

Article

A Nonlinear State-Space Model for Fatigue Attention Dynamics in Online Learning Environments

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Abstract

Behavioural analytics remain a dominant approach for modelling learner engagement and predicting performance in digital learning environments. However, existing approaches are largely retrospective, relying on observable behavioural outcomes rather than modelling the underlying cognitive state dynamics that evolve during sustained learning. This study proposes a nonlinear state-space modelling framework that formalises the interaction between cognitive fatigue, attention, and learning as a continuous-time dynamical system. Fatigue is modelled as a latent state governed by load–recovery dynamics, attention is represented as a fatigue-coupled cognitive resource, and learning accumulation is expressed as an attention-mediated process under saturation constraints. The model is discretised and empirically estimated using time-indexed webcam-derived pilot data ($N = 63$) and further validated using a large-scale intervention dataset ($N = 1245$). Parameter estimation is performed using regression-based approximation of the discretised state equations, with cluster-robust inference applied to account for intra-session dependencies. The webcam-derived features were pre-processed using temporal windowing and normalisation to ensure consistency across sessions. The swarm-optimised intervention was implemented through adaptive control of instructional load and recovery scheduling, enabling real-time regulation of fatigue progression. Empirical results demonstrate statistically significant model validity, with fatigue dynamics showing moderate *explanatory* capability ($R^2 = 0.543$, $p < 0.001$) and attention dynamics also significant ($R^2 = 0.499$, $p = 0.004$). At the system level, adaptive intervention significantly reduced fatigue and improved learning performance ($t(1244) = 14.34$, $p < 0.001$). The findings suggest a transition from retrospective behavioural modelling toward anticipatory cognitivestate regulation, contributing toward a computational foundation for fatigue-aware adaptive learning systems.

Keywords: cognitive load theory; cognitive fatigue; learning attention; systems modelling; digital learning; learning analytics; adaptive learning



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1. Introduction

Online learning has evolved beyond simple content delivery into a complex digital ecosystem that collects, processes, and analyses large-scale learner data. Modern Learning Management Systems (LMS) continuously capture user interactions, assessment results, behavioural patterns, and participation activities. According to [1], the growth of big data has significantly enhanced the development of learning analytics models capable of identifying engagement patterns and predicting academic performance with high accuracy. Despite these advancements, the dominant paradigm remains largely retrospective and

behaviour-centred, relying on observable interaction features and static machine learning architectures.

While such approaches are effective for classification and prediction, they do not account for how underlying cognitive states evolve during sustained learning. As a result, performance degradation is typically detected only after observable behavioural decline has occurred, thereby limiting adaptive systems to reactive rather than proactive interventions.

Cognitive theory suggests that learning cannot be fully understood through behavioural indicators alone [2]. Cognitive Load Theory (CLT) posits that learning efficiency is constrained by working memory capacity and influenced by intrinsic and extraneous instructional load. Furthermore, learners differ in their cognitive processing capacities, and learning effectiveness is influenced by instructional design and contextual cognitive demands [2]. However, CLT remains structurally static because it defines capacity limits without explaining how sustained instructional demand progressively accumulates into cognitive fatigue over time.

Empirical studies show that prolonged mental effort leads to cognitive fatigue, which in turn causes a gradual decline in attention and information processing capacity [3,4]. As fatigue increases, attentional stability decreases, leading to increased variability in task performance and reduced effective learning time [5]. In online learning environments, these effects are further intensified, as demonstrated in recent meta-analytic studies on videoconferencing fatigue, which confirm that fatigue is structurally embedded in digital learning contexts rather than incidental [6].

Despite this growing body of evidence, several structural limitations persist in existing research. First, many studies rely on self-report instruments that provide subjective and intermittent assessments rather than continuous representations of cognitive state over time. Second, sensing approaches often depend on unimodal indicators, such as eye tracking or facial expressions, without integrating multiple behavioural signals into a coherent model of cognition. Third, predictive frameworks typically rely on correlation-based or structural equation models to associate fatigue indicators with learning outcomes, rather than modelling fatigue as a governed internal state that evolves under load–recovery dynamics [7]. Consequently, fatigue is often identified or correlated but not mechanistically represented.

This creates a theoretical inconsistency. On one hand, learning analytics research has produced increasingly refined models for engagement prediction [1]. On the other hand, fatigue research demonstrates that cognitive exhaustion develops cumulatively under sustained demand [3]. In related domains such as human performance and transportation, state-space modelling has been successfully used to represent fatigue as a dynamic system incorporating temporal accumulation and individual variability [8]. However, in online education, fatigue is typically treated as an outcome variable rather than a dynamically evolving state within a predictive framework.

The resulting limitation is structural rather than empirical. There is currently no unified framework that integrates multimodal behavioural sensing, fatigue–attention interaction modelling and adaptive LMS intervention within a single computational architecture. Existing e-learning systems can detect engagement states, but they do not model the temporal evolution of cognitive fatigue. Consequently, adaptive mechanisms respond only after performance degradation has occurred, limiting their ability to provide anticipatory instructional regulation.

