

Article

On the Limitations of InSAR Decomposition Imposed by Displacement Direction and Coordinate System

Álvaro Hernández-Cabezudo ¹, Alfredo Fernández-Landa ^{1,*}, Jaime Sánchez-Fernández ¹,
Vrinda Krishnakumar ¹, Candela Sancho ¹, Antonio Miguel Ruiz-Armenteros ^{2,3,4}
and Miguel Marchamalo-Sacristán ⁵

¹ Detektia Earth Surface Monitoring S.L., 42005 Soria, Spain; ahernandez@detektia.com (Á.H.-C.); csancho@detektia.com (C.S.)

² Department of Cartographic, Geodetic and Photogrammetry Engineering, University of Jaén, Campus Las Lagunillas S/N, 23071 Jaén, Spain; amruiz@ujaen.es

³ Centre for Advanced Studies in Earth Sciences, Energy and Environment (CEACTEMA), University of Jaén, Campus Las Lagunillas S/N, 23071 Jaén, Spain

⁴ Research Group RNM-282 Microgeodesia Jaén, University of Jaén, Campus Las Lagunillas S/N, 23071 Jaén, Spain

⁵ Department of Land Morphology and Engineering, Universidad Politécnica de Madrid, 28040 Madrid, Spain; miguel.marchamalo@upm.es

* Correspondence: afernandez@detektia.com

Highlights

What are the main findings?

- Two-geometry InSAR decomposition is limited by the null direction of the ascending–descending observation geometry and by the validity of the physical constraint imposed on the third displacement component.
- When the true displacement violates the imposed constraint, the resulting component-wise bias is governed by the unmodeled displacement component and its geometric transfer through the null direction.

What are the implications of the main findings?

- The constrained component should be selected from the expected deformation mechanism and checked against the acquisition geometry, rather than chosen as a default processing convention.
- The proposed dashboard and synthetic sensitivity examples help identify when zero-North, vertical–planimetric, slope-aligned, or null-aligned decompositions are likely to be reliable.

Abstract

Satellite-based Interferometric Synthetic Aperture Radar (InSAR) is widely used to monitor ground deformation over large areas. However, each InSAR observation measures only the scalar projection of the three-dimensional displacement vector onto the satellite line of sight. Consequently, two independent ascending and descending line-of-sight observations can determine only two independent displacement components unless an additional physical constraint or external observation is introduced. This study examines the bias and noise sensitivity of two-geometry InSAR displacement decomposition when the additional constraint is expressed in different global and local coordinate systems. We formulate the decomposition in the East–North–Vertical and the slope-oriented Upwards–Transverse–Normal reference frames and analyze how the estimated components are affected by the true displacement direction, the orientation of the constrained axis, and its relationship with the null direction of the observation geometry. Synthetic numerical experiments are used to



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evaluate conventional zero-North, vertical–planimetric, slope-aligned, and null-direction-aligned decompositions. The results show that a constraint aligned with the null direction prevents the unobservable displacement component from biasing the two estimated components, whereas even small coordinate-system misalignments with respect to the true displacement direction may produce substantial errors when the constrained direction is poorly oriented with respect to the null direction. An interactive dashboard is provided to support geometry-specific sensitivity analysis and the selection of physically meaningful coordinate systems for practical InSAR applications.

Keywords: InSAR; displacement decomposition; line-of-sight; three-dimensional displacement; coordinate systems; slope-oriented reference frame; null direction; signal-to-noise ratio; relative error

1. Introduction

InSAR is a powerful remote sensing technique for measuring the deformation of the ground surface and infrastructure. The increasing availability of stacks of SAR images over most of the planet has triggered the development of multi-temporal InSAR (MT-InSAR), increasing the availability of consistent deformation time series for areas corresponding mainly to two types of reflectors, namely Persistent Scatterers (PS) and Distributed Scatterers (DS), based on their radar response [1,2].

The deformation direction is a three-dimensional (3D) vector that can be projected in the East, North, Vertical (ENV) coordinate system [3]. However, a fundamental limitation of InSAR is that it is only sensitive to the component of displacement projected onto the satellite's Line-of-Sight (LOS) direction [4,5]. This constraint complicates the interpretation of observed deformation patterns, as the 3D displacement components cannot be directly retrieved from single LOS measurements.

An unconstrained estimate of the three displacement components requires at least three linearly independent observation directions and a full-rank, sufficiently well-conditioned observation matrix. The availability of three SAR tracks alone does not guarantee a reliable three-component solution, because conventional near-polar SAR geometries may still provide limited sensitivity to particular displacement directions, especially the North–South component. When fewer than three independent observations are available, the problem is underdetermined and additional information must be introduced through physical constraints, displacement models, or complementary measurements. Such constrained solutions are conditional on the validity of the adopted assumptions and may contain systematic bias if those assumptions are not satisfied [3,6].

In the first decades of InSAR development, missions were not specifically designed for differential (DInSAR) or MT-InSAR processing. Therefore, the archives of missions developed from 1980s to 2010s (ERS-1, ERS-2, ENVISAT, RADARSAT, ALOS-1, ...) often provide only single-geometry, or one-track, series of images. The lack of SAR data with different views led to the direct projection of the InSAR LOS deformation in the vertical direction in many applications, ignoring the contribution of horizontal deformation to LOS displacements [7–9]. This led to heterogeneous results, depending on the geometry of each case study.

Several approaches have been developed to improve the estimation of multidimensional surface displacement by combining conventional InSAR with additional observations or physical models. These approaches include multi-track or multi-sensor InSAR [10,11], the integration of conventional interferometry with multiple-aperture interferometry or azimuth offset tracking [12,13], and the joint inversion of InSAR and GNSS

observations [14,15]. Other studies have combined single-geometry interferometric or amplitude-offset measurements with deformation models to estimate multidimensional displacement fields [16–18]. The applicability of these methods depends on the availability, spatial compatibility, temporal compatibility, precision, and independence of the additional observations, as well as on the validity of any deformation model introduced into the inversion.

The systematic acquisition strategies of recent SAR missions, particularly Sentinel-1, have substantially increased the availability of overlapping ascending and descending image series. Two independent LOS observations provide more information than a single geometry, but they still determine only two independent combinations of the three displacement components. Therefore, the resulting three-component vector can only be obtained after introducing an additional constraint, such as zero displacement along a prescribed direction, or after incorporating an external observation. A common approach is to assume negligible North–South displacement and estimate the East–West and Vertical components. More general approaches define a physically meaningful local deformation plane and estimate two components within that plane [19]. The validity and precision of either approach depend on the acquisition geometry, the actual displacement direction, the orientation of the adopted coordinate system, and the extent to which the imposed constraint represents the true deformation process.

When only one LOS geometry is available, estimating a multidimensional displacement vector requires a strong assumption about the displacement direction or a deformation model. Any discrepancy between the assumed and actual direction is transferred directly into the estimated displacement magnitude and components. With two independent geometries, a principal additional assumption is the plane in which displacement is expected to occur, or equivalently the component prescribed along the direction perpendicular to that plane [19]. This is generally less restrictive than prescribing the complete displacement direction, but the solution also requires spatially and temporally compatible observations, a consistent displacement datum, and a sufficiently well-conditioned acquisition geometry.

In many applications, it is physically reasonable to define a plane in which most of the displacement is expected to occur, for example in slopes, dams, dikes, railway embankments, or other elongated structures. A local coordinate system aligned with this plane allows the two LOS observations to be expressed as two displacement components within the assumed deformation plane, while the perpendicular component is prescribed through the additional constraint [19]. The resulting estimates remain conditional on the validity of this constraint and may be strongly biased when the actual displacement contains a non-negligible component perpendicular to the assumed plane.

Even where both ascending and descending datasets are available, displacement decomposition requires the identification of spatially compatible observations. The paired measurements should represent the same physical object, surface element, or deformation process and should be referred to a consistent spatial datum. Spatial proximity alone does not necessarily guarantee compatibility, particularly in urban areas where ascending and descending geometries may observe different façades, roofs, or scattering mechanisms. In this study, we assume that physically compatible ascending–descending point pairs have already been identified and referenced to a common displacement datum.

Ascending and descending acquisitions are generally not simultaneous. Consequently, temporal interpolation or another harmonization strategy may be required to refer both datasets to common epochs. Such temporal harmonization may affect the resulting decomposed time series, particularly when displacement changes rapidly between acquisition epochs [3]. Both datasets should also be referred to a consistent spatial and temporal datum. Therefore, the adopted temporal harmonization procedure should be clearly documented

and, where applicable, its associated uncertainty should be considered in the decomposition. A detailed discussion of the spatial and temporal compatibility requirements is provided in [3].

