

## Article

# LuTan-1 InSAR-Constrained Reconstruction of Concealed Coal-Mining Working-Face Geometry

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## Highlights

### What are the main findings?

- LuTan-1 InSAR LOS deformation combined with probability integral model constraints enables concealed working-face geometry reconstruction, achieving a deformation-fitting RMSE of 32.53 mm and accurate estimation of most geometric parameters in the case study.
- The adaptive multi-strategy differential evolution algorithm with Latin hypercube initialization and local search outperforms PSO, GA, ES, and standard DE in inversion accuracy and stability.

### What are the implications of the main findings?

- The method provides a feasible satellite-based approach for detecting concealed mining and estimating goaf parameters when mine records are incomplete, supporting mining supervision and subsidence hazard assessment.
- The constrained high-dimensional inversion framework reduces parameter uncertainty and is adaptable to other SAR datasets and mining regions for operational monitoring.

## Abstract

To address the difficulty in determining underground working face geometry when mining records are incomplete or mining activities are concealed, this study proposes an InSAR-constrained method for inverting the geometric parameters of concealed working faces. The method uses the line-of-sight (LOS) deformation field derived from LuTan-1 InSAR observations as the primary input. The search ranges of mining depth, mining thickness, coal seam dip angle, strike azimuth, working face center coordinates, strike length, and dip length are constrained using the morphology of the subsidence basin, available empirical parameters, and the Probability Integral Method (PIM). These constraints reduce the uncertainty caused by a large search space and parameter coupling in high-dimensional inversion. An adaptive multi-strategy differential evolution algorithm is then introduced. Latin hypercube sampling is used for population initialization. Dynamic parameter adjustment, multiple mutation strategies, reflective boundary handling, and local refinement are incorporated to improve optimization stability in complex parameter spaces. The simulation results show that the proposed method generally outperforms particle swarm optimization, genetic algorithm, evolutionary strategy, and standard differential evolution



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in inversion accuracy, convergence stability, and robustness to errors. It can reliably recover the main geometric parameters of a predefined working face. In the engineering case study of the 97,315 working face, the inverted active-mining geometry during the LuTan-1 observation period is generally consistent with the corresponding mine records, with a deformation-fitting RMSE of 32.53 mm and relative errors of the main geometric parameters below 7.9%. Parameter sensitivity analysis further shows that the geometric dimensions and center location of the working face have pronounced effects on the LOS deformation field. These parameters should therefore be tightly constrained during inversion. The results demonstrate that the proposed method provides a feasible remote-sensing approach for identifying concealed working faces and estimating goaf geometric parameters in mining areas with incomplete records.

**Keywords:** coal mining subsidence; parameter inversion; probability integral method; LuTan-1; mining characteristic parameters

## 1. Introduction

From 2020 to 2025, coal consistently accounted for more than 50% of China's total energy consumption [1]. As China's dominant energy source, coal must be mined safely and sustainably to support the national energy strategy [2]. However, large-scale coal extraction inevitably disturbs the original stress equilibrium of the rock mass. This disturbance induces movement and failure of the overlying strata, resulting in extensive subsidence basins. It can also damage surface infrastructure and the ecological environment and trigger a range of mining-related hazards [3,4]. Therefore, accurately determining the geometric parameters of underground goafs, including mining depth, length, width, thickness, and dip angle, is essential for reducing goaf-related hazards and accidents. However, working face records are often unavailable in historical mining areas and regions affected by illegal extraction. This lack of information poses major challenges to mine geohazard assessment, ecological restoration, and safe mining operations. Therefore, developing a method to accurately invert the geometric parameters of hidden underground working faces from surface deformation observations has important theoretical significance and practical engineering value.

Current methods for determining goaf geometric parameters mainly include historical investigation and analysis, geophysical exploration, and surface deformation-based inversion. Historical investigation and analysis rely on mine records to reconstruct the spatial geometry of goafs. However, this approach has clear limitations in historical mining areas with incomplete records and regions affected by illegal extraction [5]. Conventional geophysical techniques, such as seismic exploration [6] and electromagnetic exploration [7], obtain working face information by monitoring and analyzing changes in geophysical fields. Although these techniques provide high accuracy, their limited survey coverage and high demands on time and labor restrict their efficient application over large mining areas. Surface deformation-based parameter inversion mainly relies on the joint analysis of surface deformation observations and surface movement prediction models. Among these models, the PIM has been widely used for mining subsidence prediction and inversion because of its clear physical basis, well-established parameter system, and strong engineering applicability [8,9].

With the development of remote sensing technology, Interferometric Synthetic Aperture Radar (InSAR) has become an important tool for monitoring surface deformation in mining areas. Its main advantages include wide spatial coverage, non-contact observation,

high spatial resolution, and continuous monitoring [10–12]. Differential InSAR (D-InSAR) derives surface deformation by differencing the interferometric phases of SAR acquisitions after removing the topographic and orbital contributions, and has been widely used for mapping mining-induced deformation over specific observation intervals. In recent years, researchers have combined InSAR observations with the PIM to identify underground working face locations, invert goaf parameters, and detect illegal mining areas [13–19]. These studies have promoted a shift from conventional point-based measurements to area-wide remote sensing constraints in mining parameter inversion [20,21]. However, several limitations remain. Most existing studies rely on C-band SAR data. Such data are prone to coherence loss and missing deformation information in areas with steep subsidence gradients, dense vegetation, or complex terrain. These problems reduce the reliability of the inversion. In addition, goaf geometric parameter inversion is inherently a highly nonlinear optimization problem. Existing methods do not fully address complex parameter coupling or sensitivity to initial values. Conventional optimization algorithms can easily become trapped in local optima, and their global search capability and solution stability require further improvement. Moreover, few studies have incorporated a linearly constrained optimization framework into the inversion. The physical laws of mining subsidence and the geometric constraints of subsidence basins are rarely expressed explicitly and quantitatively. As a result, the inversion often involves an excessively large search space and strong parameter correlations, which may produce ill-posed solutions without physical meaning [22]. Previous studies have demonstrated the feasibility of inferring underground mining information from the morphology of surface subsidence basins and surface deformation fields. This approach provides a promising solution for identifying working faces in areas where complete mining records are unavailable [23–25].

Compared with conventional C-band SAR systems, such as Sentinel-1, L-band SAR has a longer wavelength and stronger penetration capability. It generally maintains better coherence in vegetated areas, mountainous terrain, and regions with steep deformation gradients. Therefore, it is more suitable for monitoring surface deformation in mining areas [26]. The study area is located in a mountainous mining region with pronounced topographic relief and complex surface conditions. These characteristics highlight the suitability of longer-wavelength LuTan-1 L-band SAR data for obtaining reliable deformation observations for subsequent working-face geometry inversion. Furthermore, coupling the LOS deformation field acquired by LuTan-1 with PIM and using an optimization algorithm with strong global search capability may overcome the limitations of conventional inversion methods in observation accuracy and solution stability under complex mountainous conditions.

To address these limitations, this study proposes an adaptive inversion method for the geometric parameters of concealed working faces. D-InSAR is used to derive the LOS deformation field during the active mining period, which serves as the observational constraint for subsequent working-face geometry inversion. The method integrates LuTan-1 L-band InSAR deformation observations, geometric constraints from the subsidence basin, empirical constraints from the mining area, and the PIM physical model. The LOS deformation field derived from LuTan-1 InSAR is used as the primary observation. The working face center coordinates, mining depth, mining thickness, coal seam dip angle, strike azimuth, strike length, and dip length are incorporated into a unified inversion framework. Reasonable search ranges are determined from the spatial morphology of the subsidence basin, empirical parameters of adjacent working faces, and the geometric relationships of mining subsidence. These constraints reduce parameter uncertainty in high-dimensional inversion. For parameter optimization, Latin hypercube sampling initialization, iteration-based dynamic parameter adjustment, multiple mutation strategies, reflective boundary

handling, and local refinement are incorporated into differential evolution. This forms an adaptive multi-strategy differential evolution algorithm (AMSDE), which improves global search capability and solution stability in complex parameter spaces. Finally, simulation experiments, an engineering case study of 97,315 working face at Sihe No. 2 mining area in Shanxi Province, parameter sensitivity analysis, and random error perturbation experiments are conducted. These experiments verify the effectiveness, stability, and parameter identifiability of the proposed method for inverting the geometric parameters of concealed working faces.

## 2. Methods

### 2.1. Generalization of Working Face Geometric Parameters and Construction of the Inversion Model

Surface movement and deformation caused by underground mining exhibit clear spatial continuity and regularity. For a subsidence basin formed by the extraction of a rectangular working face, surface subsidence, horizontal displacement, and their spatial distributions are closely related to mining depth, mining thickness, working face dimensions, coal seam dip angle, and mining direction. Therefore, surface deformation observations can be used to invert the geometric parameters of an underground working face. PIM [27,28] is widely used for mining subsidence prediction in China. Its parameters have clear physical meanings, and the model can effectively describe the surface movement and deformation caused by the extraction of regular working faces. It is therefore suitable as the physical constraint model for the inversion of concealed working face parameters.

Let  $xOy$  denote the surface coordinate system and  $sO_1t$  denote the coal seam coordinate system. For surface pixel  $i$ , PIM calculates the three-dimensional surface displacement components using the given PIM prediction parameters  $P_{PIM}$  and working face geometric parameters  $P_g$ . These components include subsidence  $W_i$ , east–west horizontal displacement  $U_{WE,i}$ , and south–north horizontal displacement  $U_{SN,i}$ . For notational convenience, the PIM forward calculation is expressed in the following compact form:

$$[W_i, U_{WE,i}, U_{SN,i}] = M_{PIM}(x_i, y_i; P_{PIM}, P_g) \quad (1)$$

where  $M_{PIM}$  represents the PIM forward operator. It does not represent a new subsidence prediction model but provides a unified expression for the integral calculation in PIM.  $x_i$  and  $y_i$  are the planar coordinates of the pixel  $i$ .  $P_{PIM}$  is the PIM prediction parameters, and  $P_g$  is the working face geometric parameters to be inverted.

The PIM formulation used in this study does not explicitly introduce a temporal response function for delayed mining subsidence. Therefore, the engineering application adopts a quasi-static equivalent representation of the active-mining geometry over the InSAR observation period.

The PIM prediction parameters and working face geometric parameters are represented separately in this study. The PIM prediction parameters are expressed as:

$$P_{PIM} = \{q, \tan\beta, b, \theta, S_d, S_u, S_l, S_r\} \quad (2)$$

where  $q$  is the subsidence coefficient,  $\tan\beta$  is the tangent of the main influence angle,  $b$  is the horizontal displacement coefficient,  $\theta$  is the mining influence propagation angle, and  $S_d$ ,  $S_u$ ,  $S_l$ ,  $S_r$  are the inflection point offsets in the down-dip, up-dip, open-off cut, and stopping-line directions. These parameters mainly describe the general characteristics of mining subsidence in the mining area. They can usually be determined from empirical statistical data and engineering analogues from adjacent or geologically similar working faces, field measurements of surface deformation, or existing subsidence-prediction parameters.

The working face geometric parameters to be inverted are expressed as:

$$P_g = \{m, \alpha, \varphi, X_0, Y_0, H_0, D_3, D_1\} \quad (3)$$

Then  $m$  is the mining thickness,  $\alpha$  is the coal seam dip angle,  $\varphi$  is the strike azimuth of the working face,  $(X_0, Y_0)$  denotes the working face center coordinates,  $H_0$  is the average mining depth, and  $D_3$  and  $D_1$  are the strike length and dip length of the working face, respectively.

$P_{PIM}$  and  $P_g$  parameters define the location, shape, and spatial geometry of the underground working face. Their geometric definitions are illustrated in Figure 1.

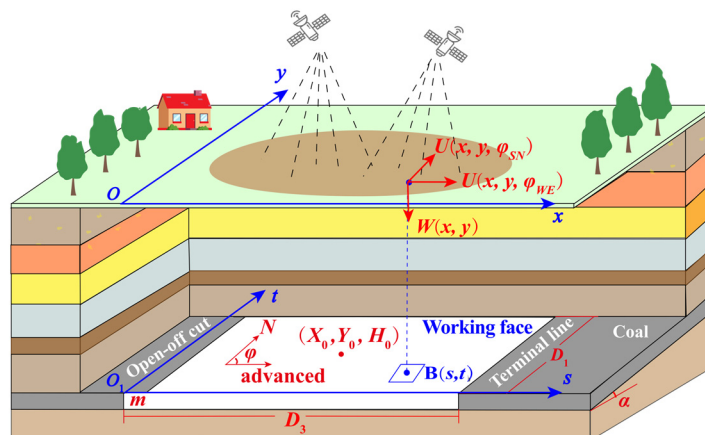


**Figure 1.** Schematic diagram of the geometric parameters of a rectangular working face. (The figure includes the following parameters, geometric parameters:  $m, \alpha, \varphi, X_0, Y_0, H_0, D_3, D_1$ , surface coordinate system  $xOy$ , coal seam coordinate system  $sO_1t$ , North direction  $N$ , Extraction unit  $B(s, t)$ , The three-dimensional components of LOS  $W(x, y), U(x, y, \varphi_{SN}), U(x, y, \varphi_{WE})$ ).

LuTan-1 InSAR measures the projection of three-dimensional surface displacement along the radar LOS rather than vertical subsidence alone. Therefore, LOS deformation cannot be directly treated as vertical subsidence in the inversion. In this study, PIM is first used to calculate the three-dimensional displacement components at each surface point. These components are then projected onto the radar LOS to obtain the predicted LOS deformation for surface pixel  $i$ :

$$d_i^{pre} = l_{WE,i} U_{WE,i} + l_{SN,i} U_{SN,i} - l_{V,i} W_i \quad (4)$$

where  $d_i^{pre}$  is the deformation predicted by PIM for surface pixel  $i$  and projected onto the LOS direction.  $l_{WE,i}$ ,  $l_{SN,i}$  and  $l_{V,i}$  are the LOS projection coefficients in the east–west, south–north, and vertical directions.

In this study, subsidence  $W_i$  is defined as positive downward. Therefore,  $-l_{V,i} W_i$  is used to represent its vertical contribution to the LOS deformation. In practice, the projection coefficients can be determined from the incidence angle and satellite heading angle of the SAR image. Their sign convention must be consistent with that of the InSAR deformation measurements. Given  $P_{PIM}$  and the LOS deformation observed by LuTan-1 InSAR, the inversion aims to find the optimal  $P_g$  that minimizes the difference between the LOS deformation predicted by PIM and that observed by InSAR. The optimization model is expressed as:

$$\hat{P}_g = \arg \min_{P_g \in \Omega} F(P_g) \quad (5)$$

where  $\hat{P}_g$  denotes the optimal working face geometric parameters obtained from the inversion,  $\Omega$  is the parameter search space jointly constrained by the geometric characteristics of

the subsidence basin, empirical information from the mining area, and the PIM physical model, and  $F(P_g)$  is the fitness function.