Addressing this gap requires reframing fatigue from a detection problem to a regulation problem. Instead of treating fatigue as an observable label, it should be conceptualised as a latent, time-evolving state governed by instructional load and recovery processes. A formal model is therefore required to represent the causal pathway through which instruc-

tional demand drives fatigue accumulation, fatigue constrains attention, and attention mediates learning outcomes.

By integrating multimodal behavioural data with nonlinear state-space modelling and fatigue-aware instructional adaptation, this study establishes a mechanistic foundation for anticipatory regulation in e-learning systems. The primary contribution is the transformation of fatigue from a descriptive classification outcome into a predictive, dynamically governed constraint within adaptive learning environments.

To address this objective, the study is guided by the following research questions:

- i. RQ1: Can cognitive fatigue, attention, and learning be jointly represented within a unified nonlinear state-space framework in online learning environments?
- ii. RQ2: Do the proposed fatigue and attention dynamical equations exhibit statistically valid state-dependent behaviour in empirical data?
- iii. RQ3: Can adaptive instructional intervention, guided by the proposed model, significantly reduce fatigue and improve learning performance?

Accordingly, the objectives of this study are:

- i. To develop a unified nonlinear state-space model representing the interaction between fatigue, attention, and learning in online environments (addresses RQ1).
- ii. To empirically validate the proposed fatigue and attention dynamical equations using time-indexed observational data (addresses RQ2).
- iii. To evaluate the effectiveness of adaptive instructional intervention in reducing fatigue and improving learning performance (addresses RQ3).

2. Theoretical Foundations and Structural Gap Analysis

Understanding of cognitive performance in e-learning environments has been shaped by three dominant research streams: Cognitive Load Theory (CLT), cognitive fatigue and sustained attention studies, and artificial intelligence-driven learning analytics. Each of these streams provides valuable insights into how learners process information, sustain effort, and interact with online platforms. However, these perspectives operate in parallel rather than as an integrated framework. While each explains a specific aspect of the learning process, none provides a unified computational architecture for modelling how cognitive states evolve under sustained instructional demand.

Cognitive Load Theory explains that working memory is limited and that learning effectiveness depends on how cognitive load is managed [9]. Subsequent studies have incorporated individual differences and contextual factors into this framework [2]. Nevertheless, CLT remains structurally static, as it does not explain how sustained instructional demand gradually accumulates into cognitive fatigue over time.

Empirical research shows that attention declines during prolonged cognitive activities [3,4]. There is further evidence that fatigue develops progressively and is intensified in online environments, particularly in video conferencing contexts [5].

Despite these findings, existing studies on fatigue largely rely on self-report instruments that capture only momentary perceptions. In addition, sensing approaches often rely on unimodal indicators, and fatigue is frequently treated as an outcome variable rather than as a dynamic internal state that evolves over time [7].

Learning analytics systems can predict engagement and dropout risk using behavioural data. However, these models remain inherently static because they treat behavioural indicators as input features rather than modelling cognition as a time-evolving state.

In contrast, research in human performance domains employs state-space models to represent fatigue as a time-dependent process governed by system dynamics [8,9]. These models enable forward prediction under varying workload conditions. A comparative sum-

mary of existing approaches is presented in Table 1, highlighting their methods, datasets, and limitations.

Table 1. Summary of existing approaches in fatigue-aware learning systems.

Author	Year	Method	Dataset	Key Finding	Limitation
[1]	2020	Learning analytics	LMS data	Predicts academic performance	Static, non-dynamic
[3]	2017	Cognitive fatigue modelling	Laboratory tasks	Fatigue accumulates over time	No LMS integration
[5]	2024	Attention modelling	Experimental data	Attention declines during prolonged tasks	No predictive control
[7]	2022	Fatigue correlation analysis	Survey + LMS data	Fatigue affects learning outcomes	No mechanistic dynamical model
[8]	2024	State-space fatigue modelling	Driving data	Models temporal fatigue dynamics	Not applied to learning

Across these research streams, the gap is structural. Cognitive Load Theory defines cognitive limits, fatigue research demonstrates cumulative depletion, and learning analytics predicts behavioural outcomes; however, none integrates dynamic fatigue–attention coupling within adaptive LMS environments.

This fragmentation has important practical consequences. Without modelling how fatigue accumulates and influences attention over time, adaptive systems respond only after performance has already declined. The limitation is therefore architectural, as current systems lack a governed representation of internal cognitive state transitions.

Addressing this discontinuity requires a conceptual shift from fatigue detection to fatigue regulation through dynamical modelling. Fatigue must be formalised as a latent, time-evolving state governed by load–recovery dynamics. Attention should be explicitly modelled as coupled to fatigue progression, and learning accumulation must be represented as an attention-mediated process within bounded cognitive resource constraints. Only through such integration can online learning environments transition from reactive performance correction to anticipatory cognitive state regulation.

The gap identified here is not the absence of research on fatigue, engagement, or adaptation. Rather, it is the absence of a unified dynamical architecture that embeds multimodal behavioural evidence within a state-space representation capable of predictive instructional control. The subsequent sections develop this formulation in detail.