Brouwer and Hanssen [3,19] developed a unified geometric framework for multidimensional InSAR displacement estimation, including the definition of the null direction, the distinction between observable and unobservable displacement components, the role of local coordinate systems, and the conditions required for spatially and temporally consistent decomposition. Their framework also demonstrated that the accuracy and interpretability of constrained solutions depend on the relationship between the observation geometry, the true displacement direction, and the selected coordinate system.

Previous studies have established the geometric basis of multidimensional InSAR displacement estimation, including the null direction, the use of local coordinate systems, and the role of physical constraints in two-geometry decompositions. The present study does not introduce a new observation model. Its contribution is to provide a compact analytical and reproducible sensitivity framework that explicitly shows how the choice of constrained axis and coordinate system transfers an unmodeled displacement component into the estimated components. The objective is therefore not to validate an operational InSAR product against field observations, but to isolate the geometric component of decomposition bias under controlled conditions in which the true displacement vector is known.

This study investigates the deterministic bias and noise sensitivity of two-geometry InSAR displacement decomposition under different global and local coordinate-system constraints. The specific objectives are to: (1) formulate constrained two-geometry decomposition consistently in the East–North–Vertical and Upwards–Transverse–Normal coordinate systems; (2) relate the deterministic bias caused by an incorrect constraint to the null direction of the ascending–descending observation geometry; (3) illustrate the sensitivity of zero–North, vertical–planimetric, slope-oriented, and null-direction-aligned decompositions through controlled synthetic examples; and (4) implement the analysis in an interactive dashboard for geometry-specific sensitivity assessment. The theoretical formulation is presented in Section 2, the controlled synthetic examples in Section 3, and the implications, limitations, and recommendations for practical applications in Sections 4 and 5.

2. Materials and Methods

InSAR measures the component of ground displacement projected onto the LOS. In this study, positive LOS displacement denotes motion towards the satellite, whereas negative LOS displacement denotes motion away from the satellite. For the synthetic analyses presented in this work, the LOS direction is described using a local flat-Earth approximation defined by the satellite heading angle ψ , measured clockwise from North, and the inclination angle θ , measured from the vertical axis of the simplified Cartesian frame, as shown in Figure 1. Under a right-looking acquisition geometry and neglecting meridian convergence, the heading angle can be approximated as $\psi \simeq \alpha_h \equiv \alpha_d + \frac{\pi}{2}$, where α_d is the azimuth of the zero-Doppler plane at the target position, measured in the direction towards the satellite. Likewise, when Earth curvature is neglected, the inclination angle approximates the local incidence angle, $\theta \simeq \theta_d$. Therefore, the formulation can be generalized to the pointwise observation geometry by substituting $\cos \psi = -\sin \alpha_d$, $\sin \psi = \cos \alpha_d$ and $\theta = \theta_d$. For precise processing of observational data, the pointwise zero-Doppler azimuth α_d and local incidence angle θ_d should be used because both depend on the target position and vary across the range direction as a consequence of the side-looking geometry, Earth curvature, and meridian convergence [3].

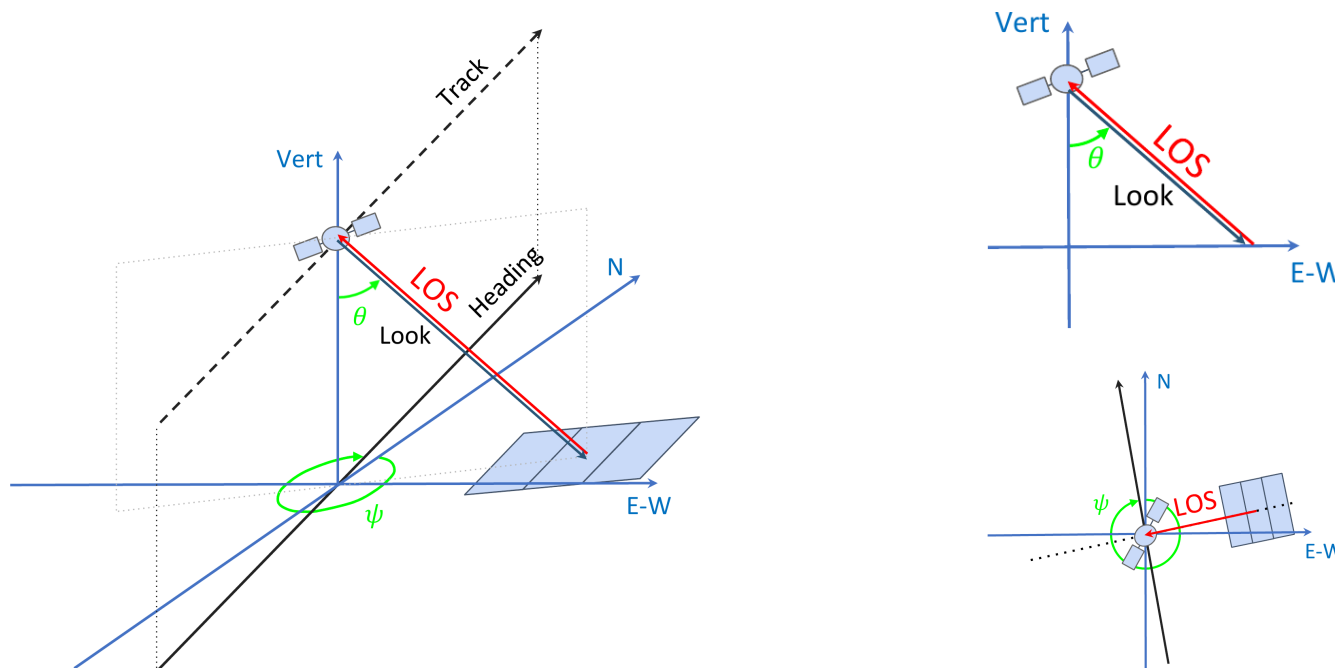


Figure 1. Ascending satellite data acquisition geometry, where θ is the inclination angle, measured from the vertical axis, and ψ is the heading angle, measured clockwise from North. The satellite moves along the track from south to north in the heading direction, while observing the surface along the look direction. The left plot shows the configuration in a 3D perspective view, the upper right plot shows a profile view, and the lower right plot shows a top view.

2.1. Decomposition in the ENV Coordinate System

In the ENV coordinate system, Figure 1, the LOS direction can be expressed as:

$$\hat{\mathbf{u}}_{LOS} = (u_1, u_2, u_3) = (-\sin \theta \cos \psi, \sin \theta \sin \psi, \cos \theta). \quad (1)$$

Here, the projection of the LOS vector in the horizontal plane is defined by $\psi - \frac{\pi}{2}$. Therefore, the deformation in the LOS direction is given by:

$$d_{LOS} = -\sin \theta \cos \psi \cdot d_{East} + \sin \theta \sin \psi \cdot d_{North} + \cos \theta \cdot d_{Vertical}, \quad (2)$$

where d_{East} , d_{North} and $d_{Vertical}$ are the displacement components in the East, North, and Vertical directions, respectively.

The three displacement components cannot be determined from two LOS observations alone. To obtain a determined system, an additional constraint or independent observation must be introduced. We express this information as

$$D = a \cdot d_{East} + b \cdot d_{North} + c \cdot d_{Vertical} = \mathbf{c}^T \mathbf{d}, \quad (3)$$

where $\mathbf{c} = (a, b, c)^T$ is the prescribed constraint direction and D is the displacement component assigned along that direction. This third equation is not derived from the two LOS observations; it represents either a physical assumption, such as zero motion along a selected axis, or an independent external measurement. The estimated three-component vector is therefore conditional on the accuracy of D and $\mathbf{c}$.

In matrix form, the system of equations can be written as:

$$\mathbf{d}_{LOS} = \mathbf{A} \cdot \mathbf{d}, \quad (4)$$

with

$$\begin{aligned} \mathbf{d}_{LOS} &\equiv (d_{LOS}^A, d_{LOS}^D, D)^T, \\ \mathbf{A} &\equiv \begin{pmatrix} -\sin \theta^A \cos \psi^A & \sin \theta^A \sin \psi^A & \cos \theta^A \\ -\sin \theta^D \cos \psi^D & \sin \theta^D \sin \psi^D & \cos \theta^D \\ a & b & c \end{pmatrix} \equiv \begin{pmatrix} u_1^A & u_2^A & u_3^A \\ u_1^D & u_2^D & u_3^D \\ a & b & c \end{pmatrix}, \\ \mathbf{d} &\equiv (d_{East}, d_{North}, d_{Vertical})^T, \end{aligned} \quad (5)$$

where the superscripts A and D refer to the ascending and descending geometries, respectively.