It should be noted that the empirical information from the mining area is used only as an auxiliary constraint rather than as an exact prior value. When mine records are incomplete or uncertain, the parameter search ranges are determined primarily from the spatial morphology of the InSAR-derived subsidence basin, available geological information, and PIM-based geometric relationships, and are appropriately broadened to avoid excluding the true solution. If the true value of a parameter falls outside the prescribed search interval, the optimizer cannot recover that value and may instead converge to the search boundary, potentially introducing additional bias into other coupled parameters.

The fitness function is defined using RMSE:

$$F(P_g) = \sqrt{\frac{1}{N} \sum_{i=1}^N (d_i^{obs} - d_i^{pre})^2} \quad (6)$$

where  $d_i^{obs}$  is the LOS deformation observed by InSAR for surface pixel  $i$ ,  $d_i^{pre}$  is the deformation predicted by PIM and projected onto the LOS direction, and  $N$  is the number of valid pixels used in the inversion.

In summary, the inversion model developed in this study is not a black-box optimization that relies solely on error minimization. Instead, it integrates LuTan-1 InSAR LOS deformation observations, the PIM physical model, the geometric characteristics of the subsidence basin, and empirical information from the mining area into a unified framework. By optimizing  $P_g$  within a physically reasonable search space, the model jointly estimates the center location, mining depth, mining thickness, coal seam orientation, and geometric dimensions of a concealed working face.

## 2.2. Adaptive Multi-Strategy Differential Evolution Algorithm

The inversion of working face geometric parameters involves multiple unknowns, including mining depth, mining thickness, coal seam dip angle, working face center coordinates, strike azimuth, and working face dimensions. This problem is highly nonlinear and involves complex parameter coupling and a large search space [29]. Conventional local optimization methods are sensitive to initial values. Traditional intelligent optimization algorithms may also suffer from premature convergence, limited global search capability, and unstable solutions [30]. Therefore, this study develops an Adaptive Multi-Strategy Differential Evolution (AMSDE) algorithm based on Differential Evolution (DE). The algorithm incorporates Latin hypercube sampling for population initialization, iteration-based adaptive parameter adjustment, multiple mutation strategies, reflective boundary handling, and local refinement. AMSDE is used to solve the geometric parameter inversion problem for concealed working face.

DE is a population-based global optimization method that consists of four basic operations: initialization, mutation, crossover, and selection [31]. Let  $D$  denote the number of parameters to be inverted and  $NP$  the population size, Individual  $i$  at generation  $t$  is represented as:

$$X_i^t = (x_{i,1}^t, x_{i,2}^t, \dots, x_{i,D}^t) \quad (7)$$

where  $i = 1, 2, \dots, NP$ . For the working face geometric parameter inversion considered in this study,  $D = 8$ . The eight parameters are mining thickness  $m$ , coal seam dip angle  $\alpha$ , strike azimuth  $\varphi$ , center coordinates  $(X_0, Y_0)$ , average mining depth  $H_0$ , strike length  $D_3$  and dip length  $D_1$ . Each individual represents a candidate combination of working face geometric parameters, and its quality is evaluated using the fitness function  $F(P_g)$ .

### 2.2.1. Latin Hypercube Sampling Initialization

Standard DE typically generates the initial population through random sampling. However, purely random initialization may result in an uneven distribution of individuals across the parameter space. This can reduce the search efficiency and solution stability during the early iterations. To improve the coverage of the search space, Latin Hypercube Sampling (LHS) is used to generate the initial population. Let  $x_{j,\min}$  and  $x_{j,\max}$  denote the lower and upper bounds of parameter  $j$ . The initial value of parameter  $j$  for individual  $i$  is expressed as:

$$x_{i,j}^0 = x_{j,\min} + r_{i,j}(x_{j,\max} - x_{j,\min}) \quad (8)$$

where  $r_{i,j}$  is the sample generated by LHS within the interval  $[0, 1]$ . Compared with conventional random initialization, LHS provides a more uniform distribution along each parameter dimension. This improves population diversity and reduces the influence of random clustering in the initial population on the inversion results.

### 2.2.2. Multi-Strategy Mutation

Mutation is a key operation in the global search process of DE. Different mutation strategies have distinct search characteristics. Random mutation helps maintain population diversity. Best-guided mutation accelerates convergence. Moving the current individual toward the best individual provides a balance between global exploration and local exploitation. Because the inversion of working face geometric parameters requires different search capabilities at different iteration stages, this study adopts a multi-strategy mutation mechanism based on the iteration stage.

DE/rand/1, DE/current-to-best/1, and DE/best/1 are commonly used mutation strategies in DE [32]. Here, “rand,” “current-to-best,” and “best” indicate how the base vector or search direction is selected, while “1” denotes the use of one difference vector. DE/rand/1 uses a randomly selected individual as the base vector and therefore maintains strong global exploration capability. DE/best/1 uses the current best individual as the base vector and improves local convergence speed. DE/current-to-best/1 guides the current individual toward the best individual while introducing a random differential perturbation, thereby balancing exploration and exploitation.

Let the current iteration count be  $g$ , the maximum number of iterations be  $G_{\max}$ , and the normalized iteration progress be  $p = g/G_{\max}$ .

During the early iterations, when  $p < 0.4$ , the algorithm enters the global exploration phase, during which it should expand its search space as much as possible to avoid prematurely converging to a local optimum. Therefore, the DE/rand/1 mutation strategy is adopted:

$$V_i^t = X_{r_1}^t + F_t(X_{r_2}^t - X_{r_3}^t) \quad (9)$$

During the middle iterations, when  $0.4 \leq p < 0.7$ , the algorithm enters a transition phase between exploration and development; during this phase, it must gradually converge toward the current optimal region while maintaining search diversity, striking a balance between optimal individual guidance and random perturbation. Therefore, the DE/current-to-best/1 mutation strategy is adopted:

$$V_i^t = X_{i_1}^t + F_t(X_{best}^t - X_{r_2}^t) + F_t(X_{r_1}^t - X_{r_2}^t) \quad (10)$$

During the late iterations, when  $p \geq 0.7$ , the algorithm enters the local development phase, at which point it has essentially identified a relatively optimal search region; at this stage, the DE/best/1 mutation strategy is employed to accelerate convergence toward the

optimal solution. Therefore, the DE/best/1 mutation strategy is adopted to strengthen local exploitation and accelerate convergence:

$$V_i^t = X_{best}^t + F_t(X_{r_1}^t - X_{r_2}^t) \quad (11)$$

where  $V_i^t$  is the mutant vector generated for individual  $i$ ,  $X_{r_1}^t$ ,  $X_{r_2}^t$  and  $X_{r_3}^t$  are distinct individuals randomly selected from the population,  $X_{best}^t$  is the best individual in the current population, and  $F_t$  is the scaling factor at generation  $t$ .

The mutation strategy is selected according to the iteration stage. Random mutation is used in the early stage to enhance global exploration. In the middle stage, the current-to-best strategy is adopted to balance exploration and exploitation. In the late stage, the best-guided strategy is used to improve local convergence efficiency. This stage-based strategy switching enables the algorithm to better handle the high-dimensional and strongly coupled inversion of working face geometric parameters.

### 2.2.3. Crossover Operation

The crossover operation combines the target and mutant individuals to generate a trial individual. This study adopts binomial crossover:

$$u_{i,j}^t = \begin{cases} v_{i,j}^t, \text{rand}_j \leq CR_t \text{ or } j = j_{\text{rand}} \\ x_{i,j}^t, \text{rand}_j > CR_t \text{ and } j \neq j_{\text{rand}} \end{cases} \quad (12)$$

where  $u_{i,j}^t$  is parameter  $j$  of the trial individual,  $v_{i,j}^t$  is parameter  $j$  of the mutant individual;  $x_{i,j}^t$  is parameter  $j$  of the target individual,  $CR_t$  is the crossover probability at generation  $t$ ,  $\text{rand}_j$  is a random number in  $[0, 1]$ , and  $j_{\text{rand}}$  is a randomly selected dimension index that ensures that at least one parameter of the trial individual is inherited from the mutant individual.

The crossover operation introduces new search directions from the mutant individual while retaining part of the information from the target individual. This helps maintain population diversity and improve search efficiency.

### 2.2.4. Selection Operation

The selection operation follows a greedy criterion. The fitness values of the target and trial individuals are compared, and the individual with the better fitness value is retained for the next generation:

$$X_i^{t+1} = \begin{cases} U_i^t, F(U_i^t) \leq F(X_i^t) \\ X_i^t, F(U_i^t) > F(X_i^t) \end{cases} \quad (13)$$

where  $U_i^t$  is the trial individual generated by crossover, and  $F(\bullet)$  is the fitness function. This selection mechanism retains better parameter combinations throughout the iterations. As a result, the difference between the predicted and observed LOS deformation gradually decreases.

### 2.2.5. Adaptive Parameter Adjustment

In standard DE, the scaling factor  $F$  and crossover probability  $CR$  are usually fixed. However, fixed parameter values cannot fully satisfy the different requirements for global exploration in the early iterations and local exploitation in the late iterations. Therefore,  $F$  and  $CR$  are dynamically adjusted according to the iteration progress.

The scaling factor  $F_t$  gradually decreases with the number of iterations:

$$F_t = F_{\max} - (F_{\max} - F_{\min}) \frac{t}{T} \quad (14)$$

The crossover probability  $CR_t$  gradually increases with the number of iterations:

$$CR_t = CR_{\min} + (CR_{\max} - CR_{\min}) \frac{t}{T} \quad (15)$$

where  $F_{\max}$  and  $F_{\min}$  are the maximum and minimum scaling factors,  $CR_{\max}$  and  $CR_{\min}$  are the maximum and minimum crossover probabilities,  $t$  is the current iteration number; and  $T$  is the maximum number of iterations.

During the early iterations, a larger  $F_t$  increases the perturbation range of individuals and enhances global exploration. A smaller  $CR_t$  helps preserve the stability of the target individuals and prevents premature convergence of the search direction. As the iterations proceed,  $F_t$  gradually decreases and  $CR_t$  gradually increases. The algorithm therefore shifts from broad global exploration to local exploitation and fine adjustment. The term “adaptive” in this study has two main aspects. First, the scaling factor and crossover probability change dynamically with the iteration progress. Second, the mutation strategy switches automatically according to the iteration stage. These mechanisms improve search stability and convergence capability in complex parameter spaces.

#### 2.2.6. Boundary Handling and Local Refinement

In working face geometric parameter inversion, each parameter has a clear physical meaning and a valid range. For example, mining depth, mining thickness, and working face dimensions must satisfy the geological and mining conditions of the study area. The center coordinates and strike azimuth must also remain within reasonable spatial ranges. Therefore, any new individual generated by mutation or crossover must be corrected if it exceeds the parameter bounds.

Conventional boundary clipping directly sets an out-of-bounds parameter to its lower or upper bound. This may cause individuals to cluster near the boundaries and reduce population diversity. This study therefore adopts a reflective boundary handling strategy:

$$x_{i,j}^t = \begin{cases} x_{j,\min} + (x_{j,\min} - x_{i,j}^t), & x_{i,j}^t < x_{j,\min} \\ x_{j,\max} - (x_{i,j}^t - x_{j,\max}), & x_{i,j}^t > x_{j,\max} \\ x_{i,j}^t, & x_{j,\min} \leq x_{i,j}^t \leq x_{j,\max} \end{cases} \quad (16)$$

If a reflected parameter still falls outside the valid range, it is restricted to the corresponding boundary. This strategy preserves the physical validity of the parameters while reducing the search bias caused by direct clipping.

To further improve the accuracy of the final solution, local refinement is performed around the current best individual after the global search. Small perturbations are applied to each parameter around the current best solution, and the fitness value is recalculated. If the perturbed parameter combination produces a lower fitness value, the current best solution is updated. This procedure further corrects the approximate optimum obtained from the global search and improves the accuracy and stability of the final inversion results.

The main procedure of AMSDE is summarized as follows. First, LHS is used to generate the initial population within the predefined parameter ranges. The LOS deformation predicted by PIM and the corresponding fitness value are then calculated for each individual. During the iterations, the mutation strategy is selected according to the current iteration stage, while  $F_t$  and  $CR_t$  are updated simultaneously. Crossover, boundary handling, and selection are then performed to update the current best individual. When the maximum number of iterations is reached or another termination criterion is satisfied, local refinement is conducted around the best solution. The optimal working face geometric parameters are then obtained. AMSDE preserves global search capability while improving

convergence stability in the late iterations. It is therefore suitable for the high-dimensional, nonlinear, and strongly coupled inversion of concealed working face parameters considered in this study.

### 2.3. Parameter Inversion Procedure and Search Space Construction

The inversion of concealed working face geometric parameters aims to determine  $P_g$  from the LOS deformation field observed by LuTan-1 InSAR and the known PIM prediction parameters  $P_{PIM}$ . This is achieved by minimizing the difference between the predicted and observed LOS deformation. The parameters to be inverted include mining thickness, coal seam dip angle, strike azimuth, center coordinates, mining depth, strike length, and dip length. These parameters are coupled to varying degrees. A direct unconstrained global search would therefore be computationally inefficient and may produce ill-posed solutions that are inconsistent with mining subsidence behavior. To address this issue, LuTan-1 InSAR deformation observations, the geometric characteristics of the subsidence basin, empirical information from the mining area, and the PIM physical model are jointly incorporated into the inversion. A physically reasonable parameter search space is then constructed to improve the stability and interpretability of the inversion results.

#### 2.3.1. Principles for Constructing the Parameter Search Space

##### 1. Determination of mining depth $H_0$

The mining depth can be estimated from the relationship between the main influence radius  $r$  and the tangent of the main influence angle  $\tan\beta$ . According to their definitions:

$$H_0 = \tan\beta \cdot r \quad (17)$$

where  $\tan\beta$  is a PIM prediction parameter that can be obtained from empirical data for the mining area. Along the principal section of the measured subsidence basin, the horizontal distance from the point with a subsidence value of  $0.16W_0$  to the point with a subsidence value of  $0.5W_0$  is  $0.4r$ . The same relationship applies between the points corresponding to  $0.5W_0$  and  $0.84W_0$ . Therefore,  $r$  can be determined from the measured subsidence profile. To estimate the main influence radius, the LOS deformation along the principal section was approximately converted to the vertical component by dividing by the cosine of the incidence angle. This approximation was used only to construct a broad mining-depth search range, while the original LOS deformation was retained for the final inversion.