3. Dynamical Formulation of the Unified Fatigue–Attention–Learning System

This section presents the formal structure of the unified fatigue–attention–learning model illustrated in the developed block diagram. The learner is represented as a continuous-time nonlinear state-space system.

Instructional demand, recovery support, and motivation are treated as interconnected inputs that jointly influence fatigue, attention, and learning outcomes. Rather than acting independently, these components evolve dynamically during sustained online learning. The formulation builds on established fatigue state theory while introducing new relationships tailored to digital learning environments. The learning environment affects the system through external instructional and support mechanisms, formally represented by the control input vector:

$$u(t) \begin{bmatrix} L(t) \\ R(t) \\ M(t) \end{bmatrix} \quad (1)$$

where

$L(t)$ = instructional load (task complexity, pace, cognitive demand);

$R(t)$ = recovery mechanisms (breaks, reduced intensity, pacing adjustments);

$M(t)$ = motivational support (feedback, engagement prompts, reinforcement).

These variables act as external drivers of cognitive state evolution.

a. Fatigue Dynamics

Cognitive fatigue is modelled as a latent state $F(t) \geq 0$ governed by load–recovery dynamics:

$$\dot{F}(t) = \alpha L(t) - \beta R(t) - \lambda F(t) \quad (2)$$

This equation represents fatigue accumulation driven by instructional load, offset by recovery processes and natural dissipation. It formalises fatigue as a governed dynamical state rather than a post hoc classification outcome.

b. Attention Dynamics

Attention is represented as a bounded cognitive resource $A(t) \in [0, 1]$, evolving as:

$$\dot{A}(t) = \gamma F(t) + \delta M(t) - \rho(A(t) - A_0) \quad (3)$$

This equation introduces the key coupling in the model, whereby fatigue negatively influences attention through the term $-\gamma F(t)$. Motivation acts as a compensatory input, enforces stability around a baseline attention level A_0 .

c. Learning Dynamics

Learning accumulation is represented as:

$$\dot{K}(t) = \eta A(t) \varnothing(L(t)) \quad (4)$$

where the saturation function is defined as:

$$\varnothing(L(t)) = \frac{L(t)}{1 + \kappa L(t)}$$

This formulation ensures that learning gains increase with instructional load but eventually saturate under high demand, reflecting cognitive capacity constraints.

Discretisation of the Continuous-Time Model

To enable empirical estimation using time-indexed observational data, the continuous-time dynamical system is discretised using a first-order forward Euler approximation. Let Δt denote the sampling interval.

$$F_{(t+1)} = F_t + \Delta t [\alpha L_t - \beta R_t - \lambda F_t]$$

$$A_{(t+1)} = A_t + \Delta t [-\gamma F_t + \delta M_t - \rho(A_t - A_0)]$$

$$K_{(t+1)} = K_t + \Delta t \left[\eta A_t \cdot \left(\frac{L_t}{(1 + \kappa L_t)} \right) \right]$$

These discrete-time equations define the computable form of the system and enable parameter estimation and empirical validation using sequential observational data. To provide a unified representation, the system is expressed in nonlinear state-space form.

We define the state vector as:

$$x(t) = \begin{bmatrix} F(t) \\ A(t) \\ k(t) \end{bmatrix},$$

And input vector:

$$U(t) \begin{bmatrix} L(t) \\ R(t) \\ M(t) \end{bmatrix}$$

The system is then written as:

$$\dot{x} = F(x_{(t)}, U_{(t)}; \Theta) \quad (5)$$

where

$$\Theta = \{\alpha, \beta, \lambda, \gamma, \delta, \rho, \eta, \kappa, A_0\}$$

Expanded Form

The expanded state equations are:

$$\dot{X}_{(t)} = \begin{bmatrix} \alpha L_t - \beta R_t - \lambda F_t - \gamma F_t + \delta M_t \\ -\rho(A_t - A_0) \eta A_t \cdot \left(\frac{L_t}{(1+\kappa L_t)} \right) \end{bmatrix}$$

The function $f(\cdot)$ corresponds to the coupled fatigue, attention, and learning dynamics defined in Equations (2)–(4).

The model was implemented in Python 3.11, NumPy 1.26, and SciPy 1.13. Parameter estimation was performed using ordinary least squares applied to the discretised state equations. Missing values in the webcam-derived features were handled using interpolation, and all variables were standardised prior to estimation. Cluster-robust standard errors were computed to account for intra-session dependencies.

d. Measurement Model

Since fatigue and attention are latent states, they are not directly observable. Instead, they are inferred through measurable behavioural indicators using an observation model:

$$y_f(t) = c_F F(t) + \epsilon_F F(t)$$

$$y_A(t) = c_A A(t) + \epsilon_A(t)$$

where

- $y_f(t)$, $y_A(t)$ are observed signals derived from webcam features;
- c_F , c_A are scaling coefficients;
- $\epsilon_F F(t)$, $\epsilon_A(t)$ represent observation noise.

e. Model Implementation Details.