By inverting matrix $\mathbf{A}$ in Equation (4), the displacement components in the ENV coordinate system can be expressed in terms of the LOS deformations and the prescribed constraint value D .

2.2. Decomposition in a Local Coordinate System

The relationship between the ENV coordinate system and a local coordinate system (see Figure 2), with Upwards, Transverse, and Normal directions (UTN) defined by the local azimuth α and the local slope β , is given by the rotation matrix $\mathbf{R}$:

$$\begin{aligned} \mathbf{R}^{-1} \\ \{East, North, Vertical\} &\longrightarrow \{Upwards, Transverse, Normal\} \\ (x, y, z) &\longrightarrow (x', y', z') \end{aligned}$$

$\mathbf{R}^{-1}$ is the inverse of the rotation matrix $\mathbf{R}$, which transforms vectors defined in the ENV to the UTN coordinate system. The columns of the matrix $\mathbf{R}$ are the unit vectors of the new coordinate system expressed in terms of the old one.

$$\mathbf{R} = (\hat{\mathbf{u}} \ \hat{\mathbf{t}} \ \hat{\mathbf{n}}), \quad (6)$$

with

$$\begin{aligned} \hat{\mathbf{n}} &= (\sin \beta \sin \alpha, \sin \beta \cos \alpha, \cos \beta)^T, \\ \hat{\mathbf{t}} &= (\cos \alpha, -\sin \alpha, 0)^T, \\ \hat{\mathbf{u}} &= \hat{\mathbf{t}} \times \hat{\mathbf{n}}. \end{aligned} \quad (7)$$

To decompose the displacement in the local coordinate system, both the LOS observations and the third constraint equation must be expressed in that same coordinate system.

$$\begin{aligned} \hat{\mathbf{u}}'_{LOS} &= \mathbf{R}^{-1} \cdot \hat{\mathbf{u}}_{LOS}, \\ D' &= a' \cdot d_{Upwards} + b' \cdot d_{Transverse} + c' \cdot d_{Normal}. \end{aligned} \quad (8)$$

Then, from (5) and (8), the system of equations used to estimate the displacement components in the UTN coordinate system is

$$\mathbf{d}_{LOS} = \mathbf{G} \cdot \mathbf{d}', \quad (9)$$

with

$$\begin{aligned} \mathbf{G} &\equiv \begin{pmatrix} \hat{\mathbf{u}}'_{LOS}{}^T \\ \hat{\mathbf{u}}'_{LOS}{}^T \\ a' \ b' \ c' \end{pmatrix}, \\ \mathbf{d}' &\equiv (d_{Upwards}, d_{Transverse}, d_{Normal})^T. \end{aligned}$$

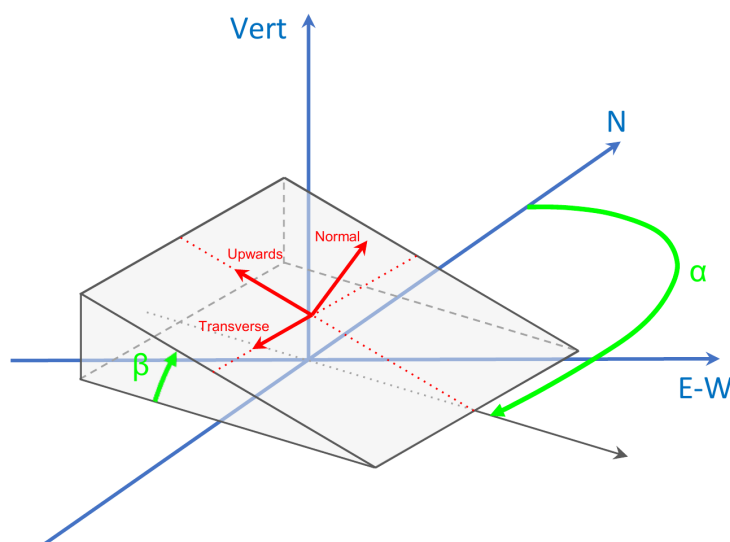


Figure 2. Local coordinate system defined by the local azimuth α and the local slope β . The Upwards direction goes along the slope in the upward direction, the Transverse direction is tangent to the contour (iso-altitude) lines of the slope, and the Normal direction is perpendicular to the slope surface.

In [3,19], more than three equations are considered as a general case, with the decomposition performed by least squares because the system is over-constrained. In contrast, the examples in this work use a determined system of three equations, which is solved by direct inversion of Equation (9). For a nonsingular system, $\mathbf{d}' = \mathbf{G}^{-1}\mathbf{d}_{LOS}$. If $\mathbf{C}_{\mathbf{d}_{LOS}}$ is the covariance matrix of the two LOS observations and the prescribed constraint, the covariance matrix of the estimated displacement vector is

$$\mathbf{C}_{\mathbf{d}'} = \mathbf{G}^{-1}\mathbf{C}_{\mathbf{d}_{LOS}}(\mathbf{G}^{-1})^T.$$

This expression describes the formal propagation of observational uncertainty through the decomposition. In the numerical examples below, the added Gaussian LOS noise and the DSNR are used as illustrative time-series diagnostics, whereas the main focus remains the deterministic bias caused by an incorrect displacement constraint. A complete observational uncertainty model including atmospheric artifacts, decorrelation, phase-unwrapping errors, reference-frame errors, and temporal interpolation errors is outside the scope of this study.

2.3. Dashboard

An interactive dashboard was developed to explore the geometric sensitivity of two-geometry InSAR displacement decomposition. The application is available at [20]. The source code, Dockerfile, tagged software release used in this study, synthetic input data, and scripts required to reproduce all manuscript figures are archived at [21].

The dashboard generates synthetic constant-velocity displacement vectors with user-defined magnitude and direction. These vectors are projected onto selected ascending and descending LOS geometries, after which the resulting synthetic LOS time series are decomposed in a user-defined UTN coordinate system. Users may prescribe the expected deformation plane, constrain one displacement component, and introduce a controlled misalignment between the assumed and actual displacement directions. The dashboard therefore provides a numerical sensitivity analysis rather than an analysis of observational InSAR measurements (see Figure 3).

First, users can select the direction of the true synthetic constant-velocity displacement by specifying the aspect and elevation angles in the ENV coordinate system. The aspect angle is measured clockwise from North, while the elevation angle is measured from the horizontal plane.

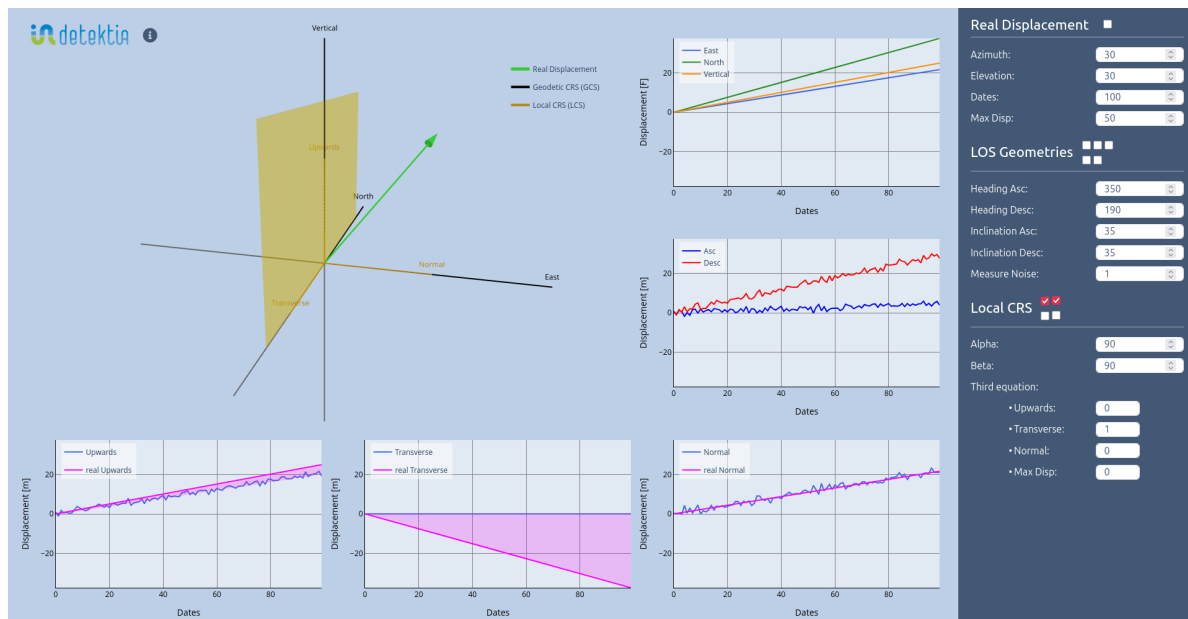


Figure 3. Detektia's decomposition dashboard [20].

