

Article

Making Trilateration Work with Imperfect Data: A Low-Cost BLE-Based Indoor Positioning Approach

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Abstract

Bluetooth Low Energy (BLE)-based trilateration is attractive for indoor positioning because it can operate with low-cost infrastructure, yet RSSI-derived distances can become geometrically incompatible in real deployments. This study characterises those failure geometries using 162 million raw RSSI records collected from Emergency Room and Office deployments at Diponegoro National Hospital, Indonesia, from which 12.9 million synchronised three-scanner observations were analysed. Four recurring Trilateration Failure Modes (TFMs) are quantified across two deployments, three LDPL exponent scenarios, and raw versus Kalman-filtered RSSI. Failure-mode prevalence depends jointly on deployment and the assumed RSSI-to-distance model: TFM-I dominates several conditions, whereas TFM-III becomes dominant under higher-attenuation assumptions, with the transition occurring at different scenarios in the two deployments. To contextualise the Radius Adjustment Intersection Method (RAIM), two deterministic geometric baselines are evaluated: a common-factor Uniform Radius Scaling (URS) baseline and a Paek-style pairwise incremental adjustment. A common scale factor is feasible for 23.63–96.34% of observations, showing that a single multiplicative correction cannot reconcile all observed circle configurations. The Paek-style adjustment and RAIM both recover near-complete three-circle feasibility, but they produce different correction magnitudes and coordinate displacements. RAIM retains the smallest RSSI-derived radius as an anchor and sequentially adjusts the remaining radii. These outcomes are interpreted as geometric constraint-satisfaction and spatial-plausibility results rather than absolute localisation accuracy because ground-truth coordinates were unavailable. Anchor-sensitivity analysis further shows that alternative anchor choices can shift estimated coordinates by several metres, while room-polygon outlier status changes by less than 0.06%. The results identify deployment-dependent failure geometry, correction-model choice, and anchor selection as considerations for low-cost BLE trilateration.



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Keywords: Bluetooth Low Energy (BLE); Georeferencing; indoor positioning (IPS); noise; Radius Adjustment Intersection Method (RAIM); RSSI; Trilateration Failure Modes (TFM)

1. Introduction

Indoor Positioning Systems (IPSs) have seen growing adoption in healthcare contexts, where reliable indoor location information can support operational monitoring, workflow analysis, and spatially aware applications [1]. In such environments, positioning systems must often operate under practical constraints on infrastructure, sensing hardware, and deployment cost.

Among positioning technologies, Bluetooth Low Energy (BLE)-based Received Signal Strength Indicator (RSSI) systems offer a promising alternative because of their cost-effectiveness, widespread availability, and ease of deployment [2]. In resource-constrained environments such as Diponegoro National Hospital (DNH), these characteristics make BLE-based IPS particularly well-suited for immediate adoption without requiring specialised or costly infrastructure.

Our previous studies on mobile (or moving) objects focused on optimising broadcast channels for data dissemination [3,4]. However, our multi-channel broadcast indexing methods assume that the data have been preprocessed and that the locations of the moving objects have been correctly identified. The present study addresses this prerequisite by focusing on reliable location identification using BLE-based IPS.

Although BLE-based IPS are widely adopted, they are susceptible to significant performance degradation caused by environmental factors such as obstructions, interference, and signal reflections [5]. Physical barriers such as walls, glass, and furniture that obstruct the line of sight between the signal transmitter (e.g., healthcare workers) and the receiver (scanner) result in signal attenuation [2]. This attenuation introduces noise into the received signal, causing rapid, high-amplitude fluctuations over short time intervals.

The central problem this paper addresses is this: In ideal conditions, the trilateration method [6–8], which estimates a target's location from its distances to three reference points, produces three circles that intersect at a common point. However, in real deployments, signal noise and environmental interference can make the three RSSI-derived distances geometrically inconsistent. As a result, conventional trilateration may fail to produce a common three-circle intersection.

Figure 1 illustrates this contrast. The left-hand image shows an ideal geometric configuration in which three circles intersect at a common point. The right-hand image shows how inconsistent RSSI-derived distances can produce a configuration in which only two circles intersect while the third remains detached. This configuration represents one recurring failure mode observed in the studied deployments.

The literature therefore provides several mechanisms for coping with noisy RSSI, but a complementary question remains less clearly characterised: what geometric forms do trilateration failures take in a long-running, low-infrastructure deployment, and how stable are those forms when the RSSI-to-distance model or filtering condition changes? Answering this question is important before selecting a correction strategy, because a correction that assumes one universal failure pattern may not transfer across propagation conditions.

This paper addresses that empirical gap using 162,092,585 raw RSSI records collected from Emergency Room and Office deployments at Diponegoro National Hospital. After timestamp synchronisation, 12,986,215 three-scanner observations were available for the complete geometric analysis. The study has at least two contributions :

- a. Large-scale empirical characterisation of trilateration failure geometry. We identify and classify four recurring Trilateration Failure Modes (TFMs) from synchronised three-scanner observations and quantify their prevalence across two real deployments, three LDPL exponent scenarios, and raw versus Kalman-filtered RSSI. The resulting cross-condition distribution shows that TFM-I and TFM-III account for most obser-

vations in the evaluated conditions, but their prevalence changes substantially with deployment and the assumed RSSI-to-distance model.

- b. A comparative geometric-correction analysis and anchor-sensitivity analysis. Guided by the observed failure geometry, we evaluate conventional Trilateration, a Paek-style pairwise incremental adjustment, a deterministic Uniform Radius Scaling (URS) baseline, and RAIM on the same synchronised observations. The comparison distinguishes pairwise incremental, common-factor, and radius-specific sequential correction strategies and quantifies geometric feasibility, spatial plausibility, correction magnitude, coordinate displacement, and anchor sensitivity. None of the correction strategies is presented as a ground-truth-validated accuracy improvement.

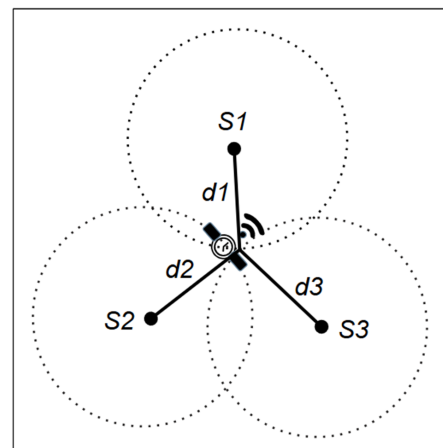
The primary contribution of the study is the empirical characterisation of real-world failure geometry; RAIM is retained as a demonstration of how that characterisation can inform a lightweight correction strategy under resource constraints. The contribution is therefore intentionally empirical and diagnostic rather than a claim of a universally superior localisation solver.

S : Fixed position of scanner

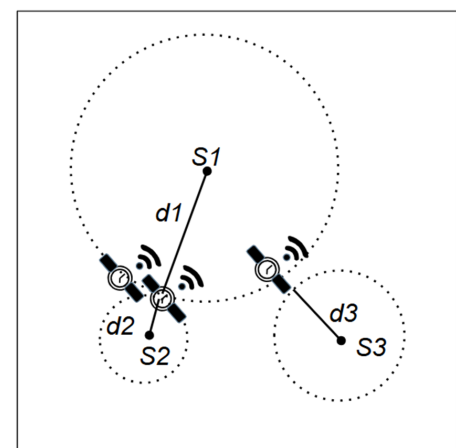
$\odot$: Signals captured by scanner

d : Distance from scanner to target (radius)

$\odot$: Target (healthcare worker / nurse)



(a) Successful trilateration



(b) Unsuccessful trilateration

Figure 1. Successful trilateration (a), where three circles intersect at a single point, and geometrically infeasible trilateration (b), where signal noise prevents a common intersection, resulting in positioning failure. The circles represent RSSI-derived distance constraints.

2. Related Work

Previous studies have sought to mitigate BLE trilateration challenges through various approaches. This section reviews these efforts and identifies the gap that this paper addresses.

2.1. Filtering-Based Approaches

Filtering-based approaches address trilateration inaccuracies by excluding unreliable measurements and prioritising signals from nearby anchor nodes, assuming that poor-quality signals are infrequent [9]. This method performs well when at least three high-quality signals remain after filtering; however, it fails when the resulting circles do not intersect, effectively discarding valuable observations.

2.2. Quality-Based Approaches

Quality-based approaches evaluate localisation accuracy using a Quality of Trilateration (QoT) score derived from the geometric configuration of reference points and the

noise characteristics of distance measurements [10]. Although this improves estimation reliability, it requires a sufficient number of reference points, which is not always available in many real deployments.

2.3. Sensor-Fusion Approaches

Sensor-fusion approaches integrate inertial sensors to track movement and enhance localisation accuracy when an insufficient number of scanners are available [11]. While effective in compensating for sparse signal readers, this approach requires additional hardware, increasing deployment cost and system complexity.

2.4. Geometric Adjustment Approaches

A different strategy is proposed in [12,13], which address trilateration through geometric adjustment. Pande and Ibwe use a circle expansion-and-reduction mechanism based on common factors applied to the radii, whereas Paek et al. [13] use pairwise geometric adjustments, primarily increasing smaller or enclosed circles in fixed increments until intersection conditions are satisfied. Thus, these studies belong to the same broad geometric-adjustment family but do not use the same correction mechanism. The present study differs by first characterising the failure geometries observed in the deployment and then using observation-specific, sequential radius adjustment rather than a common-factor or pairwise incremental rule.

2.5. Machine Learning Approaches

Other approaches frame localisation as a classification problem using machine learning techniques, which typically require ground-truth data for model training. For example, ref. [14] introduces a matrix-based method to estimate the coordinates of overlapping regions, relying on a training dataset to establish the relationship between RSSI values and distances. Similarly, several studies employ classification-based machine learning models to estimate target locations [15,16]. Decision Tree, Random Forest, and k-Nearest Neighbour (kNN) classifiers have also been explored for this purpose [17]. However, such methods require labelled ground-truth data for training, which is unavailable in many resource-constrained deployments.

2.6. The Gap

The reviewed approaches address different aspects of RSSI-based positioning as compared in Table 1, but they do not by themselves establish how trilateration failure geometry behaves across long-running real deployments and different RSSI-to-distance assumptions. This distinction matters because a geometric correction can only be interpreted appropriately if the underlying failure geometry is first characterised.

Accordingly, this study focuses on two linked questions:

- **Failure characterisation:** What recurring geometric configurations arise when three RSSI-derived circles are formed from synchronised observations, and how does their prevalence vary across deployment locations, LDPL exponent scenarios, and filtering?
- **Correction feasibility:** How do different lightweight geometric correction strategies (the pairwise incremental adjustment, common-factor scaling, and radius-specific sequential adjustment) reconcile the same observed geometry, and what evidence can be obtained without ground-truth coordinates?

These questions deliberately separate the empirical diagnosis of failure from the stronger claim of localisation accuracy. The study therefore treats geometric feasibility and spatial plausibility as observable outcomes, while recognising that absolute localisation accuracy requires independent ground-truth coordinates.

Table 1. Comparison of representative BLE positioning approaches discussed in this study.

Approach	Representative Work	Core Strategy	Additional Sensing	Training/Ground Truth	Treatment of Failed Geometry	Relation to RAIM
Filtering	[9]	Remove unreliable measurements	No	Not required for filtering	Discards observations that remain geometrically inconsistent	RAIM retains observations and adjusts selected radii
Quality-based	[10]	Quantify geometric confidence	No	Requires sufficient reference geometry	Assesses quality rather than correcting the radii	RAIM directly modifies selected radii
Sensor fusion	[11]	Combine BLE with inertial sensing	Inertial sensors	Not necessarily	Compensates for sparse or unreliable BLE observations using another modality	RAIM uses the existing BLE measurements only
Geometric adjustment	[12,13]	Common-factor expansion/reduction (Pande); pairwise incremental adjustment (Paek)	No	Study-specific validation/calibration	Adjusts radii without the TFM-based anchor/non-anchor role distinction used here	RAIM selects an anchor and adjusts the non-anchor radii sequentially
Machine learning	[14,15,17]	Learn RSSI-to-location relationship	No additional sensor necessarily	Labelled training data typically required	Learns a location mapping rather than repairing circle geometry	RAIM is training-free and geometric
RAIM	This study	TFM-guided sequential radius adjustment	No	No ground-truth coordinates required for operation	Preserves the strongest-signal radius and adjusts the middle and largest radii to a common intersection	—

3. Materials and Methods

This section describes the dataset, data collection and preprocessing procedures, the empirical trilateration failure classification, the RAIM correction method, the geometric-correction baselines, and the evaluation strategy.

3.1. Data

In this experiment, we utilised RSSI measurements. RSSI data included several attributes: *GatewayMAC* (identifying the scanner that captured the signal), *BLEMAC* (representing the transmitting device via unique MAC addresses), *RSSI* (indicating proximity between the wearer and the scanner, with stronger signals corresponding to closer distances), and high-resolution timestamps recorded to milliseconds. Table 2 lists the typical attributes of RSSI data.