The continuous-time fatigue, attention, and learning equations were transformed into discrete-time form using the first-order forward Euler method with sampling interval Δt . This yielded computable update equations for fatigue, attention, and learning across successive observational windows. Parameter estimation was performed using ordinary least squares regression applied to the discretised state equations, where lagged state variables and input variables were used as predictors of state change. Missing values in webcam-derived features were handled through interpolation, and all variables were standardised prior to estimation to ensure comparability across sessions. Cluster-robust standard errors were computed to account for within-session dependence in the time-indexed observations.

f. Webcam Feature Extraction and State Construction

Webcam video streams were captured at approximately 30 frames per second and processed using computer vision pipelines. Facial landmarks and eye regions were extracted

using computer vision methods implemented within a Python-based processing framework. Gaze behaviour was estimated from eye-region trajectories, while blink frequency and eye closure duration were computed from eyelid aspect ratio dynamics.

The following behavioural indicators were extracted:

- blink rate;
- fixation duration;
- saccadic eye movement frequency;
- gaze dispersion;
- eye closure duration (PERCLOS);
- head pose variation;
- facial activity intensity.

Features were aggregated into fixed temporal windows of 30–60 s to reduce high-frequency noise and ensure temporal consistency. Each feature was standardised using z-score normalisation prior to latent state estimation. The extracted behavioural indicators were then combined within temporal observation windows to estimate latent fatigue and attentional states. Fatigue estimation incorporated blink dynamics, PERCLOS, gaze instability, and head movement variability, while attention estimation was derived from fixation stability, gaze concentration, and interaction continuity indicators.

g. **Operationalisation of Latent Cognitive States**

i. **Cognitive fatigue**

Cognitive fatigue was defined as a gradual mental state of tiredness that develops when learners spend long periods performing demanding learning activities online. Since fatigue cannot be measured directly, it was estimated from observable behavioural patterns captured through webcam analysis. These behaviours included changes in blinking patterns, unstable eye focus, scattered gaze movement, longer eye closure duration, reduced facial expression, and irregular head movement. The behavioural data were processed and standardised over consecutive time intervals to monitor how fatigue increased or decreased during learning sessions.

ii. **Attention**

Attention was also defined as the learner's ability to maintain stable focus during online learning activities. It was estimated from behavioural indicators such as consistent gaze fixation, sustained visual concentration on learning materials, continuity in task responses, and stable interaction behaviour within the LMS environment. Learners who maintained steady gaze behaviour and consistent interactions over time were considered to have higher attention levels, while fluctuating behaviour indicated reduced attention.

iii. **Instructional load**

Instructional load referred to the level of mental effort required from learners during learning activities. It was represented by factors such as the difficulty of tasks, the amount of instructional activities, and the speed or intensity of lesson delivery within the LMS environment. Higher instructional load implied greater cognitive demand on the learner.

iv. **Recovery**

Recovery referred to conditions or activities that helped reduce cognitive strain and mental exhaustion during learning. These included scheduled breaks, reduced task intensity, slower instructional pacing, and periods of lower learning demand within the session.

v. Motivation

Motivation was defined as the learner's willingness to remain engaged and continue participating in learning activities despite cognitive demands. It was represented using LMS interaction indicators such as continued participation in tasks, responsiveness to engagement prompts, persistence in completing activities, and consistency of interaction throughout the learning session. Higher motivation was associated with sustained learner engagement and active participation over time.

h. Algorithmic Implementation

The implementation of the proposed fatigue–attention–learning system follows a structured sequential process. First, webcam-derived behavioural features and interaction data are collected and segmented into fixed temporal windows. Each window contains synchronised multimodal observations used to compute state variables including fatigue, attention, instructional load, recovery, and motivation.

Next, discrete state transitions are computed using first-difference formulations, enabling estimation of fatigue and attention dynamics. Model parameters are estimated using ordinary least squares regression applied to time-indexed observations.

The system then performs forward prediction of fatigue and attention states at subsequent time steps. Based on these predictions, an optimisation mechanism selects adaptive instructional actions aimed at minimising fatigue progression while maintaining learning performance.

Finally, instructional conditions are updated dynamically in response to the selected actions, relating to a closed-loop adaptive learning process that continuously regulates the learner's cognitive state.

Figure 1 illustrates the nonlinear state-space integration of instructional load, recovery, and motivational inputs with fatigue accumulation, attention stability, and learning formation. Solid arrows indicate structural model flow. Dashed arrows indicate intersections between adopted modelling primitives and proposed couplings that produce the unified system.

