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A Sparse Aperture ISAR Imaging Based on a Single-Layer Network Framework

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Highlights

What are the main findings?

- A fixed-point sparse-aperture ISAR imaging framework (ADnDEQ) is developed by integrating ADMM unfolding, DnCNN denoising, and the deep equilibrium model.
- ADnDEQ achieves superior reconstruction quality under extreme sparsity (10% sampling) and low SNR (0 dB), outperforming existing CS-based and deep learning methods.

What are the implications of the main findings?

- A single trained ADnDEQ model generalizes well across different sampling ratios and noise conditions, avoiding retraining under mismatched sensing configurations.
- The DEQ-based formulation offers an interpretable and memory-efficient alternative to deep stacked networks, enabling flexible and resource-friendly deployment in practical radar systems.

Abstract

Under sparse aperture (SA) conditions, inverse synthetic aperture radar (ISAR) imaging becomes a severely ill-posed inverse problem due to undersampled and noisy measurements, leading to pronounced degradation in azimuth resolution and image quality. Although deep learning approaches have demonstrated promising performance for SA-ISAR imaging, their practical deployment is often hindered by black-box behavior, fixed network depth, high computational cost, and limited robustness under extreme operating conditions. To address these challenges, this paper proposes an ADMM Denoising Deep Equilibrium Framework (ADnDEQ) for SA-ISAR imaging. The proposed method reformulates an ADMM-based unfolding process as an implicit deep equilibrium (DEQ) model, where ADMM provides an interpretable optimization structure and a lightweight DnCNN is embedded as a learned proximal operator to enhance robustness against noise and sparse sampling. By representing the reconstruction process as the equilibrium solution of a single-layer network with shared parameters, ADnDEQ decouples forward and backward propagation, achieves constant memory complexity, and enables flexible control of inference iterations. Experimental results demonstrate that the proposed ADnDEQ framework achieves superior reconstruction quality and robustness compared with conventional layer-stacked networks, particularly under low sampling ratios and low-SNR conditions, while maintaining significantly reduced computational cost.

Keywords: inverse synthetic aperture radar (ISAR); deep learning; sparse aperture (SA); deep equilibrium (DEQ)



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1. Introduction

Inverse synthetic aperture radar (ISAR) is widely used for imaging non-cooperative moving targets, such as maritime vessels and aircraft. Due to its high-resolution imaging capability under complex motion conditions, ISAR plays a crucial role in target recognition, space surveillance, and other remote sensing applications. As an advanced active microwave imaging system, ISAR can operate under all-weather conditions and around the clock. By employing pulse compression techniques, ISAR achieves high range resolution, enabling accurate discrimination of target features along the range dimension [1]. Meanwhile, azimuth resolution is obtained by exploiting the Doppler frequency modulation induced by target rotational motion, which allows the formation of two-dimensional ISAR images [2].

In radar imaging, the complete pulse signal within the observation interval is referred to as the full-aperture signal. When part of this signal is missing, the received data are known as sparse-aperture (SA) signals [3]. In ISAR systems, sparse-aperture conditions arise due to several objective factors [4].

First, ISAR targets are typically non-cooperative, and their motion trajectories are difficult to predict accurately. Second, long propagation distances and complex electromagnetic environments often result in low signal-to-noise ratios (SNRs), leading to partial loss of echo signals. Third, with the rapid development of electronic countermeasure technologies, ISAR echoes are increasingly subject to various forms of interference, which further reduces data availability.

In addition to these passive causes, sparse-aperture signals can also be intentionally employed to improve radar system efficiency. For example, intermittent signal transmission can be used to reduce storage and transmission burdens. In multifunction radar systems, target search, tracking, and imaging can be achieved by alternately transmitting narrowband and wideband signals, where narrowband signals are used for tracking and wideband signals are used for imaging.

Under sparse-aperture conditions, the conventional range-Doppler (RD) algorithm produces ISAR images with broadened main lobes and elevated sidelobes. As a result, azimuth resolution is significantly degraded, making it difficult to satisfy practical engineering requirements [5,6].

Compressed sensing (CS) plays a pivotal role in SA-ISAR imaging [7]. One of the notable advantages of CS lies in its capability to reconstruct a sparse signal from a significantly reduced number of measurements compared to those stipulated by Nyquist sampling theory. Given that the region of interest in ISAR imaging occupies only a limited portion of the entire observation scene, the received signal naturally exhibits sparsity, thereby reducing the reliance on additional sparse transformations commonly required in other imaging applications [8,9]. As a result, CS-based sparse signal recovery methods are highly suitable for SA-ISAR imaging. Zhang et al. [10] were among the first to introduce the CS framework into ISAR imaging, demonstrating that it could outperform the traditional Range-Doppler (RD) algorithm even with a limited number of measurements. Following this advancement, various sparse recovery algorithms have been adapted for SA-ISAR applications, including sparse Bayesian learning (SBL) [11], orthogonal matching pursuit (OMP) [12], gradient-based reconstruction techniques [13], and greedy Kalman filtering [14], among others. These methods exhibit superior performance compared to the traditional RD algorithm, but they often suffer from computational inefficiency because of the repetitive need for a serial processing framework or matrix inversion. In addition, various efficient convex optimization techniques have been utilized to enhance the performance of SA-ISAR imaging, including iterative shrinkage thresholding algorithm (ISTA), fast ISTA (FISTA) [15], alternating direction method of multipliers (ADMMs) [16], and approximate

message passing (AMP) [17]. These methods typically employ shrinkage thresholding functions to solve the convex relaxation problem inherent in ISAR imaging, with the least absolute shrinkage and selection operator (LASSO) being a representative approach [18]. Xu et al. [19] developed an LS-SBL-based ISAR method for maneuvering targets under compressive sensing, utilizing structured priors and parametric dictionaries to enable joint sparse imaging and rotational parameter estimation via alternating iterations, achieving high-fidelity reconstruction with cross-range scaling. Zhang et al. [20] proposed a Bayesian framework combining GaPL nonparametric priors and 2D-IFGaPL imaging to bypass matrix inversion, jointly optimizing translational compensation (range-phase estimation) and sparse reconstruction. Both methods demonstrate robustness in low-SNR/incomplete-data scenarios through simulations and experiments. However, convex optimization algorithms still demand substantial computational resources due to their inherently complex iterative procedures. Furthermore, the process of manually tuning parameters within these algorithms, often relying on empirical insights derived from prior experiments or literature, introduces an additional layer of complexity. This manual parameter adjustment can potentially lead to increased variability in performance, as well as artifacts or errors in the reconstructed images, thereby posing significant challenges with respect to both computational efficiency and precision.

In the realm of shrinkage thresholding-based methodologies, the absence of a standardized parameter refinement technique poses practical challenges for implementation. To mitigate this limitation, methodologies grounded in deep learning have garnered significant attention [21]. Notably, ISAR imaging, in contrast to optical image processing, entails rigorous mathematical constraints. Consequently, the technique of deep unfolding has been employed to transform the iterative process of CS into a deep neural network architecture with high interpretability [22]. For example, Li et al. proposed a network called CIST + Net [23], specifically designed for high-frequency feature signals, which effectively enhances the extraction of high-frequency components in sparse apertures. To minimize the dependence on high-quality labeled data, Li et al. proposed the parallel ISTA network (PIN) [24], which incorporated a multisampling matrix training approach to improve the network's robustness by leveraging symmetry constraints inherent in the parallel framework. Hu et al. demonstrate the use of a fully convolutional neural network (FCNN) to enhance ISAR imaging results [25], and discusses subsequent optimizations [26]. Wang et al. and Du et al. evidenced that autoencoder and variational autoencoder structures had been successfully applied to SA-ISAR imaging tasks [27,28]. Furthermore, Qin et al. proposed generative adversarial networks (GANs) employed to improve the resolution of SA-ISAR images and integrated pre-trained convolution denoising [29], and Chen et al. introduced a super-resolution module named AD-SRNet which was utilized for real-time super-resolution ISAR imaging [30]. Recent years have witnessed rapid progress in deep unfolding-based methods for SA-ISAR imaging. Representative works include CIST [31], which integrates CNN-based priors into iterative sparse reconstruction, TV-driven networks that embed total variation regularization into lightweight architectures [32], and block-sparse GAMP-based networks that exploit structured sparsity for computational efficiency [33]. These proposed deep neural networks exhibit substantial recovery efficacy in the context of SA-ISAR imaging. Beyond SA-ISAR imaging, deep unfolding frameworks have also been extended to other ISAR tasks, such as compressive sensing ISAR imaging and autofocusing. For example, composite networks combining adaptive sampling and reconstruction, as well as complex-valued ADMM-based networks for joint imaging and autofocusing, have demonstrated promising performance [34,35].

The engineering application of deep learning techniques in SA-ISAR imaging remains constrained by several interconnected challenges. First, the inherent black-box nature

of deep learning models introduces significant interpretability deficits, wherein opaque internal decision-making mechanisms hinder quantitative performance evaluation and undermine confidence in imaging results, particularly under complex electromagnetic interference conditions. Second, conventional approaches often rely on layer-wise stacking strategies to enhance network capacity for imaging optimization. While effective, this methodology imposes rigidity through fixed computational graphs, limiting adaptability to diverse sparse sampling patterns or dynamic target motion characteristics, while simultaneously amplifying training complexity. Third, the exponential relationship between hyperparameter scale and training data volume exacerbates computational demands, resulting in prohibitive memory consumption, prolonged convergence cycles, and compromised scalability—critical bottlenecks for real-time deployment on resource-constrained radar platforms. Fourth, under extreme conditions characterized by ultra-low signal-to-noise ratios ($\text{SNR} = 0 \text{ dB}$) and sub-Nyquist compression ratios (compression rate = 10%), ISAR imaging quality undergoes marked degradation. Collectively, these challenges highlight the difficulty of achieving a unified SA-ISAR imaging framework that simultaneously ensures interpretability, architectural flexibility, computational efficiency, and robustness under extreme operating conditions.