Next, users can define the heading and inclination angles for the ascending and descending satellite geometries. Users may specify the standard deviation of zero-mean Gaussian noise added independently to the ascending and descending synthetic LOS observations. Unless otherwise stated, the noise is assumed to be temporally uncorrelated and independent between geometries. This simplified model represents random measurement noise only and is not intended to account for spatially or temporally correlated atmospheric, orbital, reference-datum, or phase-unwrapping errors. The resulting signal-to-noise metric quantifies the effect of random noise on the decomposed time series but does not quantify deterministic bias caused by an incorrect constraint. Although the dashboard does not implement epoch-wise covariance propagation, the effect of the synthetic noise added to the LOS signal can be visualized in the resulting decomposed time series. Moreover, a measure of the signal-to-noise ratio is calculated for the decomposition results; see its definition in the next section (10).

Finally, users can choose the orientation—specified by the angles α and β (see Figure 2)—of the UTN coordinate system for the decomposition. They can also set the parameters for the third Equation (3).

The dashboard features a 3D plot of the chosen displacement vector. Additional elements, such as the LOS directions, the UTN coordinate system, and the projections of the displacement vector onto the axes of both the coordinate systems and the LOS directions among other features can also be displayed. The dashboard also includes a set of instructions to guide the user.

To provide a deeper understanding of the decomposition process, users can visualize the time series of the displacement components in the East, North, and Vertical directions, as well as the corresponding values in the LOS directions, where synthetic noise has been added. In the Up, Transverse, and Normal directions, the projections of the true synthetic displacement can be compared with the time series obtained from the decomposition.

3. Results

In this section, we analyze the sensitivity and limitations of several decomposition cases. First, we derive expressions that help us understand how neglecting a component of the deformation in the chosen reference frame affects the final solution. We then provide numerical examples to illustrate the behavior in each case.

All numerical experiments use an ascending geometry defined by $\psi^A = 349^\circ$ and $\theta^A = 32^\circ$, and a descending geometry defined by $\psi^D = 191^\circ$ and $\theta^D = 38^\circ$ (see Figure 1). Independent zero-mean Gaussian noise with standard deviation $\sigma_{LOS} = 1$ mm is added to each LOS observation. In Sections 3.1–3.3, the third equation imposes zero displacement along the Transverse direction, $d_T = 0$. Section 3.4 instead constrains the axis selected to coincide with the null direction. However, it is worth highlighting that if external information about the deformation in the constrained direction is available, it can be incorporated into (3) in order to improve the solution.

All plots in this section are obtained from the dashboard introduced in Section 2.3. Two metrics are reported: the Relative Error (RE) and the Displacement Signal-to-Noise Ratio (DSNR). These metrics are defined as:

$$\text{RE} = \left| \frac{d_{q,N}^* - d_{q,N}}{d_{q,N}} \right|, \quad \text{DSNR} = \frac{\text{var}(d^*)}{\text{var}(Y - d^*)}. \quad (10)$$

where Y is the decomposed displacement time series, d is the time series of the true displacement projected onto the q -axis, and d^* is the noise-free decomposed displacement time series. The subscript N refers to the last time step of the time series. Since the considered cases have constant-velocity displacements, the relative error is time-invariant, and N could be replaced by any other non-zero time step without changing the result.

The DSNR is used here as an illustrative time-series metric that compares the variance of the noise-free decomposed component with the variance of the perturbation introduced by the synthetic LOS noise. It should not be interpreted as a formal uncertainty estimate. Values greater than 1 indicate that the decomposed time series is signal dominated under the adopted synthetic noise model, whereas values smaller than 1 indicate that the simulated noise contribution is larger than the deterministic temporal signal. Formal component-wise uncertainty propagation is described in Section 2.2 through the covariance matrix of the decomposition.

To facilitate comparison among the different controlled synthetic examples, Table 1 summarizes the prescribed displacement direction, the selected coordinate-system orientation, the constrained component, and the resulting RE and DSNR values for the estimated components. The table is intended to provide a compact overview of the numerical examples analyzed in Sections 3.1–3.4. The reported numerical values are specific to the ascending–descending geometry used in this study. Other acquisition geometries produce different numerical transfer factors, but the governing mechanism remains the same: an unmodeled constrained component is redistributed into the estimated components according to the orientation of the constrained axis with respect to the null direction.

The examples summarized in Table 1 illustrate how the decomposition is affected by two coupled factors: the magnitude of the displacement component that is constrained or neglected, and the orientation of the constrained axis with respect to the null direction (Appendix B). The following subsections discuss these effects first in the conventional Vertical–East case, then in a general vertical–planimetric frame, then in a slope-oriented frame, and finally in a null-line-aligned coordinate system.

3.1. Vertical and East–West Planimetry

We first consider the conventional Vertical–East decomposition obtained from the general UTN system by setting $\alpha = 90^\circ$ and $\beta = 90^\circ$. With the definitions in Section 2, this configuration gives

$$\hat{\mathbf{u}} = (0, 0, 1)^T, \quad \hat{\mathbf{t}} = (0, -1, 0)^T, \quad \hat{\mathbf{n}} = (1, 0, 0)^T. \quad (11)$$

Therefore, Upwards coincides with Vertical, Transverse points towards South, and Normal coincides with East. Imposing $d_T = 0$ is consequently equivalent to imposing zero North–South displacement, with the sign of the Transverse axis being opposite to North. We evaluate how a nonzero true North component affects the estimated East and Vertical components. Starting from the system of Equation (4),

$$\begin{aligned} d_{LOS}^A &= u_1^A d_{East} + u_2^A d_{North} + u_3^A d_{Vert}, \\ d_{LOS}^D &= u_1^D d_{East} + u_2^D d_{North} + u_3^D d_{Vert}, \end{aligned}$$

and by using (A7), (A8) and (A9), and setting $\alpha = 90^\circ$ and $\beta = 90^\circ$, we get:

$$\begin{aligned} d_{East} &= \frac{u_3^D d_{LOS}^A - u_3^A d_{LOS}^D}{u_1^A u_3^D - u_3^A u_1^D} + \tan \phi d_{North}, \\ d_{Vert} &= \frac{u_1^D d_{LOS}^A - u_1^A d_{LOS}^D}{u_3^A u_1^D - u_1^A u_3^D} + \frac{\tan \zeta}{\cos \phi} d_{North}. \end{aligned} \quad (12)$$

These equations express the East and Vertical components as the sum of the two-component zero-North solution and correction terms proportional to the true North component. Therefore, when the North component is neglected, the bias in the estimated East and Vertical components is given by the second term in each equation.

Table 1. Summary of the controlled synthetic decomposition examples. RE denotes the relative error of the noise-free constrained solution. DSNR is reported only for the estimated components and is interpreted as an illustrative time-series signal-to-noise metric under the prescribed synthetic LOS noise.

Case	True Aspect (°)	True Elevation (°)	α (°)	β (°)	Fixed Component	Main Estimated Components	RE of Estimated Components (%)	DSNR
Vertical–East dominated	80	−40	90	90	$d_T = 0$	Up/Vertical Normal/East	2.74 0.38	146.76 75.19
North dominated	5	−5	90	90	$d_T = 0$	Up/Vertical Normal/East	150.85 24.72	2.17 0.58
Vertical–planimetric aligned	5	−40	5	90	$d_T = 0$	Upwards Normal	0 0	14.40 0.40
Vertical–planimetric misaligned	5	−40	3	90	$d_T = 0$	Upwards Normal	17.92 113.56	1.90 0.32
Slope-aligned	50	−30	50	20	$d_T = 0$	Upwards Normal	0 0	76.69 5.66
Slope misaligned	50	−30	45	20	$d_T = 0$	Upwards Normal	7.04 23.19	61.61 2.42
Null-line aligned	60	−20	179	8	$d_U = 0$	Transverse Normal	0.68 0.31	52.09 90.42

In the ENV coordinate system, the azimuth and elevation angles, ϕ and ζ , of the null direction are very small, Figure A3. Consequently, even if the deformation in the North direction is not negligible, the error introduced by neglecting it remains small. But if the North deformation is extremely large, the error induced will not be negligible. Thus, when there is no prior knowledge of the deformation plane, this approach provides a relatively good approximation to the true solution.