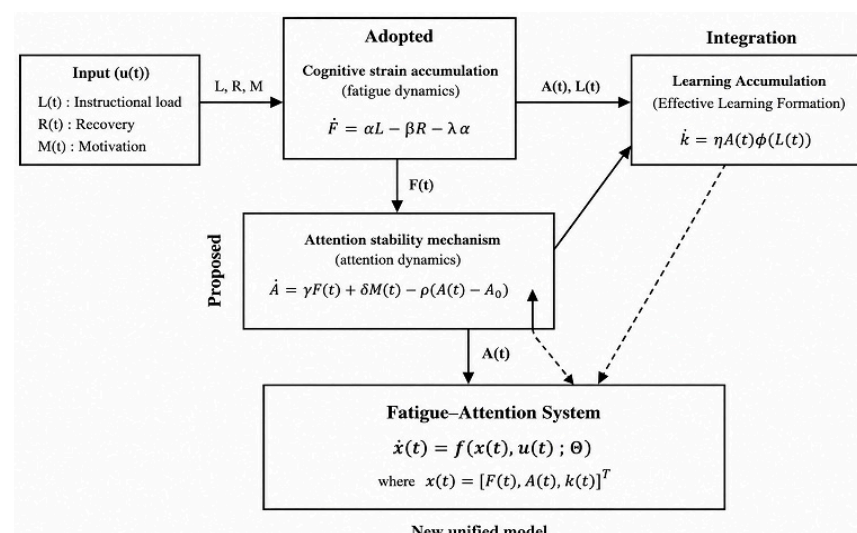


Figure 1. Architecture of the fatigue-aware adaptive learning system.

The novelty of this model lies in its integration of established load–recovery fatigue dynamics with an explicit fatigue–attention coupling and an attention-mediated learning process within a unified continuous-time state-space framework. This establishes a computational foundation for fatigue-aware adaptive learning systems.

4. Empirical Validation of the Fatigue–Attention Dynamical Model

a. Dataset Description and Pre-processing

The empirical analysis utilised two datasets: a pilot dataset ($N = 63$) and a large-scale intervention dataset ($N = 1245$). The pilot dataset was derived from webcam-based observations collected during online learning sessions, while the larger dataset was obtained from LMS-based intervention experiments.

The webcam-derived features included gaze-related and behavioural indicators such as fixation duration, blink rate, eye movement patterns, and facial activity signals. These features were extracted from video streams and aligned with interaction logs to construct time-indexed observations of learner state. Data were sampled at approximately 30 frames per second and aggregated into fixed temporal windows to enable stable estimation of fatigue and attention dynamics. Each observation window contained synchronised multi-modal features and corresponding instructional context variables. Pre-processing involved temporal windowing, standardisation of variables, and interpolation of missing values to ensure continuity of the time series. Outliers were controlled through normalisation to maintain consistency across sessions.

The intervention dataset consisted of learners engaged in structured online learning activities in which adaptive instructional strategies were applied. Although detailed demographic attributes were not explicitly modelled, the dataset reflects a heterogeneous population of learners participating in real-world LMS environments.

b. Model Estimation Procedure

The objective was to determine whether the discretised forms of the fatigue and attention differential equations were statistically supported and whether their equilibrium implications were consistent with large-sample intervention outcomes. Although nonlinear state-space systems are frequently estimated using recursive approaches such as Kalman filtering or Bayesian estimation, the present study adopted ordinary least squares estimation for three reasons.

First, the continuous-time system was discretised into approximately linear first-difference equations using a first-order forward Euler approximation, allowing direct regression-based estimation of state transitions. Second, the primary objective of this study was structural validation of the proposed fatigue–attention dynamical relationships rather than optimal recursive state tracking in real time. Third, the relatively small pilot dataset ($N = 63$) limits the stability of higher-order nonlinear recursive estimators, whereas ordinary least squares provides interpretable parameter estimates with robust asymptotic properties under clustered repeated observations. The use of cluster-robust standard errors further mitigates within-session dependence and improves inferential reliability. Future work will extend the framework using Kalman filtering and Bayesian recursive estimation for real-time adaptive deployment.

Time-indexed pilot observations were constructed by merging gaze-derived features across successive learning windows. The discretised state equations were estimated using a regression-based approach. Parameter estimation was performed using ordinary least squares applied to the time-indexed data.

To ensure robustness, variables were standardised prior to estimation, and missing observations were handled using interpolation. Cluster-robust standard errors were computed to account for intra-session dependencies. The estimated parameters of the fatigue and attention models are reported in Table 2. The overall model fit statistics for the fatigue and attention dynamical models are presented in Table 3. For each session, fatigue F_t , atten-

tion A_t , instructional load L_t , and recovery R_t were recorded across successive temporal windows. Discrete derivatives were computed as first differences within session:

$$\Delta F_t = F_t - F_{t-1}$$

$$\Delta A_t = A_t - A_{t-1}$$

$$\Delta F_t = c + \alpha L_{t-1} + \beta R_{t-1} - \lambda F_{t-1} \varepsilon_t$$

Table 2. Dynamic validation of fatigue and attention state equations.

Model	Predictor	β	SE	z	p-Value
Fatigue dynamics	Intercept	0.3108	0.0834	3.7271	<0.001
Fatigue dynamics	Instructional load lag	0.0088	0.2535	0.0348	0.972
Fatigue dynamics	Recovery lag	0.0457	0.0457	0.9990	0.318
Fatigue dynamics	Fatigue lag	−0.7995	0.1054	−7.5861	<0.001
Attention dynamics	Intercept	0.6084	0.1659	3.6672	<0.001
Attention dynamics	Fatigue lag	−0.2262	0.3490	−0.6481	0.517
Attention dynamics	Attention lag	−0.5729	0.1501	−3.8171	<0.001

Table 3. Pilot time-series validation of fatigue and attention dynamics.