3.1.1. Data Collection

To acquire the RSSI data, we collaborated with doctors and staff at Diponegoro National Hospital to deploy a BLE-based IPS during data collection. Two locations were selected to capture healthcare worker movement as illustrated in Figure 2: (1) the Emergency Room and (2) an Office area. Figure 3 presents the digital floor plans of the Office (left) and the Emergency Room (right). The red dots indicate scanner locations. Detailed latitude and longitude coordinates of the scanners are provided in Table 3. The metadata fields available in the collected log records are listed in Table 2.

Table 2. Metadata fields and their descriptions.

No	Field	Description
1	File	File name
2	Time_ms_file	Millisecond time from filename
3	Time_file	Recorded time
4	GatewayMAC	Scanner ID
5	Timestamp	Measurement time
6	Format	Message format
7	BLEMAC	Healthcare staff device identifier
8	RSSI	Received signal strength indicator
9	TxPower	Transmitted power
10	AdvInterval	Advertising interval
11	BattVoltage	Battery voltage
12	DataRate	Data rate
13	Scale	Scale metadata
14	3-axisData	Accelerometer data (X, Y, Z axes)

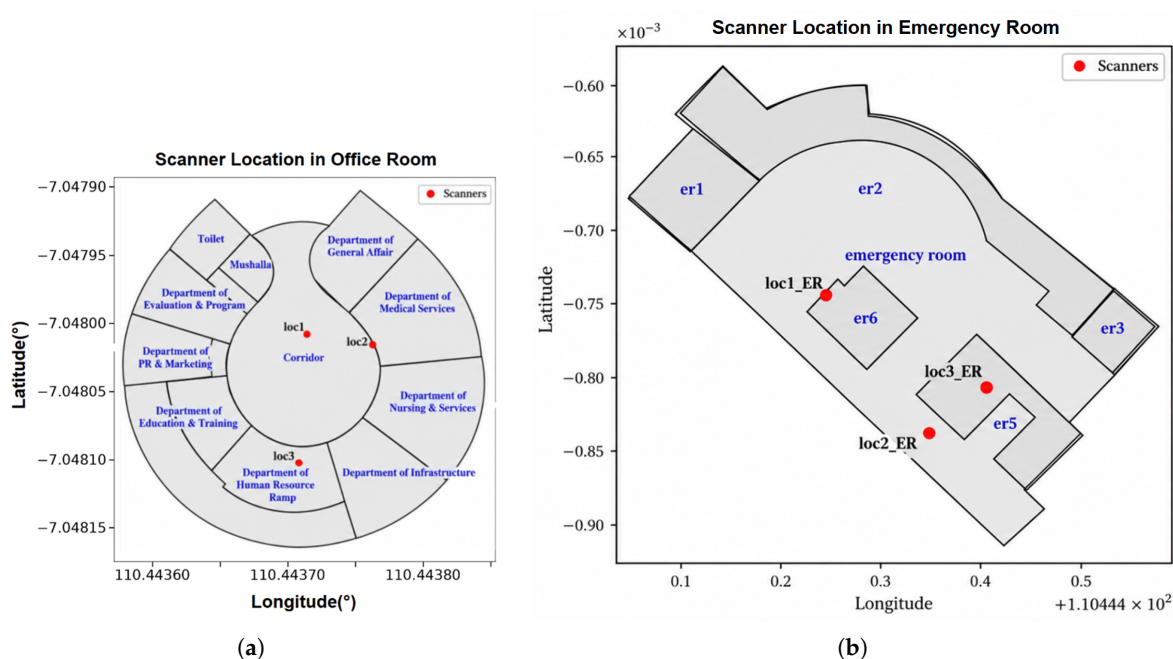
**Figure 2.** DNH Hospital.**Figure 3.** Digitalised layouts of the (a) Office and (b) Emergency Room.

Table 3. Geolocation of scanners in the Office and Emergency Room.

Location	Scanner	GatewayMAC	Latitude	Longitude
Office	loc1	0CCF8966416F	−7.048054	110.443732
	loc2	0CCF89655FF5	−7.048059	110.443770
	loc3	0CCF8965B7ED	−7.048115	110.443729
Emergency Room	loc1	0CCF896640F7	−7.047778	110.444254
	loc2	0CCF8966409F	−7.047738	110.444288
	loc3	0CCF8966425F	−7.047694	110.444305

Figure 4 presents an overview of the proposed system architecture, which integrates hardware and software components as described below:

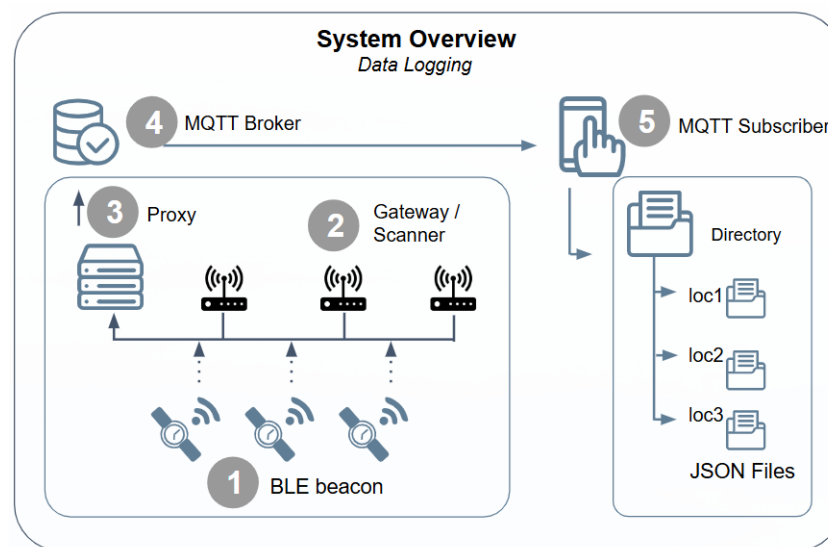


Figure 4. Environment setup. Arrows indicate the flow of RSSI signals: the BLE wristwatches transmit signals that are captured by the scanners, passed through the proxy to the MQTT broker, received by the MQTT subscriber, and finally stored as JSON files. Numbers 1–5 correspond to items a–e in the list below.

- BLE Wristwatches:** Worn by hospital staff (including nurses), these devices broadcast BLE signals every second. The study uses W5 Bluetooth Beacon Trackers with nRF52 chipsets.
- BLE Scanners/Gateways:** A set of three scanners placed in each location (Office and Emergency Room), as illustrated in Figure 2. Scanners detect BLE signals transmitted by the wristwatches. The study used MKGW1-BW Pro Bluetooth Gateways, which sent collected signal strength (RSSI) measurements to an MQTT broker.
- Proxy Network:** Since the hospital's network could not be used directly, a separate wireless network was set up for the beacons and scanners to communicate and send data to the MQTT broker.
- MQTT Broker/Server:** This central server (MQTT Eclipse Mosquitto) received data from the scanners and forwarded it to subscribers.
- MQTT Subscriber:** An application that received and processed the data from the MQTT broker.

Over 30 volunteers participated during data acquisition. Their movement within each location was captured and stored in JSON files. An extract, transform, and load (ETL) process was performed to load the JSON files into a SQL Server database using Microsoft Visual Studio Community 2022 (Microsoft Visual Studio Community 2022, version 17.11.0;

Microsoft Corporation, Redmond, WA, USA.) and SQL Server Integration Services Projects (SSIS) (SQL Server Integration Services Projects, version 1.4, build 16.0.5556.0; Microsoft Corporation, Redmond, WA, USA.). Because the loaded JSON content was not in a tabular structure, the KingswaySoft SSIS Productivity Pack (KingswaySoft SSIS Productivity Pack, 2024 Release Wave 1; KingswaySoft Inc., Oakville, ON, Canada.) was further used to parse the JSON format into structured SQL table columns. The resulting RSSI data were stored using Microsoft SQL Server 2022 Developer Edition (Microsoft SQL Server 2022 Developer Edition, version 16.x; Microsoft Corporation, Redmond, WA, USA.), with SQL Server Management Studio (SSMS) (SQL Server Management Studio, version 20.2; Microsoft Corporation, Redmond, WA, USA.) used to manage database instances and execute T-SQL queries. All subsequent data analyses in this study were performed on these structured SQL tables using Google Colab (Google Colab; Google LLC, Mountain View, CA, USA.).

As each data collection location was equipped with three scanners, ideally all signals transmitted by a single source (i.e., one healthcare worker) should be simultaneously received by all three scanners to form a single valid observation at a specific time instance.

3.1.2. Data Preprocessing

This preprocessing aimed to align timestamps to remove unsynchronised signals and to enrich the dataset using methods such as Kalman filtering.

Timestamp Alignment

The key steps for aligning simultaneous signals based on their timestamps, as illustrated in Figure 5 (Data Preprocessing), involved selecting appropriate time intervals and grouping signals into time-based sets (e.g., 1, 2, 3, 4, 5, 10, and 60 s), followed by alignment using the original timestamps. Within each group, we processed data on a per-beacon basis. Crucially, for each scanner in each interval, we retained **the last RSSI reading** detected in that window. We then assigned the selected readings to the start timestamp of the corresponding interval. This produced temporally aligned observations in which each scanner contributed at most one RSSI reading per beacon per interval. This interval-based approach also prevents the complete loss of readings when the original timestamps are not perfectly aligned across scanners.

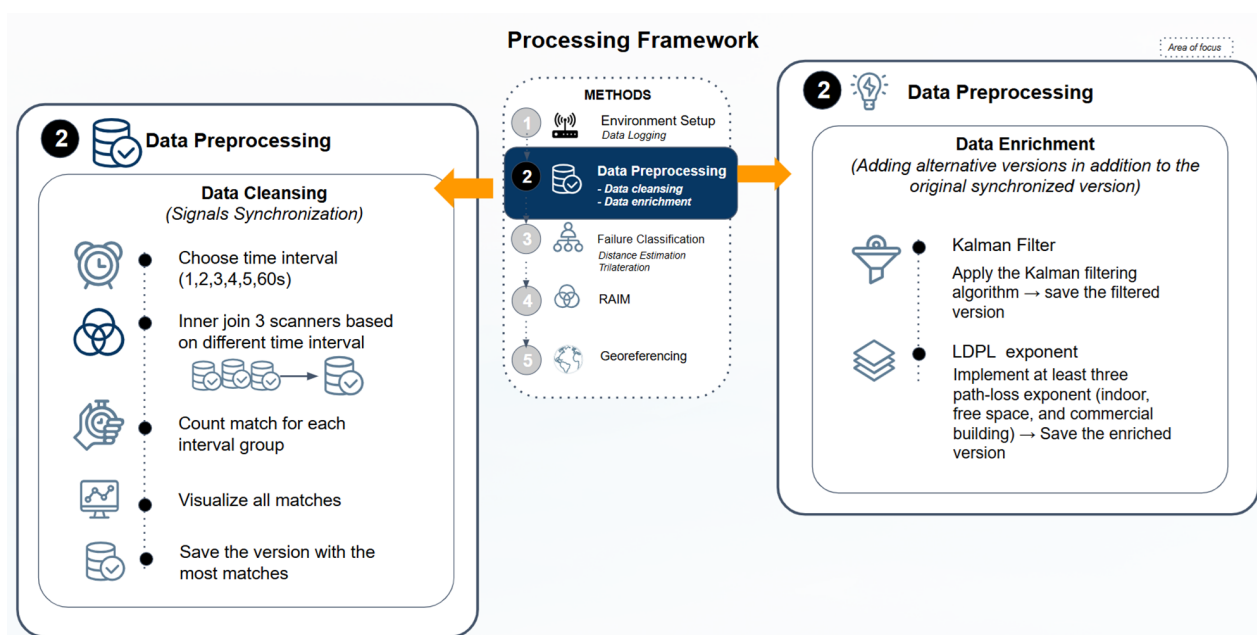


Figure 5. Data Preprocessing (left) and Data Enrichment (right).

For example, RSSI readings with varying timestamps ranging from 13:00:00.000 to 13:00:59.999 from each scanner were grouped into a 60 s interval. This resulted in one synchronised RSSI signal per scanner, with the new timestamp set to 13:00:00 (the start of the window).

This grouping was implemented using SQL's INNER JOIN operation. The query employed Common Table Expressions (CTEs), with each CTE representing data from a single scanner. In the final stage, an INNER JOIN was performed across all three CTEs, producing a set of synchronised signals with identical timestamps.

This process restricted each scanner to a single RSSI reading per beacon within each time interval at each location, resulting in clean and valid observations well suited for data analysis.

Data Enrichment

Because ground-truth data were unavailable for direct validation, we generated multiple signal variations through different parameter configurations. This enabled us to assess result consistency across scenarios. As illustrated in Figure 5 (Data Enrichment), this was achieved using:

KALMAN FILTERING ALGORITHM: Filtering techniques such as the Kalman Filter were applied primarily to mitigate the effects of noisy RSSI measurements, as scanners and sensors are inherently prone to measurement errors [18–21]. The Kalman Filter reduces noise by continuously refining measured RSSI values using predicted estimates, under the assumption that both measurement noise and system process noise follow a normal distribution.

LDPL EXPONENT SCENARIOS: Several methods can convert RSSI measurements into distance estimates, including fingerprinting, machine learning approaches, and the Log-Distance Path Loss (LDPL) model. In this study, we adopted the LDPL model because it does not require ground-truth data, unlike fingerprinting and machine learning methods.