To illustrate the range of possible outcomes in the decomposed data, we consider numerical examples in which the deformation is dominated either by the East and Vertical components, as well as cases where it is dominated by the North component.

A representative application is regional-scale urban or basin subsidence, where the displacement field may extend over heterogeneous geological and built environments and a pointwise local deformation plane is not generally available. In such cases, the zero-North approximation has frequently been used to estimate Vertical and East–West components from ascending and descending observations [22,23]. Its validity should nevertheless be evaluated against the expected deformation mechanism and, where available, independent GNSS or leveling observations.

3.1.1. Vertical–East Dominated

In this first example, the true synthetic displacement direction is set to an aspect of 80° and an elevation of -40° . Therefore, the displacement is dominated by the East and Vertical components, as shown in the upper left plot of Figure 4.

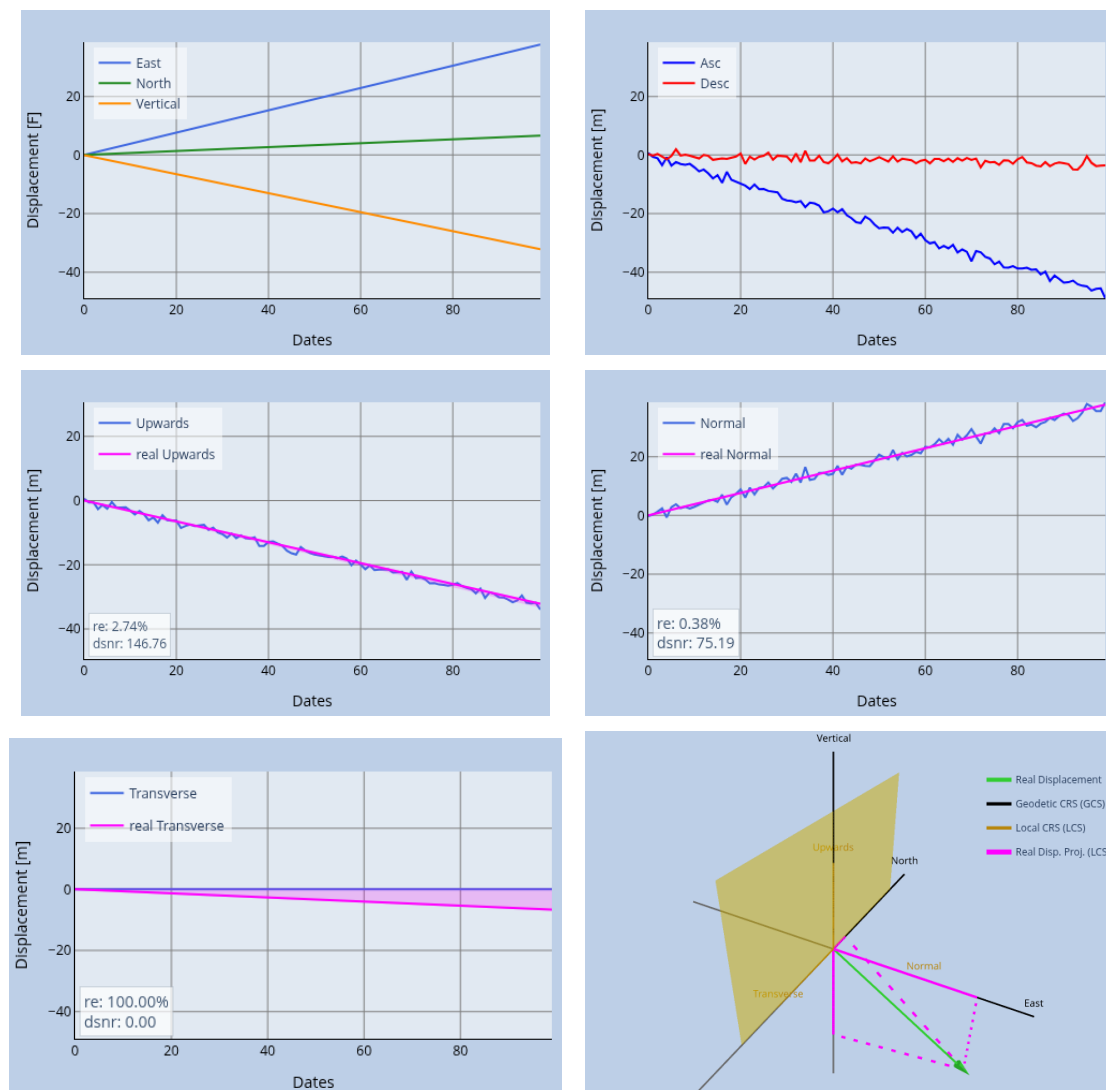


Figure 4. Displacement decomposition for the Vertical–East dominated case. **(top-left)** True synthetic displacement projections in the East, North, and Vertical directions. **(top-right)** LOS direction with added noise. **(middle-left)** Upwards, **(middle-right)** Normal, and **(bottom-left)** Transverse components of the decomposed displacement. The horizontal axis labeled “Dates” denotes synthetic acquisition epoch. The numerical solution is shown in blue, and the deviations from the true noise-free solution—if no displacement were neglected and there were no noise—are shown in pink. **(bottom-right)** 3D plot of the true synthetic displacement direction and the coordinate system.

The relative errors of the Upwards/Vertical and Normal/East components are 2.74% and 0.38%, respectively. In this example, the true North–South displacement is small relative to the Vertical and East components, and the geometric transfer factors in Equation (12) are also small for the selected observation geometry. Consequently, imposing zero Transverse displacement introduces only limited bias into the two estimated components. The relative error of the Transverse component is 100% by construction because this component is explicitly constrained to zero; this value should not be interpreted as an independent measure of the quality of the two estimated components. The DSNR values indicate that the simulated random noise is small relative to the temporal signal in both estimated components.

3.1.2. North Dominated

In this case, the true synthetic displacement direction is set to an aspect of 5° and an elevation of -5° . Therefore, the displacement is dominated by the North component, as shown in the upper left plot of Figure 5.