Some advanced deep sequence models employ a weight-tying approach to achieve a substantial reduction in parameter count while maintaining competitive performance levels. Motivated by this practice, the deep equilibrium (DEQ) model introduces an alternative paradigm by reformulating deep networks as implicit equilibrium solutions, rather than explicitly stacked layers. This design addresses the fixed-depth limitation and excessive memory consumption commonly observed in conventional deep unfolding networks. This methodology not only reduces the parameter overhead but also offers a computational framework that is inherently scalable [36,37]. The DEQ framework supports backpropagation at equilibrium points through the application of implicit differentiation, which is a memory-efficient approach that decouples the forward and backward propagation phases. This decoupling enables a more flexible optimization process, allowing users to strike an optimal balance between reconstruction time and accuracy. Furthermore, the DEQ paradigm facilitates the incorporation of various acceleration techniques, thus enhancing computational efficiency and performance. While DEQ has demonstrated its effectiveness in large-scale language modeling and image reconstruction tasks, its potential application in the domain of SA-ISAR imaging remains largely unexplored.

Building on the strong interpretability of the ADMM algorithm and the denoising capability of DnCNN [38], this study proposes an ADMM Denoising Deep Equilibrium Framework (ADnDEQ) for SA-ISAR imaging. In the proposed framework, ADMM defines the physically interpretable optimization structure, DnCNN is embedded as a learned proximal operator within this structure, and DEQ reformulates the unfolding process as an implicit equilibrium solution to achieve efficient and flexible inference. The proposed model significantly simplifies the network architecture while achieving superior imaging accuracy and robustness compared to conventional deep networks. To accelerate both training and inference, the Anderson Accelerator (AA) [39] is incorporated to optimize convergence efficiency.

The robustness of the proposed ADnDEQ framework under extremely low sampling ratios and low-SNR conditions mainly stems from its unified architectural design. First, the ADMM backbone explicitly enforces data fidelity constraints, which helps suppress noise amplification and prevents unstable updates when measurements are severely under-sampled or corrupted. Second, the embedded DnCNN acts as a learned proximal operator within the ADMM structure, enabling adaptive denoising and prior modeling beyond handcrafted regularizers, which is particularly effective under harsh noise conditions.

Finally, by reformulating the unfolding process as an implicit equilibrium solution, the DEQ framework allows the network to converge to a stable fixed point rather than relying on a fixed number of layers, thereby improving reconstruction stability and generalization across different sampling rates and SNR levels.

The main contributions of this work are summarized as follows:

1. We propose an ADnDEQ framework for sparse-aperture ISAR imaging, which reformulates an ADMM-based unfolding network as an implicit equilibrium problem. This design avoids explicit layer stacking and enables constant-memory inference with a single-layer equilibrium representation.
2. By embedding a learned denoising proximal operator within the ADMM-based equilibrium structure, the proposed framework provides a principled way to integrate data-driven priors into a physically interpretable optimization process, rather than relying on purely black-box architectures. The proposed ADnDEQ model exhibits remarkable robustness under extreme conditions, such as very low sampling ratios (10%) and harsh noise environments (e.g., 0 dB SNR), maintaining stable and accurate image reconstruction performance.
3. Owing to the equilibrium formulation and parameter-sharing mechanism, the proposed framework naturally decouples network structure from specific sampling ratios, enabling a single trained model to be flexibly applied across different sampling rates and noise levels without retraining.

Extensive experiments on both simulated and real-world ISAR data validate the superior reconstruction quality and computational efficiency of the proposed method, achieving better performance than representative methods with substantially fewer parameters and memory cost.

The structure of this paper is as follows. Section 2 reviews conventional techniques for processing SA-ISAR signals. Section 3 introduces the ADnDEQ method proposed in this study, providing a detailed explanation of its framework. Section 4 presents simulation and experimental results, followed by a comprehensive analysis. Finally, Section 5 offers the conclusions drawn from the findings.

2. Related Work

2.1. Traditional ADMM

Under ideal conditions, where no residual phase errors exist in the azimuth direction, the SA-ISAR imaging scene can be simplified as follows:

$$\mathbf{y} = \mathbf{D}\mathbf{F}\mathbf{X} + \mathbf{N} \quad (1)$$

where $\mathbf{y}$ denotes the distance and slow time domain signal, $\mathbf{D}$ denotes the undersampling matrix, $\mathbf{F}$ represents the Fourier matrix, $\mathbf{X}$ represents the imaging scene requiring reconstruction, and $\mathbf{N}$ represents random additive noise.

Under sparse aperture conditions, the azimuth dimension length l of the echo signal $\mathbf{y}$ is shorter than the azimuth dimension length M of the ISAR scene $\mathbf{X}$, resulting in a significant degradation in imaging resolution when employing conventional ISAR imaging methods, including RD techniques. This degradation fails to meet the required imaging standards.

The primary objective in tackling the challenges of SA-ISAR imaging is to reconstruct the original image from the limited, undersampled echo data. Given the sparsity of the target scene, SA-ISAR imaging can be conceptualized as a constrained optimization problem, where the introduction of regularization terms serves to induce sparsity in the

solution. Specifically, by adopting an optimization strategy within the LASSO framework, we can reformulate the problem in the following academic manner:

$$\hat{x} = \arg \min_x \|y - \mathbf{D}\mathbf{F}x\|_2^2 + \lambda \|x\|_1 \quad (2)$$

where λ is the regularization parameter. x represents the original image to be restored, and $\hat{x}$ represents the approximation of x . The problem is solved using the ADMM method, introducing an auxiliary variable z , and transforming Equation (2) into a constrained problem of the following form:

$$\hat{x} = \arg \min_x \|y - \mathbf{D}\mathbf{F}x\|_2^2 + \lambda \|z\|_1 \text{ s.t. } x = z \quad (3)$$

Then, the augmented Lagrangian function for Equation (3) can be written as follows:

$$L(x, z, u) = \|y - \mathbf{D}\mathbf{F}x\|_2^2 + \lambda \|z\|_1 + u^H(x - z) + \frac{\rho}{2} \|x - z\|_2^2 \quad (4)$$

where u and ρ represent Lagrange multipliers, then the specific iterative formula for ADMM can be written as:

$$x^{(k+1)} = \arg \min_x \|y - \mathbf{D}\mathbf{F}x\|_2^2 + \frac{\rho}{2} \|x - \tilde{x}^{(k)}\|_2^2 \quad (5)$$

$$z^{(k+1)} = \arg \min_z \|z\|_1 + \frac{\rho}{2} \|z - \tilde{z}^{(k)}\|_2^2 \quad (6)$$

$$\bar{u}^{(k+1)} = \bar{u}^{(k)} + (x^{(k+1)} - z^{(k+1)}) \quad (7)$$

In the above iterative steps, Equations (5)–(7) can all be regarded as a modular structure. Specifically, the reconstruction step is accomplished by Equation (5), the denoising step is done by Equation (6), and the parameter update step is performed by Equation (7). We define $\delta = \sqrt{\lambda/\rho}$. Equations (5)–(7) describe the standard ADMM updates for the SA-ISAR reconstruction problem. To facilitate network unfolding and equilibrium modeling, we adopt the scaled ADMM formulation by introducing the scaled Lagrange multiplier α . Accordingly, the original variables are rearranged by defining shifted auxiliary variables. With these definitions, the x , z , and u updates in Equations (5)–(7) can be equivalently rewritten in the compact form shown in Equation (8). Subsequently, each ADMM subproblem is mapped to a corresponding network operation, where the data-fidelity term leads to the linear update steps in Equations (9)–(15) and the regularization subproblem is modeled by a learned proximal operator, resulting in Equations (16)–(19). This transformation enables a one-to-one correspondence between ADMM iterations and network mappings, which further allows the unfolding process to be reformulated as an implicit equilibrium problem. By incorporating it into the practical SA-ISAR scenario, it can be rewritten in matrix form as follows:

$$\mathbf{X}^{(k+1)} = \left[\mathbf{F}^H \mathbf{D}^H \mathbf{y} + \rho (\mathbf{Z}^{(k)} - \mathbf{U}^{(k)}) \right] \% (\text{Mask} + \rho \mathbf{1}_{M \times N}) \quad (8)$$

$$\mathbf{Z}^{(k+1)} = \underset{\mathbf{Z}}{\operatorname{argmin}} \frac{1}{2\delta^2} \|\mathbf{Z}^{(k)} - \tilde{\mathbf{Z}}^{(k)}\|_2^2 + \|\mathbf{Z}^{(k)}\|_1 \quad (9)$$

$$\bar{\mathbf{U}}^{(k+1)} = \bar{\mathbf{U}}^{(k)} + \mathbf{X}^{(k+1)} - \mathbf{Z}^{(k+1)} \quad (10)$$

where $\mathbf{U}$, $\bar{\mathbf{U}}$, $\mathbf{Z}$, $\tilde{\mathbf{Z}}$, and $\mathbf{y}$ denote the matrix. The symbol $\%$ is used to denote a shifted auxiliary variable in the scaled ADMM formulation, following the notation in [22]. $\mathbf{1}_{M \times N}$ is a matrix whose elements of $M \times N$ all valued 1. The sampling mask for sparse data is

represented by a mask containing 0 or 1. For an image $\tilde{\mathbf{Z}}^{(k)}$, subproblem (9) is equivalent to a noise suppression problem. Step (8) can usually be solved using a closed-form solution for a first-order stationary point. However, due to the inherent non-convexity of l_1 regularization, gradient-based methods face significant challenges in solving step (9). Thus, incorporating neural network-based methods is essential to enhance the effectiveness of the noise suppression process.