Model	N	R ²	Adjusted R ²	F-Statistic	Prob(F)
Fatigue dynamics	63	0.5435	0.5203	61.4185	<0.001
Attention dynamics	63	0.4986	0.4819	8.4531	0.004

The lagged fatigue state was a strong negative predictor of subsequent fatigue change ($\beta = -0.7995$, $p < 0.001$). This confirms the dissipative stability mechanism predicted by the $-\lambda F(t)$ term in the theoretical model. Fatigue does not diverge indefinitely; instead, it exhibits bounded dynamic behaviour. Figure 2 presents the relationship between observed and predicted state changes for the fatigue and attention dynamical models.

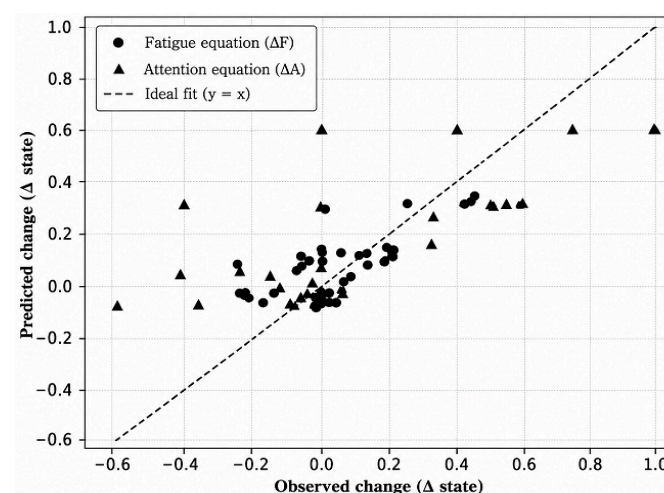


Figure 2. Observed versus predicted state changes.

This result reinforces the interpretation of attention as a bounded, dynamically regulated cognitive resource. Compared to existing models, the inclusion of fatigue–attention coupling provides a more realistic representation of cognitive state transitions during sustained learning.

The lagged attention state exhibited a significant negative association with subsequent change ($\beta = -0.5729$, $p < 0.001$, 95% CI $[-0.867, -0.279]$), confirming bounded attentional dynamics with reversion behaviour. The fatigue-to-attention coupling retained the expected negative sign but was not statistically significant ($p = 0.517$). This suggests that, although the directional relationship aligns with the theoretical model, the effect may be weak or masked by variability in the pilot dataset. It also indicates that a larger time-series sample or higher-resolution data may be required to reliably estimate the coupling parameter γ .

The combined pilot results demonstrate that both fatigue and attention exhibit stable state-dependent dynamics consistent with the mathematical formulation proposed in Section 3. The presence of significant lagged state damping terms in both equations provides empirical support for modelling these constructs as continuous-time dynamical states rather than static classification outputs.

The large-scale intervention dataset ($N = 1245$) provides equilibrium-level validation of the broader fatigue–attention–learning cascade. The adaptive intervention mechanism was implemented using a swarm-based optimisation approach to dynamically regulate instructional conditions based on predicted fatigue levels. In this framework, candidate instructional strategies were evaluated based on their ability to reduce fatigue while maintaining learning performance.

The optimisation process iteratively selected instructional actions such as load adjustment, recovery scheduling, and pacing modification to minimise fatigue progression. Rather than relying on fixed instructional sequences, the system continuously adapted learning conditions in response to the learner’s evolving cognitive state.

Participants in the large-scale dataset ($N = 1245$) were exposed to adaptive learning conditions, which were compared against baseline non-adaptive scenarios. The effectiveness of the intervention was evaluated using fatigue reduction and learning performance improvement metrics. The swarm-optimised intervention significantly reduced fatigue, $t(1244) = 14.34$, $p < 0.001$ and produced corresponding improvements in learning performance. These findings validate the monotonic implications of the learning equation: $k(t) = \eta A(t) \oslash (L(t))$

Although time derivatives of learning were not directly measured, the paired intervention outcomes confirm that reducing fatigue corresponds to improved performance, consistent with the cascade structure of the dynamical model. Taken together, the dynamic pilot validation and the large-sample intervention validation provide complementary evidence. The pilot confirms micro-level state evolution and bounded dynamics. The large dataset confirms macro-level instructional impact consistent with the proposed fatigue-constrained learning system.