To examine the sensitivity of trilateration geometry to the RSSI-to-distance conversion, three LDPL path-loss exponent scenarios were evaluated: $n = 2.0$, representing the free-space reference case; $n = 2.7$, representing a moderate indoor attenuation scenario; and $n = 4.0$, representing a relatively high indoor attenuation scenario. The free-space exponent of 2 is the theoretical reference for unobstructed propagation, whereas indoor path-loss exponents are environment-dependent and can vary substantially with line-of-sight conditions, obstruction, frequency, and building structure [22,23]. Accordingly, the three values are used here as controlled sensitivity scenarios rather than universal environment-specific parameters.

3.2. Trilateration Failure Modes Identification

BLE-based IPS signals are susceptible to environmental factors such as physical obstructions, interference, and signal reflections [2,5]. The purpose of the present classification is not to claim that the four geometric configurations are mathematically exhaustive or universally applicable. Instead, the taxonomy provides an empirical description of the recurring configurations observed in the synchronised three-scanner data and a common basis for comparing their prevalence across deployment and distance-model conditions.

Two main steps were involved in identifying and classifying potential positioning failures: (1) converting synchronised RSSI data into distance estimates using the LDPL model, and (2) attempting trilateration. When applied to real data, these steps produced distinct geometric compatibility and incompatibility patterns. Because ground truth positions were unavailable, these patterns are treated as observed failure geometries rather than verified positional errors.

3.2.1. Distance Estimation

To estimate distances, received signal strength values were converted into metric units using the Log-Distance Path Loss (LDPL) model, as described in [24,25]

$$\text{Distance} = 10^{\frac{(\text{RSSI}-A)}{-10n}} \quad (1)$$

where RSSI is the received signal strength in decibels (dB), A is the RSSI value at a reference distance of 1 metre, and n is the path loss exponent. In the analysis, $A = -72.3$ dB is used consistently across deployments; the three tested values of n are treated as controlled propagation scenarios rather than calibrated site-specific estimates.

3.2.2. Trilateration

Trilateration is a geometric positioning technique that estimates a target's location by computing its distances from at least three fixed reference points (scanners) [6–8,26]. These distances define circular radii centred at each scanner. Under ideal conditions, free from signal obstruction or interference, the circumferences of these circles intersect at a single point, yielding a unique location estimate, as illustrated in the left-hand image of Figure 1.

Trilateration was employed both for location estimation and for diagnosing positioning failures. Specifically, it enabled the detection of failures arising from noisy RSSI measurements and facilitated the calculation of occurrence rates for each failure mode.

3.2.3. Failure Mode Taxonomy

By applying trilateration to the synchronised dataset and examining the geometric relationships among the three distance circles, we identified and classified four recurring geometric failure modes, summarised in Table 4. The classification defines the geometric configurations independently of their prevalence; the prevalence of each mode is evaluated separately for each deployment, path-loss model, and RSSI filtering condition in the Results.

Table 4. Trilateration Failure Modes (TFMs). The table defines the four geometric configurations; prevalence is reported separately for each evaluated deployment, path-loss model, and filtering condition.

Category	Description
(a) TFM-I: Partial Intersection	Two circles intersect at one or two points, while the third circle does not intersect within the same region.
(b) TFM-II: Overlapping Circles	Two or more circles overlap extensively, forming a broad shared region and therefore providing weak geometric constraint.
(c) TFM-III: Separated Circles	All circles are completely disjoint and do not intersect.
(d) TFM-IV: Mixed Configuration	A hybrid configuration in which some circle pairs overlap while others intersect only partially.

The classification provides a descriptive framework for diagnosing geometric incompatibility in RSSI-derived distances. The complete results show that no single TFM dominates every deployment and propagation configuration. Instead, TFM-I and TFM-III are the two predominant modes, while TFM-II and TFM-IV are generally less prevalent. This configuration-dependent behaviour motivates a correction strategy that operates on the observed geometry rather than assuming one universal failure pattern.

- It provides a diagnostic framework for understanding why trilateration fails.
- It informs the design of correction methods that target geometrically inconsistent observations.

- c. It provides a reproducible benchmark for comparing alternative correction approaches across propagation and filtering conditions.

3.3. Radius Adjustment Intersection Method (RAIM)

The TFM results show that TFM-I and TFM-III are predominant, but their relative prevalence changes with deployment and path-loss model. RAIM is therefore formulated as a geometry-guided proof-of-concept correction mechanism rather than as a method tailored to one universal failure category. It retains the smallest RSSI-derived radius as an anchor and sequentially adjusts the remaining radii to recover a common intersection. The purpose of this step is to test whether the empirically observed incompatibility can be made geometrically feasible without discarding the observation or adding infrastructure. It does not by itself establish that the adjusted radii are physically more accurate.

RAIM uses the weaker radii as the primary correction targets because the observed geometric failures frequently involve a larger radius that is incompatible with the other two circles. This is an algorithmic assumption rather than a claim that weaker RSSI measurements are always erroneous. RSSI-derived distances can be affected by obstruction, reflection, interference, antenna orientation, and other propagation effects.

The strongest signal is therefore retained as the geometric anchor by design, while the other radii are adjusted sequentially. This asymmetric rule improves geometric feasibility but can introduce a strongest-scanner bias, which is evaluated as a limitation of the method.

Relative to the representative approaches discussed above:

- **Filtering methods** [9] discard unreliable measurements, reducing data availability.
- **Prior geometric-adjustment methods** [12,13] modify the circle radii to recover intersections; Pande and Ibwe [12] use common-factor expansion/reduction, whereas Paek et al. [13] use pairwise incremental adjustment.
- **Sensor-fusion methods** [11] require additional hardware, increasing cost.
- **Machine learning methods** [14,15] require ground-truth data for training.

RAIM preserves all observations, adjusts only the non-anchor radii, requires no additional hardware, and operates without ground-truth data. This makes it particularly suitable for resource-constrained deployments where training data is unavailable, and infrastructure changes are infeasible.

The RAIM algorithm is related to prior geometric-adjustment approaches, particularly the common-factor expansion/reduction mechanism of Pande and Ibwe [12] and the pairwise incremental adjustment scheme of Paek et al. [13]. Unlike those approaches, RAIM explicitly ranks the three RSSI-derived radii, preserves the smallest radius as an anchor, and performs two sequential adjustments to recover a feasible geometric solution. It therefore does not assume either a common multiplicative correction across all radii or the pairwise incremental rule used in Paek et al. [13].

First, the three circles are sorted by radius: the smallest circle (corresponding to the strongest signal) is anchored without modification, while the middle circle (medium signal) is evaluated for intersection with the smallest circle. If the two circles already intersect at one or two points, RAIM preserves the original radii and selects one intersection point (the first encountered in the calculation) as the anchor for the next stage. If the circles do not intersect, the radius of the middle circle is adjusted until the two circles intersect at exactly one point, and that point is recorded.

Second, the radius of the largest circle (weakest signal) is adjusted to reach the recorded intersection point from the first stage, yielding a single common intersection among all three circles by construction. This produces a geometrically feasible location estimate, regardless of which intersection point was selected in the first stage; it does not establish that the resulting point is the true device location.

For reproducibility, the implementation uses a deterministic two-stage procedure. For each observation, the three radii are sorted in ascending order; the smallest radius is the anchor, the middle radius is the second-stage input, and the largest radius is the final adjusted radius. The first two circles are tested using the standard condition $|r_1 - r_2| \leq d \leq r_1 + r_2$. If they do not intersect, the middle radius is set to $d - r_1$ when the circles are separated or to $r_1 - d$ when one circle contains the other. The first intersection returned by the implementation is then used as the target point for the third circle, whose adjusted radius is computed as the Euclidean distance from its scanner centre to that point. Thus, no iterative optimisation or learned parameter estimation is performed: the procedure consists of deterministic sorting, an intersection test, at most one correction to the middle radius, and one direct calculation for the largest radius. Ties in the radius sorting retain the original scanner order.

As illustrated in Figure 6, the RAIM method (Figure 6b) contrasts with conventional trilateration (Figure 6a). In Figure 6a, the three circles, labelled s_1 , s_2 , and s_3 , are shown in black and represent the original signal-derived distances from each scanner. Conventional trilateration yields no common intersection. However, it is apparent from Figure 6a that circle s_2 has the smallest radius, indicating the strongest signal, followed by s_3 , and then s_1 with the largest radius (weakest signal). RAIM anchors s_2 (strongest signal) without modification. The radius of s_3 (medium signal) is then adjusted, if necessary, to intersect with s_2 , and the resulting intersection point is recorded. Finally, the radius of s_1 (weakest signal) is adjusted to reach this intersection point, producing a single common intersection among all three circles.

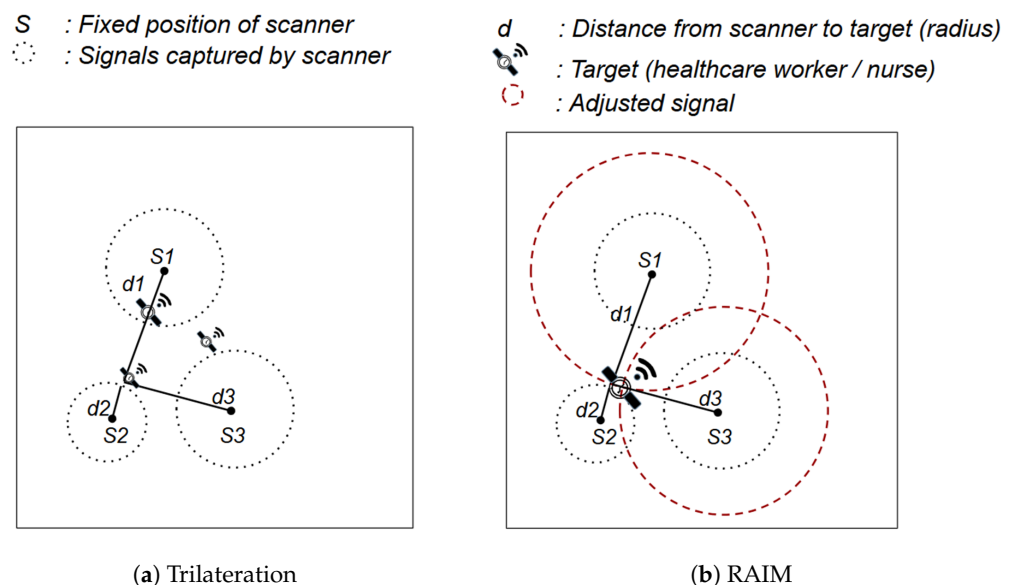


Figure 6. Comparison of trilateration (a) and RAIM methods (b).

When two intersection points exist, either choice produces a geometrically feasible solution, as the third circle is subsequently adjusted to reach the selected point. Moreover, each choice leads to a different final estimated location, as the third circle is adjusted to a different point. However, in both cases, the estimated position remains susceptible to anchor bias, since the strongest signal's circle is preserved as the geometric anchor while the weaker circles are adjusted to accommodate it. Without ground-truth data, there is no definitive way to determine which intersection point yields the more accurate estimate; the available spatial-plausibility criterion is whether the estimated location falls within one of the mapped room polygons. Consequently, both intersection points are

considered geometrically feasible, while room-polygon membership is treated only as a spatial-plausibility indicator. Figure 7 presents an overview of the RAIM process.

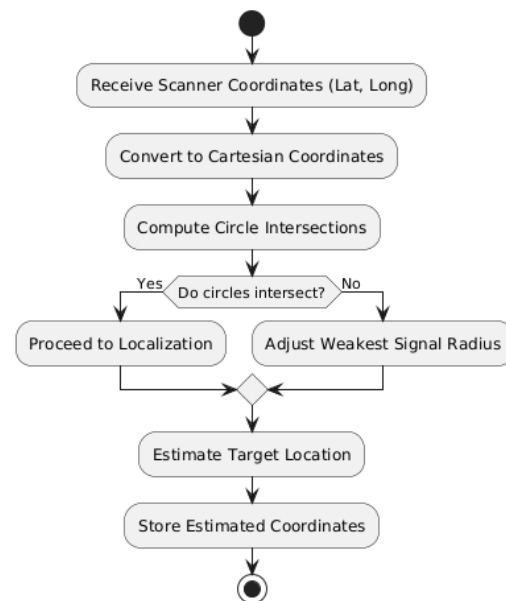


Figure 7. Process flow of the Radius Adjustment Intersection Method (RAIM). The filled black circle marks the start, the concentric circles mark the end, arrows indicate process flow, and the diamond denotes the merge point between the two decision branches.

The key steps are outlined as follows:

- a. **Cartesian Coordinate Conversion:** The RAIM method begins by converting the latitude and longitude coordinates of the scanners into Cartesian coordinates using a flat-Earth approximation:

$$x = R \cdot \Delta\lambda \cdot \cos(\phi_{\text{ref}}), \quad y = R \cdot \Delta\phi$$

where R is the Earth's radius, $\Delta\lambda$ is the difference in longitude, and $\Delta\phi$ is the difference in latitude.