2.2. DnADMM-Net

The traditional ADMM strikes a favorable balance between algorithmic efficacy and computational expense. However, the necessity for parameter tuning in ADMM when confronted with data originating from diverse observational contexts poses constraints on its practical deployment in real-world scenarios. To address this limitation, the DnADMM-Net framework incorporates supervised learning methodologies to automatically ascertain optimal parameter values, thereby enhancing overall performance.

The ADMM algorithm is highly effective for addressing a wide range of large-scale linear inverse problems. Traditional ADMM addresses the CS reconstruction problem within the equation. When applied to image processing problems, Equation (9) can be viewed as an image denoiser and is also the core of ADMM. Therefore, it can be improved by deep learning neural networks to enhance denoising operations. Thus, Equation (9) is rewritten as follows:

$$\mathbf{Z}^{(k)} = D_{\delta}(\tilde{\mathbf{Z}}^{(k-1)}) \quad (11)$$

where D_{δ} represents a neural network image denoiser.

Through this process, the ADMM algorithm is deeply unfolded into a neural network. The denoising operation in Equation (9) is replaced by a denoising network, represented by Equation (11), at each iteration step. By leveraging the deep learning capabilities of neural networks, the parameters of the denoising network are optimized during each iteration. The iterative process between Equations (8) and (10) is thus transformed into a cascaded structure within the ADMM network. The three key stages of the traditional ADMM algorithm—data reconstruction, denoising, and Lagrange multiplier updates—are each expanded into distinct layers within the ADMM network: the reconstruction layer, the denoising layer, and the Lagrange multiplier update layer. The variables $\mathbf{X}^{(k)}$, $\mathbf{Z}^{(k)}$, $\mathbf{U}^{(k)}$ and represent the outputs of these layers, respectively. Figure 1 illustrates the network block diagram of DnADMM-Net, and Figure 2 illustrates the architecture of the Denoising Convolutional Neural Network (DnCNN).

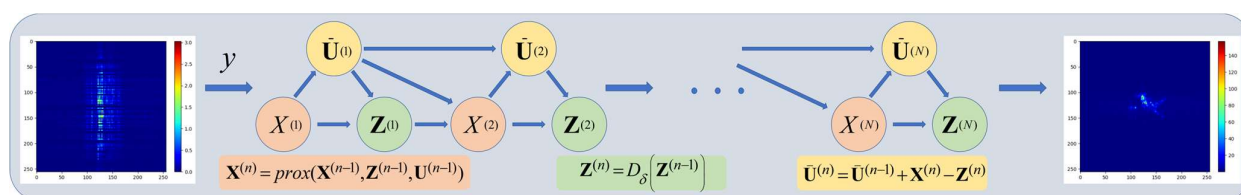


Figure 1. DnADMM-Net typically employs multilayer stacked structures.

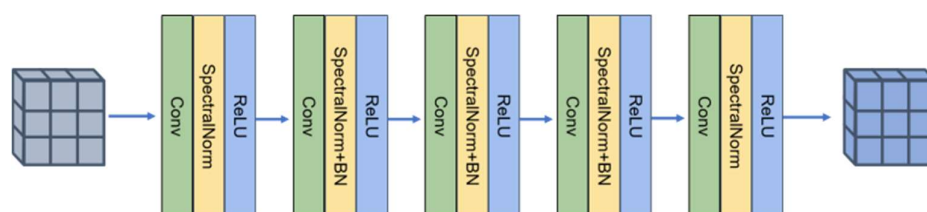


Figure 2. Schematic Diagram of DnCNN Architecture.

The detailed implementation of the DnADMM-Net is outlined below (Algorithm 1):

Algorithm 1 DnADMM-Net

```

1: Initialized:  $\mathbf{X}^{(0)} = \mathbf{Z}^{(0)} = \mathbf{U}^{(0)} = \mathbf{0}, \mathbf{D}, \mathbf{F}, \rho, \text{Mask}$ 
2: for:  $k = 1, 2, \dots$ 
3:    $\mathbf{X}^{(k+1)} = \text{prox}(\mathbf{X}^{(k)}, \mathbf{Z}^{(k)}, \mathbf{U}^{(k)})$ 
4:    $\mathbf{Z}^{(k+1)} = D_{\delta}(\tilde{\mathbf{Z}}^{(k)})$ 
5:    $\mathbf{U}^{(k+1)} = \mathbf{U}^{(k)} + \mathbf{X}^{(k+1)} - \mathbf{Z}^{(k+1)}$ 
6: End while

```

2.3. Deep Equilibrium Framework

In the context of the majority of deep learning models, the processing flow of the input is:

$$\mathbf{q}^{[i+1]} = f_{\theta}^{[i]}(\mathbf{q}^{[i]}; \mathbf{y}) \quad i = 0, 1, \dots, L \quad (12)$$

where i is utilized to signify a particular and associated network layer, $\mathbf{q}^{[i+1]}$ represents the intermediary variable that corresponds to the i -th layer, whereas $f_{\theta}^{[i]}$ denotes a nonlinear transformation parameterized by θ . The input to the model is designated as $\mathbf{y}$, and θ represents the parameters of the transformation f . As illustrated in Figure 3a, this iterative structure enables the state of preceding layers to exert a direct influence on subsequent layers, thereby fostering a hierarchical arrangement that facilitates the model's ability to discern progressively intricate patterns. However, as proposed in [10], by incorporating the concept of weight tying, the aforementioned equation can be reformulated as:

$$\mathbf{q}^{[i+1]} = f_{\theta}(\mathbf{q}^{[i]}; \mathbf{y}), i = 0, \dots, L \quad (13)$$

as i approaches infinity:

$$\lim_{i \rightarrow \infty} \mathbf{q}^{[i]} = \lim_{i \rightarrow \infty} f_{\theta}(\mathbf{q}^{[i]}; \mathbf{y}) \equiv f_{\theta}(\mathbf{q}^*; \mathbf{y}) = \mathbf{q}^* \quad (14)$$

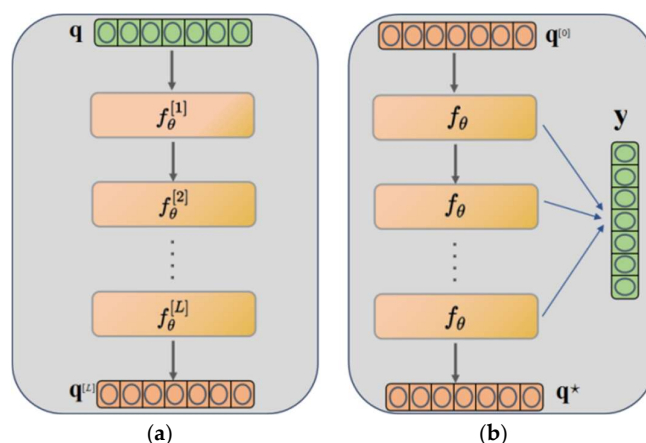


Figure 3. Principle and structural representation of the DEQ framework. (a) The structure of a conventional deep learning model obtains the final result through the multi-layer nonlinear transformation of the input variable. Typical examples include DnCNN, etc. (b) The “infinite layer” weight-tied DEQ model is equivalent to fixed-point iteration since the weights are the same for each layer of the model.

Assuming network convergence, implementing indefinite recursion in a single-layer neural network, as shown in Figure 3b, stabilizes the output at a fixed point $\mathbf{q}^*$. This method significantly reduces model parameters without compromising expressive capacity.

Recursion extracts hierarchical features and represents complex patterns through iterative transformations, ensuring effective information encoding/decoding despite reduced parametric complexity. This simplifies training, potentially decreases convergence time, and lowers overfitting risk. Preserving expressive capacity ensures model generalization to unseen data, balancing complexity and performance.

3. Methodology

This section presents the proposed ADnDEQ framework for sparse-aperture ISAR imaging. Starting from the SA-ISAR measurement model, the reconstruction problem is formulated using an ADMM-based optimization framework. The iterative ADMM updates are then unfolded and reformulated as an implicit deep equilibrium problem, where a lightweight DnCNN is embedded as a learned proximal operator. Finally, the equilibrium solution is efficiently obtained using fixed-point iterations with Anderson acceleration. Unlike conventional unfolded networks that explicitly stack multiple layers with untied parameters, the proposed ADnDEQ employs a single nonlinear mapping f_θ whose parameters are shared across all iterations. The forward inference is performed by implicitly solving for the equilibrium state of this mapping, rather than by traversing a predefined number of layers. As a result, the proposed method exhibits single-layer parameterization while achieving infinite-depth behavior in an implicit manner, leading to significant reductions in memory cost and improved flexibility during inference.

3.1. Deep Equilibrium ADMM

Based on the SA-ISAR signal model, we formulate the image reconstruction problem using an ADMM-based optimization framework. This formulation serves as the foundation for the proposed equilibrium-based unfolding network. Here, we describe choices of f_θ that explicitly account for the inverse problem. In this study, we propose the selection of the mapping function f_θ through various optimization algorithms applied to the regularized least squares problem. While this approach shares similarities with deep unfolding techniques, it uniquely enables the accommodation of an infinite number of iterations, a capability not previously achievable by existing deep unrolling architectures for solving inverse imaging problems.

The ADMM is an effective first-order algorithm designed for solving large-scale constrained optimization problems. By reformulating it as an equivalent constrained problem, ADMM can be applied to unconstrained optimization problems. Furthermore, the ADMM algorithm can be integrated with deep networks through unfolding, as illustrated in Equations (8)–(11). Consequently, we can combine the unfolded ADMM with the DEQ framework.