##### 2. Determination of strike length $D_3$ and dip length $D_1$

The strike and dip lengths of the working face are mainly estimated from the geometric relationships at the boundaries of the principal sections of the subsidence basin. Along the strike principal section, inclined lines are drawn from the left and right basin boundary points at the strike boundary angle relative to the horizontal ground surface. The distance between their intersections with the coal seam line can be approximated as the strike length  $D_3$ . Similarly, the dip length  $D_1$  can be estimated along the dip principal section using the up-dip and down-dip boundary angles. These boundary angles can be determined from empirical values for adjacent or geologically similar working faces in the mining area.

##### 3. Determination of working face center coordinates $(X_0, Y_0)$

The working face center coordinates can be estimated from the planar distribution of the subsidence basin. The basin can generally be approximated as an elliptical or elongated zone. Its geometric center is used as the initial estimate of the working face center. A buffer range is then defined around this location by considering the offset of the subsidence center. This range is used to construct the search intervals for  $(X_0, Y_0)$ .

#### 4. Determination of mining thickness $m$ and coal seam dip angle $\alpha$

Mining thickness and coal seam dip angle generally show strong spatial continuity within the same mining district. They can therefore be estimated from adjacent working faces or available geological records. When direct measurements are unavailable, their search ranges can be defined using geological exploration data and the statistical ranges of nearby working faces.

#### 5. Determination of working face strike azimuth $\varphi$

The strike azimuth can be estimated from the major-axis direction of the subsidence basin, the orientation of the dip principal section, and available mine layout records. The angle between the dip principal section and the north direction can generally be used to derive an initial estimate of the strike azimuth. A reasonable search range is then defined around this estimate.

In summary, reasonable search intervals for  $P_g$  can be established by separately constraining the mining depth, working face dimensions, center coordinates, and coal seam orientation parameters. The proposed inversion model is then used to obtain more accurate geometric parameters of the underground working face.

#### 2.3.2. Parameter Inversion Procedure

After defining the parameter search space, an inversion procedure is established for the geometric parameters of concealed working face. The procedure includes data preparation, extraction of subsidence basin geometry, construction of the parameter search space, objective function evaluation, AMSDE optimization, and back-substitution validation. The workflow is shown in Figure 2.

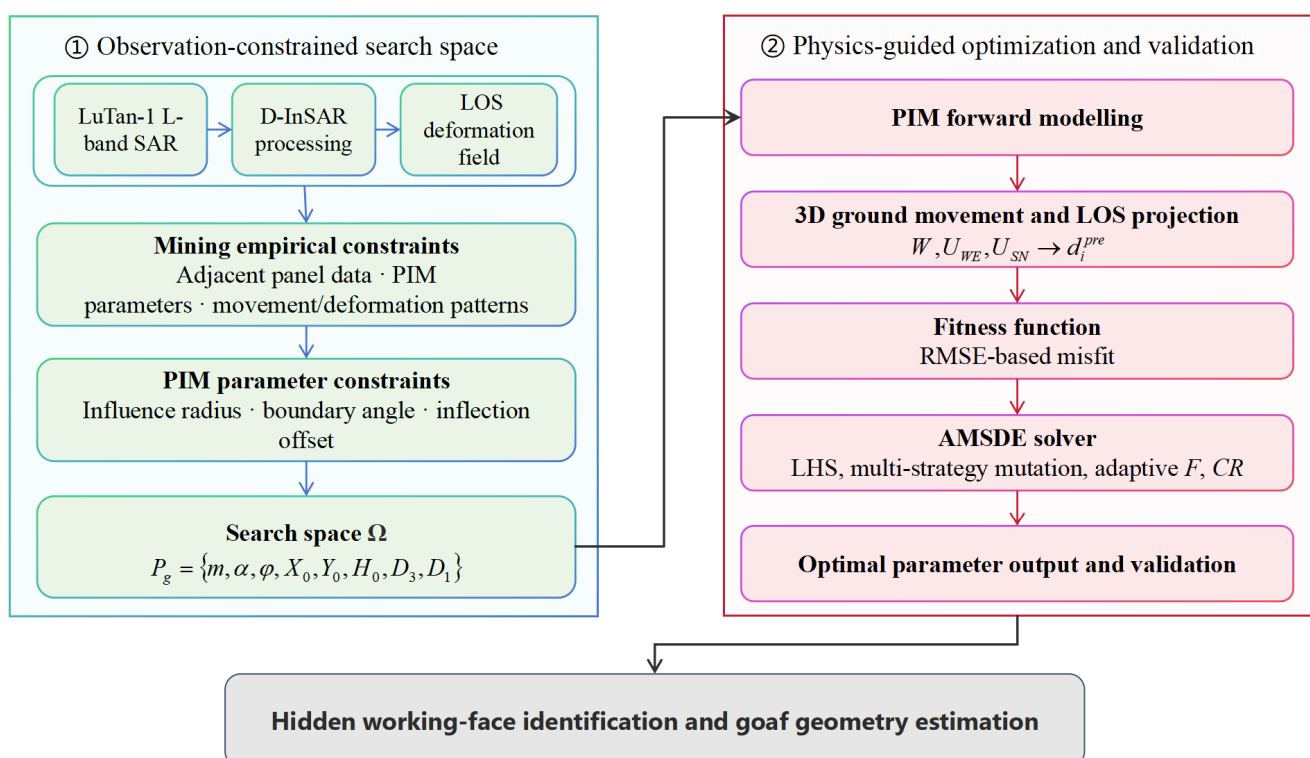


**Figure 2.** Workflow for concealed working face parameter inversion using LuTan-1 InSAR and geometric–physical constraints.

### 1. Data preparation and acquisition of prior information

LuTan-1 SAR images are acquired for the study area, and the surface LOS deformation field is obtained through Differential InSAR (D-InSAR) processing. The D-InSAR workflow includes image co-registration, interferogram generation, removal of the flat-earth and topographic phases, interferogram filtering, phase unwrapping, orbital refinement and re-flattening, phase-to-deformation conversion, and geocoding. The coordinates and LOS deformation values of valid pixels are then extracted. The coordinates and LOS deformation values of valid pixels are then extracted. Available geological and mining records for the study area are also collected. The PIM prediction parameters  $P_{PIM}$  can be determined from field measurements of adjacent working faces, empirical values for the mining area, or existing mining subsidence data.

### 2. Subsidence basin identification and geometric feature extraction

The mining-induced subsidence basin is identified from the LuTan-1 InSAR LOS deformation field. Its center, boundary, major-axis direction, and principal section locations are then extracted. These geometric features are used to constrain the search ranges of the working face center coordinates, strike azimuth, strike length, and dip length.

### 3. Construction of the parameter search space

Reasonable lower and upper bounds are assigned to the parameters in  $P_g$  using the morphology of the subsidence basin, empirical information from the mining area, and the physical relationships in PIM. This step is essential for reducing parameter non-uniqueness and avoiding physically meaningless solutions. It also distinguishes the proposed method from a purely black-box inversion approach.

### 4. Objective function evaluation and parameter optimization

For a given set of working face geometric parameters, PIM is used to calculate the corresponding three-dimensional surface displacement components. These components are projected onto the radar LOS to obtain the predicted LOS deformation field. The fitness function is then calculated from the difference between the predicted LOS deformation and the LuTan-1 InSAR observations. AMSDE is applied to iteratively search for the optimal parameter combination.

### 5. Result output and back-substitution validation

The working face geometric parameters that minimize the fitness function are selected as the inversion results. These parameters are then substituted back into PIM to generate the predicted deformation field and evaluate the inversion accuracy.

## 3. Simulation Study

### 3.1. Design of the Simulation Experiment

A simulation experiment was first conducted to verify the feasibility of the proposed geometric parameter inversion model for concealed working face constrained by LuTan-1 InSAR. The experiment also evaluated the stability of AMSDE in solving high-dimensional and strongly coupled parameter inversion problems. A regular rectangular working face was used in the simulation. Given the predefined PIM prediction parameters and working face geometric parameters, PIM was used to generate the three-dimensional surface displacement components. These components were then projected onto the radar LOS to construct a simulated LOS deformation field. The simulated field was used as the observation input for the inversion model. Different optimization algorithms were then applied to invert the working face geometric parameters. The inversion results were compared with the predefined true values to evaluate the accuracy and stability of the

proposed method. All simulations and parameter inversions were implemented in Python 3.7 (Python Software Foundation, Wilmington, DE, USA).

Based on the geological and mining conditions of a typical coal mining working face, the underground working face was assumed to have a regular rectangular geometry. The PIM prediction parameters  $P_{PIM}$  include the subsidence coefficient  $q$ , tangent of the main influence angle  $\tan\beta$ , horizontal displacement coefficient  $b$ , mining influence propagation angle  $\theta$  and inflection point offsets  $S_u$ ,  $S_d$ ,  $S_l$ , and  $S_r$  in the four boundary directions. Their values are listed in Table 1. The working face geometric parameters  $P_g$  include mining thickness  $m$ , coal seam dip angle  $\alpha$ , strike azimuth  $\varphi$ , working face center coordinates  $(X_0, Y_0)$ , average mining depth  $H_0$ , strike length  $D_3$  and dip length  $D_1$ . Their predefined values and search ranges are listed in Table 2. The search ranges were moderately expanded while ensuring that the true values remained within the feasible domain. This setting was used to evaluate the global search capability of the optimization algorithms over a relatively large parameter space.

**Table 1.** PIM prediction parameters.

| Parameters | $q$ | $\tan\beta$ | $b$  | $\theta$ (°) | $S_u$ (m) | $S_d$ (m) | $S_l$ (m) | $S_r$ (m) |
|------------|-----|-------------|------|--------------|-----------|-----------|-----------|-----------|
| Simulated  | 0.8 | 2.2         | 0.25 | 87           | 40        | 40        | 40        | 40        |

**Table 2.** Working face geometric parameters.

| Parameters | $m$ (m) | $\alpha$ (°) | $\varphi$ (°) | $X_0$ (m) | $Y_0$ (m) | $H_0$ (m) | $D_3$ (m) | $D_1$ (m) |
|------------|---------|--------------|---------------|-----------|-----------|-----------|-----------|-----------|
| Simulated  | 3       | 5            | 90            | 200       | 300       | 300       | 600       | 400       |
| Range      | 0.5~6   | 2~8          | 80~100        | 150~250   | 250~350   | 250~350   | 500~700   | 300~500   |

During the simulation, the parameters listed in Tables 1 and 2 were first used in PIM to calculate surface subsidence  $W$ , east–west horizontal displacement  $U_{WE}$  and south–north horizontal displacement  $U_{SN}$ . Then, the three-dimensional displacement components were then projected onto the LOS direction according to the radar imaging geometry to generate the simulated InSAR deformation field.

The radar incidence angle and satellite heading azimuth were set to  $44.5^\circ$ , and  $350.5^\circ$ . To simulate high-spatial-resolution InSAR deformation observations, the study area was discretized using a  $3\text{ m} \times 3\text{ m}$  grid. Each grid point was treated as a valid observation pixel for the inversion. The simulated deformation field is shown in Figure 3.

### 3.2. Inversion Results and Error Analysis

Using the simulated working face data described in Section 3.1, the LOS deformation field generated by PIM and projected onto the radar LOS was treated as the simulated observation. The working face geometric parameters were then inverted within the search ranges listed in Table 2. It should be noted that LOS deformation, rather than vertical subsidence, was used as the observation in the simulation. This treatment is consistent with the actual InSAR observation geometry.

To evaluate the effectiveness and stability of AMSDE in working face geometric parameter inversion, Particle Swarm Optimization (PSO), Genetic Algorithm (GA), Evolution Strategy (ES), and standard DE were selected for comparison. To reduce the randomness associated with a single optimization run, the maximum number of iterations and population size were set to 200 and 50 for all algorithms. Each algorithm was independently run 10 times. The mean results from the 10 runs were used as the final inversion results. The relative error of each parameter, RMSE, and fitness value were also calculated to evaluate

the inversion accuracy and solution stability of the different algorithms. The specific parameters for each algorithm are determined based on commonly recommended values in relevant literature: for the PSO algorithm, the inertia weight is set to 0.7, and the cognitive and social coefficients are both set to 2; for the GA algorithm, the crossover probability is 0.8, the mutation probability is 0.1, and the number of elite individuals is 2; for the ES algorithm, the number of parent and offspring individuals are 15 and 50, respectively; for the DE algorithm, the classic DE/rand/1/bin strategy is adopted, with a scaling factor of 0.5 and a crossover probability of 0.9; for the AMSDE algorithm in this study, the lower and upper limits of the scaling factor are set to 0.4 and 0.9, while the upper and lower limits of the crossover probability are set to 0.7 and 0.95, respectively. The inversion results are presented in Table 3.

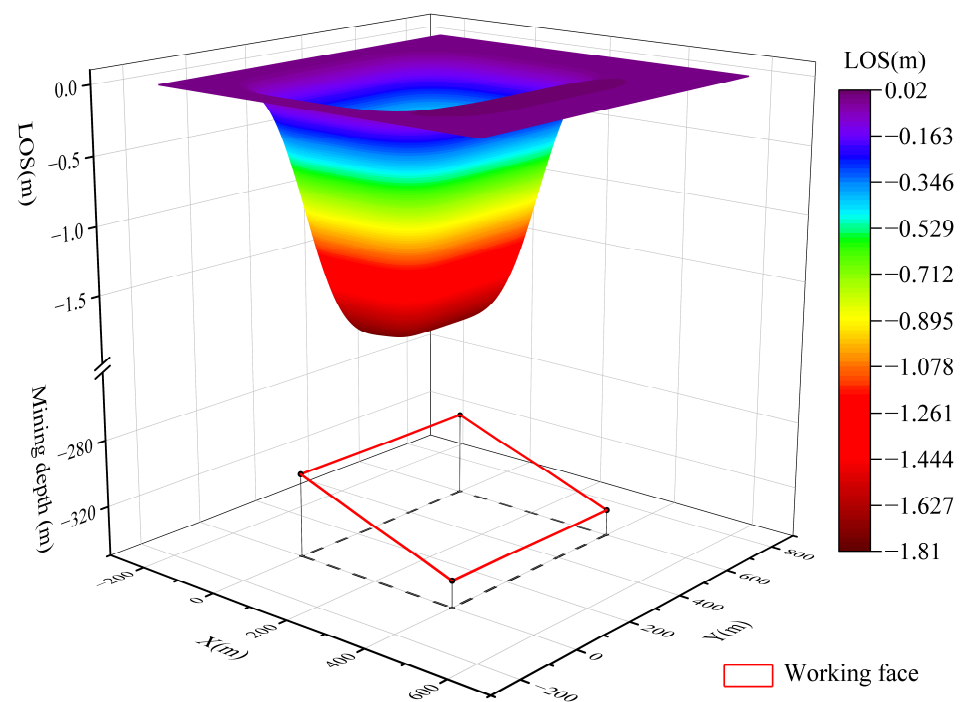


**Figure 3.** Simulated deformation field and location of the working face.

As shown in Table 3, PSO, GA, ES, DE, and AMSDE all recovered working face geometric parameters close to the predefined true values within the specified search ranges. This result confirms the feasibility of inverting mining geometric parameters using the PIM forward model and LOS deformation constraints. Compared with PSO, GA, and ES, standard DE produced smaller inversion errors for most parameters and a lower fitness value. This indicates that DE has strong global search capability for continuous multi-parameter optimization problems. After incorporating LHS initialization, iteration-based adaptive parameter adjustment, multi-strategy mutation, reflective boundary handling, and local refinement, AMSDE produced results that were generally closer to the predefined true values. Its fitness value decreased from 0.466 mm for standard DE to 0.238 mm. These results demonstrate that the proposed improvements effectively enhance parameter search accuracy and convergence stability.