- b. **Circle Intersection Calculation:** For each observation, the three radii are sorted in ascending order. The smallest radius is designated as the radius-adjustment anchor, while the middle and largest radii are processed sequentially. For two circles, the distance between their centres is calculated as

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

where d is the Euclidean distance between the circle centres. The circles intersect when:

$$|r_1 - r_2| \leq d \leq r_1 + r_2.$$

If this condition is not satisfied, the second circle's radius is adjusted to establish tangency with the first circle.

- c. **Adjust Radii to Force Intersection:** If the two circles are externally separated ($d > r_1 + r_2$), the second radius is adjusted to:

$$r'_2 = d - r_1.$$

If the second circle contains the first without intersection ($d < r_2 - r_1$), the second radius is adjusted to:

$$r'_2 = r_1 + d.$$

These adjustments establish external or internal tangency, respectively.

- d. **Intersection Point Calculation:** Once the two circles intersect, the intersection point is calculated as:

$$a = \frac{r_1^2 - r_2^2 + d^2}{2d}, \quad h = \sqrt{r_1^2 - a^2}.$$

The coordinates of the intersection points are:

$$x_3 = x_1 + a \frac{x_2 - x_1}{d} \pm h \frac{y_2 - y_1}{d}$$

$$y_3 = y_1 + a \frac{y_2 - y_1}{d} \mp h \frac{x_2 - x_1}{d}.$$

Here, a is the distance from the centre of the first circle to the point where the common chord intersects the line connecting the two circle centres, while h is the perpendicular distance from this point to either circle-intersection point. The quantities r_1 and r_2 denote the circle radii, d is the Euclidean distance between the circle centres, and (x_1, y_1) and (x_2, y_2) are their respective centre coordinates. The resulting (x_3, y_3) represents one of the two possible circle-intersection points.

- e. **Convert Intersection Point Back to Latitude/Longitude:** Once the intersection point is determined in Cartesian coordinates, it is converted back to latitude and longitude:

$$\phi = \phi_{\text{ref}} + \frac{y}{R}, \quad \lambda = \lambda_{\text{ref}} + \frac{x}{R \cos(\phi_{\text{ref}})}.$$

3.4. Paek-Style Pairwise Geometric Adjustment

To provide a second geometric baseline, we implemented the pairwise adjustment rule, where it increases a smaller or enclosed circle in 1 m increments until a pair of circles satisfies the intersection condition; when equal-sized circles remain non-intersecting, both radii are increased. The published method describes the resulting pairwise intersections but does not specify a unique global aggregation rule for the final three-scanner position.

For reproducibility, the present implementation applies the described 1 m incremental rule sequentially to scanner pairs (1, 2), (1, 3), and (2, 3). After the pairwise adjustments, the available pairwise intersection points are combined using their Cartesian centroid as an operationalisation of the final estimate. This deterministic implementation should therefore be interpreted as a *Paek-style reimplement* of the described geometric adjustment rule, not as an exact reproduction of every implementation detail of the original study. The purpose is to compare correction geometry on the same hospital observations used for RAIM and URS.

Because the pairwise adjustment explicitly increases radii until intersection conditions are satisfied, its resulting GFR is a constraint-satisfaction outcome rather than an independent performance measure. The more informative comparison therefore considers correction magnitude, coordinate displacement relative to conventional Trilateration, and room-polygon spatial plausibility.

3.5. Uniform Radius Scaling Baseline

To provide a simple geometric reference for the RAIM demonstration, we implemented a deterministic Uniform Radius Scaling baseline inspired primarily by the common-factor expansion/reduction mechanism. The pairwise incremental adjustment is included as a related geometric-adjustment reference, but its published procedure uses pairwise incre-

mental radius changes rather than a single common multiplicative factor. The present URS implementation is therefore not an exact reproduction of either published method. Instead, it asks a narrower question: for a given observation, can all three RSSI-derived radii be reconciled by multiplying them by one common positive factor?

For each scanner pair (i, j) with centre separation d_{ij} and original radii r_i and r_j , a common scale factor $s > 0$ is feasible when

$$s|r_i - r_j| \leq d_{ij} \leq s(r_i + r_j).$$

This gives a feasible interval for s for each pair. The three pairwise intervals are intersected; if their common intersection is non-empty, the positive scale factor closest to $s = 1$ is selected, and conventional trilateration is applied to the uniformly scaled radii. If no common factor exists, the observation is classified as uniform-scaling infeasible. The primary URS metric is therefore the *uniform-scaling feasibility rate*, defined as the proportion of valid observations admitting at least one common scale factor. The selected scale factor and the resulting room-polygon outlier status are reported as secondary descriptive indicators.

This baseline deliberately preserves the relative proportions among the three RSSI-derived radii, whereas RAIM permits radius-specific sequential adjustment. The comparison is consequently a comparison of correction geometry, not a comparison of absolute localisation accuracy.

3.6. Georeferencing

Georeferencing is the process of aligning geospatial data with a defined coordinate system on the Earth's surface. This method was implemented by developing room-level digital maps of the two DNH Hospital deployment areas based on the coordinates of the six deployed scanners. The digital maps were developed using QGIS Desktop (QGIS Desktop, version 3.34.9; QGIS.org, Grüt, Switzerland.). Georeferencing was used to compensate for the absence of ground-truth data for direct validation. During data collection, all healthcare workers were known to be located within the mapped hospital areas; therefore, estimated locations were expected to fall within one of the mapped room polygons. A spatial outlier is consequently defined as an estimated point that is not within any room polygon. The resulting outlier count is used only as a spatial-plausibility indicator, not as a measure of localisation accuracy.

3.7. Evaluation Strategy

Because ground-truth coordinates were unavailable, the evaluation distinguishes three concepts that are often conflated in indoor-positioning studies: *geometric feasibility*, *localisation consistency*, and *localisation accuracy*. Geometric feasibility refers to whether the RSSI-derived circles admit a common intersection. Localisation consistency refers to spatial plausibility indicators such as whether estimated points fall within the mapped hospital area. Localisation accuracy would require comparison with known ground-truth coordinates and is therefore not measured in this study.

3.7.1. Geometric Feasibility and Spatial Plausibility Indicators

- a. **Geometric Feasibility Rate (GFR):** The primary geometric measure is the proportion of observations for which all three circles have a common intersection

$$\text{GFR} = \frac{N_{3C}}{N_{\text{obs}}} \times 100\%,$$

where N_{3C} is the number of observations with a common three-circle intersection, and N_{obs} is the number of synchronised observations evaluated. GFR measures geometric feasibility, not localisation accuracy.

- b. **Total Number of Outliers:** This counts estimated locations that do not fall within any mapped room polygon. Outlier counts are treated as a spatial-plausibility indicator rather than an accuracy measure because a point inside a room polygon is not necessarily close to the true device location.
- c. **Uniform-Scaling Feasibility Rate:** For the URS baseline, this is the proportion of valid observations for which one common positive scale factor satisfies all three pairwise circle-intersection conditions. The selected scale factor is the feasible value closest to one. This metric characterises whether a common-factor correction can reconcile the observed geometry; it is not a localisation-accuracy measure.

3.7.2. Radius-Adjustment Analysis

To explain how RAIM changes the input geometry, we computed the radius correction for every synchronised observation in both deployment locations:

$$\Delta r_i = r_i^{\text{RAIM}} - r_i^{\text{RSSI}}, \quad i \in \{1, 2, 3\}.$$

The analysis reports the number of observations, mean, standard deviation, minimum, first quartile, median, third quartile, maximum, percentage of adjusted radii, and percentage of negative corrections. For the four-method comparison, counts, feasibility/outlier rates, and means are exact; median and P95 statistics are estimated using a deterministic bounded reservoir sample of up to 100,000 observations per condition to control memory use. Results are reported both by scanner circle and by geometric role (anchor, middle, and largest radius). The role is assigned from the original RSSI-derived radii for each observation, with the smallest radius corresponding to the strongest signal and serving as the RAIM anchor. This role-based analysis is used to examine the extent and direction of RAIM corrections, not an independent accuracy measure.

3.7.3. Anchor-Selection Sensitivity Analysis

Because the default RAIM procedure retains the smallest RSSI-derived radius as the anchor, an additional full-dataset sensitivity experiment was conducted to quantify the dependence of the estimated coordinates on this choice. For each deployment, LDPL scenario, and filtering condition, RAIM was rerun three times with each physical scanner designated in turn as the fixed anchor. When a fixed scanner was used as the anchor, the other two radii retained their original values initially and were ordered as the middle and largest radii for the same deterministic sequential correction procedure. The default solution remained the reference condition.

The sensitivity analysis reports the Euclidean displacement between each fixed-anchor RAIM estimate and the default RAIM estimate. Mean, median, and 95th-percentile displacement were calculated, together with the percentages of observations whose displacement exceeded 1 m, 2 m, and 5 m. The proportion of observations for which the fixed anchor matched the default strongest-RSSI anchor was also recorded. Because no ground-truth coordinates were available, these displacements measure *sensitivity of the RAIM solution*, not localisation error. The room-polygon outlier rule described above was additionally applied to each fixed-anchor solution to determine whether anchor changes altered spatial-plausibility classification. GFR was not used as a discriminating sensitivity metric because RAIM is explicitly designed to enforce geometric feasibility.

3.7.4. Evaluation Sampling and Scope

The TFM classification, geometric evaluation, spatial-plausibility analysis, and radius-adjustment analysis use the complete synchronised datasets available at the selected synchronisation intervals: 2,670,728 Emergency Room observations at the 3 s interval and 10,315,487 Office observations at the 1 s interval. The synchronisation interval was deployment-specific; consequently, the observation counts should not be interpreted as directly comparable measures of activity or data density.

4. Results

This section presents the results of our analysis. We first describe the raw and pre-processed data characteristics and then present the empirical trilateration failure classification derived from the dataset; finally, we evaluate RAIM against conventional trilateration using the defined indicators.

4.1. Data Characteristics

4.1.1. Raw Data

Data were collected between November 2021 and April 2022. A total of 263,511 gateway log files were summarised across both deployment locations, amounting to 33.2 GB of data, as summarised in Table 5.

Table 5. Log files of ER and Office locations.

Location	Files	Size (GB)
ER	131,397	16.5
Office	132,114	16.7
Total	263,511	33.2

Table 6 summarises the characteristics of the collected data, including the total number of log files, the number of records, the recording duration in minutes, and the throughput performance of each gateway (measured in readings per minute) for both the Emergency Room and Office locations.

Table 6. Throughput statistics by location.

Location	GatewayMAC	Files	Throughput (Readings/min)				Mins	Records
			Avg	Min	Max	Std		
Emergency Room	0CCF8966409F	12,049	65.02	1	262	40.22	104,159	6,772,375
	0CCF8966425F	24,566	175.48	1	744	72.18	83,027	14,569,926
	0CCF896640F7	94,782	568.64	1	1514	120.04	105,788	60,155,806
	Avg:		269.71	1.00	840.00	77.48		
	Total Files:	131,397						81,498,107
Office	0CCF8965B7ED	39,841	288.05	1	504	65.62	83,846	24,151,867
	0CCF8966416F	41,762	321.18	5	1915	81.06	79,245	25,451,960
	0CCF89655FF5	50,511	369.95	5	638	76.35	83,769	30,990,651
	Avg:		326.06	3.67	1019.00	74.34		
	Total Files:	132,114						80,594,478
	Grand Total :	263,511					–	162,092,585

As shown in Table 6 and Figure 8, the Emergency Room dataset (a) contains a total of 81,498,107 signal records, while the Office dataset (b) comprises 80,594,478 records. Despite the comparable total volumes, the distribution of signals across scanners, particularly in the Emergency Room, is highly imbalanced, as illustrated by the bar chart. Because

a single valid observation requires three signals that can be aligned within the selected synchronisation window, the signal counts across scanners should ideally be equal or nearly equal to ensure consistent and complete data acquisition.

The Emergency Room throughput varied considerably across the three deployed gateways. Gateway 0CCF896640F7 achieved the highest throughput, with an average of 568.64 readings per minute, a peak of 1514 readings within a single minute, and a cumulative total exceeding 60 million records. In contrast, gateway 0CCF8966409F exhibited a substantially lower average throughput of 65.02 readings per minute. Overall, the Emergency Room gateways recorded a mean throughput of approximately 269.71 readings per minute. The observed variability in signal readings and throughput is attributed to factors such as irregular power supply, human error, or device malfunction.

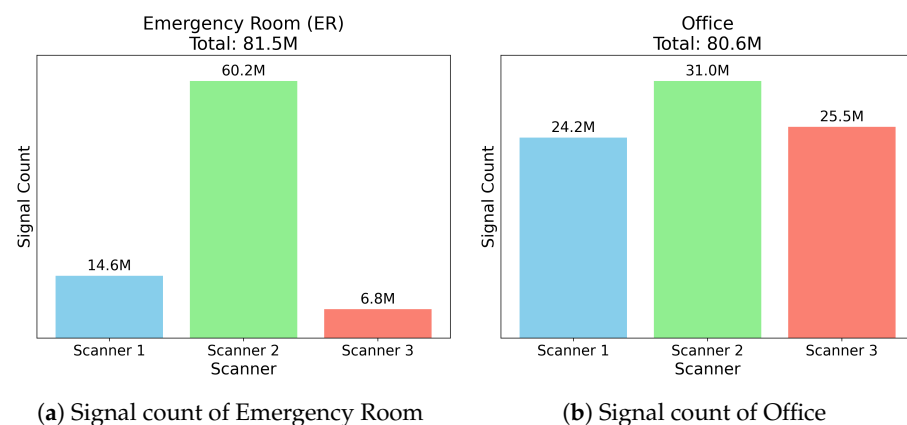


Figure 8. Signal count summary of (a) Emergency Room and (b) Office.