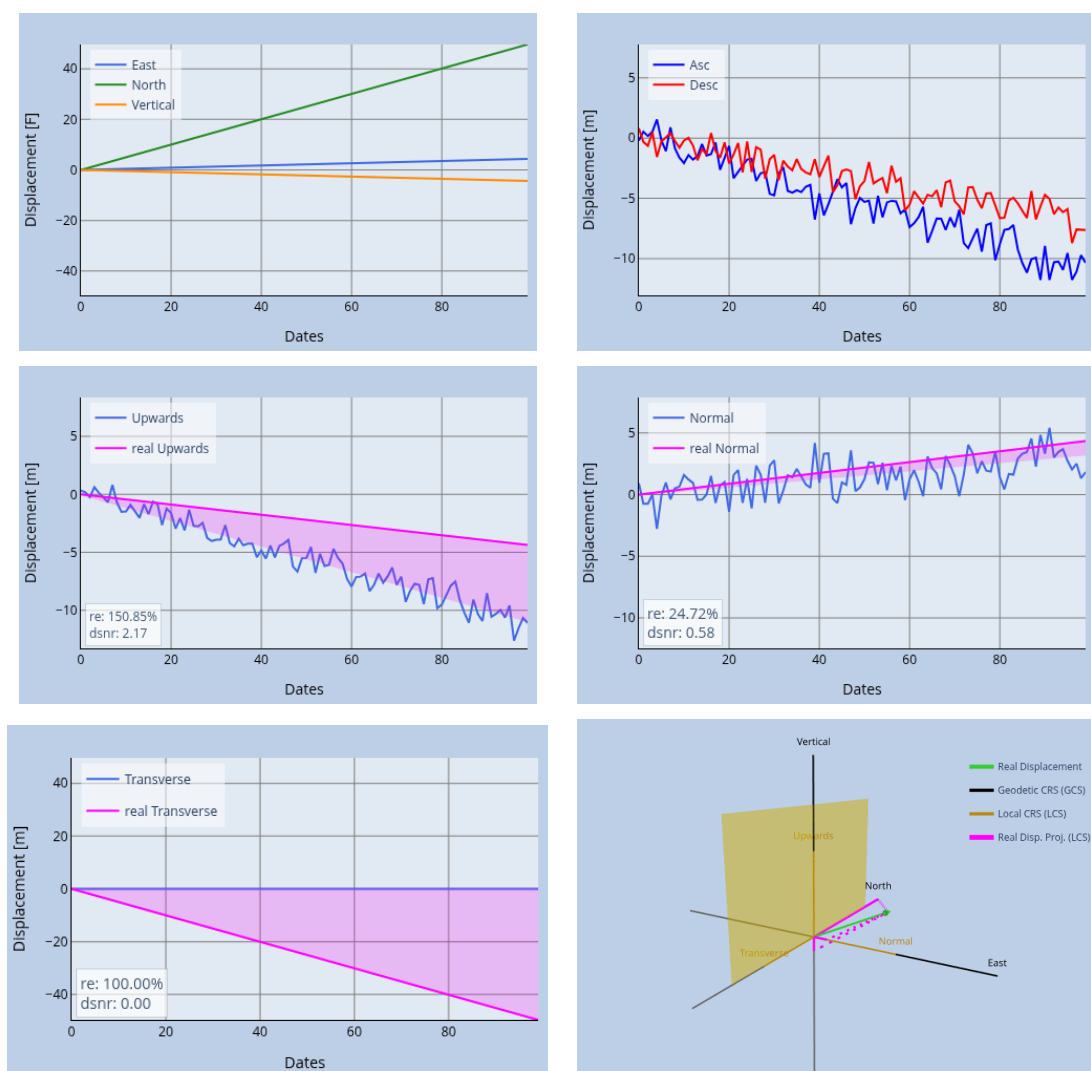


Figure 5. Displacement decomposition for the North dominated case. **(top-left)** True synthetic displacement projections in the East, North, and Vertical directions. **(top-right)** LOS direction with added noise. **(middle-left)** Upwards, **(middle-right)** Normal, and **(bottom-left)** Transverse components of the decomposed displacement. The horizontal axis labeled “Dates” denotes synthetic acquisition epoch. The numerical solution is shown in blue, and the deviations from the true noise-free solution—if no displacement were neglected and there were no noise—are shown in pink. **(bottom-right)** 3D plot of the true synthetic displacement direction and the coordinate system.

The relative errors of the Upwards and Normal components are substantial, 150.85% and 24.72%, respectively. This is expected, because the neglected Transverse component is much larger than in the Vertical–East dominated case, leading to larger correction terms in Equation (12).

The DSNR values of the Upwards/Vertical and Normal/East components are 2.17 and 0.58, respectively. The low DSNR of the East component results from the weak LOS sensitivity to a displacement vector dominated by the North direction. As the true displacement approaches the null direction, its projections onto both LOS vectors decrease, while the imposed measurement-noise level remains unchanged. The decomposition is therefore affected by two distinct mechanisms: first, the zero-North constraint produces deterministic bias because the true North component is large; second, the weak LOS signal increases the relative influence of random measurement noise. These mechanisms must not be conflated. In this example, the large relative errors of 150.85% and 24.72% show that the deterministic bias is not small, while the DSNR values describe the additional degradation caused by noise.

3.2. Vertical and General Planimetry

We next consider a more general case in which the coordinate system has an arbitrary planimetric orientation (Figure 2). This configuration is obtained by setting $\beta = 90^\circ$, with α defining the orientation of the Normal direction. As in the previous example, the third Equation (8) imposes zero displacement in the Transverse direction. We therefore solve for d_{Upwards} and d_{Normal} as functions of $d_{\text{Transverse}}$, in order to quantify the bias introduced when this component is neglected. Starting from the system of Equations (9),

$$\begin{aligned} d_{\text{LOS}}^A &= u_1^A d_{\text{Up}} + u_2^A d_{\text{Transverse}} + u_3^A d_{\text{Normal}} , \\ d_{\text{LOS}}^D &= u_1^D d_{\text{Up}} + u_2^D d_{\text{Transverse}} + u_3^D d_{\text{Normal}} , \end{aligned}$$

using (A7), (A8) and (A9) we get:

$$\begin{aligned} d_{\text{Up}} &= \frac{u_3^D d_{\text{LOS}}^A - u_3^A d_{\text{LOS}}^D}{u_1^A u_3^D - u_3^A u_1^D} + \tan \phi' d_{\text{Transverse}} , \\ d_{\text{Normal}} &= \frac{u_1^D d_{\text{LOS}}^A - u_1^A d_{\text{LOS}}^D}{u_3^A u_1^D - u_1^A u_3^D} + \frac{\tan \zeta'}{\cos \phi'} d_{\text{Transverse}} . \end{aligned} \quad (13)$$

These two equations present the complete solution of the Upwards and Normal components of the true displacement in terms of the Transverse component and the LOS deformations. Therefore, as in the previous case, the difference of our solution in which the Transverse component is neglected with respect to the real solution is given by the second terms in both equations. These equations are a generalization of the equations in (12).

For the cases presented in this section, we consider a true synthetic displacement direction with an aspect of 5° and an elevation of -40° . To illustrate how the solution is affected by the neglected component, we use a coordinate system defined by $\beta = 90^\circ$ and vary α around 5° . From Figure A3, a coordinate system with $\alpha = 5^\circ$ and $\beta = 90^\circ$ has $\tan \phi' \sim 1$ and $\tan \zeta' / \cos \phi' \gg 1$. Therefore, a misalignment with respect to the true synthetic displacement direction, that is, neglecting a nonzero Transverse component, is expected to substantially affect the solution.

A representative application is the monitoring of dams, dikes, retaining structures, or elongated embankments for which deformation is expected to occur mainly within a vertical plane perpendicular to the structural axis. In this configuration, Upwards is Vertical, Normal is the horizontal direction perpendicular to the axis, and Transverse is the horizontal direction parallel to the axis. Setting $d_T = 0$ is physically meaningful only when longitudinal movement along the structure is expected to be negligible or is independently constrained. Applications

to elongated infrastructure should therefore justify the assumed deformation plane using structural geometry, geotechnical information, or independent monitoring data [24].

3.2.1. Aligned Coordinate System

Here, we consider a coordinate system with $\alpha = 5^\circ$ and $\beta = 90^\circ$, whose Upwards–Normal plane is perfectly aligned with the true synthetic displacement direction.

The relative errors of the Upwards and Normal components are 0%, indicating that the noise-free estimated components are unbiased (Figure 6). This occurs because the coordinate system is perfectly aligned with the true synthetic displacement direction, so the neglected Transverse component is zero. The DSNR values for the Upwards and Normal components are 14.40 and 0.40, respectively. Thus, under the prescribed synthetic noise model, the Upwards component is signal dominated, whereas the Normal component is noise dominated. This behavior is consistent with cases in which the dominant displacement component is close to the null direction, as shown in Figures 5 and 6.