The fatigue equation demonstrated moderate explanatory capability ($R^2 = 0.543$, $F(3, 59) = 61.42$, $p < 0.001$), with the lagged fatigue term exhibiting significant negative damping ($\beta = -0.7995$, $p < 0.001$, 95% CI $[-1.006, -0.593]$). The reported confidence intervals further confirm the stability and reliability of the estimated parameters, as they do not cross zero for the significant predictors. The explanatory power of the fatigue model ($R^2 = 0.543$) indicates moderate explanatory capability when compared with prior fatigue modelling studies, which typically report lower explanatory performance due to reliance on static or unimodal indicators [4,8]. Similarly, the attention model ($R^2 = 0.499$) reflects substantial explanatory capacity given the inherently high variability associated with attention processes in experimental settings [6].

In contrast to existing approaches that primarily establish correlation relationships between fatigue and performance, the proposed model captures state-dependent temporal dynamics, thereby extending prior work from static prediction to dynamic system modelling.

At the system level, the intervention effect ($t(1244) = 14.34, p < 0.001$) represents a statistically significant and practically relevant improvement compared to traditional non-adaptive learning systems, where intervention effects are typically incremental and reactive. This result demonstrates that the proposed framework not only models cognitive states but also enables effective regulation of fatigue, leading to measurable gains in learning performance.

Overall, these findings suggest that the proposed approach extends conventional behavioural and correlation models by combining explanatory dynamical modelling with actionable adaptive intervention, thereby advancing the state of the art in fatigue-aware learning systems.

Taken together, the pilot time-series validation confirms micro-level state evolution and bounded dynamics, while the large-sample intervention confirms macro-level adaptive effectiveness aligned with the proposed fatigue-constrained learning mechanism. This dual-layer validation strengthens the structural credibility of the dynamical formulation.

This study extends Cognitive Load Theory from a structurally static framework into a governed temporal system. The work also progresses learning analytics by developing a mechanistic alternative to conventional feature prediction architectures. This represents a conceptual shift from outcome prediction toward cognitive trajectory modelling.

c. Algorithmic Implementation and Data Structure

The time-indexed dataset was organised as sequential observations for each learner session. Each observation window t contains:

$$X_t = \{F_t, A_t, L_t, R_t, M_t, Y_t\}$$

where F_t represents fatigue, A_t , attention, L_t instructional load, R_t recovery input, M_t motivational input, and Y_t observed behavioural features derived from webcam data. These observations form a multivariate time series used for model estimation and validation.

Algorithmic Steps

The implementation of the fatigue–attention–learning system follows a sequential pipeline:

- Collect webcam-derived behavioural features and interaction data;
- Segment data into fixed temporal windows;
- Pre-process data through normalisation and interpolation;
- Compute state variables (fatigue, attention, load, recovery);
- Estimate discretised state equations using regression;
- Predict fatigue and attention evolution over time;
- Apply swarm-based optimisation to select adaptive instructional actions;
- Update instructional parameters dynamically based on optimisation output.

d. Swarm Optimisation Configuration

The adaptive intervention mechanism was implemented using Particle Swarm Optimisation (PSO). Each particle represented a candidate instructional strategy consisting of instructional load adjustment, recovery scheduling, pacing modification, and reinforcement intensity.

The optimisation objective was:

$$J(u_t) = w_f F_{t+1} - w_p P_{t+1} + w_c C(u_t)$$

where

F_{t+1} denote predicted fatigue,

P_{t+1} predicted learning performance,

$C(u_t)$ intervention cost

The optimisation process used:

- swarm size = 20;
- maximum iterations = 50;
- inertia weight $w = 0.7$;
- cognitive coefficient $c_1 = 1.5$;
- social coefficient $c_2 = 1.5$.

The algorithm iteratively updated instructional parameters to minimise fatigue progression while preserving learning effectiveness.

5. Discussion, Limitation and Conclusions

a. Discussion

The findings of this study provide empirical support for modelling fatigue and attention as dynamically evolving cognitive states within online learning environments. Unlike conventional learning analytics approaches that primarily rely on static behavioural indicators for prediction, the proposed framework models fatigue, attention, and learning as interconnected state-dependent processes evolving over time.

Compared with the approaches summarised in Table 1, the proposed model integrates fatigue, attention, and learning within a unified nonlinear state-space framework. Existing learning analytics systems such as [1] primarily use behavioural interaction data to predict performance outcomes without explicitly modelling the temporal evolution of cognitive states. In contrast, the present study demonstrates that fatigue and attention exhibit bounded dynamical behaviour over successive learning intervals, with fatigue dynamics showing moderate explanatory capability ($R^2 = 0.543$) and attention dynamics also demonstrating meaningful explanatory performance ($R^2 = 0.499$).

From a modelling perspective, previous fatigue studies such as [3] describe fatigue accumulation conceptually but do not formalise it within a governed dynamical system. Similarly [6] establishes relationships between fatigue and learning outcomes but does not represent fatigue as a continuously evolving latent state. In contrast, the present study formulates fatigue, attention, and learning as a coupled nonlinear state-space system in which fatigue evolves under load–recovery dynamics, attention is influenced by fatigue progression, and learning is represented as an attention-mediated process operating under bounded cognitive constraints.

Compared to state-space approaches in other domains, such as [8] in driving fatigue modelling, the present work extends state-space fatigue representation into adaptive educational environments by integrating instructional control variables, attention dynamics, and learning regulation within a unified adaptive framework. This enables not only prediction of cognitive fatigue progression but also real-time adaptive instructional regulation.