In the Office environment, throughput improved substantially and was more evenly distributed across gateways. The three gateways recorded average throughputs of 288.05, 321.18, and 369.95 readings per minute, respectively. The highest single-minute throughput was observed at gateway 0CCF8966416F, with 1915 readings. Overall, the Office deployment achieved a mean throughput of 326.06 readings per minute, slightly higher than that of the Emergency Room.

These results confirm the practical challenges of real-world BLE signal acquisition, where environmental conditions, gateway placement, and human activity significantly influence data volume and consistency. This observation provides the baseline justification for employing the data synchronisation method. The synchronisation window should also be distinguished from exact simultaneity: measurements from different scanners may occur at different points within the selected interval. In the Emergency Room, the 3 s window can therefore associate measurements that were not acquired at exactly the same instant. For moving wearables, such temporal staleness can contribute to geometric inconsistency alongside radio-propagation effects. The present data do not provide sufficiently fine-grained ground-truth trajectories to separate these effects, so the reported failure modes describe the combined behaviour of propagation variability and the deployed temporal sampling scheme.

4.1.2. Synchronised Data

Figure 9 compares the cleaned and synchronised signals for the **Emergency Room (a)** and **Office (b)** locations, which are defined as signals successfully matched across all three scanners. The corresponding numerical summaries are provided in Tables 7 and 8.

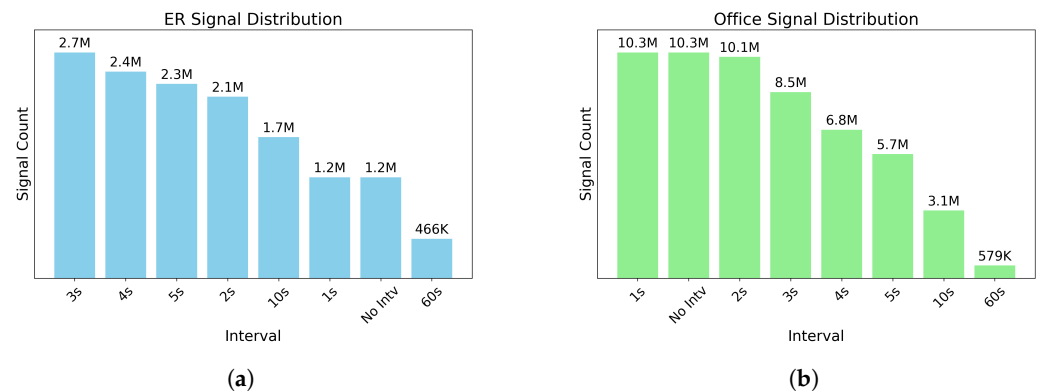


Figure 9. Cleaned signals by interval: (a) Emergency Room and (b) Office.

Table 7. Cleaned ER Signal by interval.

Interval	Location	Total Count
3 Sec Interval	ER	2,670,728
4 Sec Interval	ER	2,443,900
5 Sec Interval	ER	2,298,010
2 Sec Interval	ER	2,146,976
10 Sec Interval	ER	1,668,044
1 Sec Interval	ER	1,194,034
No Interval	ER	1,194,034
60 Sec Interval	ER	465,960

Table 8. Cleaned Office Signal by interval.

Interval	Location	Total Count
1 Sec Interval	Office	10,315,487
No Interval	Office	10,315,487
2 Sec Interval	Office	10,110,577
3 Sec Interval	Office	8,504,484
4 Sec Interval	Office	6,781,209
5 Sec Interval	Office	5,667,771
10 Sec Interval	Office	3,093,988
60 Sec Interval	Office	579,418

As expected, all versions of the joined datasets contained substantially fewer signal records than the original raw datasets. For example, in the Emergency Room deployment, the raw dataset comprised approximately 81 million signal records. When disaggregated by scanner, the scanner with the lowest volume still recorded at least 6 million signals. However, after applying the synchronisation and cleansing procedure, only up to 2.7 million matched signals were retained with a 3 s time interval, with even fewer matches obtained using the original millisecond-level timestamps.

A similar trend was observed in the Office deployment. Although the raw dataset contained approximately 80.5 million signals across all scanners, corresponding to at least 25 million signals per scanner, the cleaned and synchronised dataset was reduced to approximately 10 million matched signals.

Figures 10 and 11 present a daily breakdown of matched signal counts for both datasets.

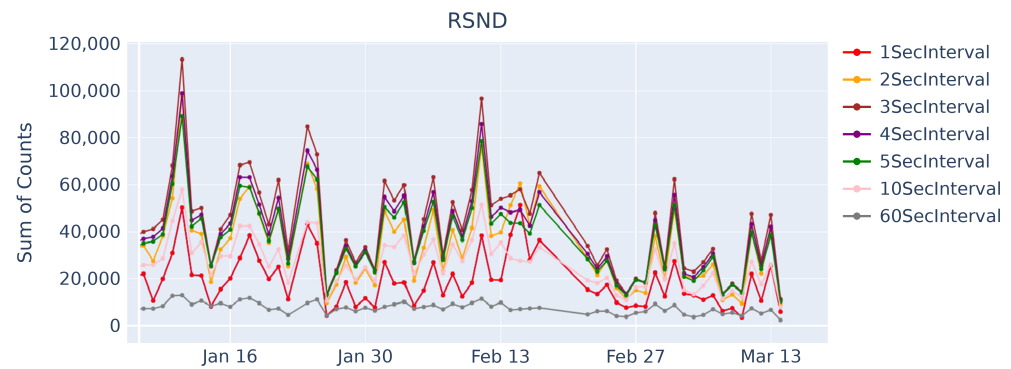


Figure 10. Number of matching signals at Emergency Room location grouped by 1, 2, 3, 4, 5, 10 and 60 s.

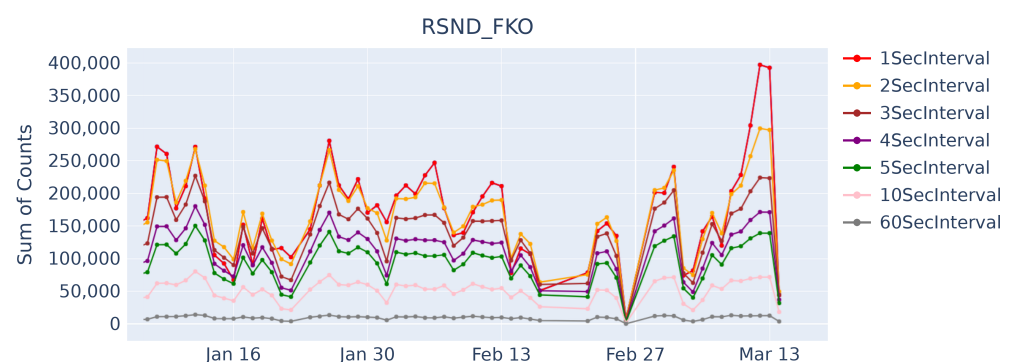


Figure 11. Number of matching signals at Office location grouped by 1, 2, 3, 4, 5, 10 and 60 s.

These summaries were used to compare matching-signal yields across candidate synchronisation intervals; the final analysis retained the deployment-specific intervals of 3 s for the Emergency Room and 1 s for the Office. The final TFM classification and comparative RAIM evaluation reported in this manuscript use the complete synchronised datasets.

4.1.3. Enriched Data

We used Kalman filtering and three path-loss values to convert signals to distances. The key observations are:

- Effect of Kalman Filtering:** As illustrated in Figure 12, the noisy RSSI signal (blue) became significantly less fluctuating after Kalman filtering was applied (yellow). The y-axis represents signal strength in decibels (dB), while the x-axis denotes time in milliseconds.
- Effect of LDPL Exponent Scenarios:** Using three distinct values ($n = 2.0$, $n = 2.7$, and $n = 4.0$) for the LDPL model produced three additional signal versions.

As illustrated in Figure 13, combining Kalman filtering with the three LDPL exponent scenarios produced six different versions of observations (1 observation $\times$ 2 filtering types $\times$ 3 path loss exponents). Multiplying by 4 geometric processing methods (Trilateration, URS, pairwise, and RAIM) produces 24 possible method-condition combinations for each location.

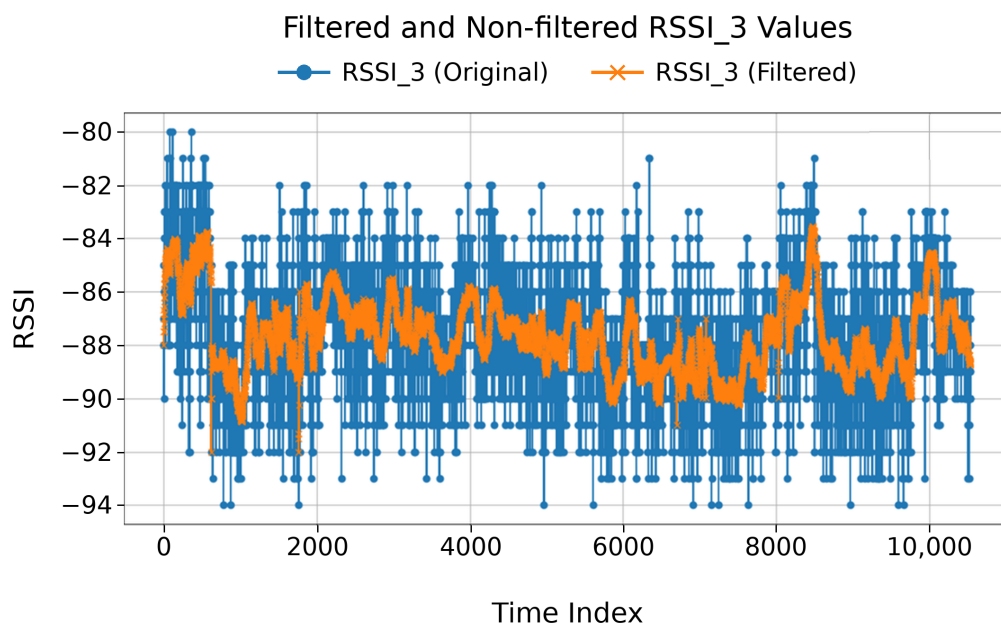


Figure 12. Original vs noise-reduced signals using Kalman filtering.

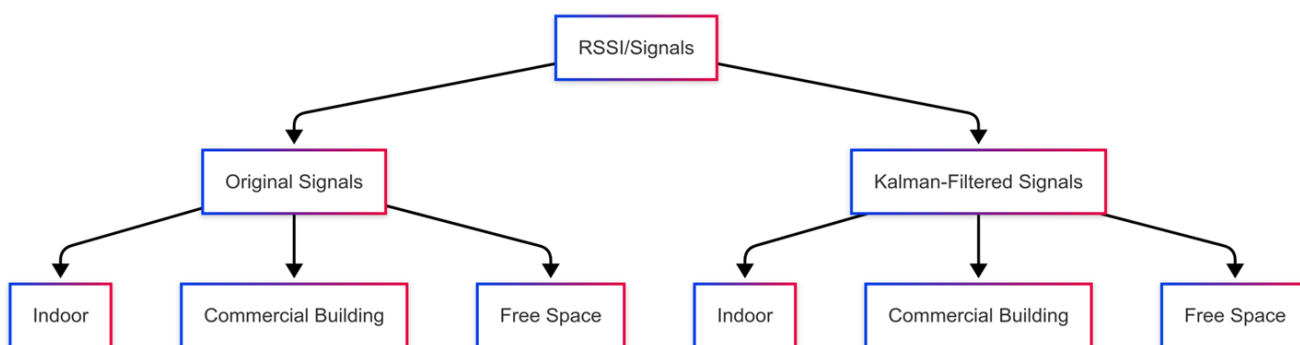


Figure 13. Data Enrichment.

4.2. Trilateration Failure Modes

Figure 14 illustrates the four Trilateration Failure Modes (TFMs) identified from the synchronised datasets. The complete TFM distributions were calculated for both deployment locations, three path-loss models, and both raw and Kalman-filtered RSSI. The resulting 12 configurations are summarised in Table 9.

