpha)Q_o > (1-\alpha)\widetilde{Q}_o. \end{cases}$$

□

Proof of Proposition A2. The contribution margin is unaffected by quality improvements in the copyrighted good, i.e., $\nu_q^c = \nu^c = \beta$. If the joining margin ν_q^J is lower than the contribution margin, i.e., $\nu_q^J \leq v^c$, so that every user who joins $\mathcal{N}$ always contributes platform quality, $\delta_q = 1$, and the payoff to the marginal user-contributor with cost ν_q^J from joining $\mathcal{N}$ is $U^N = \alpha(Q_o + \phi(x)) + 1 + \beta - \nu_q^J$. Since this consumer is indifferent between joining and contributing in $\mathcal{N}$ and buying the proprietary good of quality $Q_o + \phi(x)$, ν_q^J must solve, $\nu_q^J = 1 + \beta - (1-\alpha)(Q_o + \phi(x)) + P_q$. This equilibrium holds if $\nu_q^J \leq v^c$, i.e., $1 + \beta - (1-\alpha)(Q_o + \phi(x)) + P_q \leq \beta$, or $P_q \leq (1-\alpha)(Q_o + \phi(x)) - 1$. If $P_q > (1-\alpha)(Q_o + \phi(x)) - 1$, then $\nu_q^J > \nu^c$ where $\nu_q^J = 1$ and $\delta_q = \beta$.

The demand for the copyrighted good with higher quality, $Q_o + \phi(x)$, given price P_q is

$$1 - \nu_q^J = \begin{cases} 1 + \beta - (1-\alpha)(Q_o + \phi(x)) + P_q \text{ if } P_q \leq (1-\alpha)(Q_o + \phi(x)) - 1, \\ 0 \text{ if } P_q > (1-\alpha)(Q_o + \phi(x)) - 1 \end{cases}$$

If $(1-\alpha)Q_o \in [0, \beta]$ and $(1-\alpha)\phi(x) < \beta - (1-\alpha)Q_o$, then the demand for the copyrighted good continues to be zero, and investment in quality does not improve profits. If $\beta - (1-\alpha)Q_o < (1-\alpha)\phi(x) < (1-\alpha)\widetilde{Q}_o - (1-\alpha)Q_o$, demand increases from zero to $1-\beta$ with price $P_q^* = (1-\alpha)(Q_o + \phi(x)) - \beta$, so that the increase in profits is $\pi_q^* - \pi_0^* = [(1-\alpha)(Q_o + \phi(x)) - \beta](1-\beta)$. Finally, if $\phi(x) \geq \widetilde{Q}_o - Q_o$, then demand changes from zero to 1 with all consumers buying the copyrighted good at price $P_q^* = (1-\alpha)(Q_o + \phi(x)) - 1 - \beta$. The corresponding increase in profit is $\pi_q^* - \pi_0^* = (1-\alpha)(Q_o + \phi(x)) - 1 - \beta$.

If the initial quality was $Q_o \in \left[\frac{\beta}{1-\alpha}, \widetilde{Q}_o\right]$, then an increase in quality of $\phi(x) \leq \widetilde{Q}_o - Q_o$ increases profit by $\pi_q^* - \pi_0^* = (1-\alpha)\phi(x)$, while an increase in the quality of $\phi(x) \geq \widetilde{Q}_o - Q_o$ increases profit by $\pi_q^* - \pi_0^* = (1-\alpha)(Q_o + \phi(x)) - 1 - \beta - ((1-\alpha)Q_o - \beta)(1-\beta) = (1-\alpha)\phi(x) - 1 + \beta((1-\alpha)Q_o - \beta)$. Finally, if initial quality was $Q_o \geq \widetilde{Q}_o$, then profits from a quality improvement of $\phi(x)$ increase by $\pi_q^* - \pi_0^* = (1-\alpha)\phi(x)$. □

Proof of Lemma A1. For any targeting strategy let us consider the cases where the equilibrium is characterized by the presence of free-riding and where it is not. It can be shown that the profit function of the proprietor given P, λ^H and λ^L is described as follows

$$\pi_L(P) = \begin{cases} P \text{ if } 0 \leq P \leq P_1 \\ \left(1 - \nu_L^J(P)\right) P \text{ if } P_1 \leq P \leq P_2 \\ (1 - \nu_L^c) P \text{ if } P_2 \leq P \leq P_3 \\ 0 \text{ otherwise.} \end{cases}$$

where

$$\begin{aligned} P_1 &= (1-\alpha)Q_o - 1 - \beta + D\left(\rho\lambda^H + (1-\rho)\lambda^L\right) \\ P_2 &= (1-\alpha)Q_o - 1 + D\left(\rho\lambda^L + (1-\rho)\lambda^H\right) \\ P_3 &= (1-\alpha)Q_o - \nu_L^c + D\left(\rho\lambda^L + (1-\rho)\lambda^H\right) \\ \nu_L^c &= \beta - D(2\rho - 1)\left(\lambda^H - \lambda^L\right) \\ \nu_L^J(P) &= 1 + \beta - D\left(\rho\lambda^H + (1-\rho)\lambda^L\right) - (1-\alpha)Q_o + P. \end{aligned}$$