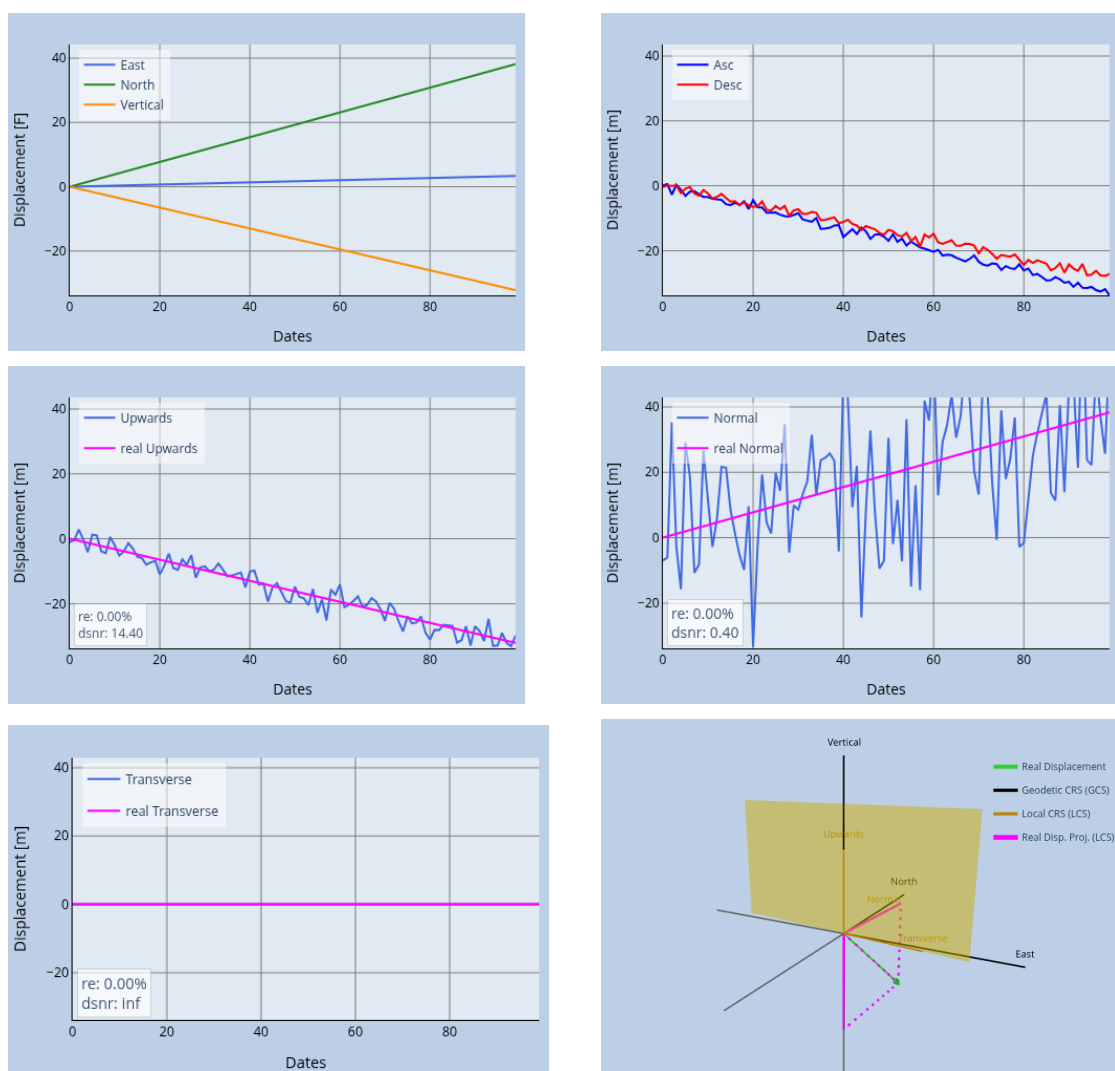


Figure 6. Displacement decomposition for the aligned coordinate system case. **(top-left)** True synthetic displacement projections in the East, North, and Vertical directions. **(top-right)** LOS direction with added noise. **(middle-left)** Upwards, **(middle-right)** Normal, and **(bottom-left)** Transverse components of the decomposed displacement. The horizontal axis labeled “Dates” denotes synthetic acquisition epoch. The numerical solution is shown in blue, and the deviations from the true noise-free solution—if no displacement were neglected and there were no noise—are shown in pink. **(bottom-right)** 3D plot of the true synthetic displacement direction and the coordinate system.

3.2.2. Misaligned Coordinate System

We next introduce a small coordinate-system misalignment by setting $\alpha = 3^\circ$ and $\beta = 90^\circ$ (Figure 2). This represents a case in which the expected deformation direction is prescribed, but the true synthetic deformation deviates slightly from it.

In this case, the relative error of the Upwards component is 17.92%, whereas the relative error of the Normal component reaches 113.56% (Figure 7). Although the nonzero Transverse component is small because the angular misalignment is small, the transfer factors are large for $\alpha \approx 5^\circ$ and $\beta = 90^\circ$, with $\tan \phi' \simeq -2.02$ and $\tan \zeta' / \cos \phi' \simeq -15.21$. Therefore, the neglected component has a large effect on the estimated components (Equation (13) and Figure A3). The DSNR values are 1.90 and 0.32 for the Upwards and Normal components, respectively, indicating that the Upwards component remains weakly signal dominated, whereas the Normal component is noise dominated under the prescribed synthetic noise model.

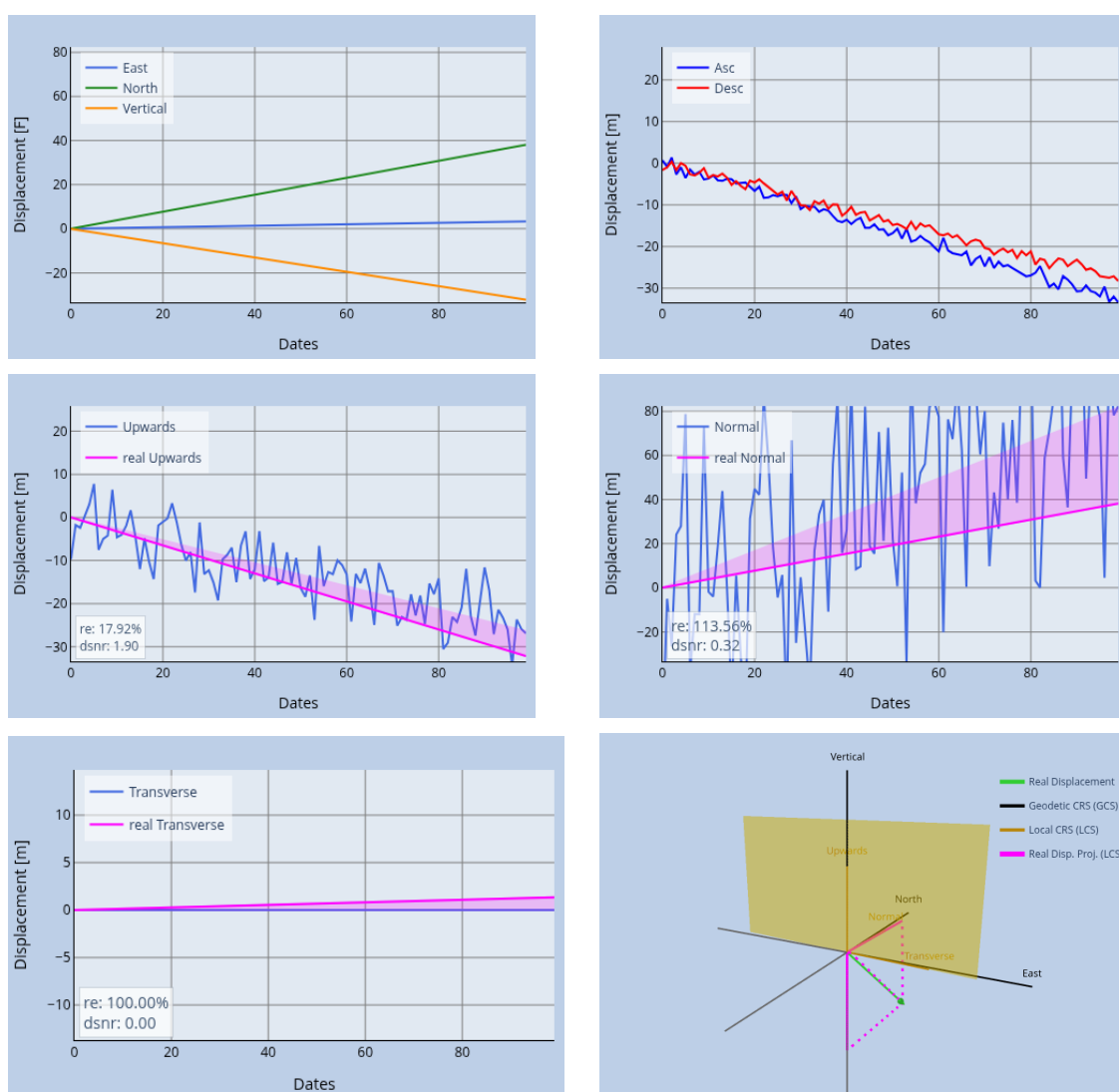


Figure 7. Displacement decomposition for the misaligned coordinate system case. **(top-left)** True synthetic displacement projections in the East, North, and Vertical directions. **(top-right)** LOS direction with added noise. **(middle-left)** Upwards, **(middle-right)** Normal, and **(bottom-left)** Transverse components of the decomposed displacement. The horizontal axis labeled “Dates” denotes synthetic acquisition epoch. The numerical solution is shown in blue, and the deviations from the true noise-free solution—if no displacement were neglected and there were no noise—are shown in pink. **(bottom-right)** 3D plot of the true synthetic displacement direction and the coordinate system.

3.3. Slope Coordinate System

We next use the slope-oriented coordinate system as a controlled synthetic benchmark. The slope geometry, the true displacement vector, and the acquisition geometry are prescribed, so the effect of using an aligned or misaligned local coordinate system can be quantified directly against the known input displacement. This benchmark is representative of applications in which deformation is expected to occur mainly within the vertical plane defined by the direction of maximum slope, such as landslides, engineered slopes, embankments, or open-pit mine walls. In this coordinate system, the Upwards direction points uphill, the Transverse direction is tangent to the contour line, and the Normal direction is perpendicular to the slope surface, see Figure 2.

The true synthetic displacement is set to an aspect of 50° and an elevation of -30° . The local slope is assigned an aspect of 50° and a slope angle of 20° , so that the displacement contains both a downslope component and a component normal to the slope surface. We first consider the aligned case and then introduce a 5° aspect mismatch to evaluate the sensitivity of the decomposition to coordinate-system misalignment.