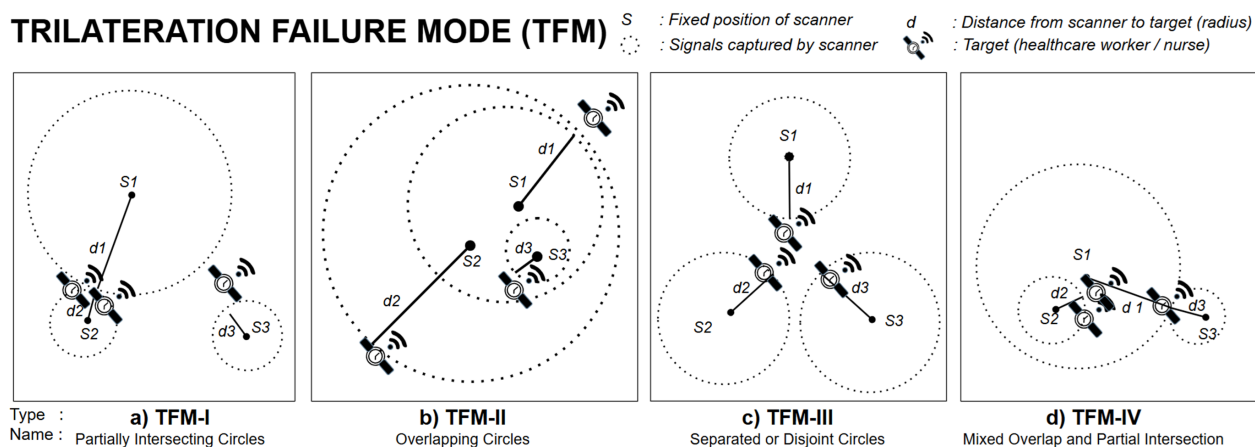


Figure 14. Trilateration Failure Modes (TFMs).

Table 9. Full-dataset distribution of Trilateration Failure Modes across deployment locations, path-loss models, and RSSI filtering. Percentages are calculated over all synchronised observations for each configuration.

Location	Path-Loss Model	Filter	TFM-I	TFM-II	TFM-III	TFM-IV
Emergency Room	Free-space scenario ($n = 2.0$)	Raw	94.41	4.31	0.99	0.29
Emergency Room	Free-space scenario ($n = 2.0$)	Kalman	98.05	1.28	0.67	0.00
Emergency Room	Moderate-indoor scenario ($n = 2.7$)	Raw	96.53	0.00	3.47	0.00
Emergency Room	Moderate-indoor scenario ($n = 2.7$)	Kalman	96.98	0.00	3.02	0.00
Emergency Room	High-attenuation indoor scenario ($n = 4.0$)	Raw	3.08	0.00	96.92	0.00
Emergency Room	High-attenuation indoor scenario ($n = 4.0$)	Kalman	0.44	0.00	99.56	0.00
Office	Free-space scenario ($n = 2.0$)	Raw	62.55	3.09	28.41	5.95
Office	Free-space scenario ($n = 2.0$)	Kalman	69.64	0.39	27.51	2.46
Office	Moderate-indoor scenario ($n = 2.7$)	Raw	42.19	0.02	57.77	0.01
Office	Moderate-indoor scenario ($n = 2.7$)	Kalman	41.78	0.00	58.22	0.00
Office	High-attenuation indoor scenario ($n = 4.0$)	Raw	2.05	0.00	97.95	0.00
Office	High-attenuation indoor scenario ($n = 4.0$)	Kalman	0.59	0.00	99.41	0.00

The TFM distribution changes substantially across path-loss configurations. In the Emergency Room, TFM-I dominates under the Free-space scenario (94.41%) and Moderate-indoor scenario (96.53%), whereas TFM-III dominates under the High-attenuation indoor scenario (96.92%). The Office shows the same broad transition in a different form: TFM-I dominates the Free-space scenario (62.55%), while TFM-III dominates the Moderate-indoor scenario (57.77%) and High-attenuation indoor scenario (97.95%). Kalman filtering preserves these dominant patterns and, in most configurations, increases the prevalence of the dominant category. For example, TFM-I increases from 94.41% to 98.05% in the Emergency Room Free-space scenario condition, while TFM-III increases from 96.92% to 99.56% in the Emergency Room High-attenuation indoor scenario condition and from 97.95% to 99.41% in the Office High-attenuation indoor scenario condition. The Office Moderate-indoor scenario condition is an exception: TFM-III remains dominant and increases from 57.77% to 58.22%, while TFM-I changes slightly from 42.19% to 41.78%.

These results indicate that the observed geometric failure structure is configuration-dependent rather than a single universal failure mode. The classification therefore serves primarily as a diagnostic description of geometric incompatibility; it does not identify which scanner distance is physically incorrect and should not be interpreted as a measure of localisation accuracy.

Path-Loss Model Sensitivity

The cross-condition results provide an additional empirical finding beyond the identification of individual failure modes. Changing the LDPL exponent changes the geometry of the inferred circles and, consequently, the observed failure-mode distribution. In the Emergency Room, TFM-I remains predominant at $n = 2.0$ and $n = 2.7$, but the predominant category changes to TFM-III at $n = 4.0$. In the Office, the predominant category changes from TFM-I at $n = 2.0$ to TFM-III at both $n = 2.7$ and $n = 4.0$.

For the Office, TFM-I falls from 62.55% under the Free-space scenario to 42.19% under the Moderate-indoor scenario and 2.05% under the High-attenuation indoor scenario, while TFM-III rises from 28.41% to 57.77% and 97.95%, respectively. This pattern does not identify the correct path-loss exponent because no ground-truth distances are available. It does, however, show that the choice of RSSI-to-distance transformation is not a neutral preprocessing detail: it materially changes the geometry presented to the trilateration stage. The TFM distribution can therefore be used as a diagnostic sensitivity analysis when selecting or evaluating propagation assumptions. The point is not that changing n changes the geometry but that the resulting failure-mode transition differs between deployments.

The deployment-dependent transition therefore indicates that the propagation assumption cannot be evaluated independently of the scanner geometry and observed RSSI structure.

4.3. Correction Comparison: Paek-Style Adjustment, Uniform Scaling, and RAIM

The correction comparison evaluates four methods on the complete synchronised datasets: conventional Trilateration without radius correction, the Paek-style pairwise incremental adjustment, the deterministic Uniform Radius Scaling (URS) baseline, and RAIM. The full-dataset geometric feasibility and room-polygon outlier comparison is reported in Table 10. For each location, the same six signal variants (raw and Kalman-filtered RSSI under the three LDPL exponent scenarios) were evaluated. Three-circle intersections are interpreted as geometric feasibility, while room-polygon outliers are treated as a spatial-plausibility outcome. For URS, the primary correction metric is the proportion of observations admitting a common scale factor. For Paek-style adjustment and RAIM, near-complete GFR is largely a consequence of their geometric construction, so GFR is reported descriptively rather than used as an accuracy ranking. Correction magnitude and coordinate displacement relative to conventional Trilateration provide complementary information about how strongly each correction changes the original geometry.

Table 10. Full-dataset geometric comparison across the four correction strategies. Paek GFR is reported as the resulting three-circle feasibility of the Paek-style reimplementation; URS feasible is the percentage admitting one common positive scale factor. Outlier rates are based on room-polygon membership.

Deployment	Scenario/Filter	Tri GFR	Paek GFR	URS Feasible	RAIM GFR	Paek Outlier	URS Outlier	RAIM Outlier
ER	Free-space/Raw	0.055	100.000	24.55	99.996	0.002	6.354	0.011
ER	Free-space/Kalman	0.000	100.000	23.63	100.000	0.000	5.347	0.000
ER	Moderate-indoor/Raw	0.002	100.000	50.57	100.000	0.000	8.883	0.000
ER	Moderate-indoor/Kalman	0.000	99.9997	52.39	100.000	0.000	5.944	0.000
ER	High-attenuation/Raw	0.000	100.000	88.73	100.000	0.000	3.952	0.000
ER	High-attenuation/Kalman	0.000	99.9999	94.08	100.000	0.000	2.260	0.000
Office	Free-space/Raw	2.522	100.000	44.56	98.743	0.003	0.756	0.0004
Office	Free-space/Kalman	1.201	99.9999	56.57	99.586	0.000	1.811	0.010
Office	Moderate-indoor/Raw	0.001	100.000	66.76	100.000	0.000	1.024	0.000
Office	Moderate-indoor/Kalman	0.000	99.9999	78.93	100.000	0.000	1.540	0.005
Office	High-attenuation/Raw	0.000	100.000	89.70	100.000	0.000	1.011	0.000
Office	High-attenuation/Kalman	0.000	99.9998	96.34	100.000	0.000	0.940	0.000

Table 11 summarises representative raw-RSSI conditions to illustrate the correction magnitudes and coordinate displacements produced by the three geometric correction strategies.

Table 11. Representative raw-RSSI conditions for correction magnitude and coordinate displacement relative to conventional trilateration. Median and P95 values use the bounded reservoir described in the Methods section. Radius adjustment is the total absolute change across the three radii. For URS, N is the number of observations for which a common scale factor was feasible; for Paek-style adjustment and RAIM, N equals all synchronised observations.

Deployment	Scenario/Filter	Method	Median $ \Delta r $ (m)	P95 $ \Delta r $ (m)	Mean Shift (m)	Median Shift (m)	P95 Shift (m)	N
ER	Free-space/Raw	Paek	10.177	13.591	5.571	5.188	9.196	2,670,728
ER	Free-space/Raw	URS	14.931	19.586	5.717	5.626	9.719	655,675
ER	Free-space/Raw	RAIM	14.873	19.056	6.964	6.598	10.451	2,670,728
ER	Moderate-indoor/Raw	Paek	14.660	17.021	3.619	3.532	4.824	2,670,728
ER	Moderate-indoor/Raw	URS	20.408	23.526	7.103	6.653	12.013	1,350,635

Table 11. Cont.

Deployment	Scenario/Filter	Method	Median Δr (m)	P95 Δr (m)	Mean Shift (m)	Median Shift (m)	P95 Shift (m)	N
ER	Moderate-indoor/Raw	RAIM	20.248	22.342	6.703	6.780	7.316	2,670,728
ER	High-attenuation/Raw	Paek	17.752	19.000	2.941	2.920	3.287	2,670,728
ER	High-attenuation/Raw	URS	22.809	26.320	7.123	6.282	13.457	2,369,765
ER	High-attenuation/Raw	RAIM	23.193	24.254	6.854	6.930	7.154	2,670,728
Office	Free-space/Raw	Paek	8.000	13.698	3.194	2.779	6.584	10,315,487
Office	Free-space/Raw	URS	9.339	16.923	3.179	2.825	7.520	4,596,177
Office	Free-space/Raw	RAIM	9.178	15.968	4.769	4.731	7.892	10,315,487
Office	Moderate-indoor/Raw	Paek	11.032	14.291	2.287	2.161	3.608	10,315,487
Office	Moderate-indoor/Raw	URS	12.485	17.799	3.838	3.503	8.083	6,886,279
Office	Moderate-indoor/Raw	RAIM	12.263	17.157	4.694	4.779	5.509	10,315,487
Office	High-attenuation/Raw	Paek	13.000	14.787	2.006	1.982	2.463	10,315,487
Office	High-attenuation/Raw	URS	14.244	18.436	3.535	3.105	7.969	9,252,667
Office	High-attenuation/Raw	RAIM	14.381	17.978	4.918	4.997	5.419	10,315,487

The four methods exhibit distinct correction geometries. Conventional Trilateration leaves the RSSI-derived radii unchanged. The Paek-style baseline modifies radii through pairwise 1 m increments, URS preserves their relative proportions while applying one common multiplicative factor, and RAIM preserves one anchor radius while adjusting the remaining radii sequentially. The comparison therefore controls the input observations and distance model while changing only the geometric correction strategy.

4.3.1. Full-Dataset Radius Adjustment Analysis

The full-dataset analysis quantifies how RAIM changes each RSSI-derived radius. The correction is defined as $\Delta r_i = r_i^{\text{RAIM}} - r_i^{\text{RSSI}}$, so positive values indicate an expansion and negative values indicate a contraction.

For the Emergency Room, the corrections are concentrated on circles 2 and 3. Under the Moderate-indoor scenario condition, for example, the median corrections for circles 2 and 3 are 9.142 m and 11.043 m with raw RSSI, and 9.167 m and 11.126 m after Kalman filtering (Table 12). Under the High-attenuation indoor scenario condition, the corresponding medians are 10.821 m and 12.329 m for raw RSSI and 10.823 m and 12.354 m after filtering. Circle 1 has a median correction of 0 m in all Emergency Room conditions and is modified in only approximately 1.1% of raw observations and 0.03% of Kalman-filtered observations.

The Office results show a different correction magnitude. Median corrections for the three circles under the raw Free-space scenario conditions are 2.570 m, 2.790 m, and 0 m, respectively, while the corresponding Kalman-filtered medians are 3.309 m, 4.203 m, and 0 m (Table 13). Under the High-attenuation indoor scenario conditions, the raw medians are 6.497 m, 7.581 m, and 0 m, increasing to 7.182 m, 8.223 m, and 0 m after Kalman filtering. Unlike the Emergency Room, the fixed scanner-circle statistics in the Office do not map directly to a fixed geometric role because the anchor, middle, and largest roles are assigned separately for each observation.

The role-based analysis clarifies this behaviour. The anchor radius has $\Delta r = 0$ by construction. In the Emergency Room, the median corrections for the middle and largest roles range from 6.128–12.314 m and 6.144–10.959 m, respectively, across LDPL scenarios and filtering conditions. In the Office, the corresponding ranges are 4.021–8.323 m and 4.021–8.009 m. The predominance of positive corrections on the non-anchor radii confirms that RAIM generally expands those radii to accommodate the anchor-defined intersection. These corrections describe the operation of the algorithm and should not be interpreted as evidence that the adjusted distances are closer to the true physical distances.