In each range of prices, $\frac{\partial \pi_L(P)}{\partial \lambda^H} \geq 0$. Thus, it is profit maximizing to set $\lambda^H = 1$. □

Proof of Lemma A2. Given $\lambda^L = 1$, $\nu_L^c = \beta = \nu^c$. The payoff to a contributor from the platform is $\delta_L + \beta - \nu - D$ and the payoff to a free-rider is $\delta_L - D$. If the joining margin, ν_L^J, is lower than the contribution margin, i.e., $\nu_L^J \leq v^c$, so that every user who joins $\mathcal{N}$ always contributes platform quality, $\delta_L = 1$, and the payoff to the marginal user-contributor with cost ν^J from joining $\mathcal{N}$ is $U^N = 1 + \alpha Q + \beta - \nu^J$. Since this consumer is indifferent between joining and contributing in $\mathcal{N}$ and buying the proprietary good, ν^J must solve $\nu^J = 1 + \beta - U^o - D$. This equilibrium holds if $\nu^J \leq v^c$, i.e., $1 + \beta - U^o - D \leq \beta$, or $U^o \geq 1 - D$. If $U^o \leq 1 - D$, then $\nu_L^J > \nu^c$ where $\nu^J = 1$ and $\delta_L = \beta$.

Thus, demand for the copyrighted good with $\lambda^L = 1$ given price P is

$$1 - \nu_L^J = \begin{cases} 1 + \beta - (1-\alpha)Q_o - D + P \text{ if } P \leq (1-\alpha)Q_o + D - 1, \\ 0 \text{ if } P \geq (1-\alpha)Q_o + D - 1 \end{cases}$$

This is identical to the demand for the copyrighted good derived in the proof of Proposition A2 where $(1-\alpha)\phi(x) = D$. Hence, when $\lambda^L = 1$, $\nu_L^J = \nu_q^J$ and $\pi_L^* = \pi_q^*$ for $(1-\alpha)\phi(x) = D$. $\square$

Proof of Lemma A3. Let us first describe the platform equilibrium by considering the following different parametrizations of ρ and U^o.

(i) First, suppose $U^o + D \leq 1 + \alpha Q_o + \beta$. Define

$$\lambda_1^L = \frac{1 - U^o + \alpha Q_o - D(1-\rho)}{D\rho}.$$

It can be checked that as long as $D \geq 1$, $\lambda_1^L \leq 1$. Let us assume this to be the case. Now suppose we consider an equilibrium where $\nu_L^J < \nu_L^c$, so that $\delta_L = 1$. Then, the marginal consumer that uses the shared good is a contributor. This gives the cut-off marginal cost, ν_L^J as

$$\nu_L^J = \nu_L^{nf} = 1 + \beta - D\Big(\rho + (1-\rho)\lambda^L\Big) - U^o + \alpha Q_o.$$

For $U^o + D \leq 1 + \alpha Q_o + \beta$, the above expression is always positive. Moreover, for $\lambda^L > \lambda_1^L$, it is also less than ν_L^c, confirming our equilibrium that $\delta_L = 1$. However, if $\lambda^L \leq \lambda_1^L$, then a user with marginal cost ν_L^c obtains a strictly positive payoff from joining the platform. In this case, the platform equilibrium is characterized by the presence of free-riding and $\nu_L^J \geq \nu_L^c$. Since the payoff to a free-rider from the shared good is independent of ν_i, if $\nu_L^J > \nu_L^c$, then it must be that $\nu_L^J = 1$. When this is so, $\delta_L = \nu_L^c$, and the payoff to a free-rider from the shared good is given by

$$\nu_L^c - D\Big((1-\rho)\lambda^L + \rho\Big).$$

It can be shown that the above expression is negative. The maximum value for this expression is when $\rho = \frac{1}{2}$. In this case, the expression reduces to $\beta - \frac{D}{2} \leq 0$ given that $\beta \leq \frac{1}{2}$ and $D \geq 1$. This means that $\nu_L^J > \nu_L^c$ cannot be true. Hence, $\nu_L^J = \nu_L^c$.

Finally, we have to make sure that ν_L^c is positive. This is true for $\lambda^L \geq 1 - \frac{\beta}{D(2\rho-1)}$. It can also be checked that $1 - \frac{\beta}{D(2\rho-1)} \leq \lambda_1^L$. This is because $U^o + D \leq 1 + \alpha Q_o + \beta \Rightarrow U^o + D \leq 1 + \alpha Q_o + \frac{\beta\rho}{2\rho-1}$. Thus, the joining cut-off is given by the following

$$\nu_L^J\Big(\lambda^L\Big) = \begin{cases} \nu_L^{nf} \text{ if } \lambda^L > \lambda_1^L \\ \nu_L^c \text{ if } 1 - \frac{\beta}{D(2\rho-1)} \leq \lambda^L \leq \lambda_1^L \\ 0 \text{ otherwise.} \end{cases}$$