Hence, the aforementioned update rules can be conceptualized as fixed-point iterations applied to the combined variable set $\mathbf{q} = (\mathbf{z}, \mathbf{u})$. In this context, the iteration mapping $f_\theta(\mathbf{q}; \mathbf{y})$ is implicitly characterized as the function that fulfills the condition specified by the update equations.

$$\mathbf{q}^{(k+1)} = f_\theta(\mathbf{q}^{(k)}; \mathbf{y}) \text{ with } \mathbf{q}^{(k)} = (\mathbf{z}^{(k)}, \mathbf{u}^{(k)}) \quad (15)$$

As the incremental contribution of each subsequent layer to the final output diminishes with an increasing number of iterations, the pursuit of infinite iterations to attain a fixed point becomes unnecessary. However, achieving convergence through straightforward fixed-point iterations is frequently associated with considerable computational demands and susceptibility to instability. To address these challenges, advanced methods can be used. Specifically, as illustrated in Equation (15), root-finding algorithms, such as the Anderson Acceleration (AA) and Broyden's method, present viable alternatives. These algorithms

can be employed to expedite the convergence process towards the forward propagation equilibrium point, starting from an initial estimate $\mathbf{q}^{[0]}$.

$$\begin{aligned} \mathbf{q}^* &= \text{RootFind}(h_\theta; \mathbf{y}) \\ \text{s.t. } h_\theta(\mathbf{q}^*; \mathbf{y}) &= f_\theta(\mathbf{q}^*; \mathbf{y}) - \mathbf{q}^* \end{aligned} \quad (16)$$

To solve the equilibrium point efficiently, we define the residual operator h_θ , which converts the fixed-point problem into a standard root-finding problem, enabling the use of Anderson acceleration. However, the utilization of a black-box solver to simulate an indefinitely large number of fixed-point iterations within the forward propagation framework, with the intent of directly attaining the fixed point, poses a significant obstacle to the implementation of explicit backpropagation. Specifically, this methodology hinders the straightforward application of traditional gradient-based optimization techniques during the backward pass, which are crucial for training deep learning models. To circumvent this intricate issue, the DEQ framework adopts the principle of implicit differentiation as an alternative to explicit backpropagation. This strategic shift not only streamlines the computational process but also ensures a consistent conservation of memory resources, thereby enhancing the overall efficiency and scalability of the training procedure.

Implicit differentiation, in the context of DEQ, facilitates the computation of gradients without the need to explicitly store and manipulate the entire sequence of fixed-point iterations. Instead, it leverages the implicit function theorem to derive the gradient of the solution to the fixed-point equation with respect to the model parameters directly. This approach circumvents the potential memory explosion that would otherwise occur if one were to attempt to store the intermediate states of an infinitely long iteration sequence. Moreover, implicit differentiation harmonizes with the continuous-time dynamics inherent in differential equations.

Firstly, denote $\ell(\mathbf{q}^*; \mathbf{q}^{\text{label}})$ as ℓ . The gradient of ℓ with respect to the network parameters θ can be expressed as:

$$\frac{d\ell}{d\theta} = \frac{d\mathbf{q}^{*T}}{d\theta} \frac{d\ell}{d\mathbf{q}^*} \quad (17)$$

In the ADnDEQ model, which uses mean squared error (MSE) loss, the second term in (17) is represented as:

$$\frac{d\ell}{d\mathbf{q}^*} = \mathbf{q}^* - \mathbf{q}^{\text{label}} \quad (18)$$

Taking the derivative of both sides of $f_\theta(\mathbf{q}^*; \mathbf{y}) = \mathbf{q}^*$ and solving the resultant equation yields:

$$\frac{d\mathbf{q}^*}{d\theta} = \left(\mathbf{I} - \left. \frac{df_\theta(\mathbf{q}; \mathbf{y})}{d\mathbf{q}} \right|_{\mathbf{q}=\mathbf{q}^*} \right)^{-1} \frac{df_\theta(\mathbf{q}^*; \mathbf{y})}{d\theta} \quad (19)$$

This observation underscores the feasibility of conceptualizing the backpropagation process in the context of an “infinite” superposition of neural network layers as a streamlined, single-step operation that leverages matrix multiplication. This matrix multiplication inherently involves the utilization of a balanced Jacobian determinant, thereby encapsulating the complex interplay of gradients across the entire network architecture in a more manageable form. Specifically, the rightmost term within (19) can be efficiently computed through the deployment of an advanced automatic differentiation framework, which automates the gradient computation process and facilitates the handling of intricate mathematical expressions.

However, it is worth noting that the computation of the inverse of the Jacobian matrix remains a computationally intensive and resource-demanding task. This challenge arises due to the high dimensionality and complexity of the Jacobian matrix, which often

necessitates significant computational resources and time for its inversion. To circumvent this issue, an indirect approach to solving for the Jacobian matrix can be devised by focusing on the solution of a specific equation. This equation serves as a proxy for the direct computation of the Jacobian inverse, thereby enabling the extraction of the required gradient information in a more efficient and computationally feasible manner.

$$g = \left(\frac{df_{\theta}(\mathbf{q}^*; \mathbf{y})}{d\mathbf{q}^*} \right)^T g + \frac{d\ell}{d\mathbf{q}^*} \quad (20)$$

The solution to Equation (20) corresponds in part to the product of the Jacobian and MSE gradient as shown in Equation (18):

$$g^* = \left(\mathbf{I} - \frac{df_{\theta}(\mathbf{q}; \mathbf{y})}{d\mathbf{q}} \bigg|_{\mathbf{q}=\mathbf{q}^*} \right)^{-T} \frac{d\ell}{d\mathbf{q}^*} \quad (21)$$

where $-T$ denotes the inverse of the transposed matrix. To further elevate the efficiency of the solution methodology, the resolution of Equation (20) can be significantly accelerated through the utilization of the AA. This advanced technique facilitates a more streamlined and efficient handling of both the forward and backward propagation phases, thereby minimizing resource consumption while concurrently maintaining high computational efficiency.

3.2. ADnDEQ Model

DnCNN is a typical deep network structure, which employs CNNs as denoisers; we select a layer of DnCNN to serve as the nonlinear transformation f_{θ} in DEQ:

$$f(\mathbf{z}) = \mathbf{z} - \text{DnCNN}(\mathbf{z}) \quad (22)$$

where $\text{DnCNN}(\mathbf{z})$ is a multi-layer convolutional network, with the operations of each layer described as follows:

$$f_1(\mathbf{z}) = \text{ReLU}(W_1 * \mathbf{z}) \quad (23)$$

$$f_i(\mathbf{z}) = \text{ReLU}(\text{SpectralNorm}(W_i * f_{i-1}(\mathbf{z}))) \quad (24)$$

$$f_L(\mathbf{z}) = W_L * f_{L-1}(\mathbf{z}) \quad (25)$$

where W_1 , W_i , and W_L denote convolutional layers configured with a kernel size of 3 and comprising 64 output channels. The ReLU serves as the nonlinear activation function employed within the network, whereas SpectralNorm facilitates batch normalization. To accommodate variations in the number of channels at the network's input and output, channel-tuning convolutional layers are strategically positioned at both ends. These layers employ a kernel size of 3 and are designed to transition the channel dimension from 1 to 64 at the entrance and revert it back to 1 at the exit.

Given the inherent complexity of ISAR data, preprocessing steps are essential to facilitate network processing. Consequently, the ISAR data is decomposed into its real and imaginary components prior to network input and subsequently recombined post-processing.

$$\mathbf{y} = [\text{Re}(\mathbf{y}); \text{Im}(\mathbf{y})] \quad (26)$$

The reconstructed network then generates complex ISAR data by recombining these real and imaginary components. Figure 4 illustrates the training process of the DEQ framework, and Figure 5 depicts the network architecture of the ADnDEQ model. The detailed

implementation of the ADnDEQ is outlined below (Algorithm 2):

Algorithm 2 ADnDEQ Method

- 1: Input: observed data $\mathbf{y}$; transformation matrix $\Phi = \mathbf{D}\mathbf{F}$; auxiliary variable $\mathbf{u}^{[0]}$ set to zero; initial value $\mathbf{z}^{[0]} = \Phi^T \mathbf{y}$, fixed-point iteration equation f_θ , storage size $m = 5$, minimum relative error 0.01, calculated from $\|\mathbf{z}^{[k]} - \mathbf{z}^{[k-1]}\| / \|\mathbf{z}^{[k-1]}\|$
 - 2: Initialization: Concatenate $\mathbf{z}^{[0]}$ and $\mathbf{u}^{[0]}$ into a single vector $\mathbf{q}^{[0]}$.
 - 3: DEQ Fixed-Point Iteration: Begin the iterative process to find the fixed point of the DEQ-inspired system.
 - 4: Within Each Iteration:
 ADMM Basic Block:
 x-update: Compute the updated value of $\mathbf{x}$ based on the current estimates of $\mathbf{z}$ and $\mathbf{u}$.
 z-update: Compute the updated value of using a DnCNN denoiser.
 u-update: Update the auxiliary variable $\mathbf{u}$.
 Anderson Acceleration:
 Apply Anderson Acceleration to accelerate convergence by minimizing the residual error in the fixed-point iteration. Formulate and solve the least squares problem to find optimal coefficients α_i such that $\underset{\alpha}{\text{minimize}} \|\mathbf{A}\alpha\|_2^2$ s.t. $\sum_{i=1}^m \alpha_i = 1$.
 Update $\mathbf{q}^{k+1}$ using the weighted average of previous iterates and function evaluations:

$$\mathbf{q}^{k+1} = \sum_{i=1}^m \alpha_i f(\mathbf{q}^{k-i+1}, \mathbf{y}).$$
 - 5: Convergence Detection: After each iteration, check the relative error and the number of iterations against the predefined convergence criteria.
 - 6: End: If the convergence criteria are met, terminate the iteration and output the final recovered signal or image $\mathbf{z}$. If not, continue to the next iteration until the maximum number of iterations is reached or convergence is achieved.
-

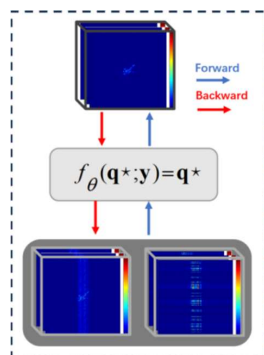


Figure 4. Training Process of the DEQ framework.