The pilot time-series validation demonstrates that both fatigue and attention exhibit stable state-dependent behaviour consistent with the theoretical dynamical formulation. The significant negative lagged fatigue coefficient supports the existence of bounded fatigue dynamics rather than uncontrolled fatigue accumulation. Similarly, the significant lagged attention term indicates that attention behaves as a dynamically regulated cognitive resource with reversion behaviour over time.

Although the fatigue-to-attention coupling retained the expected negative direction, the relationship was not statistically significant in the pilot dataset. This suggests that the interaction may be influenced by variability in behavioural attention measurements or by limitations associated with the relatively small pilot sample. The result should therefore be interpreted cautiously and viewed as preliminary evidence rather than definitive con-

firmation of the coupling mechanism. Future studies using larger longitudinal datasets and higher-resolution sensing modalities may provide more reliable estimation of the coupling parameter.

At the system level, the adaptive intervention mechanism demonstrated statistically significant improvement over baseline non-adaptive conditions ($t(1244) = 14.34$, $p < 0.001$). The optimisation-based intervention continuously adjusted instructional conditions through load regulation, pacing modification, and recovery scheduling in response to predicted fatigue progression. This indicates that the proposed framework extends beyond passive monitoring toward actionable adaptive regulation of learner cognitive state.

The novelty of the proposed framework lies in three principal contributions. First, fatigue is represented as a governed latent dynamical state rather than as a post hoc classification outcome. Second, the model explicitly integrates fatigue, attention, and learning within a unified nonlinear state-space architecture. Third, the framework combines dynamical cognitive modelling with adaptive instructional optimisation, thereby enabling anticipatory fatigue-aware learning regulation in online educational environments.

Overall, the findings suggest that the proposed framework contributes toward a computational foundation for fatigue-aware adaptive learning systems. By integrating multimodal behavioural sensing, nonlinear dynamical modelling, and optimisation-based intervention within a unified architecture, the study advances learning analytics from retrospective behavioural prediction toward predictive cognitive state regulation.

b. Limitations

The pilot dataset was comparatively small, which may affect parameter estimation precision and stability of the estimated coupling relationships. Webcam-derived behavioural features may also introduce variability due to environmental and recording conditions. In addition, learning accumulation was evaluated indirectly through intervention outcomes rather than direct measurement of its temporal derivative. The intervention study may further be influenced by confounding factors such as learner motivation, prior knowledge, and individual learning differences. Future work will focus on large-scale longitudinal validation, higher-resolution sensing modalities, and real-time LMS integration.

c. Ethical Considerations

This study adhered to established ethical standards for research involving human participants. Ethical approval exemption was granted because the study involved non-invasive behavioural observation within an educational environment and did not involve medical experimentation or clinical intervention. All procedures followed institutional research and data protection guidelines.

Participants were informed about the nature and purpose of the study, and informed consent was obtained prior to data collection. The use of webcam-based data for gaze and facial feature extraction was conducted with full awareness of participants, and no covert recording was involved.

No raw facial video recordings were publicly shared or externally distributed. Behavioural features were anonymised immediately after extraction, and all datasets were stored on password-protected systems accessible only to authorised research personnel.

The analysis focused solely on behavioural and cognitive indicators relevant to learning, without any attempt to infer sensitive personal attributes.

The study design ensured that participants were not exposed to any form of harm, discomfort, or undue risk. Participation was voluntary, and participants had the right to withdraw at any stage without penalty.

Overall, the research complies with ethical principles of confidentiality, transparency, and responsible data use in technology-enhanced learning environments.

d. Conclusions

This research develops and evaluates a unified dynamical framework to describe how fatigue, attention, and learning work together in online environments. The study moves learning analytics beyond simple engagement tracking and provides a clearer theoretical and computational foundation for fatigue-aware instructional support.

By putting together cognitive theory, state-space modelling, and adaptive educational technology within a unified architecture, the research redirects online learning analytics from behavioural classification toward a more dynamic understanding of cognitive processes.

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Informed Consent Statement: Informed consent was obtained from all participants involved in the study prior to data collection. Participants were informed about the use of webcam-derived behavioural indicators and their right to withdraw at any stage without penalty.

Data Availability Statement: The datasets generated and analysed during the current study are not publicly available due to privacy and ethical restrictions involving behavioural participant data but may be made available from the corresponding author upon reasonable request and subject to institutional approval.

Conflicts of Interest: The authors declare no conflicts of interest.

Abbreviations

The following abbreviations are used in this manuscript:

LMS	Learning Management System
CLT	Cognitive Load Theory
AI	Artificial Intelligence
N	Sample Size
R ²	Coefficient of Determination
p	p-Value
T	t-Statistic
RQ	Research Question
OLS	Ordinary Least Squares
AIC	Akaike Information Criterion
BIC	Bayesian Information Criterion
CI	Confidence Interval
SE	Standard Error
Z	z-score

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