(ii) Next, suppose $1 + \alpha Q_o + \beta < U^o + D \leq 1 + \alpha Q_o + \frac{\beta\rho}{2\rho-1}$. Let us define

$$\lambda_2^L = \frac{1 + \beta - U^o + \alpha Q_o - D\rho}{D(1-\rho)}.$$

It can be checked that for $U^o + D > 1 + \alpha Q_o + \beta$, $\lambda_2^L < 1$. For $\lambda^L \leq \lambda_2^L$, $\nu_L^{nf} \geq 0$. On the other hand, for $\lambda^L > \lambda_2^L$, it is negative, implying that even the lowest marginal cost does not join the platform, even if everyone contributes in equilibrium. Here, $\nu_L^J = 0$. Finally, for $U^o + D \leq 1 + \alpha Q_o + \frac{\beta\rho}{2\rho-1}$, $1 - \frac{\beta}{D(2\rho-1)} \leq \lambda_1^L \leq \lambda_2^L$. Thus, we have

$$\nu_L^J\Big(\lambda^L\Big) = \begin{cases} 0 \text{ if } \lambda^L > \lambda_2^L \\ \nu_L^{nf} \text{ if } \lambda_1^L < \lambda^L \leq \lambda_2^L \\ \nu_L^c \text{ if } 1 - \frac{\beta}{D(2\rho-1)} \leq \lambda^L \leq \lambda_1^L \\ 0 \text{ otherwise.} \end{cases}$$

(iii) Finally, let $U^o + D > 1 + \alpha Q_o + \frac{\beta\rho}{2\rho-1}$. In this case, it can be shown that $\nu_L^J = 0$. Let us consider two ranges for λ^L. Suppose, $\lambda^L \leq 1 - \frac{\beta}{D(2\rho-1)}$, then $\nu_L^c \leq 0$, so that no one joins the platform and $\nu_L^J = 0$. If $\lambda^L > 1 - \frac{\beta}{D(2\rho-1)}$, then $\lambda^L > \lambda_2^L$. This is because $1 - \frac{\beta}{D(2\rho-1)} > \lambda_2^L$ when $U^o + D > 1 + \alpha Q_o + \frac{\beta\rho}{2\rho-1}$. This means that $\nu_L^J = 0$.

From case (i), it is clear that when $U^o + D \leq 1 + \alpha Q_o + \beta$, $\widehat{\lambda}^L = \lambda_1^L$. If $\lambda^L > \lambda_1^L$, $\nu_L^J < \nu^c$, so that every user in the platform obtains strictly higher payoff from contributing relative to free-riding. Conversely, $\nu_L^J < \nu_L^c$ is true only when $\lambda^L > \lambda_1^L$. □

Proof of Proposition A3. (a) From cases (ii) and (iii) in Lemma A3, it is clear that if $\lambda^L = 1$, the proprietor can capture the entire market when $U^o + D > 1 + \alpha Q_o + \beta$. From case (ii), $\nu_L^J(\lambda^L) = 0$ for any $\lambda_2^L \leq \lambda^L \leq 1$. From case (iii), $\nu_L^J(\lambda^L) = 0$ for every $0 \leq \lambda^L \leq 1$.

For the case where $U^o + D \leq 1 + \alpha Q_o + \beta$, let us define

$$\widehat{\rho} = \frac{1}{2} + \frac{\beta}{2D}.$$

Then, we can consider two cases. First, when $\rho \geq \widehat{\rho}$, then the proprietor can capture the entire market through a targeted litigation strategy by setting $0 \leq \lambda^L \leq 1 - \frac{\beta}{D(2\rho-1)}$. If $\rho < \widehat{\rho}$, then from case (i), we can see that $\nu_L^J(\lambda^L)$ is increasing in λ^L for $0 \leq \lambda^L \leq \lambda_1^L$ and then decreasing in λ^L for $\lambda_1^L \leq \lambda^L \leq 1$ since $\frac{\partial \nu_L^c}{\partial \lambda^L} = D(2\rho - 1) \geq 0$ and $\frac{\partial \nu_L^{nf}}{\partial \lambda^L} = -D(1-\rho) \leq 0$. The proprietor chooses the litigation strategy that minimizes $\nu_L^J(\lambda^L)$. Since $\nu_L^J(\lambda^L)$ is concave in λ^L, she chooses $Min\left\{\nu_L^J(0), \nu_L^J(1)\right\}$. Thus, we have to compare $\nu_L^J(0) = \nu_L^c$ with $\nu_L^J(1) = 1 + \beta - D - U^o$.