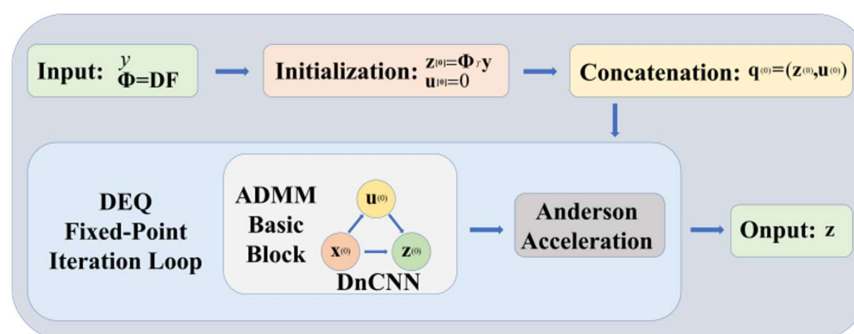


Figure 5. ADnDEQ.

4. Experiments

4.1. Simulation Data Generation

Due to the lack of publicly accessible ISAR datasets caused by data confidentiality, the application of deep learning techniques in this domain is significantly constrained. To address this challenge, we propose a simulation-based method for generating ISAR data using 3D point cloud data from ShapeNet. This approach is grounded in the principle that ISAR imaging fundamentally involves the linear projection of three-dimensional scattering point echoes onto a two-dimensional plane. Considering that the scattering points in ISAR targets are more sparsely distributed compared to those in the point cloud dataset, and their echo intensities exhibit variability, we have designed a preprocessing technique for point cloud data, as outlined in Equation (27):

$$\begin{cases} \eta_i \sim |\text{Mask}_i N(0, \beta_i)|, i = 1, 2, \dots, L \\ \text{Mask}_i \sim B(L, \rho) \end{cases} \quad (27)$$

We utilize a Bernoulli–Gaussian distribution to create a random sparse matrix, denoted as Mask_i , with binary values of 0 or 1. Here, L represents the total number of point clouds, while ρ signifies the density of the point clouds, which is configured as 0.07 in our simulations. At this density level, the ISAR imaging outcomes of the simulated data exhibit a closer resemblance to actual measured data. Assuming an initial scattering point intensity of 1 in the point cloud dataset, the echo intensity of the scattering points is randomly drawn from a standard normal distribution, expressed as $\beta_i = 1$. The final point cloud mask matrix is symbolized by η .

The point cloud data used in this study are generated based on a standard ISAR echo simulation model, where the target is represented by a set of discrete scattering centers. Each scattering point is characterized by its spatial location in the range–azimuth plane and a complex-valued reflectivity coefficient. Sparse-aperture observations are obtained by applying random azimuthal undersampling to the simulated full-aperture echo signals. The corresponding full-aperture ISAR image is reconstructed using the conventional RD algorithm and serves as a reference benchmark. To ensure numerical stability and compatibility with the ADMM-based unfolding framework, a preprocessing step is applied as described in Equation (27), which performs normalization and structural reorganization of the sparse echo-derived representation without introducing additional prior information.

The training and test datasets are constructed independently. The training data are generated from synthetic point cloud models, where scattering centers are randomly distributed with varying sparsity levels and complex amplitudes. This strategy enables the network to learn a target-agnostic sparse reconstruction prior. The test data are derived from measured ISAR echoes of the YK-42 aircraft and are used exclusively for evaluation. No YK-42 samples are involved in the training process.

In the radar imaging simulation framework, both azimuth dimensions are set to 256. The system operates with a carrier frequency of 9 GHz and a bandwidth of 400 MHz. The pulse width is 0.0256 ms, and the pulse repetition frequency is fixed at 100 Hz. Assuming that the translational motion of the target is sufficiently compensated, the target rotates with a constant angular velocity of 0.02 rad/s at a reference distance of 5 km. To enhance the realism of the simulation, the scattering points are subjected to minor angular deviations along the three-dimensional axes. High-resolution ISAR images are derived by applying the RD algorithm to the full aperture data, which are subsequently used as ground truth labels for model training. Furthermore, to simulate practical noise scenarios, 20 dB of noise

is incorporated into the simulated signals during the training phase. Figure 6a–c show the generation process of simulation data from point cloud to radar data.

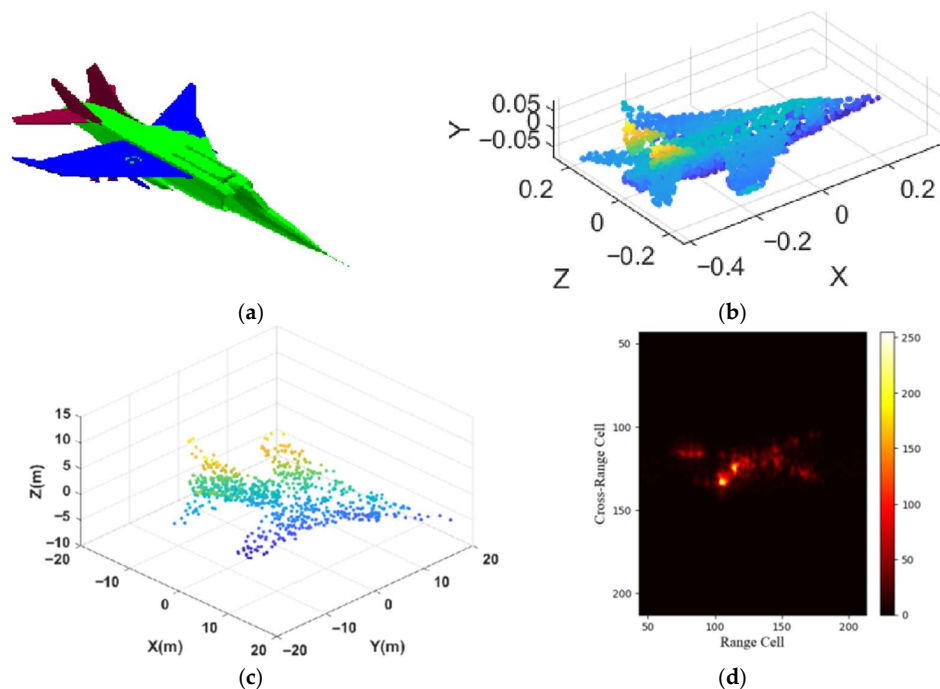


Figure 6. (a) Structural model. (b) ShapeNet 3D point cloud model. (c) Sparsely and randomly deflected point cloud model. (d) The full-aperture (FA) RD imaging results.

4.2. Experimental Details

The proposed ADnDEQ model is composed of three core components: an ADMM-based iterative module (BasicBlock), a denoising sub-network (DnCNN), and a deep equilibrium (DEQ) framework. Specifically, the denoising sub-network is implemented as a compact DnCNN structure with five convolutional layers, including an input layer, three intermediate layers with batch normalization and ReLU activations, and one output layer. The total number of learnable parameters is approximately 112,130, which includes the DnCNN sub-module and two scalar parameters: regularization weight and step-size factor used in the ADMM update. The DEQ module utilizes an AA solver with a maximum of 50 iterations and a convergence tolerance of 0.01. The software used in the experiment is Python 3.8.0 under PyTorch.

The model was trained from scratch using the Adam optimizer with an initial learning rate of 0.001. A StepLR scheduler was employed to reduce the learning rate by a factor of 0.5 every 10 epochs. The training lasted for 20 epochs due to computational constraints, with each epoch consisting of a full pass through the dataset. The batch size was set to 1 owing to the large memory footprint of 512×256 ISAR images. The MSE loss function was used throughout the training process. A total of 1200 data samples were used, with 90% randomly selected for training and 10% for validation.

To ensure a fair performance comparison, all baseline models (ADMM-Net, PIN, and AD-SRNet) were retrained under the same data partitioning and hardware environment. The same optimizer (Adam) and initial learning rate were adopted for all models. Their learning rate adjustment strategies followed their respective original implementations. All comparison models were faithfully reproduced based on their original architectures and hyperparameters as described in their respective publications.

Comparative analyses were performed between ADnDEQ, conventional SA-ISAR imaging techniques, and alternative deep learning architectures such as the RD method,

ADMM optimization framework, ADMM-Net, PIN, and AD-SRNet. To ensure experimental consistency across different approaches, all deep learning implementations adopted the MSE loss function. Identical training datasets and hardware configurations were maintained throughout all comparative experiments to guarantee methodological fairness. Figure 6d illustrates the full aperture (FA) RD imaging results.