Based on the results of parameter inversion, the AMSDE algorithm achieved high inversion accuracy for  $m$ ,  $H_0$ ,  $X_0$ ,  $Y_0$ ,  $D_3$  and  $D_1$ , with low relative errors for the main parameters. The inverted  $X_0$ ,  $Y_0$ ,  $H_0$ ,  $D_3$  and  $D_1$  were close to their predefined values. This indicates that the LOS deformation field provides strong constraints on the spatial location and geometric extent of the working face. The inversion error of  $\alpha$  was relatively large. This may be because the simulated working face had a small dip angle, and variations in the dip

angle had a limited effect on the LOS deformation field. Therefore, under near-horizontal coal seam conditions, the dip angle is less identifiable than geometric parameters such as the working face dimensions and center location.

Figure 4 compares the RMSE values obtained using different optimization algorithms. Overall, AMSDE produced lower or comparable RMSE values for most parameters, indicating good stability across multiple independent runs. Compared with GA and PSO, AMSDE maintained strong global search capability while improving the quality of the optimal solution through local refinement during the late iterations. This reduced the variability caused by random initialization and convergence to local optima.

**Table 3.** Inversion results of simulated working face geometric parameters using different optimization algorithms.

| Parameters |                  | $m$ (m) | $\alpha$ (°) | $\varphi$ (°) | $X_0$ (m) | $Y_0$ (m) | $H_0$ (m) | $D_3$ (m) | $D_1$ (m) | Fitness (mm) |
|------------|------------------|---------|--------------|---------------|-----------|-----------|-----------|-----------|-----------|--------------|
| Simulated  |                  | 3       | 5            | 90            | 200       | 300       | 300       | 600       | 400       | /            |
| PSO        | Estimated        | 2.999   | 4.662        | 89.701        | 200.248   | 299.907   | 299.872   | 599.928   | 400.100   | 0.648        |
|            | Relative error/% | 0.033   | 6.758        | 0.332         | 0.124     | 0.031     | 0.043     | 0.012     | 0.025     | /            |
|            | RMSE             | 0.004   | 1.295        | 0.950         | 0.756     | 0.297     | 0.409     | 0.228     | 0.303     | /            |
| GA         | Estimated        | 3.002   | 4.814        | 87.950        | 201.591   | 299.602   | 299.882   | 599.654   | 400.447   | 1.709        |
|            | Relative error/% | 0.067   | 3.714        | 2.277         | 0.795     | 0.133     | 0.039     | 0.058     | 0.112     | /            |
|            | RMSE             | 0.016   | 2.224        | 3.771         | 2.568     | 0.596     | 1.416     | 0.912     | 1.062     | /            |
| ES         | Estimated        | 3.001   | 4.286        | 90.401        | 199.805   | 300.011   | 300.069   | 600.016   | 400.041   | 1.304        |
|            | Relative error/% | 0.033   | 14.286       | 0.446         | 0.098     | 0.004     | 0.023     | 0.003     | 0.010     | /            |
|            | RMSE             | 0.003   | 1.557        | 1.641         | 0.927     | 0.063     | 0.252     | 0.070     | 0.138     | /            |
| DE         | Estimated        | 2.998   | 5.201        | 89.959        | 199.910   | 299.991   | 299.978   | 599.992   | 400.027   | 0.466        |
|            | Relative error/% | 0.067   | 4.024        | 0.005         | 0.045     | 0.003     | 0.007     | 0.001     | 0.007     | /            |
|            | RMSE             | 0.006   | 0.532        | 0.457         | 0.554     | 0.068     | 0.093     | 0.050     | 0.238     | /            |
| AMSDE      | Estimated        | 2.999   | 5.029        | 90.001        | 199.990   | 300.002   | 299.999   | 600.002   | 399.979   | 0.238        |
|            | Relative error/% | 0.033   | 0.580        | 0.000         | 0.005     | 0.000     | 0.000     | 0.000     | 0.005     | /            |
|            | RMSE             | 0.003   | 0.468        | 0.064         | 0.109     | 0.040     | 0.058     | 0.026     | 0.219     | /            |

Figure 5 presents violin plots of the parameter estimates obtained from 10 independent inversion runs for the simulated working face using each algorithm. These plots illustrate the distribution and dispersion of the inversion results. For each violin plot, the upper and lower limits represent the maximum and minimum estimates from the 10 runs. The width of the violin indicates the data density at a given value. A wider section represents a higher concentration of estimates. The upper and lower edges of the gray box denote the third quartile Q3 and first quartile Q1. Q3 and Q1 correspond to the 75th and 25th percentiles. The hollow marker inside the gray box indicates the mean value. The lines extending from the box represent the range of non-outlier values, defined from  $Q1 - 1.5 * IQR$  to  $Q3 + 1.5 * IQR$ , where  $IQR = Q3 - Q1$ . The IQR describes the dispersion of the middle 50% of the data. The red line indicates the reference true value of each parameter. As shown in Figure 5, except for  $m$  and  $D_1$ , the violin plots obtained by AMSDE have shorter vertical ranges than those produced by the other algorithms. Their widest sections are also concentrated near the true values. This indicates lower dispersion, smaller parameter fluctuations, and better inversion stability. The improvement is particularly clear for  $\varphi$ , for which all estimates are concentrated near  $90^\circ$  with only minor variation. In addition, the mean estimates obtained by AMSDE are generally closer to the true values. These results demonstrate that AMSDE provides more accurate and robust parameter inversion.

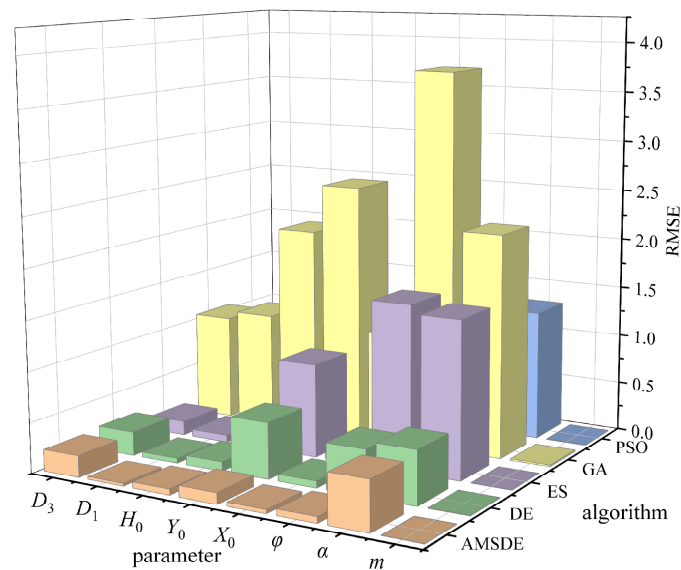


**Figure 4.** Comparison of RMSE values obtained by inverting different optimization algorithm parameters. (Different colors represent different algorithms, while varying heights of the same color indicate the magnitude of the RMSE values for different feature parameters).

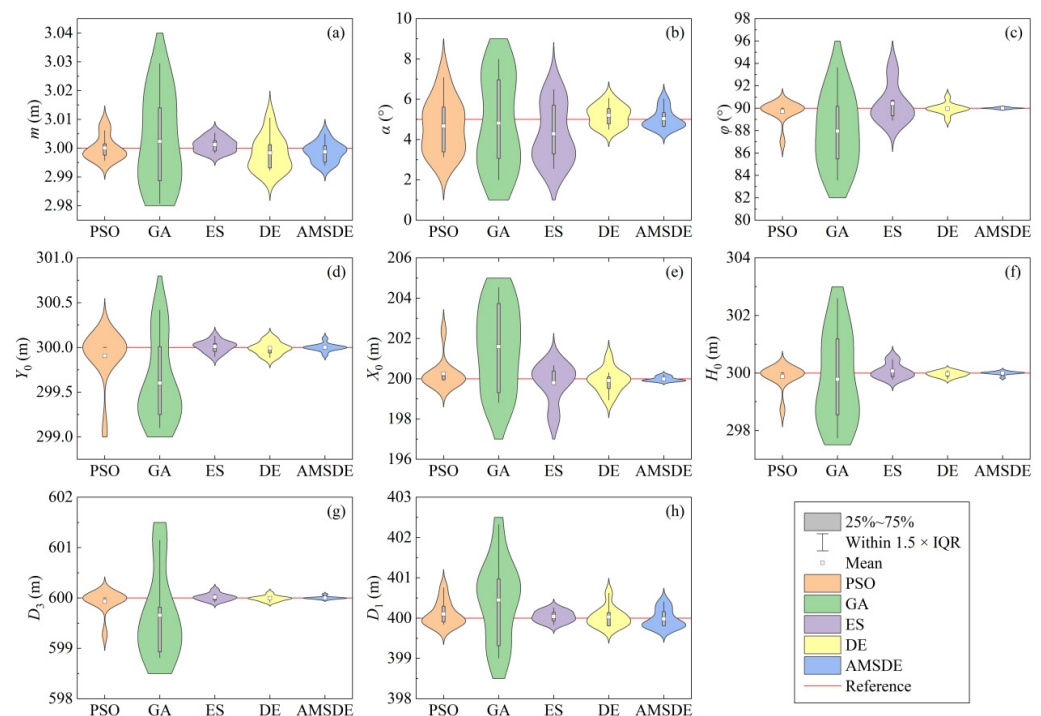


**Figure 5.** Fluctuation ranges of various algorithm parameters for the simulated working face over 10 inversion iterations: (a)  $m$ ; (b)  $\alpha$ ; (c)  $\varphi$ ; (d)  $Y_0$ ; (e)  $X_0$ ; (f)  $H_0$ ; (g)  $D_3$ ; and (h)  $D_1$ . Different colors denote different algorithms; the red line is the reference accurate value. Different colors represent different algorithms, and the red line represents the reference accurate value.

To further evaluate the spatial plausibility of the inverted parameters, the optimal parameters obtained by AMSDE were substituted into PIM to recalculate the predicted LOS deformation field. The result was compared with the simulated LOS deformation field and the predefined working face location, as shown in Figure 6. The AMSDE-predicted LOS deformation field agrees well with the simulated observations in terms of subsidence basin morphology, deformation center, and spatial extent. In addition, the inverted working face boundary nearly overlaps the predefined boundary. This indicates that the

proposed method can accurately recover the spatial location and geometric dimensions of the underground working face.

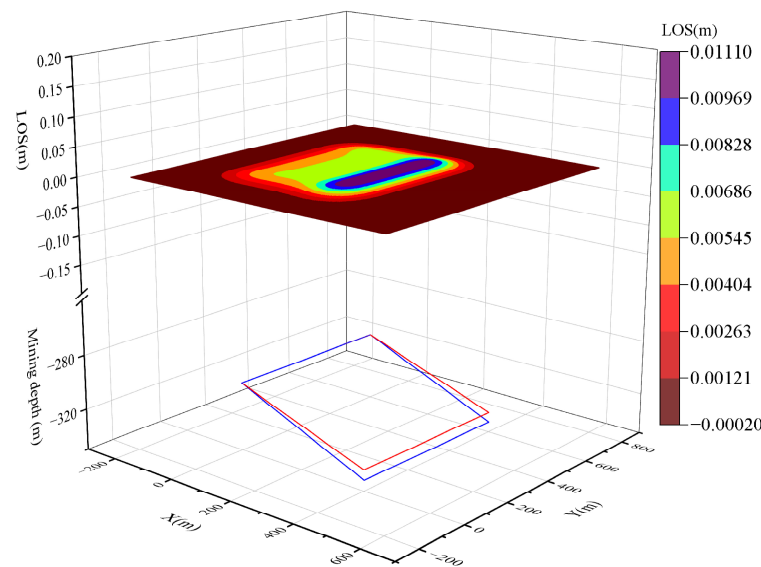


**Figure 6.** Spatial distribution of the LOS residual between the simulated and AMSDE-predicted deformation fields. (The red frame indicates the actual working face, while the blue frame indicates the predicted working face).

In summary, the simulation results show that the PIM-LOS deformation-constrained inversion model effectively represents the relationship between underground working face geometric parameters and the surface LOS deformation field. Compared with PSO, GA, ES, and standard DE, AMSDE provides better overall performance in inversion accuracy, fitness value, and stability across repeated runs. These results demonstrate the feasibility of the proposed method for inverting concealed working face geometric parameters under regular working face conditions. They also provide a methodological basis for the subsequent engineering case study.

## 4. Engineering Case Study

### 4.1. Study Area and Data

#### 4.1.1. Overview of the Study Area

The engineering case study selected a working face in Jincheng City, Shanxi Province as the subject of investigation. The working face is located northwest of Jincheng, and the mine field extends across Qinshui, Yangcheng, and Zezhou. The study area is located in a mountainous mining region with pronounced topographic relief, local bedrock exposure, and complex surface conditions. It is therefore suitable for evaluating the applicability of LuTan-1 L-band InSAR to mining-induced deformation monitoring and concealed working face geometric parameter inversion in mountainous areas. The coal seam has a relatively simple structure and favorable mining conditions. The 97,315 working face has a designed total strike length of 1145 m and a dip length of 220 m. During the LuTan-1 observation period, however, only part of the working face had been extracted. According to the mining records, the cumulative advance distances were 0 m, 134 m, and 284 m on 28 January, 25 February, and 24 March 2024, respectively. Therefore, the geometric parameter  $D_3$  used in the engineering-case inversion represents the active-mining strike extent at the corresponding acquisition time rather than the complete designed strike length of 1145 m. The average coal seam dip angle, mining thickness, and mining depth are  $3^\circ$ , 1.64 m, and 471 m. An overview of the study area is shown in Figure 7.