$$\nu_L^J(0) - \nu_L^J(1) = U^0 + 2D(1-\rho) - 1.$$

The LHS of the above inequality is decreasing in ρ. At $\rho = \frac{1}{2}$ the difference is positive, so that $\lambda^{L*} = 1$. If $\rho = \widehat{\rho}$, the difference is negative since $U^0 \leq 1 + \beta - D$. Hence, there exists a $\overline{\rho} \in \left[\frac{1}{2}, \widehat{\rho}\right]$ such that $\lambda^{L*} = 0$ if and only if $\rho \geq \overline{\rho}$. Thus, for $\rho \geq \overline{\rho}$ the average contribution in the platform, $\delta_L < 1$, and the equilibrium is characterized by the presence of free-riding, whereas, for $\rho \leq \widehat{\rho}$, $\delta_L = 1$, and everyone in the platform contributes. More specifically,

$$\delta_L = \begin{cases} 1 \text{ if } 0 \leq \rho \leq \overline{\rho} \\ \nu_L^c \text{ if } \overline{\rho} \leq \rho \leq \widehat{\rho} \\ 0 \text{ if } \widehat{\rho} \leq \rho \leq 1. \end{cases}$$

□

Proof of Proposition A4. Let π_q and π_L denote the profit under quality investment and targeted litigation, respectively. From Proposition A3, we know that if $U^o + D > 1 + \alpha Q_o + \beta \Rightarrow P < (1-\alpha)Q_o + D - \beta - 1$, then $\lambda^{L*} = 1 \Rightarrow \pi_q = P$. On the other hand, for $P \geq (1-\alpha)Q_o + D - \beta - 1$, $\pi_q = \left(1 - \nu_L^{nf}\right)P$ and $\pi_L = \left(1 - \nu_L^c\right)P$.

Let us first find the optimal π_q.

$$\pi_q = \begin{cases} P \text{ if } P < (1-\alpha)Q_o + D - \beta - 1 \\ ((1-\alpha)Q_o + D - \beta - P)P \text{ if } (1-\alpha)Q_o + D - \beta - 1 \leq P < (1-\alpha)Q_o + D - \beta \\ 0 \text{ otherwise.} \end{cases}$$

We can consider different ranges for Q_o to obtain P_q, ν_q and π_q.

$$P_q = \begin{cases} \frac{1}{2}((1-\alpha)Q_o + D - \beta) \text{ if } (1-\alpha)Q_o + D \leq 2 + \beta \\ (1-\alpha)Q_o + D - \beta - 1 \text{ otherwise.} \end{cases}$$

$$1-\nu_q^J=\begin{cases} 0 \text{ if } (1-\alpha)Q_o+D\le\beta \\ \frac{1}{2}((1-\alpha)Q+D-\beta) \text{ if } \beta<(1-\alpha)Q_o+D\le 2+\beta \\ 1 \text{ otherwise.}\end{cases}$$

$$\pi_q^*=\begin{cases} 0 \text{ if } (1-\alpha)Q_o+D\le\beta \\ \frac{1}{4}((1-\alpha)Q_o+D-\beta)^2 \text{ if } \beta<(1-\alpha)Q_o+D\le 2+\beta \\ ((1-\alpha)Q_o+D-\beta-1) \text{ otherwise.}\end{cases}$$

Since π_L is increasing in P, if the proprietor adopts a targeted litigation strategy, then she sets $P_L^*=(1-\alpha)Q_o\ge(1-\alpha)Q_o+D-1-\beta$, and $\nu_L^{J*}=\nu_L^c(1)$, so that $\pi_L^*=(1-\beta-D(2\rho-1))(1-\alpha)Q_o$.

(a) Since litigation induces a positive cost of using the shared good, the proprietor can charge a higher price as well as increase demand for the copyrighted good. Moreover, since a targeted litigation strategy allows the proprietor to set the monopoly price, it is higher than the price that can be charged with quality investment. Finally, we know that for $P>(1-\alpha)Q_o+D-\beta-1$, $\nu_q^J=\nu_L^{nf}$ and $\nu_L^J=\nu_L^c>\nu_L^{nf}$. If $P\le(1-\alpha)Q_o+D-\beta-1$, then from cases (ii) and (iii) in Lemma 3, $\nu_L^J(0)=\nu_L^J=\nu_L^J(1)=\nu_q^J=0$. Thus, we have that $P_L^*\ge P_q^*\ge P_0^*$ and $\nu_0^{J*}\ge\nu_L^{J*}\ge\nu_q^{J*}$.

(b) π_L^* is increasing in ρ. Moreover, it can be checked that for every Q_o, at $\rho=\frac{1}{2}$, $\pi_q^*>\pi_L^*$ and for $\rho=\widehat{\rho}$, $\pi_q^*<\pi_L^*$. Thus, for every Q_o, there exists, $\widetilde{\rho}\in\left(\frac{1}{2},\widehat{\rho}\right)$ such that $\pi_L^*\ge\pi_q^*$ if and only if $\rho\ge\widetilde{\rho}$, so that the proprietor chooses a targeted litigation strategy with $P^*=P_L^*$ and ν_L^{J*} if and only if $\rho\ge\widetilde{\rho}$.

(c) Let us first describe some properties of the function $\pi_q^*-\pi_L^*$.