Table 12. Full-dataset radius correction ($\Delta r_i = r_i^{\text{RAIM}} - r_i^{\text{RSSI}}$) by scanner circle—Emergency Room. Mean $\pm$ SD, median [Q1, Q3], and range are reported in metres.

LDPL Scenario	Filter	Circle	N	Mean $\pm$ SD	Median [Q1,Q3]	Min–Max	% Adj.	% Neg.
Free-space scenario	Raw	1	2,670,728	0.091 $\pm$ 0.949	0.000 [0.000, 0.000]	−43.005–14.467	1.116	0.013
Free-space scenario	Raw	2	2,670,728	5.911 $\pm$ 2.193	6.179 [4.411, 7.385]	−17.157–12.554	99.617	0.923
Free-space scenario	Raw	3	2,670,728	8.631 $\pm$ 2.135	8.841 [7.468, 10.043]	−14.004–14.817	99.050	0.038
Free-space scenario	Kalman	1	2,670,728	0.003 $\pm$ 0.163	0.000 [0.000, 0.000]	0.000–11.783	0.026	0.000
Free-space scenario	Kalman	2	2,670,728	6.268 $\pm$ 1.304	6.128 [5.305, 7.209]	−2.115–10.437	99.994	0.001
Free-space scenario	Kalman	3	2,670,728	9.049 $\pm$ 1.147	8.978 [8.273, 9.702]	−1.202–14.008	99.978	0.000
Moderate-indoor scenario	Raw	1	2,670,728	0.118 $\pm$ 1.127	0.000 [0.000, 0.000]	−6.532–14.273	1.129	0.001
Moderate-indoor scenario	Raw	2	2,670,728	9.034 $\pm$ 1.273	9.142 [8.488, 9.772]	−2.546–12.526	99.689	0.044
Moderate-indoor scenario	Raw	3	2,670,728	10.917 $\pm$ 1.474	11.043 [10.420, 11.645]	0.000–14.614	99.077	0.000
Moderate-indoor scenario	Kalman	1	2,670,728	0.003 $\pm$ 0.187	0.000 [0.000, 0.000]	0.000–12.550	0.026	0.000
Moderate-indoor scenario	Kalman	2	2,670,728	9.244 $\pm$ 0.605	9.167 [8.802, 9.660]	0.000–11.323	99.996	0.000
Moderate-indoor scenario	Kalman	3	2,670,728	11.166 $\pm$ 0.584	11.126 [10.810, 11.459]	0.000–13.930	99.979	0.000
High-attenuation indoor scenario	Raw	1	2,670,728	0.135 $\pm$ 1.264	0.000 [0.000, 0.000]	0.000–14.178	1.129	0.000
High-attenuation indoor scenario	Raw	2	2,670,728	10.711 $\pm$ 1.009	10.821 [10.514, 11.111]	0.000–12.518	99.752	0.000
High-attenuation indoor scenario	Raw	3	2,670,728	12.196 $\pm$ 1.324	12.329 [12.038, 12.623]	0.000–14.359	99.119	0.000
High-attenuation indoor scenario	Kalman	1	2,670,728	0.003 $\pm$ 0.202	0.000 [0.000, 0.000]	0.000–13.198	0.026	0.000
High-attenuation indoor scenario	Kalman	2	2,670,728	10.856 $\pm$ 0.302	10.823 [10.655, 11.039]	0.000–11.858	99.996	0.000
High-attenuation indoor scenario	Kalman	3	2,670,728	12.368 $\pm$ 0.332	12.354 [12.213, 12.502]	0.000–13.829	99.979	0.000

Table 13. Full-dataset radius correction ($\Delta r_i = r_i^{\text{RAIM}} - r_i^{\text{RSSI}}$) by scanner circle—Office. Mean $\pm$ SD, median [Q1, Q3], and range are reported in metres.

LDPL Scenario	Filter	Circle	N	Mean $\pm$ SD	Median [Q1,Q3]	Min–Max	% Adj.	% Neg.
Free-space scenario	Raw	1	10,315,487	2.485 $\pm$ 3.458	2.570 [0.000, 5.093]	−23.205–9.960	83.112	12.458
Free-space scenario	Raw	2	10,315,487	3.421 $\pm$ 3.572	2.790 [0.000, 7.075]	−23.678–11.543	58.897	1.384
Free-space scenario	Raw	3	10,315,487	2.773 $\pm$ 3.418	0.000 [0.000, 5.678]	−12.465–11.482	48.096	1.101
Free-space scenario	Kalman	1	10,315,487	3.102 $\pm$ 2.585	3.309 [0.775, 5.189]	−12.212–8.904	87.842	7.510
Free-space scenario	Kalman	2	10,315,487	3.747 $\pm$ 3.444	4.203 [0.000, 7.013]	0.000–10.375	58.237	0.000
Free-space scenario	Kalman	3	10,315,487	2.851 $\pm$ 3.422	0.000 [0.000, 5.943]	−2.867–10.495	45.759	0.072
Moderate-indoor scenario	Raw	1	10,315,487	4.083 $\pm$ 2.865	4.273 [1.369, 6.667]	−12.097–9.758	87.787	1.748
Moderate-indoor scenario	Raw	2	10,315,487	4.286 $\pm$ 3.913	5.308 [0.000, 8.205]	−1.910–11.367	59.792	0.011
Moderate-indoor scenario	Raw	3	10,315,487	3.665 $\pm$ 3.899	0.000 [0.000, 7.463]	−1.168–11.290	48.496	0.000
Moderate-indoor scenario	Kalman	1	10,315,487	4.446 $\pm$ 2.620	5.640 [2.055, 6.674]	−0.464–8.979	93.458	0.027
Moderate-indoor scenario	Kalman	2	10,315,487	4.468 $\pm$ 3.954	6.747 [0.000, 8.146]	0.000–10.243	58.274	0.000
Moderate-indoor scenario	Kalman	3	10,315,487	3.556 $\pm$ 3.943	0.000 [0.000, 7.519]	0.000–10.340	45.818	0.000
High-attenuation indoor scenario	Raw	1	10,315,487	5.024 $\pm$ 2.907	6.497 [2.555, 7.699]	0.000–9.517	90.825	0.000
High-attenuation indoor scenario	Raw	2	10,315,487	4.820 $\pm$ 4.251	7.581 [0.000, 8.930]	0.000–11.090	60.651	0.000
High-attenuation indoor scenario	Raw	3	10,315,487	4.221 $\pm$ 4.379	0.000 [0.000, 8.674]	0.000–11.006	48.496	0.000
High-attenuation indoor scenario	Kalman	1	10,315,487	5.347 $\pm$ 2.680	7.182 [2.878, 7.707]	0.000–8.995	95.845	0.000
High-attenuation indoor scenario	Kalman	2	10,315,487	4.930 $\pm$ 4.325	8.223 [0.000, 8.899]	0.000–10.146	58.337	0.000
High-attenuation indoor scenario	Kalman	3	10,315,487	4.038 $\pm$ 4.412	0.000 [0.000, 8.762]	0.000–10.305	45.818	0.000

4.3.2. Geometric Feasibility and Spatial Plausibility Results

Geometric Feasibility Rate

The three-circle intersection results are interpreted as geometric feasibility rather than localisation accuracy. The geometric feasibility rate (GFR) is calculated as $N_{3C} / N_{\text{obs}} \times 100\%$. Across the complete Office dataset, RAIM achieved 98.74% GFR under the unfiltered Free-space scenario, 99.59% after Kalman filtering, and 100% under both the Moderate-indoor scenario and High-attenuation indoor scenario conditions. Conventional Trilateration produced only 2.52% and 1.20% GFR under the unfiltered and Kalman-filtered Free-space scenario, respectively, and essentially no full intersections under the Moderate-indoor scenario and High-attenuation indoor scenario conditions (Table 10).

For the complete Emergency Room dataset, RAIM achieved 100% GFR under the Moderate-indoor scenario and High-attenuation indoor scenario conditions and 100.00% after Kalman filtering under the Free-space scenario, with 99.996% under the unfiltered Free-space scenario. Conventional Trilateration produced 0.055% GFR under the unfiltered Free-space scenario, 0% after Kalman filtering, and approximately 0% under the Moderate-indoor scenario and High-attenuation indoor scenario conditions (Table 10). These results show the geometric effect of the RAIM construction.

Spatial Outliers

Outlier counts provide a spatial-plausibility check based on membership within the mapped room polygons. Table 10 reports the corresponding outlier rates for the Paek-style, URS, and RAIM correction strategies, allowing comparison across deployments with different observation counts. For the Office dataset, RAIM reduced the unfiltered Free-space scenario outliers from 92,475 to 39, corresponding to a reduction from approximately 0.897% to 0.0004%. Under Kalman filtering, RAIM reduced the Free-space scenario outliers from 26,446 to 1017 (0.010%), while the Moderate-indoor scenario decreased from 2077 to 547 (0.005%). The High-attenuation indoor scenario produced zero outliers for both conventional Trilateration and RAIM. In the Emergency Room, RAIM reduced the unfiltered Free-space scenario outliers from 22,305 to 292 (0.011%) and eliminated the 77 outliers observed after Kalman filtering. All other Emergency Room conditions produced zero outliers for both methods. These results indicate differences in spatial plausibility under the room-polygon criterion; however, the criterion is not a substitute for coordinate-level ground-truth validation.

4.3.3. Anchor-Selection Sensitivity

The fixed-anchor experiment shows that the RAIM coordinate solution can be sensitive to the scanner designated as the anchor, even when room-polygon spatial plausibility changes very little. The summary in Table 14 reports ranges across the three LDPL scenarios for each deployment, filtering condition, and fixed scanner. In the Emergency Room, forcing Scanner 2 or Scanner 3 as the anchor produced mean shifts of 5.977–10.803 m and 8.642–12.301 m, respectively, relative to the default strongest-RSSI anchor solution. The corresponding 95th-percentile shifts reached 11.526 m for Scanner 2 and 13.057 m for Scanner 3. In contrast, forcing Scanner 1, which is the default anchor in approximately 99% of observations, produced mean shifts below 0.14 m.

The Office showed a different pattern. All three fixed-anchor choices produced non-negligible coordinate changes, with mean shifts ranging from 2.858 to 5.335 m across the tested LDPL scenarios and filtering conditions. The default strongest-RSSI anchor was more distributed across the three scanners: Scanner 1 matched the default in 9.163% of raw observations and 4.811% after Kalman filtering, Scanner 2 in 39.333% and 41.103%, and Scanner 3 in 51.504% and 54.086%, respectively. Thus, unlike the Emergency Room, no single scanner overwhelmingly represented the default anchor in the Office.

The coordinate-level sensitivity did not translate into comparable changes in room-polygon classification. Across all tested anchor configurations, the proportion of observations whose outlier status changed was at most 0.020% in the Emergency Room and 0.058% in the Office. This indicates that the mapped-room plausibility indicator is comparatively insensitive to anchor choice, whereas the underlying coordinates can vary by several metres. The result does not establish which anchor is more accurate; without ground-truth positions, it demonstrates only that the strongest-RSSI anchor rule is an influential modelling assumption. GFR is not used to distinguish the anchor configurations because RAIM is designed to enforce geometric feasibility.

Table 14. Full-dataset anchor-selection sensitivity. Ranges are across the three LDPL scenarios for each deployment and filtering condition. Mean and P95 shift are Euclidean displacements from the default strongest-RSSI RAIM solution; >5 m is the percentage of observations exceeding a 5 m shift; match is the percentage for which the fixed scanner was also the default anchor; outlier change is the maximum percentage of observations whose room-polygon outlier status changed across the three LDPL scenarios.

Deployment	Filter	Fixed Anchor	Mean Shift (m)	P95 Shift (m)	>5 m (%)	Match Default (%)	Outlier Status Changed (%)
Emergency Room	Raw	Scanner 1	0.095–0.135	0.000–0.000	0.921–1.129	98.871	0.020
Emergency Room	Raw	Scanner 2	5.977–10.713	9.163–11.526	71.318–98.985	0.248	0.006
Emergency Room	Raw	Scanner 3	8.642–12.196	11.723–13.057	95.881–98.886	0.881	0.020
Emergency Room	Kalman	Scanner 1	0.047–0.056	0.000–0.000	0.440–0.443	99.557	0.000
Emergency Room	Kalman	Scanner 2	6.136–10.803	8.889–11.432	74.857–99.557	0.072	0.000
Emergency Room	Kalman	Scanner 3	8.884–12.301	11.364–12.929	99.470–99.557	0.370	0.000
Office	Raw	Scanner 1	3.579–5.047	7.785–8.478	30.399–51.500	9.163	0.009
Office	Raw	Scanner 2	3.665–4.825	9.241–9.631	39.377–51.505	39.333	0.004
Office	Raw	Scanner 3	2.893–4.226	9.207–9.637	30.861–48.493	51.504	0.027
Office	Kalman	Scanner 1	3.693–5.335	7.473–8.391	31.440–54.086	4.811	0.058
Office	Kalman	Scanner 2	3.753–4.934	8.673–9.497	45.555–54.086	41.103	0.000
Office	Kalman	Scanner 3	2.858–4.040	9.230–9.580	31.488–45.914	54.086	0.014

5. Discussion

5.1. The Primary Contribution: Real-World Failure Geometry

The central empirical finding is that BLE trilateration failures in the studied hospital deployment are concentrated in a small number of recurring geometric configurations rather than being randomly distributed. More importantly, the prevalence of these configurations changes with deployment and the assumed RSSI-to-distance model, as detailed in Table 9. Kalman filtering generally preserves these broad patterns. The evidence therefore supports a diagnostic interpretation: the geometry of failure is structured, but it is not universal.