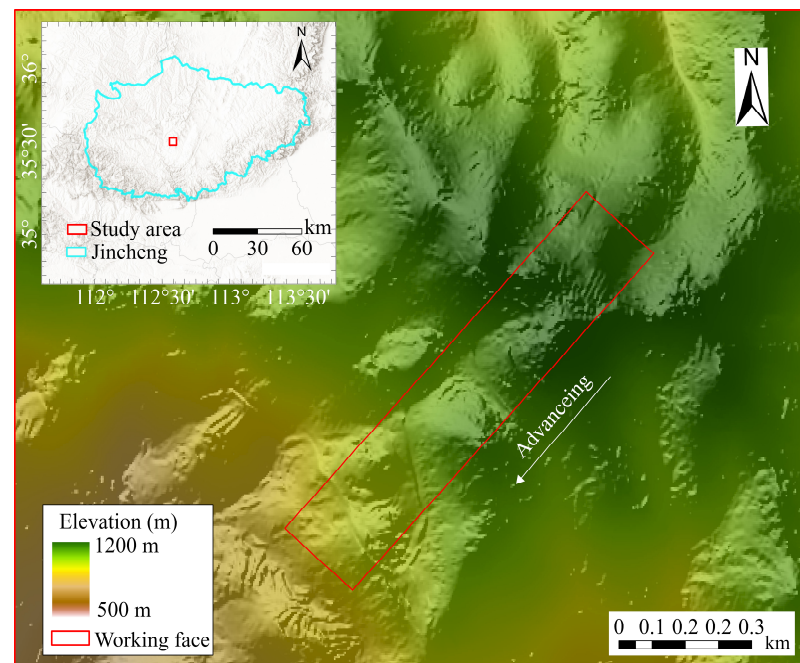


**Figure 7.** Overview of the study area.

#### 4.1.2. LuTan-1 SAR Data and D-InSAR Results

To obtain surface deformation during the extraction of the 97,315 working face, three ascending-track LuTan-1 L-band SAR images covering the Sihe No. 2 mining area were used. The images were acquired on 28 January, 25 February, and 24 March, 2024. Two interferometric pairs were formed from consecutive acquisitions, each with a temporal baseline of 28 days. The spatial resolution of the SAR images was 3 m. The basic information on the datasets is listed in Table 4. Table 5 summarizes the eight geometric parameters of the active-mining area at the three LuTan-1 acquisition dates.

**Table 4.** SAR image data and interferometric combination information of LuTan-1 in the study area.

| Master Image     | Secondary Image  | Time Baseline | Spatial Resolution | Master Image Incidence Angle | Master Image Orientation |
|------------------|------------------|---------------|--------------------|------------------------------|--------------------------|
| 28 January 2024  | 25 February 2024 | 28 d          | 3 m                | 30.02°                       | 347.37°                  |
| 25 February 2024 | 24 March 2024    | 28 d          | 3 m                | 30.20°                       | 347.37°                  |

**Table 5.** Geometric parameters of the active-mining area at the three LuTan-1 acquisition dates

| Parameters       | $m$ (m) | $\alpha$ (°) | $\varphi$ (°) | $X_0$ (m) | $Y_0$ (m) | $H_0$ (m) | $D_3$ (m) | $D_1$ (m) |
|------------------|---------|--------------|---------------|-----------|-----------|-----------|-----------|-----------|
| 28 January 2024  | 1.64    | 3            | 138           | /         | /         | 471       | 0         | 220       |
| 25 February 2024 | 1.64    | 3            | 138           | *5831     | *5019     | 471       | 134       | 220       |
| 24 March 2024    | 1.64    | 3            | 138           | *5782     | *4963     | 471       | 284       | 220       |

Note: The asterisk (\*) indicates that the leading digits of the coordinates have been masked to protect sensitive mine-location information; this treatment does not affect the relative geometric relationships.

Following the D-InSAR processing workflow described in Section 2.3.2. The LOS deformation fields for 28 January to 25 February 2024, and 25 February to 24 March 2024, were obtained, as shown in Figure 8a,b. These two deformation fields were then accumulated to derive the LOS deformation field from 28 January to 24 March 2024 (Figure 8c). A total of 29 sample points were selected along the strike and dip directions of the working face. Their LOS deformation values were used to invert the working face geometric parameters. The results are shown in Figure 8c.

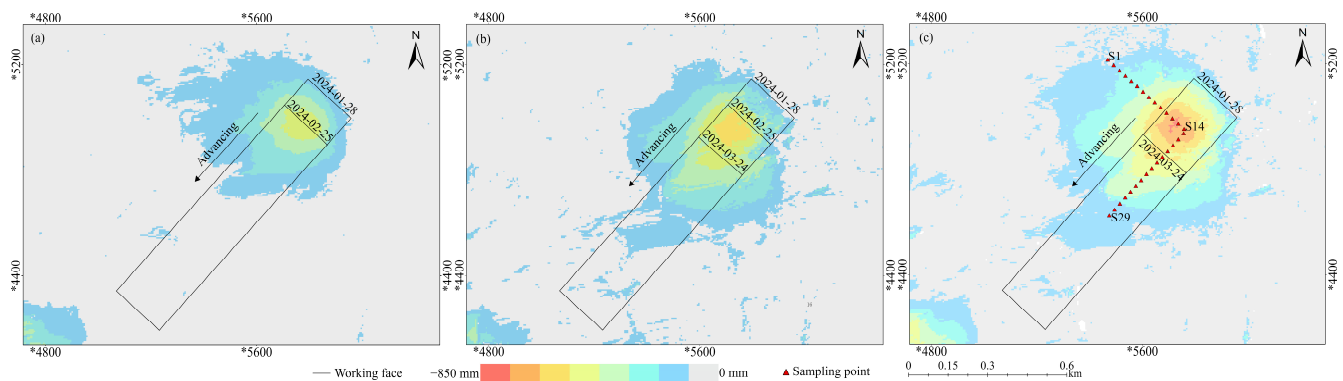


**Figure 8.** Spatial distribution of LuTan-1 InSAR-derived LOS deformation during different observation periods: (a) LOS deformation from 28 January to 25 February 2024; (b) LOS deformation from 25 February to 24 March 2024; and (c) cumulative LOS deformation from 28 January to 24 March 2024. The black outlines indicate the working-face geometry, and the red triangles in (c) represent the 29 sampling points used for geometric parameter inversion. The map coordinates are expressed in meters in the Gauss–Krüger projected coordinate system, with their leading digits masked to protect sensitive mine-location information. The asterisk (\*) indicates that the leading digits of the coordinates have been masked to protect sensitive mine-location information; this treatment does not affect the relative geometric relationships.

#### 4.2. Experimental Results and Analysis

Before estimating the geometric parameters of the underground working face, the PIM parameters for the study area must be determined. This study primarily utilizes measured data from mountainous working faces in the Sihe Mining Area—where geological and mining conditions are comparable to those in Table 6—as its foundational basis. In conjunction with the actual mining geological and operational conditions of the Sihe Mining Area, we comprehensively determine the dynamic probability integral estimation parameters for the 97,315 working face. Among these parameters, the subsidence coefficient  $q$ , the tangent of the primary influencing angle  $\tan\beta$ , and the offset values at inflection points in four directions  $S_u$ ,  $S_d$ ,  $S_l$  and  $S_r$  are primarily derived from measured subsidence basin data obtained at surface deformation observation stations of adjacent similar working faces; these parameters were determined through fitting and inversion, statistical analogy, and boundary morphology analysis. The horizontal displacement coefficient  $b$  and the mining influence propagation angle  $\theta$ , on the other hand, were selected primarily based on the coal seam dip angle, overlying stratum lithology, topographic conditions, and local mining experience in the Sihe Mining Area, with subsequent verification conducted using measured data from neighboring working faces. The PIM prediction parameters for 97,315 working face were determined as follows:  $q = 0.77$ ,  $b = 0.3$ ,  $\tan\beta = 1.78$ ,  $\theta = 88^\circ$ ,  $S_u$ ,  $S_d$ ,  $S_l$  and  $S_r$  were 32.64 m,  $-6.83$  m, 11.52 m,  $-63.75$  m, respectively.

The approximate search ranges of the working-face geometric parameters were determined using information from adjacent or geologically similar working faces, together with the constraint principles described in Section 2.3.1. Mining thickness and coal-seam dip angle generally show strong spatial continuity within the same mining district. Therefore, based on mining information from working faces adjacent to the 97,315 working face in the Sihe mining area, the mining thickness was constrained to 1–3 m and the coal-seam dip angle to  $1\text{--}6^\circ$ . The mining depth was estimated from the relationship between the main influence radius and the tangent of the main influence angle. Based on the determined PIM parameters and the geometric characteristics of the observed subsidence basin, the search range of mining depth was set to 400–600 m. The strike length and dip length were constrained according to the boundary geometry of the principal sections of the InSAR-derived subsidence basin and the PIM-based geometric relationships described in Section 2.3.1.

Accordingly, their search ranges were set to 200–400 m and 150–300 m, respectively. The geometric center of the InSAR-derived subsidence basin was used only to provide an initial estimate of the working-face center coordinates. Based on the center coordinates shown in Figure 8c, the search ranges of  $X_0$  and  $Y_0$  were set to \*5600–\*5900 m and \*4800–\*5100 m, respectively. The initial strike azimuth was estimated from the orientation of the principal section of the subsidence basin relative to north, and its search range was set to 100–150°. These values represent physically admissible initial search intervals rather than statistically exact quantities, and their uncertainty is represented by the corresponding ranges listed in Table 7.

**Table 6.** Parameter information of working faces in mountainous areas of China.

| Mining Area | $q$  | $\tan\beta$ | $b$  | $\theta$ (°) | $S_u$ (m) | $S_d$ (m) | $S_l$ (m) | $S_r$ (m) | $m$ (m) | $\alpha$ (°) | $H_0$ (m) | $D_3$ (m) | $D_1$ (m) |
|-------------|------|-------------|------|--------------|-----------|-----------|-----------|-----------|---------|--------------|-----------|-----------|-----------|
| 1           | 0.67 | 2.50        | 0.26 | 87.00        | −18.20    | /         | /         | /         | 2.20    | 8.00         | 120       | 198       | 90        |
| 2           | 0.87 | 2.30        | 0.23 | 87.00        | −11.70    | /         | /         | /         | 1.80    | 9.00         | 110       | 341       | 70        |
| 3           | 0.65 | 2.20        | 0.30 | 87.50        | 17.00     | 47.00     | 4.00      | 20.00     | 2.00    | 6.00         | 210       | 340       | 118       |
| 4           | 0.78 | 2.34        | 0.37 | 88.00        | 21.00     | 25.00     | 15.00     | /         | 1.85    | 4.00         | 130       | 540       | 136       |
| 5           | 0.72 | 1.82        | 0.30 | 89.00        | −33.60    | /         | /         | −26.40    | 1.85    | 2.50         | 115       | 350       | 180       |
| 6           | 0.69 | 1.64        | 0.34 | 86.00        | −12.85    | /         | −8.30     | /         | 1.90    | 4.50         | 106       | 419       | 105       |
| 7           | 0.64 | 1.60        | 0.30 | 85.20        | 36.00     | 36.00     | 36.00     | 36.00     | 1.55    | 10.00        | 450       | 710       | 135       |
| 8           | 0.82 | 2.19        | 0.35 | 81.00        | 20.00     | /         | 15.00     | /         | 1.40    | 9.00         | 105       | 380       | 134       |
| 9           | 0.83 | 2.30        | /    | /            | 29.00     | /         | /         | /         | 1.80    | 5.00         | 100       | 400       | 145       |
| 10          | 0.80 | /           | 0.34 | 88.70        | /         | /         | /         | /         | 1.80    | 6.00         | 589       | 1800      | 220       |
| 11          | 0.73 | 2.50        | 0.36 | 88.00        | 24.00     | 24.00     | 36.00     | 26.00     | 1.93    | 8.00         | 203       | 575       | 105       |
| 12          | 0.60 | 2.16        | 0.15 | 83.00        | −11.00    | −29.00    | /         | −36.00    | 1.70    | 6.00         | 87        | 380       | 86        |
| 13          | 0.80 | 2.00        | 0.35 | 88.00        | 20.00     | 20.00     | 20.00     | 20.00     | 2.80    | 3.00         | 760       | 1260      | 200       |
| 14          | 0.62 | 1.68        | 0.29 | 84.00        | −8.40     | 9.90      | −4.60     | /         | 1.60    | 7.00         | 45        | 300       | 160       |
| 15          | 1.09 | 2.50        | 0.39 | 85.00        | 9.50      | 1.60      | 3.80      | /         | 1.40    | 6.00         | 66        | 247       | 55        |
| 16          | 0.82 | 1.85        | 0.30 | /            | 26.00     | 26.00     | 26.00     | 26.00     | 1.70    | 8.00         | 266       | 475       | 170       |
| 17          | 0.71 | 2.30        | 0.31 | /            | 1.20      | 1.20      | 1.20      | 1.20      | 1.65    | 3.50         | 167       | 1320      | /         |

**Table 7.** Parameter range for the inversion working face.

| Parameters   | $m$ (m) | $\alpha$ (°) | $\varphi$ (°) | $X_0$ (m)   | $Y_0$ (m)   | $H_0$ (m) | $D_3$ (m) | $D_1$ (m) |
|--------------|---------|--------------|---------------|-------------|-------------|-----------|-----------|-----------|
| Actual value | 1.64    | 3            | 138           | *5782       | *4963       | 471       | 284       | 220       |
| Range        | 1~3     | 1~6          | 100~150       | *5600~*5900 | *4800~*5100 | 400~600   | 200~400   | 150~300   |

Note: The asterisk (\*) indicates that the leading digits of the coordinates have been masked to protect sensitive mine-location information; this treatment does not affect the relative geometric relationships.

The LOS deformation observations derived from LuTan-1 data were used to invert the working face geometric parameters with AMSDE. Its performance and accuracy were compared with those of PSO, GA, ES, and standard DE. To ensure a fair comparison, all algorithms were run for 200 iterations with a population size of 50. Each algorithm was independently run 10 times, and the mean parameter estimates were taken as the final results. The inversion results are listed in Table 8.

As shown in Table 8, AMSDE produced the smallest absolute errors for all parameters, and its estimates were closest to the reference values. It also achieved lower parameter RMSE values than the other algorithms. The relative errors were within 7.9%, and the RMSE of the deformation fitting was 32.53 mm, indicating good agreement between the predicted and observed deformation.