$\pi_q^*-\pi_L^*$ is convex in Q_o. Moreover, at $Q_o=0$, $\pi_q^*=\pi_L^*=0$ and $\pi_q^*-\pi_L^*$ is decreasing in Q_o. Furthermore, $\pi_q^*-\pi_L^*$ is increasing in Q_o for $(1-\alpha)Q_o>2+\beta-D$. Hence, there exists, $Q_m^u>0$, such that $\pi_q^*\le\pi_L^*$ if and only if $Q_o\le Q_m^u$. Now, $\pi_q^*-\pi_L^*$ is decreasing in ρ. Moreover, at $Q_o=0$, $\frac{\partial}{\partial Q}\left(\pi_q^*-\pi_L^*\right)$ also decreases as ρ increases. This means that Q_m^u increases as ρ increases. Now, since $\widetilde{\rho}$ is just the inverse function corresponding to Q_m^u and since $\pi_q^*-\pi_L^*$ is decreasing in ρ, this must mean that $\widetilde{\rho}$ is decreasing in Q_m^u. □

Proof of Proposition A5. The profit function with an allocation of $x=I-D$ in quality and D in litigation with $\lambda^L=0$, $\lambda^H=1$ is

$$\pi_L(P)=\begin{cases} P \text{ if } 0\le P\le P_1 \\ ((1-\alpha)Q_o+(1-\alpha)\phi(I-D)-\beta+D\rho-P)P \text{ if } P_1\le P\le P_2 \\ (1-\beta+D(2\rho-1))P \text{ if } P_2\le P\le P_3 \\ 0 \text{ otherwise.}\end{cases}$$

$$\frac{d}{dP}\pi_L(P)=\begin{cases} 1 \text{ if } 0\le P\le P_1 \\ (1-\alpha)Q_o+(1-\alpha)\phi(I-D)-\beta+D\rho-2P \text{ if } P_1\le P\le P_2 \\ (1-\beta+D(2\rho-1)) \text{ if } P_2\le P\le P_3 \\ 0 \text{ otherwise.}\end{cases}$$

$$\begin{aligned} P_1 &= (1-\alpha)Q_o+(1-\alpha)\phi(I-D)-1-\beta+D\rho \\ P_2 &= (1-\alpha)Q_o+(1-\alpha)\phi(I-D)-1+D(1-\rho) \\ P_3 &= (1-\alpha)Q_o+(1-\alpha)\phi(I-D)-\nu_L^c+D(1-\rho) \\ \nu_L^c &= \beta-D(2\rho-1) \\ \nu_L^J(P) &= 1+\beta-D\rho-(1-\alpha)Q_o-(1-\alpha)\phi(I-D)+P. \end{aligned}$$

Taking the first derivative of the profit function

$$\frac{d}{dP}\pi_L(P) = \begin{cases} 1 \text{ if } 0 \leq P \leq P_1 \\ (1-\alpha)Q_o + (1-\alpha)\phi(I-D) - \beta + D\rho - 2P \text{ if } P_1 \leq P \leq P_2 \\ (1-\beta+D(2\rho-1)) \text{ if } P_2 \leq P \leq P_3 \\ 0 \text{ otherwise.} \end{cases}$$

In all cases, $\frac{d^2}{dP^2}\pi_L(P) \leq 0$, so that the second order condition is satisfied. Let us denote the profit in the range where $P_1 \leq P \leq P_2$ as π_L^{nf} (the nf superscript denotes the fact that there is no free-riding in the platform) and similarly the profit in the range where there is free-riding when $P_2 \leq P \leq P_3$ is represented by π_L^f. The stationary point for π_L^{nf} is $P_L^{nf*} = \frac{(1-\alpha)Q_o+(1-\alpha)\phi(1-D)-\beta+D\rho}{2}$. The maximized value of π_L^f is $\pi_{nf}^* = \frac{[(1-\alpha)Q_o+(1-\alpha)\phi(1-D)-\beta+D\rho]^2}{4}$

π_L^f is increasing in P, and hence the highest value of profit in this range is at $P_L^{f*} = P_3$. The maximum value of profits in this range is $\pi_L^{f*} = (1-\beta+D(2\rho-1))[(1-\alpha)Q_o + (1-\alpha)\phi(I-D) - \beta + D\rho]$.

In order to obtain closed form solutions, let us derive the equilibrium by assuming $\phi(x) = \sqrt{x}$ and setting $I = 1$. Furthermore, in order to restrict our equilibrium to the case where the proprietor does not capture the entire market, I assume that $\frac{d}{dP}\pi_L(P)\Big]_{P_1} > 0$. Sufficient conditions that ensure this are $(1-\alpha)Q_o < \beta + \frac{3}{4}$ and $\frac{1}{2} > \beta > \frac{1}{4}$.

