Supplementary Information

1. Fresnel Coefficients

The reflection and refraction of a plane electromagnetic wave at a plane surface which separates two homogeneous media are described by Fresnel's equations. Their polarimetric properties are best described by decomposing the incident wave into two parts: the first one (horizontal h, or TE) has its E-field perpendicular to the normal vector of the plane, the second one (vertical v, or TM) has its H-field perpendicular to the normal vector [1].

The $[T_{mn}]$ term in the model contains the two-way transmission coefficients $T_{v,m}$ and $T_{h,m}$, which pertain to the horizontal and the vertical part, respectively. In these expressions m refers to the acquisition. These coefficients describe the change in amplitude that the E-field of the incident wave experiences when it (i) penetrates into the soil; and (ii) is transmitted back from the soil into the upper halfspace (with $\varepsilon = 1$). Assuming nonmagnetic media, these can be expressed as [1]

$$T_{v,m} = \frac{2\cos\theta^i}{\cos\theta^i + \sqrt{\varepsilon_m - \sin^2\theta^i}} \cdot \frac{2\cos\theta^t}{\cos\theta^t + \sqrt{\varepsilon_m^{-1} - \sin^2\theta^t}}$$

$$T_{h,m} = \frac{2\sqrt{\varepsilon_m}\cos\theta^i}{\varepsilon_m\cos\theta^i + \sqrt{\varepsilon_m - \sin^2\theta^i}} \cdot \frac{2\sqrt{\varepsilon_m^{-1}}\cos\theta^t}{\varepsilon_m^{-1}\cos\theta^t + \sqrt{\varepsilon_m^{-1} - \sin^2\theta^t}}$$

In each of this terms, the first part represents the transmission into the soil, and the second one the one back into the air.

2. Phase Terms

The interferometric phase ϕ_{mn} of a point scatterer embedded in the half-space consists of two terms (after linearization of (7) with respect to the antenna position). The first one $\tilde{\phi}_{mn}$ is due to the change in the background dielectric constant; if there is no such change, this term will vanish.

$$\tilde{\phi}_{mn} = 2k_0 \left[w_m \sqrt{z^2 + (y_s - y_{i,m})^2} - w_n \sqrt{z^2 + (y_s - y_{i,m})^2} \right] \tag{1}$$

The second term $\bar{\phi}_{mn}$ accounts for the spatial baseline B, *i.e.*, the variability of the antenna position between the acquisitions [2]. The spatial baseline can be decomposed into its horizontal $\Delta y_a = B\cos(\alpha)$ and vertical $\Delta H = B\sin(\alpha)$ components, where α is the angle between the baseline and the horizontal [3].

$$\begin{aligned} \bar{\phi}_{mn} &= 2k_0 \left[\sin\theta^i \Delta y_a - \cos\theta^i \Delta H \right] \\ &= 2k_0 B \sin(\theta^i - \alpha) \end{aligned} \tag{2}$$

Both these terms depend on the position of the point scatterer ($\bar{\phi}_{mn}$ implicitly via the incidence angle). As volume scattering consists of contributions from different positions, it is expedient to study their dependence on the position. For small penetration depths (see the main document for details) only the first derivatives are important. We will evaluate these just below the surface, *i.e.*, $z = 0$, by parameterizing the location of the point scatterer in terms of y_s and z. The coordinates of the antenna and the surface are fixed parameters. All additional quantities, in particular the incidence and transmission angles as well as y_i, are thus determined too.

2.1. Total Differentials

The total differentials of ϕ_{mn} with respect to y_s and z follow from Snell's law and the geometric definitions given in the main document. Starting with $\tilde{\phi}_{mn}$, we find that the partial derivative with respect to y_s vanishes. As the partial derivative with respect to the respective y_i cancels in each of the terms (Snell's law), we have

$$\begin{aligned}\left(\frac{\partial\tilde{\phi}_{mn}}{\partial y_s}\right)_z &= 2k_0\left(w_m\frac{y_s-y_{i,m}}{\sqrt{z^2+(y_s-y_{i,m})^2}}-w_n\frac{y_s-y_{i,n}}{\sqrt{z^2+(y_s-y_{i,n})^2}}\right)\\ &= 2k_0\left(\sin\theta^i-\sin\theta^i\right)\\ &= 0\end{aligned}\tag{3}$$

where the second line follows from Snell's law as well.

The derivative with respect to z is computed using the same identities:

$$\begin{aligned}\left(\frac{\partial\tilde{\phi}_{mn}}{\partial z}\right)_{y_s} &= 2k_0\left(w_m\frac{z}{\sqrt{z^2+(y_s-y_{i,m})^2}}-w_n\frac{z}{\sqrt{z^2+(y_s-y_{i,n})^2}}\right)\\ &= -2k_0\left(\sqrt{w_m^2-\sin^2\theta^i}-\sqrt{w_n^2-\sin^2\theta^i}\right)\end{aligned}\tag{4}$$

We now turn to the part that depends on the baseline, *i.e.*, $\bar{\phi}_{mn}$. Keeping y_s fixed (implicit in the following equations), we have via the chain rule

$$\left(\frac{\partial\bar{\phi}_{mn}}{\partial z}\right)_{y_s}=\frac{\partial\bar{\phi}_{mn}}{\partial\sin\theta^i}\frac{\partial\sin\theta^i}{\partial y_i}\frac{\partial y_i}{\partial z}\tag{5}$$

In turn this terms evaluate to:

$$\frac{\partial\bar{\phi}_{mn}}{\partial\sin\theta^i}=2k_0B\frac{1}{\cos(\theta^i)}\cos(\theta^i-\alpha)$$

whereas

$$\frac{\partial\sin\theta^i}{\partial y_i}=H\cos^{-3}(\theta^i)$$

as H is held constant. The last term follows from Figure S1a

$$\frac{\partial y_i}{\partial z}=\tan\theta^t$$

Collecting these terms and expressing θ^t in terms of θ^i, one finds

$$\left(\frac{\partial\bar{\phi}_{mn}}{\partial z}\right)=\left[-2k_0\frac{B_\perp}{R}\frac{\cos\theta^i\sin\theta^i}{\sqrt{w_m^2-\sin^2\theta^i}}\right]\tag{6}$$

where $B_\perp=\cos(\theta^i-\alpha)$.