**Table 8.** Results of feature parameter inversion.

| Parameters   |                  | $m$ (m) | $\alpha$ (°) | $\varphi$ (°) | $X_0$ (m) | $Y_0$ (m) | $H_0$ (m) | $D_3$ (m) | $D_1$ (m) | Fitness (mm) |
|--------------|------------------|---------|--------------|---------------|-----------|-----------|-----------|-----------|-----------|--------------|
| Actual value |                  | 1.64    | 3            | 138           | *5782     | *4963     | 471       | 284       | 220       | /            |
| PSO          | Estimated        | 2.190   | 4.005        | 142.517       | *5770.826 | *4970.501 | 499.157   | 319.013   | 170       | 36.195       |
|              | Relative error/% | 33.537  | 33.500       | 3.273         | /         | /         | 5.979     | 12.329    | 22.727    | /            |
|              | RMSE             | 0.551   | 1.199        | 5.138         | 11.207    | 9.549     | 28.535    | 35.228    | 50.000    | /            |
| GA           | Estimated        | 1.812   | 3.972        | 140.933       | *5771.858 | *4969.621 | 488.329   | 302.635   | 208.693   | 35.684       |
|              | Relative error/% | 10.488  | 32.4         | 2.125         | /         | /         | 3.679     | 6.562     | 5.140     | /            |
|              | RMSE             | 0.229   | 1.596        | 3.950         | 10.299    | 7.478     | 21.968    | 26.174    | 19.539    | /            |
| ES           | Estimated        | 1.792   | 3.722        | 139.657       | *5770.757 | *4970.705 | 482.997   | 302.207   | 204.118   | 35.190       |
|              | Relative error/% | 9.268   | 24.067       | 1.201         | /         | /         | 2.547     | 6.411     | 7.219     | /            |
|              | RMSE             | 0.157   | 1.011        | 2.607         | 11.294    | 8.505     | 12.399    | 20.256    | 16.509    | /            |
| DE           | Estimated        | 1.821   | 3.413        | 138.676       | *5770.783 | *4970.324 | 481.089   | 302.146   | 211.219   | 34.509       |
|              | Relative error/% | 11.037  | 13.767       | 0.490         | /         | /         | 2.142     | 6.389     | 3.991     | /            |
|              | RMSE             | 0.295   | 0.794        | 2.604         | 11.263    | 7.790     | 11.921    | 18.475    | 14.470    | /            |
| AMSDE        | Estimated        | 1.727   | 3.236        | 137.529       | *5772.264 | *4967.799 | 478.248   | 292.647   | 211.673   | 32.526       |
|              | Relative error/% | 5.305   | 7.867        | 0.341         | /         | /         | 1.539     | 3.045     | 3.785     | /            |
|              | RMSE             | 0.105   | 0.574        | 1.066         | 9.774     | 5.498     | 8.729     | 11.740    | 9.722     | /            |

Note: The asterisk (\*) indicates that the leading digits of the coordinates have been masked to protect sensitive mine-location information; this treatment does not affect the relative geometric relationships.

Given the large coordinate values of the working face, the percentage error in map coordinates depends on the choice of the coordinate origin. To simplify the calculation of errors, this study no longer employs percentage errors in coordinates; instead, we uniformly utilize horizontal displacement relative to a reference center and the displacement azimuth to evaluate the positional error of the center. The reference center is defined as  $X_0 = *5782$  m and  $Y_0 = *4963$  m. The positional error of the center is calculated as  $\sqrt{\Delta X^2 + \Delta Y^2}$ , while the displacement azimuth is computed using  $\text{atan2}(\Delta X, \Delta Y)$ , with due north defined as  $0^\circ$  and angles increasing clockwise; negative values are adjusted by adding  $360^\circ$ . Table 9 presents the error results obtained by inverse-calculating the center coordinates using each algorithm.

As shown in Table 9, all inversion centers are located to the northwest-west of the reference center, with displacement azimuths ranging from  $296.246^\circ$  to  $304.430^\circ$ . Among them, the horizontal displacement of the AMSDE inversion center is 10.85 m, which is significantly smaller than the displacements of the other results (12.112–13.630 m). The average horizontal displacement of the other four results is approximately 13.15 m; AMSDE reduces this value by about 2.30 m, representing a decrease of approximately 17.5%. This indicates that AMSDE achieves higher positioning accuracy in center location inversion. The azimuths of the other results are concentrated around  $303\text{--}304^\circ$ , suggesting that their center offset directions are largely consistent, albeit with larger offset magnitudes; in contrast, AMSDE exhibits an azimuth of  $296.246^\circ$ , which is slightly westward, yet possesses the smallest absolute offset, thereby demonstrating optimal overall positioning accuracy.

**Table 9.** Center Position Error Table.

| Parameters | $X_0$ (m) | $Y_0$ (m) | $\Delta X$ (m) | $\Delta Y$ (m) | Centre Location Errors (m) | Orientation Errors ( $^{\circ}$ ) |
|------------|-----------|-----------|----------------|----------------|----------------------------|-----------------------------------|
| PSO        | *5770.826 | *4970.501 | −11.174        | 7.501          | 13.458                     | 303.873                           |
| GA         | *5771.858 | *4969.621 | −10.142        | 6.621          | 12.112                     | 303.118                           |
| ES         | *5770.757 | *4970.705 | −11.243        | 7.705          | 13.630                     | 304.430                           |
| DE         | *5770.783 | *4970.324 | −11.217        | 7.324          | 13.396                     | 303.129                           |
| AMSDE      | *5772.264 | *4967.799 | −9.736         | 4.799          | 10.854                     | 296.246                           |

Note:  $\Delta X = X_0 - *5782$ ,  $\Delta Y = Y_0 - *4963$ ; the horizontal displacement is  $\sqrt{\Delta X^2 + \Delta Y^2}$ ; the displacement azimuth is measured from due north at  $0^{\circ}$  and increases clockwise. The asterisk (\*) indicates that the leading digits of the coordinates have been masked to protect sensitive mine-location information; this treatment does not affect the relative geometric relationships.

To evaluate the reliability of the proposed method, the estimated geometric parameters and PIM parameters were substituted into the model to calculate the predicted surface LOS deformation. The fitting results and box plots of the signed LOS residuals are shown in Figure 9c. The deformation field predicted from the AMSDE inversion agrees well with the observed deformation in terms of basin morphology, spatial extent, and subsidence center. The box plots also show that AMSDE produces a smaller error range and more accurate results. The location and geometry of the working face were reconstructed using the geometric parameters estimated by AMSDE, as shown in Figure 10. The red rectangle represents the recorded reference geometry of the active-mining area, whereas the blue rectangle represents the geometry reconstructed by AMSDE. The predicted and actual working faces agree well in location, shape, and dimensions. These results indicate that AMSDE can accurately recover the location and geometry of the underground working face.

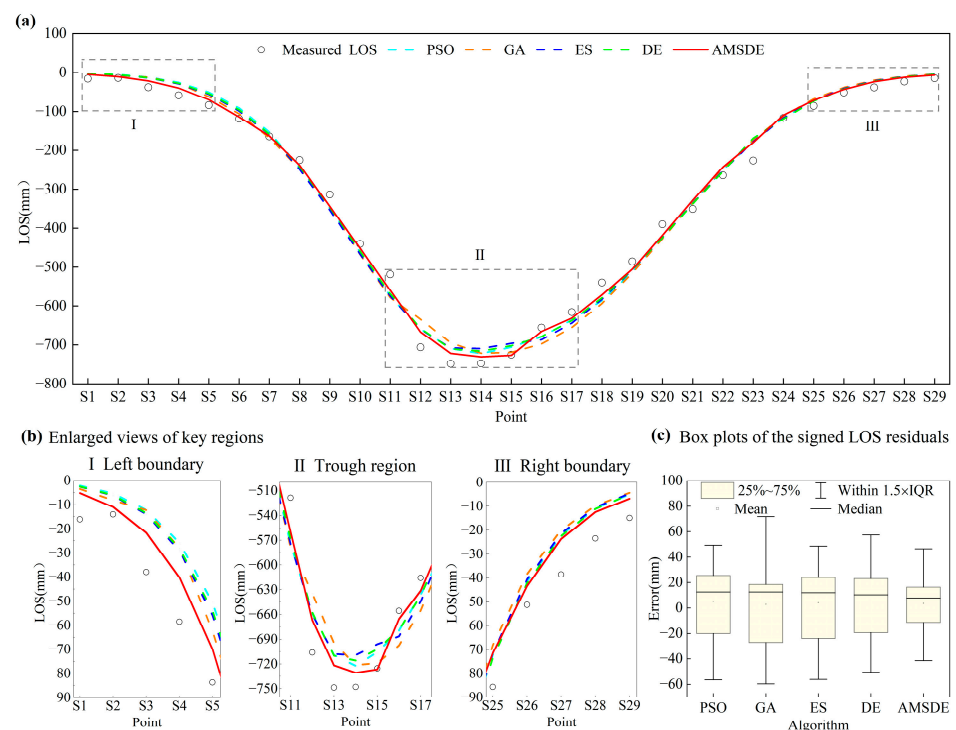


**Figure 9.** Comparison chart between the actual and predicted LOS values at the Sihe working face: (a) Comparison of the fitted LOS values predicted by each algorithm with the actual values; (b) Enlarged views of key regions, include left boundary, trough region and right boundary; (c) Box plots of the signed LOS residuals.

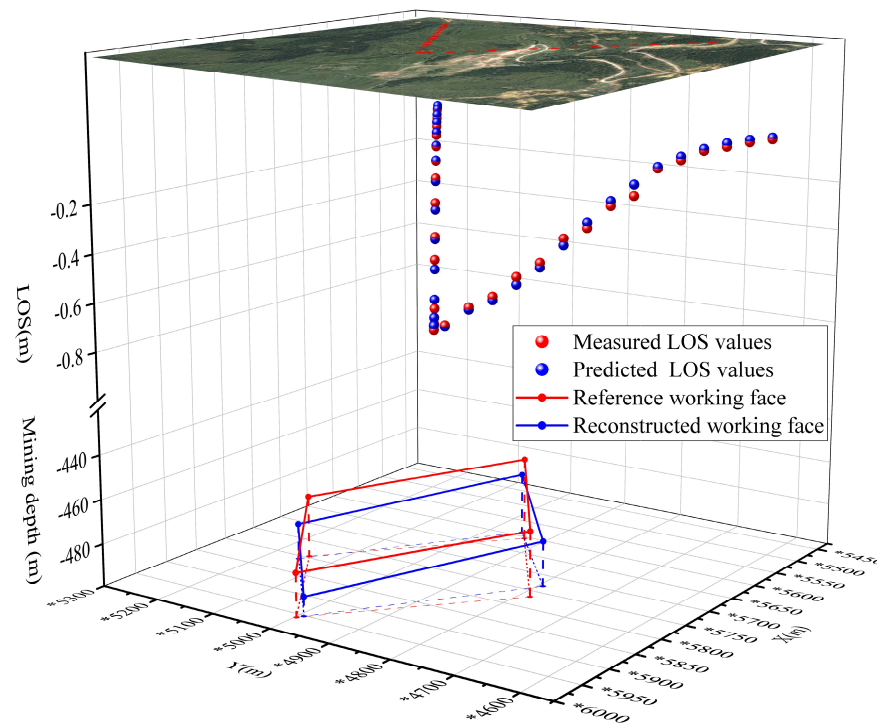


**Figure 10.** Comparison of measured and predicted LOS deformation values and working-face geometries. The measured LOS values represent the cumulative LOS deformation derived from LuTan-1 InSAR observations at the selected sampling points, whereas the predicted LOS values are obtained by projecting the three-dimensional displacements calculated using the PIM with the inverted geometric parameters onto the radar LOS direction. The red and blue frames indicate the reference and reconstructed working-face geometries, respectively. The asterisk (\*) indicates that the leading digits of the coordinates have been masked to protect sensitive mine-location information; this treatment does not affect the relative geometric relationships.

Figure 11 presents violin plots of the parameter estimates obtained from 10 independent inversion runs for Working Face 97,315 at the Sihe Mine using each algorithm. For some parameters, PSO produces shorter violin ranges. However, its estimates are concentrated near the parameter bounds. This suggests that PSO may become trapped in boundary local optima and therefore produce inaccurate parameter estimates. In contrast, most violin plots obtained by AMSDE have shorter ranges than those of the other algorithms. The  $1.5 \times \text{IQR}$  intervals also show narrower ranges of non-outlier values. These results indicate lower dispersion and smaller fluctuations in the parameter estimates. In addition, the widest sections of the AMSDE violin plots are closer to the reference values, demonstrating better robustness. The mean estimates obtained by AMSDE are also generally closer to the reference values. Overall, AMSDE provides more accurate and stable inversion results. It should be noted that the  $X_0$  estimates obtained by all algorithms exhibit systematic deviations from the reference value (Figure 11e). Unlike the idealized simulation experiments, the engineering application involves actual LuTan-1 InSAR observations and a simplified PIM, which may introduce model–observation discrepancies. In addition, the quasi-static approximation of mining geometry and parameter coupling may contribute to the positioning bias, although AMSDE achieves the smallest center-position error among the compared algorithms.

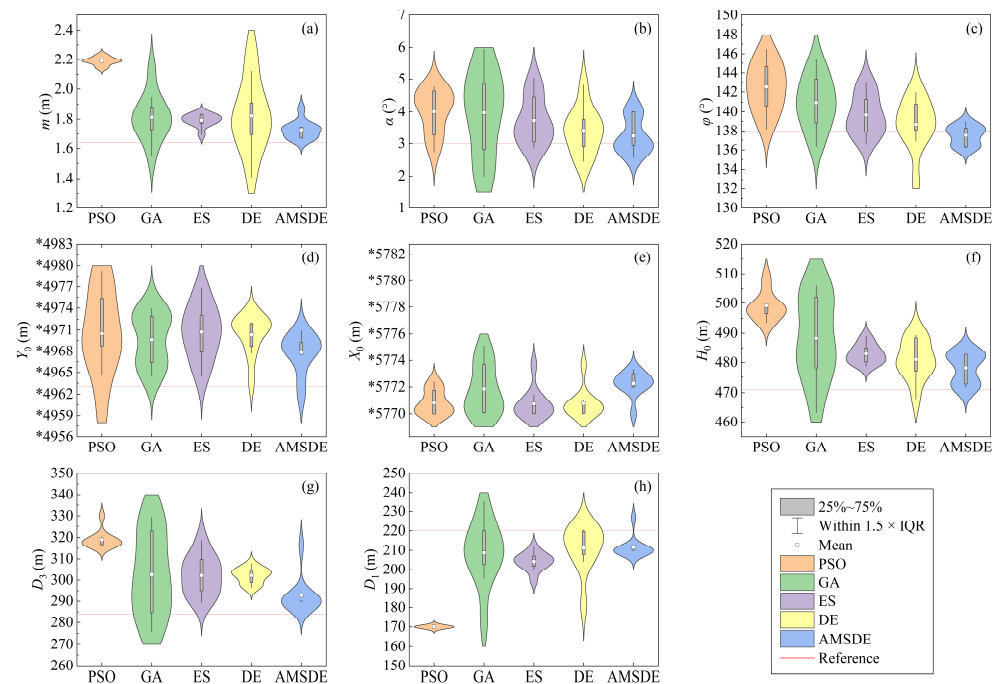


**Figure 11.** Range of variation in characteristic parameters obtained using different algorithms at the Sihe working face over 10 inversion iterations: (a)  $m$ ; (b)  $\alpha$ ; (c)  $\varphi$ ; (d)  $Y_0$ ; (e)  $X_0$ ; (f)  $H_0$ ; (g)  $D_3$ ; and (h)  $D_1$ . The asterisk (\*) indicates that the leading digits of the coordinates have been masked to protect sensitive mine-location information; this treatment does not affect the relative geometric relationships. Different colors denote different algorithms, and the red line represents the reference accurate value.