Next, let us see if there can be a local maximum in the range where $P_1 \leq P \leq P_2$. $P_L^{nf*} < P_2$ if and only if $(1-\alpha)Q_o + (1-\alpha)(1-D)^{\frac{1}{2}} - \beta + D\rho - 2P_2 < 0$ or $(1-\alpha)(1-D)^{\frac{1}{2}} + D(2-3\rho) - 2 + \beta + (1-\alpha)Q_o > 0$. The LHS is concave in D and it is positive at $D = 0$ as along as $(1-\alpha)Q_o > \beta$, which I assume to hold. At $D = 1$, it is positive if and only if $(1-\alpha)Q_o > 3\rho - \beta$. If $\rho > \frac{11}{12}$, then $(1-\alpha)Q_o < \beta + \frac{3}{4} < 3\rho - \beta$. In that case, there exists $D_1 \in (0,1)$ which solves

$$(1-\alpha)(1-D_1)^{\frac{1}{2}} + D_1(2-3\rho) - 2 + \beta + (1-\alpha)Q_o = 0.$$

If $D \leq D_1$, then $P_L^{nf*} < P_2$, and we have to compare $\pi_L^{nf*}$and π_L^{f*}. If $D \geq D_1$, $P_L^{f*} = P_3$ and $\pi_L^{f*} = (1-\beta+D(2\rho-1))\Big[(1-\alpha)Q_o + (1-\alpha)(1-D)^{\frac{1}{2}} - \beta + D\rho\Big]$.

If $\frac{1}{2} < \rho \leq \frac{11}{12}$ and $\beta + \frac{3}{4} < (1-\alpha)Q_o \leq 3\rho - \beta$, as above, for $D \leq D_1$ we compare π_L^{nf*} and π_L^{f*}, and for $D \geq D_1$, $P_L^{f*} = P_3$.

If $(1-\alpha)Q_o > 3\rho - \beta$, we compare π_L^{nf*} and π_L^{f*}.

In the cases where we have to compare maximized profit in the two regions, $\pi_{nf}^* - \pi_f^* \geq 0$ if and only if $(1-\alpha)Q_o + (1-\alpha)(1-D)^{\frac{1}{2}} - 4 + 3\beta - D(7\rho - 4) > 0$

LHS is concave. At $D = 0$, $(1-\alpha)Q_o - 3(1-\beta) > 0$. Assume that $(1-\alpha)Q_o > \frac{9}{4}$. Then, this is true. At $D = 1$, LHS is negative.

So, define $\widehat{D}$ which solves,

$$(1-\alpha)Q_o + (1-\alpha)\Big(1-\widehat{D}\Big)^{\frac{1}{2}} - 4 + 3\beta - \widehat{D}(7\rho - 4) = 0.$$

Thus, in every case, if $D < \widehat{D}$, $P_L^* = P_L^{nf*}$ and we are in the range where the platform equilibrium is characterized by no free-riding. If $D \geq \widehat{D}$, then $P_L^* = P_L^{f*}$, and we are in the range where the platform equilibrium is characterized by free-riding. $\square$

Proof of Proposition A6. (a) From the price equilibrium derived in Proposition A5, the profit function is

$$\pi_L^*(D) = \begin{cases} \frac{\left[(1-\alpha)Q_o+(1-\alpha)(1-D)^{\frac{1}{2}}-\beta+D\rho\right]^2}{4} \text{ if } 0 \le D \le \widehat{D} \\ (1-\beta+D(2\rho-1))\left[(1-\alpha)Q_o+(1-\alpha)(1-D)^{\frac{1}{2}}-\beta+D\rho\right] \text{ if } \widehat{D} \le D \le 1. \end{cases}$$

Taking the derivative with respect to D,

$$\frac{d}{dD}\pi_L^*(D) = \begin{cases} \frac{\left[(1-\alpha)Q_o+(1-\alpha)(1-D)^{\frac{1}{2}}-\beta+D\rho\right]}{2}\left[-\frac{1}{2}(1-\alpha)(1-D)^{-\frac{1}{2}}+\rho\right] \text{ if } 0 \le D \le \widehat{D}, \\ (2\rho-1)\left[(1-\alpha)Q_o+(1-\alpha)(1-D)^{\frac{1}{2}}-\beta+D\rho\right]+(1-\beta+D(2\rho-1))\left[-\frac{1}{2}(1-\alpha)(1-D)^{-\frac{1}{2}}+\rho\right] \text{ if } \widehat{D} \le D \le 1. \end{cases}$$

The slope of the profit function at $\widehat{D}$ in the second segment is greater than in the first segment. The first segment is characterized by no free-riding, so I denote it to be $\pi_L^{nf}(D)$, and in the second segment there is free-riding in the platform, and hence I denote that by $\pi_L^f(D)$.

So, if profit is decreasing in the first segment at $\widehat{D}$, then it is possible for the slope to increasing in the second segment at $\widehat{D}$. In that case, we check the profit in the two regions.

The stationary $D^* = D^*_{nf}$ in the first profit region is

$$D^*_{nf} = 1 - \frac{(1-\alpha)^2}{4\rho^2}$$

The stationary point in the second profit region is $D^* = D^*_f$, which solves

$$(2\rho-1)\left[(1-\alpha)Q_o+(1-\alpha)\left(1-D^*_f\right)^{\frac{1}{2}}-\beta+D^*_f\rho\right]+\left(1-\beta+D^*_f(2\rho-1)\right)\left[-\frac{1}{2}(1-\alpha)\left(1-D^*_f\right)^{-\frac{1}{2}}+\rho\right]=0$$

$1-\frac{(1-\alpha)^2}{4\rho^2} < \widehat{D}$ if and only if $(1-\alpha)Q_o+3\beta-7\rho+2\frac{(1-\alpha)^2}{\rho}-\frac{(1-\alpha)^2}{\rho^2} > 0$.