4.3. Comparison of Imaging Results

This section evaluates the imaging performance of the aforementioned methods on YK-42 ISAR echo data under two noise environments (20 dB regular SNR and 0 dB low SNR) with three sampling conditions: 50% regular sparsity, 25% regular sparsity, and 10% extreme sparsity. The quantitative metrics used for evaluation include peak signal-to-noise ratio (PSNR) and structural similarity index (SSIM), with higher values indicating better image quality, as well as normalized mean square error (NMSE) and image entropy (ENT), where lower values signify improved reconstruction accuracy. The parameter count of each deep learning model is also analyzed. As shown in Figure 7a,g,m and Figure 8a,g,m, the RD imaging results exhibit pronounced strip-like artifacts due to the phase coherence degradation caused by SA. In comparison, Figure 7b,h,n and Figure 8b,h,n demonstrate that the CS-based ADMM method partially suppresses these artifacts under regular sparsity conditions, but results in noticeable degradation of critical target features. Notably, under 10% extreme sparsity and 0 dB low-SNR scenarios, the insufficient noise suppression capability leads to severe information loss, rendering the results impractical for engineering applications.

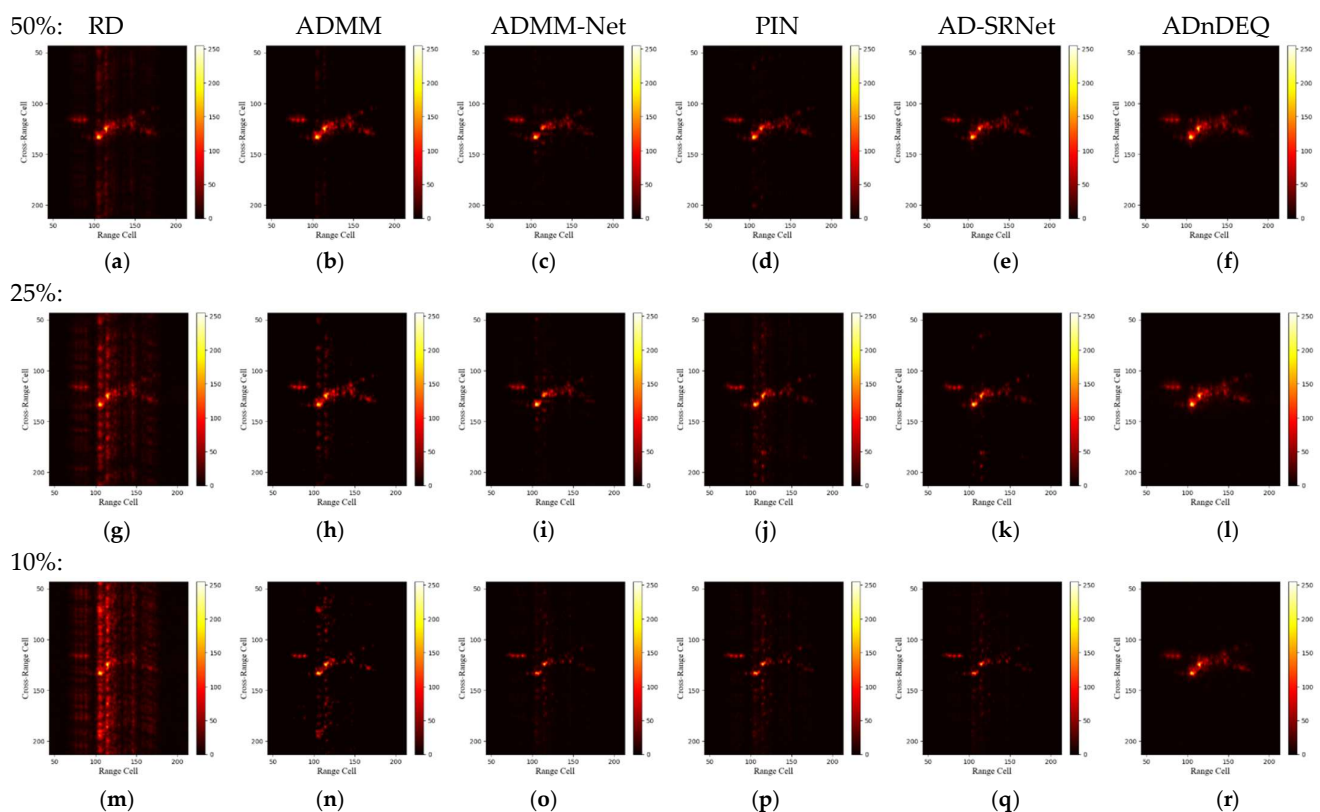


Figure 7. SA-ISAR imaging results under different conditions. (SNR = 20 dB). In this paper, the percentages (50%, 25%, and 10%) denote the azimuth sampling ratios, meaning that only the corresponding proportion of azimuth pulses from the full-aperture data is retained through random undersampling.

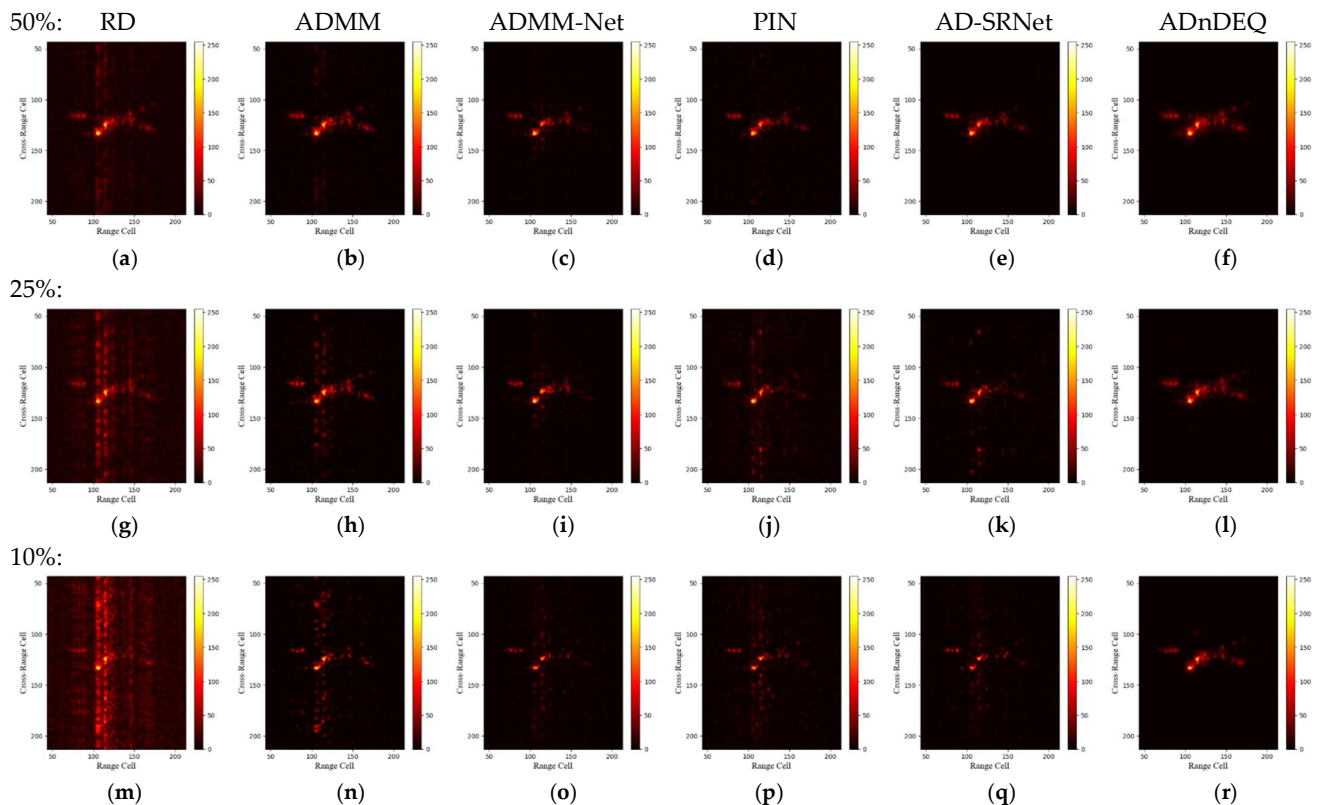


Figure 8. SA-ISAR imaging results under different conditions. (SNR = 0 dB).

Among deep learning approaches, ADMM-Net, PIN, and AD-SRNet achieve satisfactory imaging quality under regular sparsity and SNR conditions, as depicted in Figures 7 and 8. However, three principal limitations are identified:

ADMM-Net (Figure 7c,i,o and Figure 8c,i,o) suffers from structural prior constraints, requiring high-quality training data to maintain performance. Its rigid mathematical formulation struggles to model non-structured targets effectively, often producing incomplete reconstructions or phantom artifacts when handling complex scattering scenarios.

PIN (Figure 7d,j,p and Figure 8d,j,p), despite its parallel iterative framework reducing dependency on pristine training data, shows limited robustness under extreme sparsity and low SNR conditions. Additionally, its large parameter count imposes significant computational overhead and training convergence challenges.

AD-SRNet (Figure 7e,k,q and Figure 8e,k,q) exhibits interpretability limitations due to its black-box architecture, making it difficult to track error sources during imaging. Moreover, the dual module series connection leads to a lengthy process and increases structural complexity. These hinder system optimization and reliability verification in critical task applications.

In contrast, Figure 7f,l,r and Figure 8f,l,r present the imaging results of ADnDEQ, which demonstrates superior capability in suppressing stripe noise while preserving critical features under low-SNR conditions. Remarkably, it maintains high imaging fidelity even at 10% compression rates. Quantitative validation in Tables 1 and 2 confirms ADnDEQ's advantages in reconstruction accuracy, and robustness.

Table 1. Performance Evaluation of Yk42 ISAR Imaging Results Across Various Metrics (SNR = 20 dB). (Bold font indicates optimal performance).