## 5. Discussion

### 5.1. Exploration of AMSDE Algorithm Performance

AMSDE dynamically adjusts the scaling factor  $F$  and crossover probability  $CR$  according to the iteration progress. This adaptive parameter adjustment provides strong global exploration in the early stage and gradually strengthens local exploitation in the late stage. AMSDE also switches among three mutation strategies during the iterations. This multi-strategy mutation mechanism maintains population diversity and reduces the risk of premature convergence.

Although both mechanisms offer clear advantages, their synergistic effect requires further verification. Therefore, an ablation study was conducted using the simulated working face. AMSDE1 used only adaptive parameter adjustment, AMSDE2 used only multi-strategy mutation, and AMSDE combined both mechanisms. All algorithms used the same parameter settings as those described above. Each algorithm was independently run 10 times, and the mean results were used for comparison. The parameter estimates are listed in Table 10, while the convergence curves and computation times are shown in Figure 12. As shown in Table 10, AMSDE achieved smaller or equal absolute errors than AMSDE1 and AMSDE2 for seven of the eight parameters. Only for  $D_3$  was the AMSDE error (0.002 m) slightly larger than that of AMSDE1 (0.001 m). AMSDE also achieved the lowest fitness value of 0.238 mm, indicating the best overall performance. Figure 12 further shows that AMSDE converged faster and required less computation time. These results confirm that adaptive parameter adjustment and multi-strategy mutation have a synergistic effect. Their combined use further improves the overall performance of the algorithm.

**Table 10.** Algorithm Performance Comparison Results.

| Parameters     | $m$ (m) | $\alpha$ (°) | $\varphi$ (°) | $X_0$ (m) | $Y_0$ (m) | $H_0$ (m) | $D_3$ (m) | $D_1$ (m) | Fitness (mm) |
|----------------|---------|--------------|---------------|-----------|-----------|-----------|-----------|-----------|--------------|
| Simulated      | 3       | 5            | 90            | 200       | 300       | 300       | 600       | 400       | /            |
| AMSDE1         | 2.998   | 4.911        | 90.025        | 199.979   | 300.006   | 300.0267  | 600.001   | 399.972   | 0.385        |
| Absolute Error | 0.002   | 0.089        | 0.025         | 0.021     | 0.006     | 0.0267    | 0.001     | 0.028     | /            |
| AMSDE2         | 2.999   | 4.926        | 90.008        | 200.071   | 300.003   | 300.001   | 600.003   | 400.097   | 0.316        |
| Absolute Error | 0.001   | 0.074        | 0.008         | 0.071     | 0.003     | 0.001     | 0.003     | 0.097     | /            |
| AMSDE          | 2.999   | 5.029        | 90.001        | 199.990   | 300.002   | 299.999   | 600.002   | 399.979   | 0.238        |
| Absolute Error | 0.001   | 0.029        | 0.001         | 0.010     | 0.002     | 0.001     | 0.002     | 0.021     | /            |

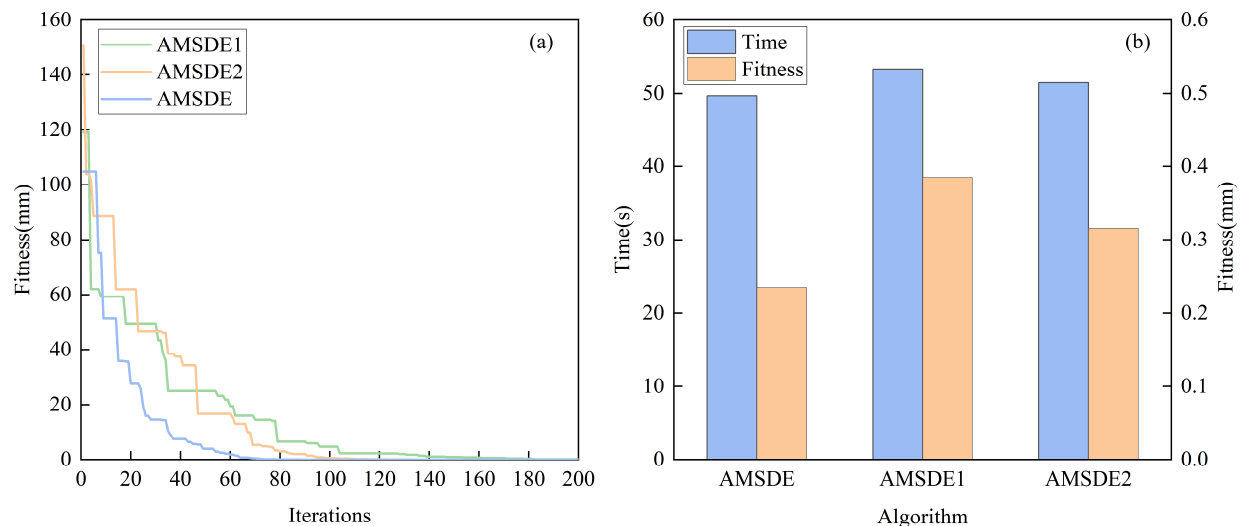


**Figure 12.** Comparison of algorithm iteration, computation time, and fitness performance: (a) Comparison of iteration curves for different AMSDE algorithms; (b) Comparison of inversion times and fitness values for different AMSDE algorithms.

## 5.2. Uncertainty and Sensitivity Analysis of the Inverse Calculation Results

Uncertainty in the inversion mainly arises from model error, parameter sensitivity, and observation error. These factors jointly affect the reliability and interpretability of the inversion results.

Model error results from the simplification of complex strata movement and surface response processes in PIM. PIM treats the overlying rock mass as a stochastic medium and represents its complex mechanical behavior using a mathematical model. This simplification introduces structural bias and approximation errors. As a result, systematic differences may arise when the model is applied to heterogeneous media, nonlinear responses, or local instability. Such structural errors may affect both parameter estimates and the evaluation of prediction uncertainty.

Parameter sensitivity can be evaluated by perturbing one parameter while keeping the others fixed and then examining the resulting change in the fitness value. In this experiment, perturbations of  $\pm 5\%$  and  $\pm 10\%$  were separately applied to each parameter. The corresponding fitness values are listed in Table 11. When a 5% perturbation was applied, the fitness value for the coal seam dip angle was less than 1 mm, whereas that for the strike azimuth was less than 10 mm. The results show that the strike length, dip length, and center coordinates have the strongest effects on the deformation field and are therefore the most sensitive parameters. In contrast, when the coal seam is nearly horizontal, the LOS deformation field is less sensitive to the strike azimuth  $\varphi$ . These findings suggest that stronger prior constraints or narrower search ranges should be assigned to highly sensitive parameters. This can reduce parameter instability and interaction-induced uncertainty during inversion.

**Table 11.** Results of parameter sensitivity analysis.

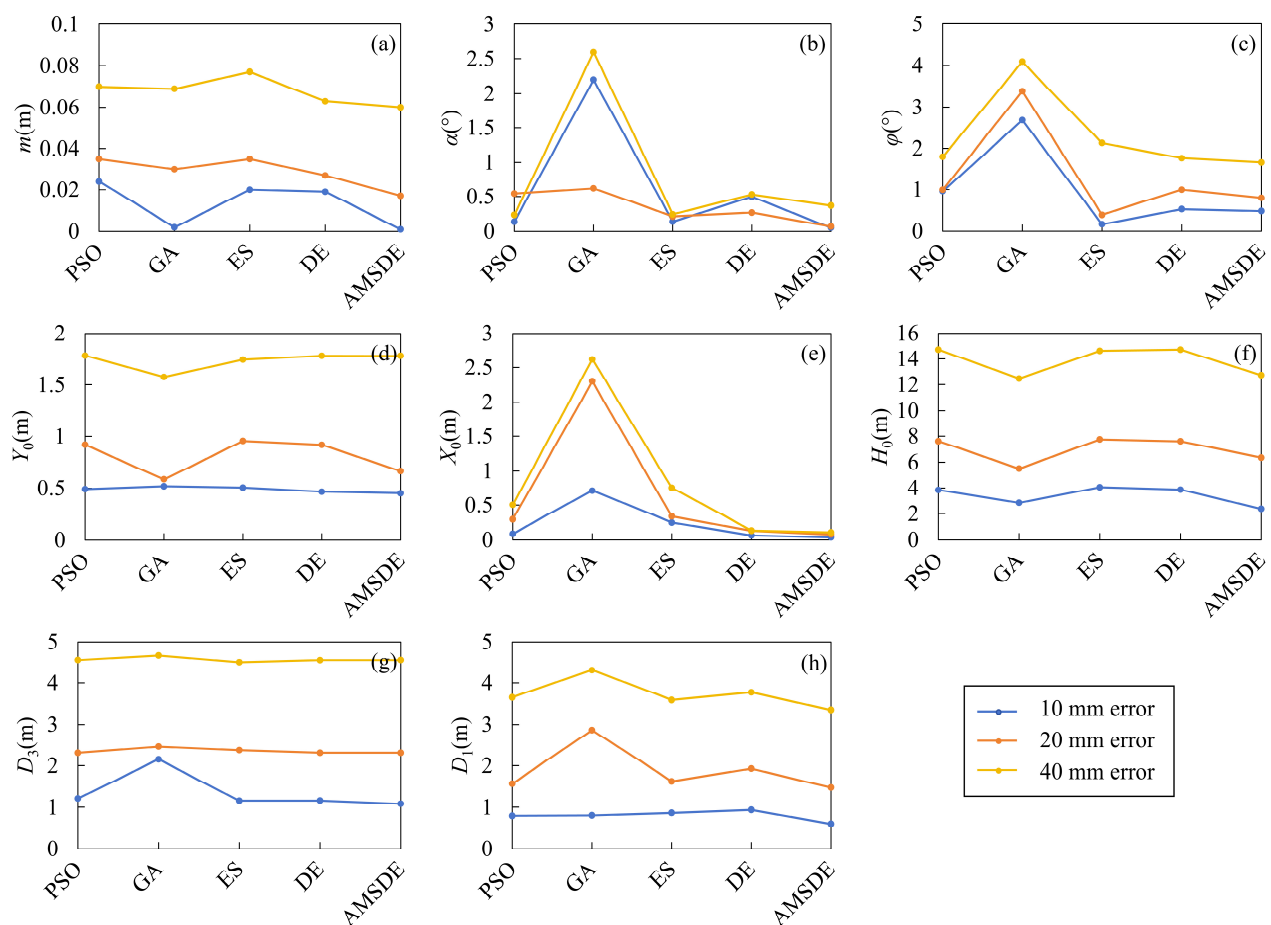
| Parameters               | Simulated | Error                   | Fitness (mm)       |
|--------------------------|-----------|-------------------------|--------------------|
| $m$ (m)                  | 3         | $\pm 5\%$<br>$\pm 10\%$ | 35.006<br>70.012   |
| $\alpha$ ( $^{\circ}$ )  | 5         | $\pm 5\%$<br>$\pm 10\%$ | 0.376<br>0.780     |
| $\varphi$ ( $^{\circ}$ ) | 90        | $\pm 5\%$<br>$\pm 10\%$ | 7.936<br>15.891    |
| $X_0$ (m)                | 200       | $\pm 5\%$<br>$\pm 10\%$ | 33.413<br>66.683   |
| $Y_0$ (m)                | 300       | $\pm 5\%$<br>$\pm 10\%$ | 70.171<br>139.505  |
| $H_0$ (m)                | 300       | $\pm 5\%$<br>$\pm 10\%$ | 12.529<br>24.729   |
| $D_3$ (m)                | 600       | $\pm 5\%$<br>$\pm 10\%$ | 100.584<br>199.570 |
| $D_1$ (m)                | 400       | $\pm 5\%$<br>$\pm 10\%$ | 74.583<br>148.211  |

InSAR observations contain errors from multiple sources, including residual tropospheric and ionospheric delays, orbit correction errors, coherence loss, temporal and spatial decorrelation, and phase unwrapping errors. These error sources reduce the accuracy of the derived LOS deformation. To evaluate their effects on the estimated geometric parameters, random errors were added to the simulated LOS deformation field. Random errors of 10 mm, 20 mm, and 40 mm were separately added to the simulated LOS deformation data. The perturbed data were then used to invert the working face geometric parameters using the different algorithms. The results are presented in Table 12. Overall, AMSDE produced the smallest parameter errors after the addition of observation errors. It provided higher accuracy for mining thickness, coal seam dip angle, and strike azimuth than the other algorithms. The algorithms achieved similar accuracy for the working face dimensions, center coordinates, and mining depth. Nevertheless, AMSDE showed the best overall accuracy, indicating greater robustness to observation errors. Figure 13 shows the absolute errors of the inverted geometric parameters under different levels of LOS deformation error. The parameter errors generally increased as the observation error increased. Similar error trends were observed for all algorithms. However, AMSDE produced the smallest overall inversion errors and showed better stability.

In the inversion of underground working face geometric parameters, the PIM prediction parameters  $P_{PIM} = \{q, \tan\beta, b, \theta, S_d, S_u, S_l, S_r\}$  are initially assumed to be known and accurate. In practice, however, the PIM parameters obtained for an actual working face may contain errors, which can affect the accuracy of the inverted geometric parameters. To evaluate the effects of different PIM parameters and error levels, the simulated working face data described above were used. The LOS deformation data were assumed to be error-free. Errors of  $\pm 10\%$ ,  $\pm 20\%$ , and  $\pm 30\%$  were separately added to each PIM parameter. Only one parameter was perturbed at a time, while all other parameters remained unchanged. The perturbed PIM parameters were then used to invert the working face geometric parameters. The resulting inversion errors are shown in Figure 14.