The *LHS* is decreasing in ρ. At $\rho = \frac{1}{2}$, $(1-\alpha)Q_o+3\beta-\frac{7}{2} > 0$ if and only if $(1-\alpha)Q_o > \frac{7}{2}-3\beta$.

If $(1-\alpha)Q_o \le \frac{7}{2}-3\beta$, then, $1-\frac{(1-\alpha)^2}{4\rho^2} > \widehat{D}$ for all ρ. This means that $D^* = D^*_f \ge \widehat{D}$ and we are in the free-riding range.

If $(1-\alpha)Q_o > \frac{7}{2}-3\beta$, then there exists ρ', which solves

$$(1-\alpha)Q_o+3\beta-7\rho'+2\frac{(1-\alpha)^2}{\rho'}-\frac{(1-\alpha)^2}{\rho'^2}=0,$$

such that for $\rho > \rho'$, we have $D^* = D^*_f \ge \widehat{D}$ in the free-riding range.

If $\rho \le \rho'$, then we have to compare $\pi_L^{nf*}\left(D^*_{nf}\right)$ and $\pi_L^{f*}\left(D^*_f\right)$. $\pi_L^{nf*}\left(D^*_{nf}\right)-\pi_L^{f*}\left(D^*_f\right)$ is decreasing in ρ and decreasing in Q_o. At $\rho = \rho'$, $\pi_L^{nf*}\left(D^*_{nf}\right)-\pi_L^{f*}\left(D^*_f\right) < 0$.

At $\rho = \frac{1}{2}$, $D^*_{nf} = D^*_f = \alpha[2-\alpha]$.

$$\pi_L^{nf*}\left(D^*_{nf}\right)-\pi_L^{f*}\left(D^*_f\right) = \frac{\left[(1-\alpha)Q_o+(1-\alpha)(1-D^*)^{\frac{1}{2}}-\beta+D^*\rho\right]^2}{4}-(1-\beta)\left[(1-\alpha)Q_o+(1-\alpha)(1-D^*)^{\frac{1}{2}}-\beta+D^*\rho\right]$$

$\pi_L^{nf*}\left(D_{nf}^*\right) - \pi_L^{f*}\left(D_f^*\right) > 0$ if and only if $\left[(1-\alpha)Q_o + (1-\alpha)^2 - \beta + \frac{\alpha[2-\alpha]}{2}\right] > 4(1-\beta)$. At $(1-\alpha)Q_o = \frac{7}{2} - 3\beta$, this inequality holds if and only if $-\alpha(7-6\beta) + (1-\alpha)^2 > 0$. LHS is decreasing in α. At $\alpha = 0$, it is positive, and at $\alpha = 1$, it is negative. Define $\widehat{\alpha}$ as

$$-\widehat{\alpha}(7-6\beta) + (1-\widehat{\alpha})^2 = 0$$

If $\alpha \leq \widehat{\alpha}$, then $\pi_L^{nf*}\left(D_{nf}^*\right) - \pi_L^{f*}\left(D_f^*\right) > 0$ for all $(1-\alpha)Q_o > \frac{7}{2} - 3\beta$ at $\rho = \frac{1}{2}$. Define $\widehat{\rho}$ as

$$\frac{\left[(1-\alpha)Q_o + (1-\alpha)\left(1-D_{nf}^*\right)^{\frac{1}{2}} - \beta + D_{nf}^*\widehat{\rho}\right]^2}{4} - \left(1-\beta+D_f^*(2\widehat{\rho}-1)\right)\left[(1-\alpha)Q_o + (1-\alpha)\left(1-D_f^*\right)^{\frac{1}{2}} - \beta + D_f^*\widehat{\rho}\right] = 0.$$

If $\frac{1}{2} < \rho \leq \widehat{\rho}$, then $\pi_L^{nf*}\left(D_{nf}^*\right) - \pi_L^{f*}\left(D_f^*\right) > 0$ and $D^* = D_{nf}^* < \widehat{D}$. If $\widehat{\rho} < \rho < \rho'$, then $D^* = D_f^* > \widehat{D}$.

If $\alpha > \widehat{\alpha}$, then define

$$Q_o' = \frac{6(1-\beta) + 1 - (1-\alpha)^2}{2(1-\alpha)}$$

If $\frac{7}{2(1-\alpha)} - \frac{3\beta}{(1-\alpha)} < Q_o \leq Q_o'$, then $\pi_L^{nf*}\left(D_{nf}^*\right) - \pi_L^{f*}\left(D_f^*\right) < 0$ for all $\frac{1}{2} < \rho \leq \rho'$.

If $Q_o > Q_o'$, then there exists $\widehat{\rho}$, such that if $\frac{1}{2} < \rho \leq \widehat{\rho}$, then $\pi_L^{nf*}\left(D_{nf}^*\right) - \pi_L^{f*}\left(D_f^*\right) > 0$ and $D^* = D_{nf}^* < \widehat{D}$. If $\widehat{\rho} < \rho \geq \rho'$, then $D^* = D_f^* > \widehat{D}$.