CS Ratio	Image Method	PSNR/ dB	SSIM	NMSE	ENT
50%	RD	35.1567	0.7514	0.9107	2.4335
	ADMM	42.6714	0.8261	0.5123	1.9436
	ADMM-Net	53.7163	0.9002	0.1261	1.0381
	PIN	53.3367	0.9127	0.0644	0.9467
	AD-SRNet	54.1265	0.9141	0.0902	0.8701
	ADnDEQ	56.7429	0.9883	0.0013	0.5153
25%	RD	30.3020	0.6480	2.7842	3.0700
	ADMM	37.5128	0.7866	1.7850	2.3670
	ADMM-Net	42.8362	0.8899	0.8423	1.9987
	PIN	41.3004	0.8743	0.9881	1.5127
	AD-SRNet	43.6712	0.8632	0.7563	1.3536
	ADnDEQ	49.0503	0.9754	0.0370	0.6221
10%	RD	26.4564	0.5389	6.7540	3.5672
	ADMM	32.4877	0.7466	4.3601	2.8102
	ADMM-Net	37.5044	0.8210	3.0089	2.2344
	PIN	38.7963	0.8611	3.1307	1.3910
	AD-SRNet	39.1931	0.8554	3.0119	1.4101
	ADnDEQ	45.7846	0.9471	0.3883	0.8404

Table 2. Performance Evaluation of Yk42 ISAR Imaging Results Across Various Metrics (SNR = 0 dB). (Bold font indicates optimal performance).

CS Ratio	Image Method	PSNR/ dB	SSIM	NMSE	ENT
50%	RD	30.3687	0.1789	2.2729	3.9083
	ADMM	37.1845	0.5424	1.5463	2.3889
	ADMM-Net	44.9324	0.8727	0.2298	1.6560
	PIN	45.6258	0.9034	0.1002	1.1165
	AD-SRNet	46.5982	0.9100	0.1133	1.0006
	ADnDEQ	49.4561	0.9705	0.0039	0.5777
25%	RD	26.3670	0.0913	6.8764	4.5233
	ADMM	33.6012	0.5583	3.0276	3.0133
	ADMM-Net	39.9050	0.7733	1.0006	2.3914
	PIN	40.5130	0.7941	1.2145	1.7345
	AD-SRNet	42.8292	0.7391	0.9171	1.5207
	ADnDEQ	46.8851	0.9499	0.0611	0.7533
10%	RD	22.6111	0.0439	16.3187	5.1021
	ADMM	29.3047	0.3341	10.7100	3.6519
	ADMM-Net	33.4061	0.7214	4.0423	2.8140
	PIN	33.3951	0.6993	5.5312	1.7948
	AD-SRNet	34.0194	0.7140	3.1467	1.7882
	ADnDEQ	42.1329	0.9270	0.4827	0.9349

4.4. Balance Between Efficiency and Performance

Notably, the DEQ framework, grounded in convex optimization algorithms, enables ADnDEQ to converge within dozens of training epochs—significantly fewer than the hundreds of epochs required by conventional models. This advantage is particularly critical, as increasing model depth and parameter count in traditional architectures substantially escalates training demands, including training duration and computational resource con-

sumption. As illustrated in Figure 9, ADnDEQ demonstrates remarkable training efficiency, achieving convergence with significantly fewer training epochs compared to conventional methods.

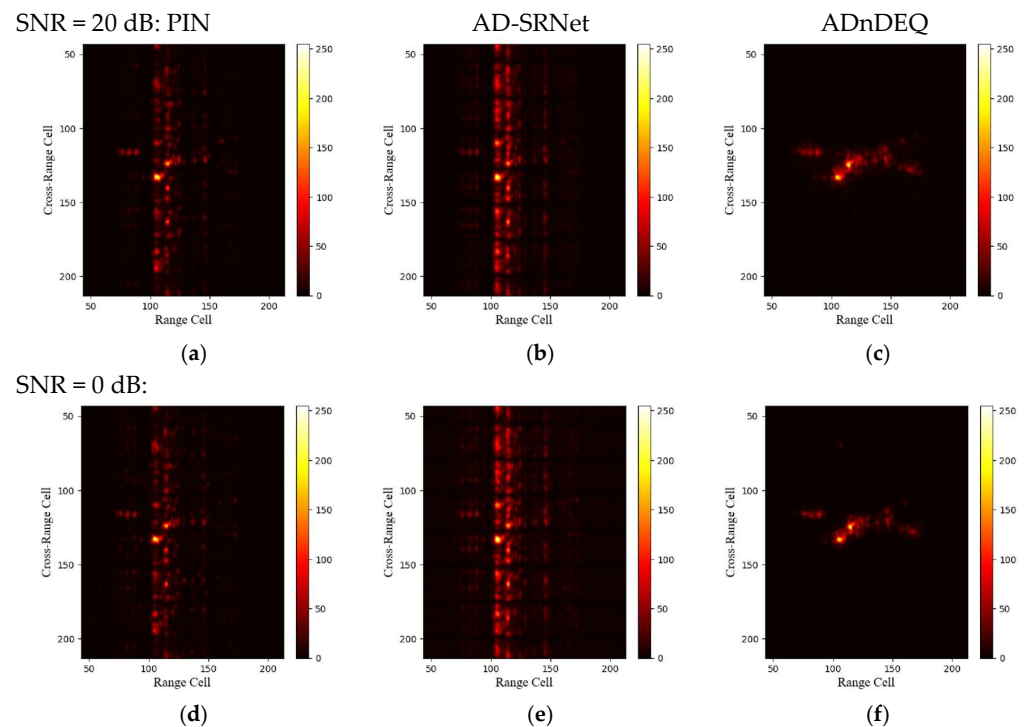


Figure 9. Reconstruction results of different deep networks under the condition of 20 epoch. (CS ratio = 10%).

To rigorously evaluate its robustness under constrained computational resources, we designed experiments with the following parameters: 20 training epochs (simulating limited computational budgets), a 10% undersampling ratio (extreme sparsity), and SNR levels of 20 dB and 0 dB (high-noise environments). Under these adversarial conditions, we comprehensively assessed the imaging performance of PIN, AD-SRNet, and ADnDEQ. As shown in Figure 9, under limited training epochs, only ADnDEQ maintained high reconstruction fidelity, whereas other models suffered severe performance degradation. This empirical validation confirms that ADnDEQ can achieve high-quality imaging under extreme sparsity and high noise levels without excessive computational overhead.

Beyond its advantages in model training, ADnDEQ also exhibits efficiency benefits during inference. Unlike deep learning models with fixed computational graphs, ADnDEQ allows users to dynamically adjust iteration strategies to balance computational efficiency with imaging quality. As illustrated in Figure 10, incremental iterations progressively enhance imaging performance, yielding improved noise suppression and reconstruction fidelity. The results indicate a significant denoising effect from 5 to 50 iterations. However, as shown in Figure 10h, further increasing the runtime leads to negligible quality improvements. This observation is further substantiated by the data presented in Figure 11. Therefore, within a reasonable range of iteration counts, more iterations lead to better imaging performance when computational efficiency is not a primary constraint, whereas fewer iterations enhance efficiency when imaging quality requirements are lower.

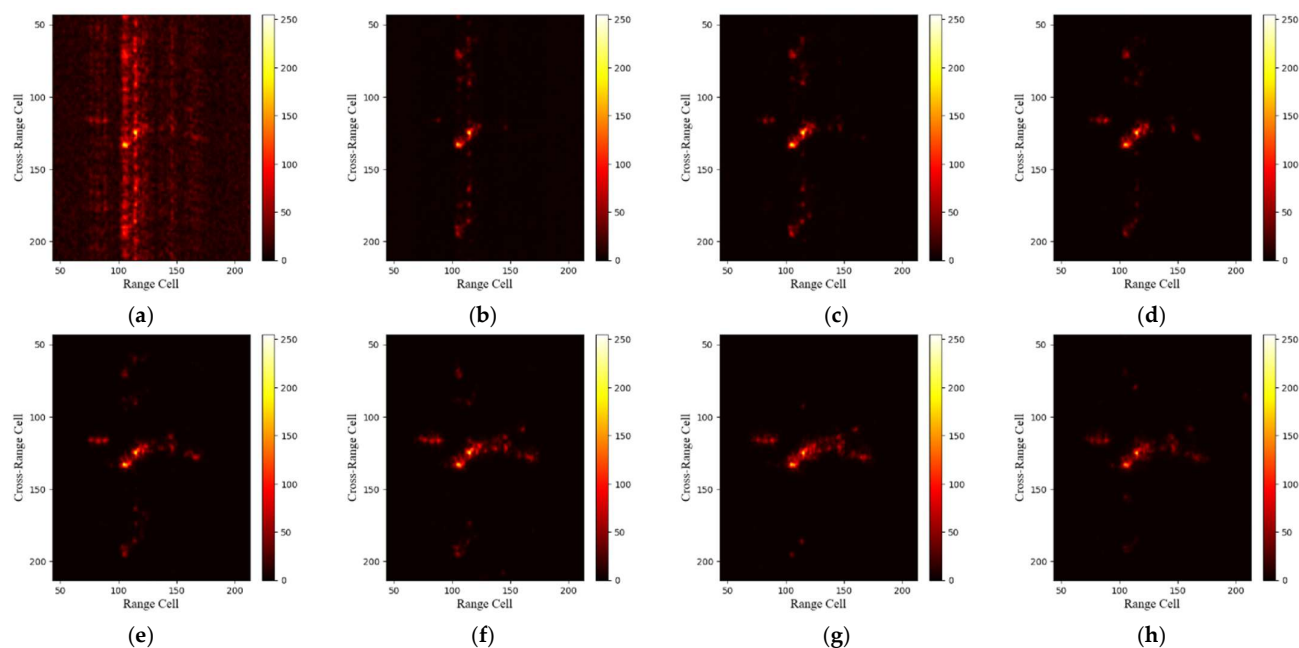


Figure 10. Reconstruction results of ADnDEQ with AA under 10% undersampling ratio, SNR = 0 dB, evaluated across varying iteration counts. (a) RD reconstruction; (b–h) reconstructed outputs after 5, 10, 15, 20, 25, 30 and 50 iterations, respectively.