The result does not imply that a detached circle is physically incorrect or that the strongest RSSI measurement is necessarily the most accurate. RSSI-derived distances can be affected by obstruction, multipath propagation, interference, antenna orientation, and other environmental factors. The TFM classification describes the geometry generated by the adopted distance model; it does not establish which scanner measurement is erroneous.

5.2. What the Paek-Style Comparison Adds

The Paek-style reimplementations provides an important reference because it uses a different correction geometry from both URS and RAIM. Across the evaluated conditions, Paek-style adjustment produced approximately complete three-circle feasibility, but this outcome is expected from its incremental intersection-forcing rule and therefore is not treated as an independent accuracy measure. More informative differences appear in correction magnitude and coordinate displacement. For example, in the Office Free-space raw condition, the median total absolute radius adjustment was 8.00 m for Paek-style adjustment, compared with 9.34 m for URS and 9.18 m for RAIM; the mean coordinate shifts were 3.19 m, 3.18 m, and 4.77 m, respectively. In the Emergency Room High-attenuation raw condition, Paek-style adjustment produced a median total adjustment of 17.75 m and a mean coordinate shift of 2.94 m, whereas RAIM produced 23.19 m and 6.85 m. These differences show that the methods can achieve similar geometric feasibility while producing materially different corrected geometries.

The room-polygon outlier results are mixed rather than uniformly favouring one correction. Paek-style adjustment has lower outlier rates than RAIM in several conditions, while RAIM has lower rates in others, such as the unfiltered Office Free-space condition. Because room-polygon membership is only a coarse spatial-plausibility criterion and no

ground-truth coordinates are available, these differences cannot be converted into an accuracy ranking. The Paek comparison therefore strengthens the methodological interpretation of the study: RAIM is a distinct radius-specific correction strategy, but the present evidence does not establish that it is more accurate than pairwise incremental adjustment.

5.3. What the Uniform Scaling Comparison Adds

The URS baseline provides a simple reference for interpreting the correction demonstration. A common positive scale factor was feasible for only 23.63–96.34% of observations across the evaluated conditions. This range is important because it shows that the observed geometric incompatibility is not uniformly resolvable by preserving the relative proportions of all three RSSI-derived radii. The feasibility increase with the LDPL scenario also differs between deployments, reinforcing that the correction problem depends on both the distance-conversion model and the deployment geometry.

The URS scale factors are also substantial. Median factors range from approximately 1.92 to 3.77 in the Office and 2.03 to 4.89 in the Emergency Room across the tested scenarios and filtering conditions. These values should be understood as geometric correction factors, not calibrated propagation parameters or estimates of physical ranging error. The URS results therefore complement the Δr analysis: both show that substantial changes to the RSSI-derived radii may be required to obtain mutually compatible circles, but they do not establish which correction is physically correct.

The comparison also clarifies the limited claim that can be made about RAIM. RAIM permits scanner-specific sequential adjustments and can therefore construct a feasible configuration in observations for which no common URS factor exists. Unlike the Paek-style rule, RAIM does not increment radii pairwise in fixed 1 m steps; instead, it uses the scanner-specific radius ordering and a direct final-radius calculation. The comparison demonstrates a difference in geometric correction flexibility, not a validated accuracy advantage. Ground-truth positions would be required to determine whether either correction strategy improves absolute localisation.

5.4. Why the Predominant Failure Modes Matter

The prevalence of TFM-I in several configurations indicates that many failed observations retain partial geometric compatibility: two circles can intersect even though the third does not. TFM-III, however, dominates other configurations and represents a different geometric incompatibility in which the circles are separated. RAIM therefore targets the broader problem of incompatible radii rather than assuming that every failure has the TFM-I structure. It uses the strongest-signal radius as an anchor and modifies the other radii to construct a common intersection.

The full-dataset Δr results provide quantitative evidence of this mechanism. The anchor role has $\Delta r = 0$ by construction, while the middle and largest roles receive predominantly positive corrections. In the Emergency Room, the median correction of the middle role ranges from 6.128 to 12.314 m and that of the largest role from 6.144 to 10.959 m across the tested environments and filtering conditions. In the Office, the corresponding ranges are 4.021–8.323 m and 4.021–8.009 m. The correction magnitudes therefore depend on the deployment environment and signal-processing condition rather than representing a single fixed adjustment.

5.5. What RAIM Demonstrates and What It Does Not

The increase in three-circle intersections shows that RAIM achieves its immediate geometric objective of restoring circle compatibility. Across the complete synchronised datasets, RAIM produced near-complete GFR in every evaluated condition and 100% GFR in all Emergency Room conditions except the unfiltered Free-space scenario, where GFR

was 99.996%. The Office results ranged from 98.74% to 100%. However, these results should be interpreted carefully because producing a common intersection is the explicit geometric objective of RAIM; a high GFR is therefore evidence of successful constraint satisfaction, not independent evidence of positional correctness.

These results must be interpreted according to the distinction between geometric feasibility and localisation accuracy. A common intersection means that the adjusted circles are mutually compatible; it does not mean that the intersection coincides with the physical location of the BLE device. Similarly, an estimate inside a mapped room polygon is spatially plausible but not necessarily accurate. Because no ground-truth coordinates were available, the study cannot quantify absolute positioning error, such as mean absolute error, median positioning error, or root mean square error.

The Δr analysis should also be interpreted as mechanism evidence rather than accuracy evidence. A positive correction indicates that RAIM expanded a radius relative to the original RSSI-derived value. It does not demonstrate that the expanded radius is closer to the true distance.

5.6. Strongest-Scanner Bias and Anchor Selection

The fixed-anchor experiment provides direct evidence that the strongest-RSSI anchor is an influential modelling assumption. The default RAIM procedure retains the smallest RSSI-derived radius and adjusts the other two radii. In the Emergency Room, the default anchor was Scanner 1 in approximately 98.9% of raw observations and 99.6% after Kalman filtering. Replacing it with Scanner 2 or Scanner 3 changed the RAIM coordinates by mean values of approximately 6.0–10.8 m and 8.6–12.3 m, respectively, across the tested LDPL scenarios. In the Office, where default-anchor selection was distributed more evenly across the three scanners, fixed-anchor alternatives still produced mean shifts of approximately 2.9–5.3 m.

The same experiment also shows why spatial plausibility should not be treated as a substitute for coordinate-level validation. Although the coordinate solutions changed by several metres, the room-polygon outlier classification changed for at most 0.020% of Emergency Room observations and 0.058% of Office observations across the evaluated conditions. Thus, a position can move substantially while remaining inside the same mapped room. This distinction matters for the intended application scope: room-level occupancy, room presence, and other coarse spatial indicators may remain stable when the coordinate shifts stay within the same room, whereas fine-grained applications such as sub-room trajectory reconstruction or precise contact-distance estimation would be directly affected by metre-scale coordinate changes. Without ground-truth coordinates, the experiment cannot determine whether the default or an alternative anchor is closer to the true device position. It does, however, demonstrate that the anchor-selection rule should be reported as an explicit source of model sensitivity rather than treated as an innocuous implementation detail.

Alternative anchor strategies, such as selection based on signal stability, scanner geometry, or multi-anchor consistency, remain potential directions for future work. Such strategies should be evaluated against independent ground truth when absolute localisation accuracy is the objective.

5.7. The Importance of Data Synchronisation

A critical practical lesson is that raw BLE data volume does not equate to usable localisation observations. The original deployment contained 162,092,585 RSSI records, whereas timestamp synchronisation reduced the data to 2,670,728 Emergency Room observations at

the 3 s interval and 10,315,487 Office observations at the 1 s interval, for 12,986,215 analysed observations in total.

This reduction underscores the importance of scanner synchronisation in real-world BLE IPS. Large volumes of unsynchronised observations cannot be used for three-circle geometric analysis because the three RSSI values must correspond to the same observation window. Consequently, preprocessing and temporal alignment are important components of the overall localisation pipeline, not merely data-cleaning steps.

6. Conclusions

This study investigated BLE-based trilateration using 162,092,585 raw RSSI records collected from Emergency Room and Office deployments at Diponegoro National Hospital. The primary contribution is an empirical characterisation of the geometric configurations produced when RSSI measurements are converted into distances and used for three-scanner trilateration. Four recurring Trilateration Failure Modes were identified and quantified over the complete synchronised datasets.

The failure-mode distribution is configuration-dependent. In the Emergency Room, TFM-I accounted for 94.41% of observations under Free-space conditions and 96.53% under Moderate-indoor conditions, whereas TFM-III accounted for 96.92% under High-attenuation conditions. In the Office, TFM-I accounted for 62.55% under Free-space conditions, while TFM-III accounted for 57.77% under Moderate-indoor conditions and 97.95% under High-attenuation conditions. The change in prevalence across LDPL assumptions demonstrates that the RSSI-to-distance model materially affects the geometry presented to trilateration. This is a diagnostic finding rather than evidence that any particular path-loss exponent is physically correct.

The URS experiment provides a simple geometric reference for the correction problem. A common positive scale factor was feasible for 23.63–96.34% of observations across the evaluated conditions. The feasibility range and the required scale factors varied with deployment, LDPL scenario, and filtering, demonstrating that a single multiplicative correction does not reconcile all observed circle configurations. This baseline is not a claim about the accuracy of prior uniform-adjustment methods; it is a deterministic common-factor comparison implemented for the present dataset.

Guided by the failure characterisation, the study retained RAIM as a secondary geometry-guided demonstration and evaluated it alongside Paek-style pairwise adjustment and URS. RAIM retains the smallest RSSI-derived radius as an anchor and sequentially adjusts the remaining radii to recover a common intersection. Across the complete synchronised datasets, this procedure produces near-complete three-circle geometric feasibility, while room-polygon outlier rates decrease in several conditions. The full-dataset Δr analysis shows that non-anchor radii generally receive positive corrections, with magnitudes that vary across deployments and signal-processing conditions. The comparison with Paek-style adjustment and URS shows that sequential radius-specific, pairwise incremental, and common-factor corrections impose different geometric constraints and produce different correction magnitudes and coordinate displacements. In particular, URS cannot admit a common factor for a substantial fraction of observations in several conditions, whereas Paek-style adjustment and RAIM achieve very high geometric feasibility under the implemented adjustment rules.

These results establish geometric and spatial-plausibility behaviour, not absolute localisation accuracy. A common intersection can be constructed even when the adjusted radii do not correspond to the physical distances of the BLE device. The anchor-sensitivity experiment further shows that alternative scanner anchors can shift estimated coordinates by several metres while changing room-polygon outlier status only marginally. With-

out ground-truth positions, however, the analysis cannot establish whether selecting the strongest-RSSI scanner as the anchor provides the most accurate estimate.

Overall, the study shows that a large real-world BLE deployment can reveal structured trilateration failure geometries that vary with the assumed distance model and that geometric correction strategies impose different constraints on how those failures can be reconciled. TFM characterisation provides the primary empirical contribution, while Paek-style adjustment, URS, and RAIM provide complementary correction demonstrations. Ground-truth measurements remain necessary to determine whether any correction strategy improves absolute localisation accuracy.

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Nomenclature

The following abbreviations and symbols are used in this manuscript:

Abbreviations

BLE	Bluetooth Low Energy
DNH	Diponegoro National Hospital
ER	Emergency Room
ETL	Extract, Transform, and Load
GFR	Geometric Feasibility Rate
IPS	Indoor Positioning System
JSON	JavaScript Object Notation
LDPL	Log-Distance Path Loss
MQTT	Message Queuing Telemetry Transport
QGIS	Quantum Geographic Information System
RAIM	Radius Adjustment Intersection Method
RSSI	Received Signal Strength Indicator
SSIS	SQL Server Integration Services
SSMS	SQL Server Management Studio
TFM	Trilateration Failure Modes
URS	Uniform Radius Scaling

Symbol	Description
R	Earth's radius
A	RSSI reference value at 1 m
n	LDPL path-loss exponent
$\Delta\lambda$	Difference in longitude
$\Delta\phi$	Difference in latitude
ϕ_{ref}	Reference latitude
λ_{ref}	Reference longitude
x, y	Cartesian coordinates
d_{ij}	Euclidean distance between scanners i and j
(x_1, y_1)	Coordinates of first scanner
(x_2, y_2)	Coordinates of second scanner

r_i	RSSI-derived distance/radius associated with scanner i
r_i^{RAIM}	RAIM-adjusted radius for scanner i
r_i^{RSSI}	Original RSSI-derived radius for scanner i
Δr_i	Radius adjustment, $r_i^{\text{RAIM}} - r_i^{\text{RSSI}}$
s	Common radius-scaling factor in URS
s^*	Selected common radius-scaling factor in URS
a	Horizontal distance to chord
h	Perpendicular distance to intersection
(x_3, y_3)	Intersection point coordinates
ϕ	Calculated latitude
λ	Calculated longitude

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