Thus, in every case, there exists $\widehat{Q}$, such that for $Q < \widehat{Q}$, $D^* = D_f^* \geq \widehat{D}$. For $Q \geq \widehat{Q}$, there exists $\widehat{\rho}$, such that $D^* = D_{nf}^* < \widehat{D}$ if $\frac{1}{2} < \rho \leq \widehat{\rho}$ and $D^* = D_f^* \geq \widehat{D}$ if $\widehat{\rho} \leq \rho < 1$.

(b) Comparative statics with α.

$$\frac{d}{d\alpha}D_{nf}^* = \frac{(1-\alpha)}{2\rho^2} \geq 0$$

Let us check if $\frac{d}{d\alpha}D_f^* > 0$. This is true if and only if

$$-(2\rho-1)\left[Q_o + \left(1-D_f^*\right)^{\frac{1}{2}}\right] + \frac{1}{2}\left(1-\beta+D_f^*(2\rho-1)\right)\left(1-D_f^*\right)^{-\frac{1}{2}} > 0$$

Substituting from the FOC for D_f^* and simplifying, $\frac{d}{d\alpha}D_f^* > 0$ if and only if

$$-3\rho\beta + \beta + 2(2\rho-1)D_f^*\rho + \rho > 0$$

$$-\beta(3\rho-1) + 2(2\rho-1)D_f^*\rho + \rho > 0$$

Since $\beta < \frac{1}{2} < \frac{\rho}{(3\rho-1)}$ is true, the above condition holds. □

Proof of Proposition A7.

(a)

$$W_L^*(D) = \begin{cases} \int_0^{\nu_L^J}(1+\alpha Q_o+\beta-\nu-D\rho)d\nu + \int_{\nu_L^J}^1\left[Q_o+(1-D)^{\frac{1}{2}}\right]d\nu \text{ if } 0 \leq D \leq \widehat{D} \\ Q_o+(1-D)^{\frac{1}{2}} \text{ if } \widehat{D} \leq D \leq 1. \end{cases}$$

$$\nu_L^J = \frac{2+\beta-D\rho-(1-\alpha)Q_o-(1-\alpha)(1-D)^{\frac{1}{2}}}{2}.$$

$$\frac{d}{dD}W_L^*(D) = \begin{cases} -\rho\nu_L^J + \left(\frac{(1-\alpha)}{4\sqrt{1-D}} - \frac{1}{2}\rho\right)\left(1+\alpha Q_o + \beta - \nu_L^J - D\rho\right) - \frac{(1-\nu_L^J)}{2\sqrt{1-D}} - \left[Q_o + (1-D)^{\frac{1}{2}}\right]\left(\frac{(1-\alpha)}{4\sqrt{1-D}} - \frac{1}{2}\rho\right) \text{ if } 0 \le D \le \widehat{D} \\ -\frac{1}{2\sqrt{1-D}} \text{ if } \widehat{D} \le D \le 1 \end{cases}$$

It can be checked that for α and β low enough, $\frac{d}{dD}W_L^*(D)\Big|_{D=0} > 0$, and hence $D^o > 0$. We restrict our attention to an interior optimum here.

Next, note that $D^o \le \widehat{D}$, since welfare is decreasing in the range where $\widehat{D} \le D \le 1$.

Finally, if $0 \le D \le \widehat{D}$, at $D^* = D^*_{nf} = 1 - \frac{1}{4\rho^2}$, $\frac{d}{dD}W_L^*(D) = -\rho\nu_L^J - \frac{\left(1-\nu_L^J\right)}{2\sqrt{1-D^*}} < 0$. This means that $D^o < D^*$.

(b) We have already shown that D^* is increasing in α. To see the efficient incentive, let us look at how $\frac{d}{dD}W_L^*(D)$ changes with α.

After some simplification, D^o solves

$$-\frac{(1+\alpha)}{8\sqrt{1-D^o}}(\beta - D^o\rho - (1-\alpha)Q_o) + \frac{(1-\alpha)^2}{8} - \frac{1}{4}\rho\left(4 + 3\beta - 3D^o\rho - 3(1-\alpha)Q_o - (3-\alpha)(1-D^o)^{\frac{1}{2}}\right) = 0.$$

The LHS is $\frac{d}{dD}W_L^*(D)$. Differentiating LHS of the inequality with respect to α and, simplifying, we see that

$$\frac{\partial^2 W_L^*(D)}{\partial\alpha\partial D^o} = -\frac{1}{8\sqrt{1-D^o}}(\beta - D^o\rho + 2\alpha Q_o) - \frac{(1-\alpha)}{4} - \frac{1}{4}\rho\left(4 + 3\beta - 3D^o\rho + 3Q_o + (1-D^o)^{\frac{1}{2}}\right) < 0.$$

Given that due to the Second Order Condition, $\frac{\partial^2 W_L^*(D)}{\partial {D^o}^2} < 0$, we see that D^o is decreasing in α. This means that, given the result in Proposition A6(b), $D^* - D^o$ increases as α increases. □

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