**Table 12.** Results against random errors.

| Parameters    | Simulated | Error | PSO     | Absolute Error | GA      | Absolute Error | ES      | Absolute Error | DE      | Absolute Error | AMSDE   | Absolute Error |
|---------------|-----------|-------|---------|----------------|---------|----------------|---------|----------------|---------|----------------|---------|----------------|
| $m$ (m)       | 3         | 10    | 2.976   | 0.024          | 3.002   | 0.002          | 2.980   | 0.020          | 2.981   | 0.019          | 2.999   | 0.001          |
|               |           | 20    | 2.965   | 0.035          | 2.970   | 0.030          | 2.965   | 0.035          | 2.973   | 0.027          | 2.983   | 0.017          |
|               |           | 40    | 2.930   | 0.070          | 2.931   | 0.069          | 2.923   | 0.077          | 2.937   | 0.063          | 2.941   | 0.060          |
| $\alpha$ (°)  | 5         | 10    | 4.859   | 0.141          | 7.198   | 2.198          | 4.858   | 0.142          | 4.505   | 0.495          | 5.055   | 0.055          |
|               |           | 20    | 5.543   | 0.543          | 5.619   | 0.619          | 5.216   | 0.216          | 5.270   | 0.270          | 4.929   | 0.072          |
|               |           | 40    | 4.763   | 0.237          | 2.405   | 2.595          | 5.245   | 0.245          | 5.529   | 0.529          | 4.631   | 0.369          |
| $\varphi$ (°) | 90        | 10    | 90.959  | 0.959          | 87.319  | 2.681          | 89.829  | 0.171          | 90.527  | 0.527          | 90.482  | 0.482          |
|               |           | 20    | 90.995  | 0.995          | 93.388  | 3.388          | 89.607  | 0.393          | 90.996  | 0.996          | 90.796  | 0.796          |
|               |           | 40    | 91.800  | 1.800          | 94.091  | 4.091          | 92.128  | 2.128          | 91.769  | 1.769          | 91.665  | 1.665          |
| $X_0$ (m)     | 200       | 10    | 199.913 | 0.087          | 200.712 | 0.712          | 199.752 | 0.248          | 199.934 | 0.066          | 199.963 | 0.037          |
|               |           | 20    | 199.704 | 0.296          | 202.298 | 2.298          | 200.337 | 0.337          | 199.868 | 0.132          | 200.080 | 0.080          |
|               |           | 40    | 199.497 | 0.504          | 197.382 | 2.618          | 200.754 | 0.754          | 199.868 | 0.133          | 199.891 | 0.109          |
| $Y_0$ (m)     | 300       | 10    | 299.509 | 0.491          | 299.481 | 0.519          | 299.494 | 0.506          | 299.534 | 0.466          | 299.549 | 0.451          |
|               |           | 20    | 299.082 | 0.918          | 300.588 | 0.588          | 299.046 | 0.954          | 299.083 | 0.917          | 299.332 | 0.668          |
|               |           | 40    | 298.218 | 1.783          | 298.426 | 1.574          | 298.256 | 1.744          | 298.218 | 1.782          | 298.219 | 1.781          |
| $H_0$ (m)     | 300       | 10    | 296.156 | 3.844          | 297.158 | 2.842          | 295.942 | 4.058          | 296.127 | 3.873          | 297.635 | 2.365          |
|               |           | 20    | 292.400 | 7.600          | 294.483 | 5.517          | 292.261 | 7.739          | 292.404 | 7.596          | 293.649 | 6.351          |
|               |           | 40    | 285.285 | 14.715         | 287.546 | 12.454         | 285.372 | 14.628         | 285.284 | 14.717         | 287.285 | 12.716         |
| $D_3$ (m)     | 600       | 10    | 598.789 | 1.212          | 597.849 | 2.151          | 598.842 | 1.158          | 598.840 | 1.160          | 598.918 | 1.082          |
|               |           | 20    | 597.690 | 2.310          | 597.539 | 2.461          | 597.626 | 2.374          | 597.690 | 2.310          | 597.691 | 2.309          |
|               |           | 40    | 595.44  | 4.562          | 595.327 | 4.673          | 595.493 | 4.507          | 595.440 | 4.560          | 595.439 | 4.561          |
| $D_1$ (m)     | 400       | 10    | 399.203 | 0.797          | 399.192 | 0.808          | 399.135 | 0.865          | 399.064 | 0.936          | 399.392 | 0.608          |
|               |           | 20    | 398.429 | 1.571          | 397.139 | 2.861          | 398.371 | 1.629          | 398.070 | 1.930          | 398.519 | 1.481          |
|               |           | 40    | 396.347 | 3.653          | 395.686 | 4.314          | 396.409 | 3.591          | 396.228 | 3.772          | 396.644 | 3.356          |

**Figure 13.** Absolute errors of feature parameters under inversion with different line-of-sight (LOS) deformation error levels and different algorithms: (a)  $m$ ; (b)  $\alpha$ ; (c)  $\varphi$ ; (d)  $Y_0$ ; (e)  $X_0$ ; (f)  $H_0$ ; (g)  $D_3$ ; and (h)  $D_1$ . Different colors indicating different noise datasets.

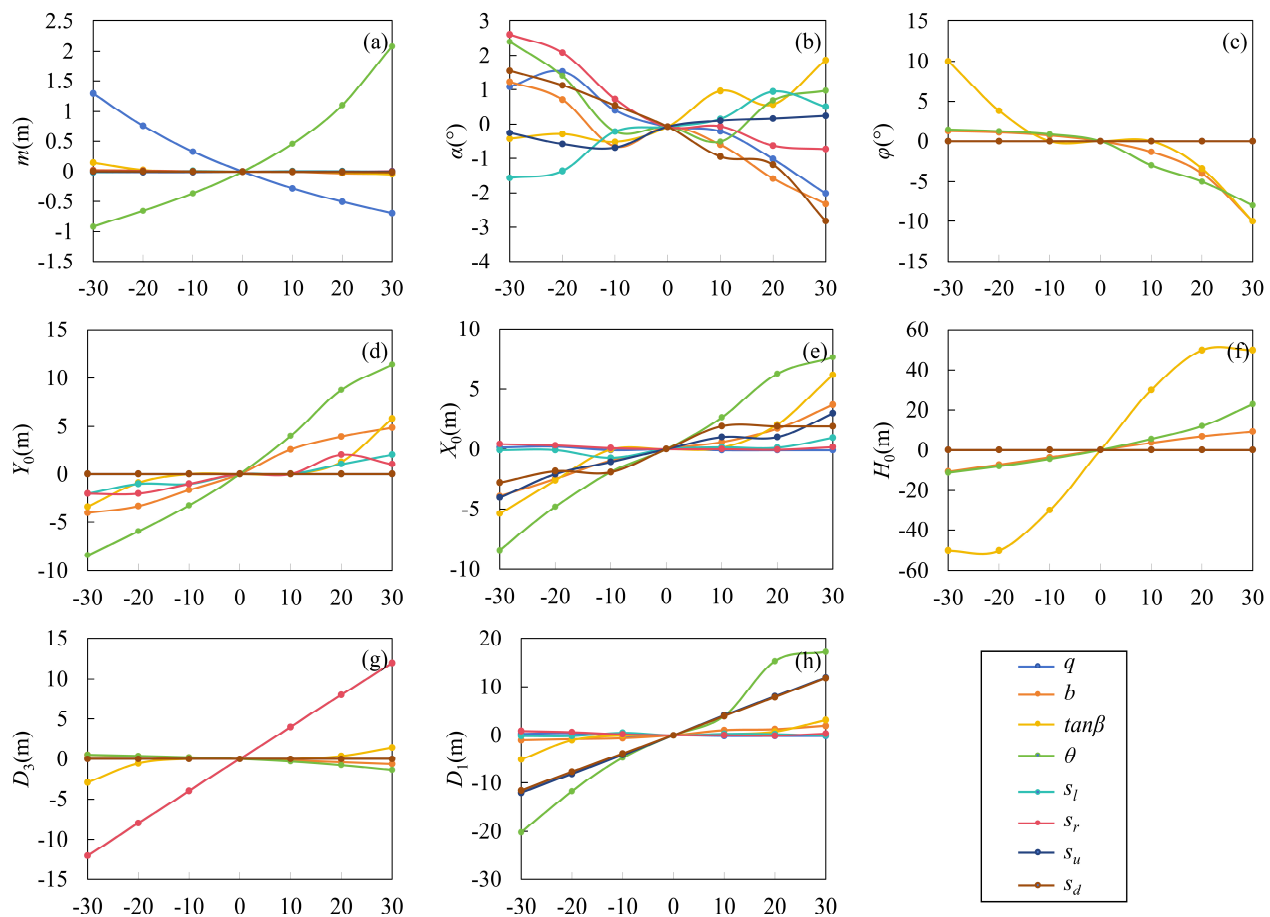


**Figure 14.** Impact of errors in probability integration method (PIM) parameters on characteristic parameters: (a)  $m$ ; (b)  $\alpha$ ; (c)  $\varphi$ ; (d)  $Y_0$ ; (e)  $X_0$ ; (f)  $H_0$ ; (g)  $D_3$ ; and (h)  $D_1$ . Different colors indicating the influence of distinct PIM parameters.

As shown in Figure 14, the inverted mining thickness  $m$  is most sensitive to errors in the subsidence coefficient  $q$  and the mining influence propagation angle  $\theta$ . Its inversion error is negatively correlated with the error in  $q$  and positively correlated with the error in  $\theta$ . Errors in all eight PIM parameters affect the inversion accuracy of the coal seam dip angle  $\alpha$ . The strike azimuth  $\varphi$  and mining depth  $H_0$  are most sensitive to errors in  $\tan\beta$ . Their inversion errors show linear positive and negative correlations with the error in  $\tan\beta$ . The working face center coordinates  $X_0$  and  $Y_0$  are most strongly affected by errors in  $\theta$ . Their inversion errors increase approximately linearly with the error in  $\theta$ . The strike length  $D_3$  and dip length  $D_1$  are mainly influenced by the inflection point offsets and  $\theta$ .

These results show that errors in the PIM parameters can substantially affect the accuracy of the inverted working face geometric parameters. Therefore, the PIM parameters should be constrained using field measurements of surface deformation and available geological and mining data to reduce inversion errors. The inversion accuracy of the coal seam dip angle  $\alpha$  is affected by errors in all PIM parameters. In particular,  $q$ ,  $\theta$  and  $\tan\beta$  influence the inversion of nearly all working face geometric parameters. Their accuracy should therefore be carefully constrained. In contrast, the inflection point offsets have relatively small effects on the inversion accuracy and can be assigned using empirical values.

Time-series InSAR methods are valuable for characterizing the long-term temporal evolution of mining-induced deformation and for reducing the influence of individual interferometric errors. In vegetated mining areas, the density of persistent scatterers may be limited, while distributed scatterer (DS)-based approaches can provide additional coherent observations. However, the main objective of this study is to reconstruct working-face

geometry from the spatial deformation field during a specific active-mining period rather than to characterize long-term deformation evolution. In addition, only three LuTan-1 acquisitions were available for the engineering case, which is insufficient for a robust long-term time-series analysis. Therefore, D-InSAR was adopted to obtain the interval and cumulative LOS deformation fields required for the inversion. With the accumulation of additional LuTan-1 observations, DS-based time-series InSAR will be considered to investigate the temporal evolution of mining subsidence and further improve the reliability of the inversion.

## 6. Conclusions

This study addresses the identification of unknown geological and mining parameters by developing and systematically validating a physics-informed intelligent inversion framework. The framework integrates a physical model, high-resolution satellite remote sensing observations, and advanced optimization strategies. It provides substantial improvements in both methodology and engineering application.

- (1) This paper proposes a method for reconstructing the geometric morphology of concealed coal mining faces using LuTan-1 InSAR observation data, PIM, and the AMSDE algorithm; the effectiveness of the proposed method is validated through simulation experiments and real-world engineering case studies. The improved differential evolution algorithm proposed in this paper effectively mitigates the common drawback of conventional intelligent optimization algorithms—namely, their tendency to converge to local minima—through the synergistic effect of multiple strategy mechanisms. In numerical simulation experiments, this algorithm demonstrates superior solution accuracy and robustness compared to the standard particle swarm optimization algorithm and the genetic algorithm. In the simulation experiments, AMSDE achieved higher solution accuracy and robustness than standard PSO and GA. The inverted parameters were nearly identical to the predefined true values, with negligible errors. In the engineering case study, the inversion results agreed well with the mine records. The relative errors of all key parameters were within 7.9%. These results demonstrate the reliability, effectiveness, and practical value of the proposed method under real mining conditions.
- (2) After LOS deformation errors of 10–40 mm were introduced, AMSDE generally produced smaller parameter errors than the comparison algorithms. This indicates that the improved algorithm is more robust to observation errors. Parameter sensitivity analysis showed that the strike length, dip length, and center coordinates had the strongest effects on the deformation field and were the most sensitive parameters. Therefore, stronger prior constraints or narrower search ranges should be assigned to these parameters during inversion.
- (3) The proposed method in this study is primarily applicable to concealed mining areas characterized by limited geological and mining data, yet where high-quality full-field SAR deformation observations are available—particularly when the working face geometry is relatively regular and the target inversion parameters can be constrained within a reasonable search interval. This approach leverages remote sensing observations as its primary constraint, enabling the collaborative identification of key parameters for unknown goaf areas with minimal reliance on downhole measurement data. However, the method still exhibits certain limitations. First, the inversion accuracy is constrained by the fidelity of the physical model; the approximation employed by the probabilistic integration method under complex geological and mining conditions may introduce systematic deviations. Second, current validation studies have predominantly focused on relatively regular rectangular working faces; whether the

method remains stable and accurate under more complex scenarios—such as irregular working faces, multiple working faces, multiple coal seams, pronounced structural features, lithological heterogeneity, or steeply dipping coal seams—requires further validation. Third, the coherence, coverage area, deformation gradient, and external errors of SAR observations can all influence the inversion results; although the AMSDE method demonstrates superior robustness against errors, it cannot entirely eliminate such uncertainties.

- (4) The proposed method provides an efficient and practical approach for identifying hidden mine hazard sources, supporting mine safety supervision, and assessing environmental impacts. Unlike conventional methods that rely on detailed underground records or point-based measurements, this method mainly uses accessible satellite remote sensing data to detect unknown underground mining activities. It therefore has strong potential for wider application. Future work may improve the physical subsidence model, integrate multi-source observations, and introduce uncertainty quantification to extend the applicability of the method and increase confidence in the inversion results. Surrogate models, multi-fidelity strategies, and parallel computing may also be used to reduce computational cost and support real-time or near-real-time monitoring.

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## Abbreviations

The following abbreviations are used in this manuscript:

|       |                                                          |
|-------|----------------------------------------------------------|
| LOS   | Line-of-sight                                            |
| InSAR | Interferometric Synthetic Aperture Radar                 |
| PIM   | Probability Integral Method                              |
| SAR   | Synthetic Aperture Radar                                 |
| AMSDE | adaptive multi-strategy differential evolution algorithm |
| DE    | Differential Evolution                                   |
| RMSE  | Root Mean Square Error                                   |
| LHS   | Latin Hypercube Sampling                                 |
| PSO   | Particle Swarm Optimization                              |
| GA    | Genetic Algorithm                                        |
| ES    | Evolution Strategy                                       |

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