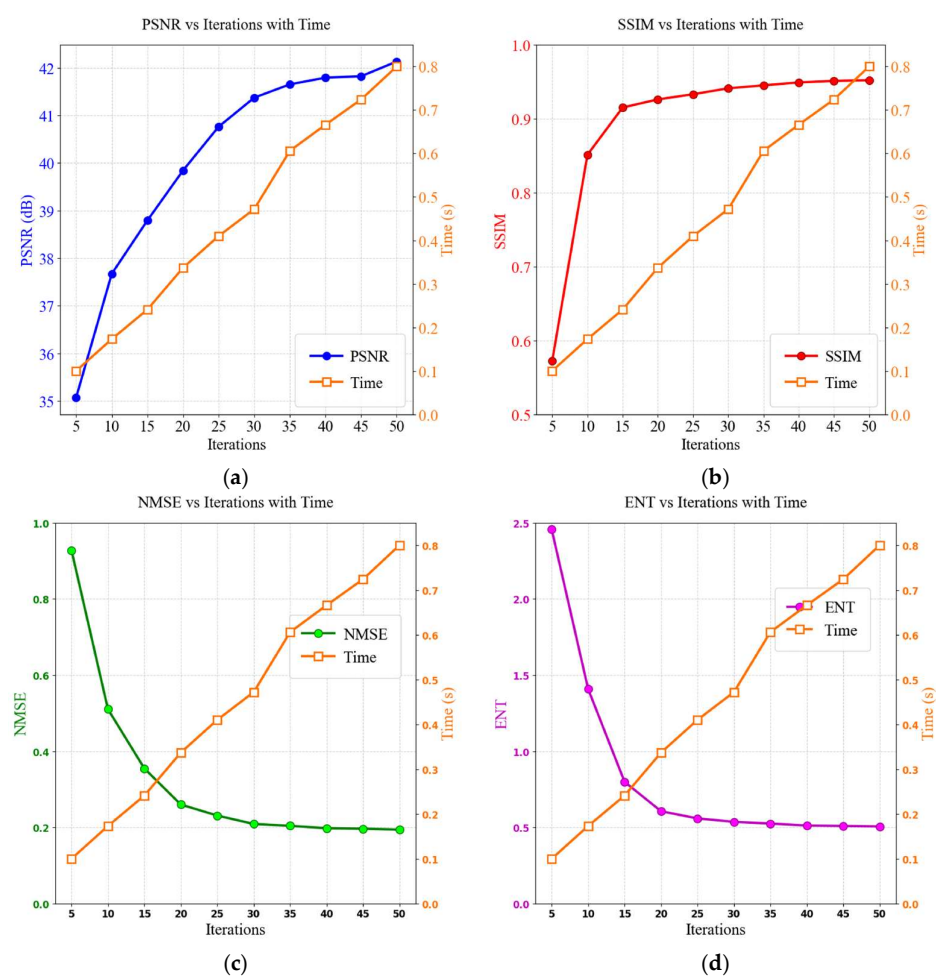


Figure 11. Variation in ADnDEQ runtime with imaging results for different number of iterations. (a–d) correspond to the PSNR, SSIM, NMSE and ENT, respectively.

This efficiency makes ADnDEQ a practical solution for resource-constrained imaging systems.

4.5. The Flexibility and Robustness

Unlike conventional deep networks applied in SA-ISAR, DnDEQ exhibits inherent adaptability to variations in the observation matrix. We trained a unified ADnDEQ model at a 10% undersampling ratio, which generalizes to distinct compression rates for individualized imaging tasks. As demonstrated in Figure 12 and Table 3, the framework was rigorously evaluated under adversarial conditions by intentionally employing mismatched measurement models during testing (50% and 25% undersampling ratios) despite training exclusively on 10% undersampled data. Remarkably, ADnDEQ maintains high imaging fidelity under mismatched undersampling conditions. These results empirically validate that ADnDEQ, grounded in deep equilibrium principles, achieves superior robustness and operational flexibility in dynamic compressive sensing scenarios.

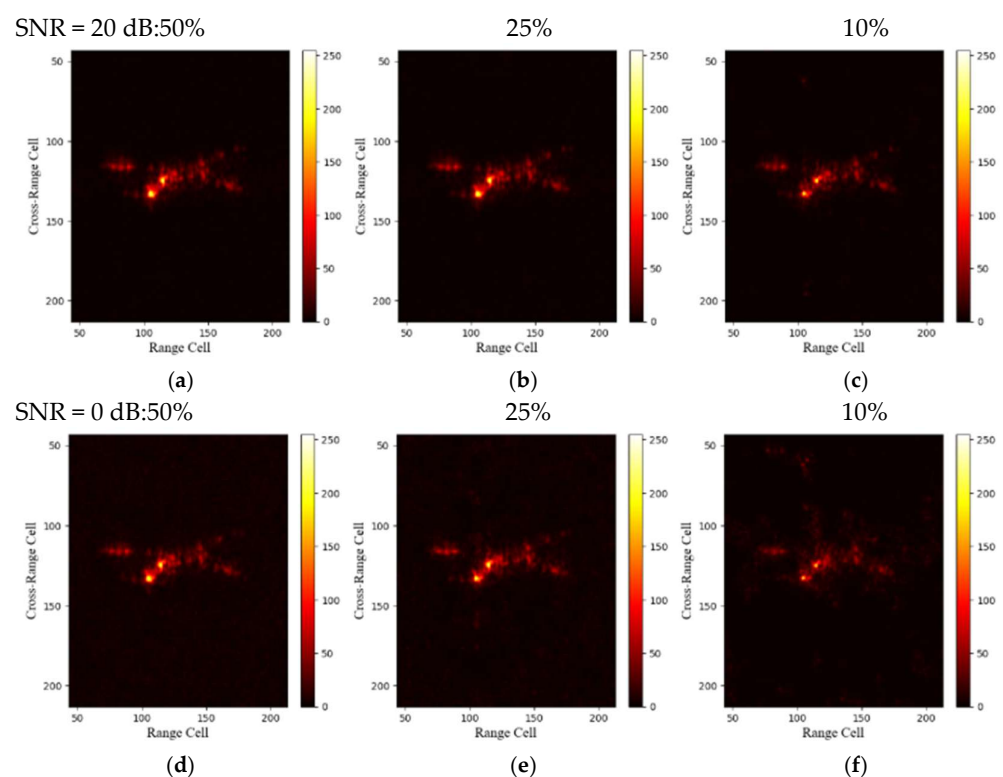


Figure 12. Verification experiments on robustness and flexibility at different CS ratio.

Table 3. Validation experiments on robustness and flexibility under different SNR and CS ratios.

SNR/ dB	CS Ratio	PSNR/ dB	SSIM	NMSE	ENT
20	50%	46.9636	0.8314	0.0943	1.1674
	25%	44.8517	0.8272	0.0969	1.2646
	10%	41.3957	0.8084	0.2164	1.4574
0	50%	39.3967	0.7510	0.5690	2.1900
	25%	36.8105	0.6678	0.6217	2.3699
	10%	34.8514	0.6136	0.9759	2.7153

Section 4.5 investigates the generalization capability of the proposed ADnDEQ framework under different compression ratios. Specifically, a single ADnDEQ model is trained

using data with a fixed sampling configuration and is directly evaluated under unseen compression ratios of 50%, 25%, and 10%, without any retraining or parameter adjustment.

As shown in Figure 12 and summarized in Table 3, ADnDEQ consistently maintains stable reconstruction performance across all tested compression ratios. In contrast, most comparison methods exhibit noticeable performance degradation as the sampling rate decreases, particularly under extreme sparsity conditions. This observation highlights the strong adaptability of the proposed framework to varying undersampling levels.

The superior generalization ability of ADnDEQ can be attributed to its implicit deep equilibrium formulation combined with the ADMM-based unfolding structure. By modeling the reconstruction process as the equilibrium solution of a single-layer network with shared parameters, ADnDEQ avoids explicit dependence on a fixed network depth or sampling pattern. Moreover, the ADMM structure enforces data fidelity and sparsity priors in a principled manner, while the embedded denoising module enhances robustness against noise and missing data.

These results demonstrate that ADnDEQ is well suited for practical SA-ISAR imaging scenarios, where sampling conditions may vary dynamically and retraining is often infeasible.

5. Conclusions

This paper introduces ADnDEQ, a DnCNN network based on ADMM deep unfolding that utilizes the DEQ framework for SA-ISAR imaging. Compared to traditional CS algorithms, DnDEQ eliminates the need for manual parameter tuning while enhancing imaging accuracy, and robustness. In contrast to layered deep learning models, ADnDEQ, based on ADMM deep unfolding, retains interpretability and, through an iterative network approach, demonstrates shorter training times, faster convergence rates, and reduced memory footprints while achieving superior reconstruction capabilities, particularly under extremely sparse and low-SNR conditions. Furthermore, ADnDEQ exhibits strong robustness and flexibility to variations in the observation matrix, generalizing to different compression rates and challenging the assumption that “universal priors are more robust,” thereby offering new insights into the application of deep models in dynamic measurement scenarios. Additionally, the integration of the DEQ framework with deep unfolding models in SA-ISAR imaging shows promise, facilitating the further fusion of traditional iterative algorithms with deep learning networks, a research direction that we are currently exploring.

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