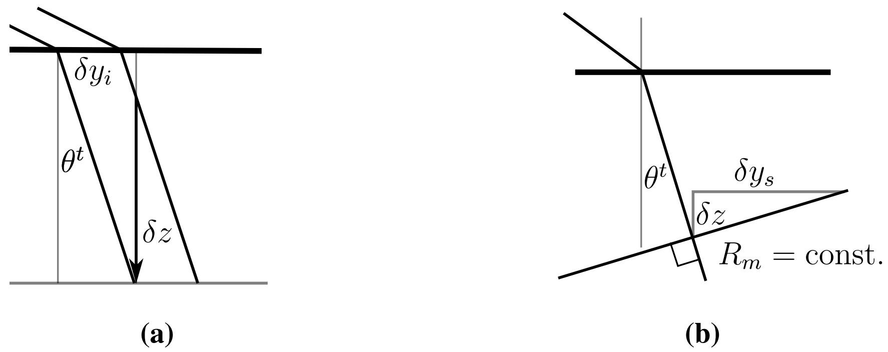

Figure S1. Derivation of geometric quantities: (**a**) change in y_i with z for constant y_s; (**b**) change in y_s with z for constant R_1.

The partial derivative with respect to y_s, $\left(\frac{\partial\bar{\phi}_{mn}}{\partial y_s}\right)_z$, can be evaluated by similar means. As we evaluate z just below the surface

$$\left(\frac{\partial y_i}{\partial y_s}\right)_z \bigg|_{\frac{z}{R}\to 0} \to 1$$

i.e., the horizontal coordinate of the piercing point coincides with that of the scatterer. Omitting the z subscript for clarity, we get

$$\begin{aligned}\frac{\partial\bar{\phi}_{mn}}{\partial y_s} &= \frac{\partial\bar{\phi}_{mn}}{\partial \sin\theta^i}\frac{\partial\sin\theta^i}{\partial y_i}\frac{\partial y_i}{\partial y_s} \\ &= \left(-2k_0 B\cos(\theta^i-\alpha)\frac{1}{\cos\theta^i}\right)\left(\frac{1}{R}\cos^2\theta^i\right) \\ &= \left(-2k_0\frac{B_\perp}{R}\cos\theta^i\right) \end{aligned} \tag{7}$$

2.2. Interferometric Wavenumbers

These are the partial derivatives with respect to range R_m and z, keeping the other one constant. They follow from the chain rule using the geometric relation shown in Figure S1b . According to the highlighted triangle, a small change δz entails a change δy_s if R_m is to remain constant:

$$\left(\frac{\partial y_s}{\partial z}\right)_{R_m} = \frac{\delta y_s}{\delta z} = \cot\theta^t \tag{8}$$

where this is again evaluated just below surface.

The vertical interferometric wavenumber follows from the terms due to $\bar{\phi}_{mn}$ and $\tilde{\phi}_{mn}$ and simplifying:

$$\begin{aligned}\left(\frac{\partial\phi_{mn}}{\partial z}\right)_{R_m} &= \left(\frac{\partial\bar{\phi}_{mn}}{\partial z}\right)_{y_s}\left(\frac{\partial z}{\partial z}\right)_{R_m} + \\ &\quad + \left(\frac{\partial\tilde{\phi}_{mn}}{\partial z}\right)_{y_s}\left(\frac{\partial z}{\partial z}\right)_{R_m} + \left(\frac{\partial\bar{\phi}_{mn}}{\partial y_s}\right)_{z}\left(\frac{\partial y_s}{\partial z}\right)_{R_m} + \left(\frac{\partial\tilde{\phi}_{mn}}{\partial y_s}\right)_{z}\left(\frac{\partial y_s}{\partial z}\right)_{R_m} \\ &= -2k_0\left[\sqrt{w_m^2-\sin^2\theta^i} - \sqrt{w_n^2-\sin^2\theta^i} + \frac{B_\perp}{R\sin\theta^i}\frac{w_m^2\cos\theta^i}{\sqrt{w_m^2-\sin^2\theta^i}}\right]\end{aligned} \tag{9}$$

and similarly for the range interferometric wavenumber

$$\left(\frac{\partial\phi}{\partial R_m}\right)_z = -2k_0\frac{B_\perp \cos\theta^i}{R\sin\theta^i} \tag{10}$$

References

1. Jackson, J. *Classical Electrodynamics*, third ed.; John Wiley & Sons: Hoboken, NJ, USA, 1999.
2. Oveisgharan, S.; Zebker, H. Estimating Snow Accumulation From InSAR Correlation Observations. *IEEE Trans. Geosci. Remote Sens.* **2007**, *45*, 10–20.
3. Dall, J. InSAR Elevation Bias Caused by Penetration Into Uniform Volumes. *IEEE Trans. Geosci. Remote Sens.* **2007**, *45*, 2319–2324.