3.3.1. Aligned Slope Coordinate System

In this example, the coordinate system is aligned with the true synthetic displacement direction; therefore, we set $\alpha = 50^\circ$ and $\beta = 20^\circ$. The estimated displacement contains a downslope component, represented by negative Upwards values, and a component normal to the slope surface. Because the coordinate system is aligned with the true synthetic displacement direction, the neglected Transverse component is zero and the noise-free Upwards and Normal components are unbiased; their relative errors are 0% (Figure 8). The DSNR values for the Upwards and Normal components are 76.69 and 5.66, respectively, indicating that both components are signal dominated under the prescribed synthetic noise model.

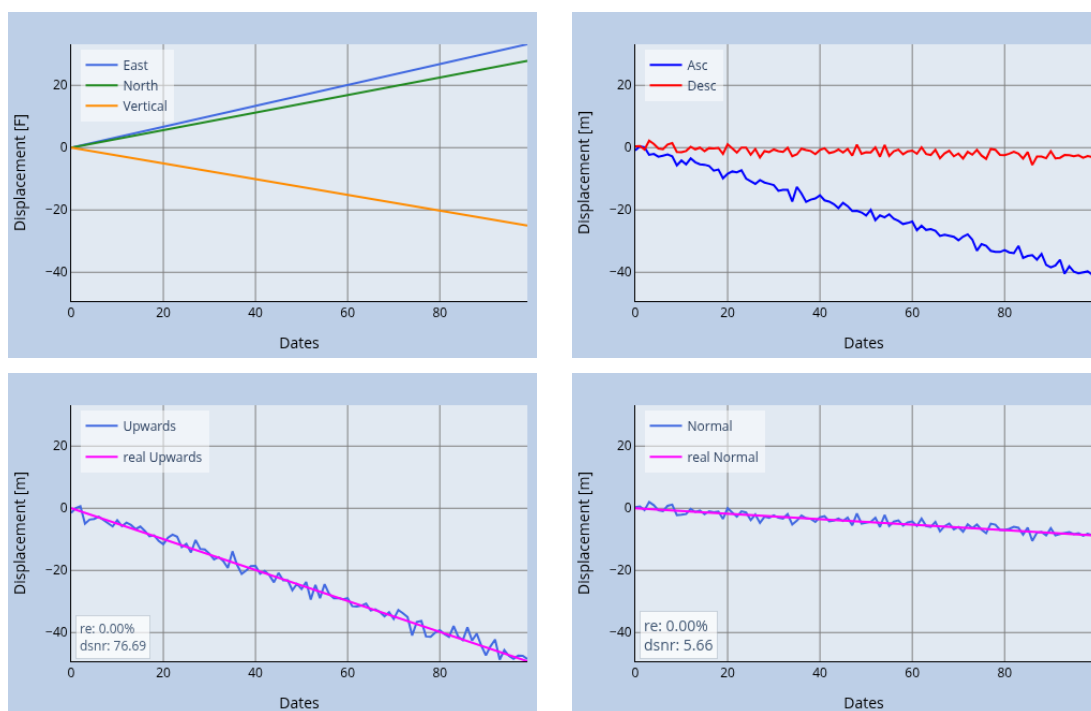


Figure 8. Cont.

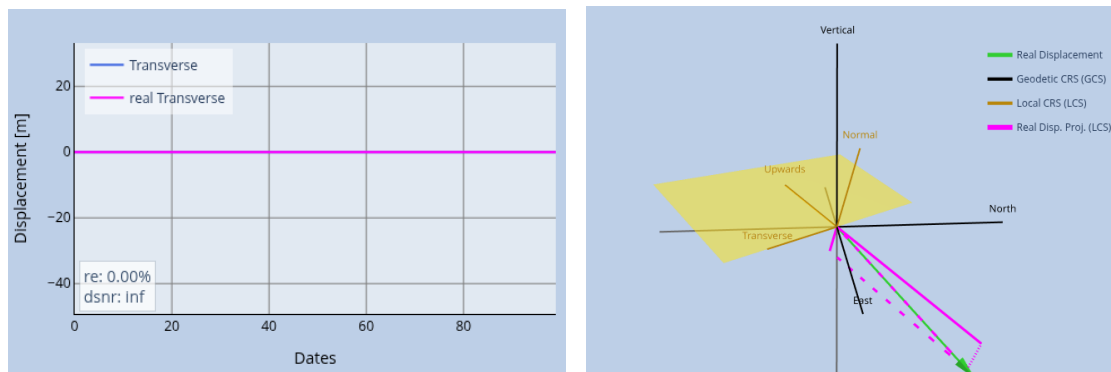


Figure 8. Displacement decomposition for the aligned slope coordinate system case. (top-left) True synthetic displacement projections in the East, North, and Vertical directions. (top-right) LOS direction with added noise. (middle-left) Upwards, (middle-right) Normal, and (bottom-left) Transverse components of the decomposed displacement. The horizontal axis labeled “Dates” denotes synthetic acquisition epoch. The numerical solution is shown in blue, and the deviations from the true noise-free solution—if no displacement were neglected and there were no noise—are shown in pink. (bottom-right) 3D plot of the true synthetic displacement direction and the coordinate system.

3.3.2. Misaligned Slope Coordinate System

We then introduce a coordinate-system misalignment by setting $\alpha = 45^\circ$ and $\beta = 20^\circ$. In this case, the neglected Transverse component is expected to affect the estimated components. For this coordinate system, the values of $\tan \zeta' / \cos \phi'$ and $\tan \phi'$ indicate that the geometric transfer of the neglected Transverse component is moderate but non-negligible (Figure A3).

In this case, the estimated displacement again contains a downslope component and a component normal to the slope surface. However, because the coordinate system is not aligned with the true synthetic displacement direction, the nonzero Transverse component introduces bias in the estimated Upwards and Normal components; their relative errors are 7.04% and 23.19%, respectively (Figure 9). The DSNR values for the Upwards and Normal components are 61.61 and 2.42, respectively, indicating that the solutions are strongly and moderately signal dominated under the prescribed synthetic noise model.

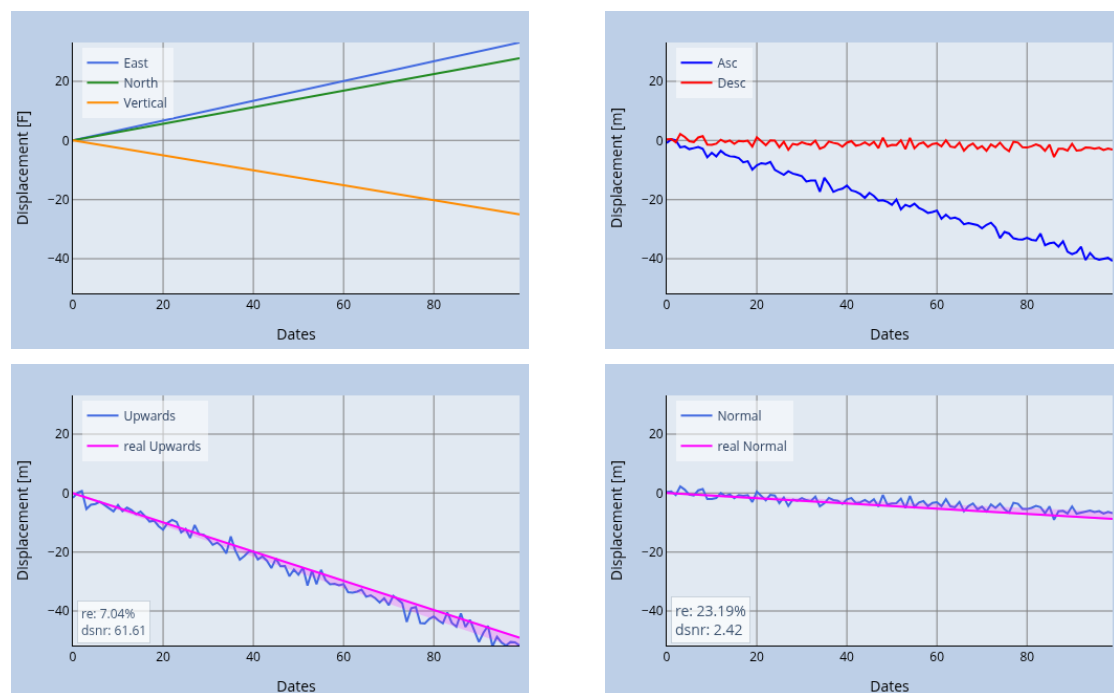


Figure 9. Cont.

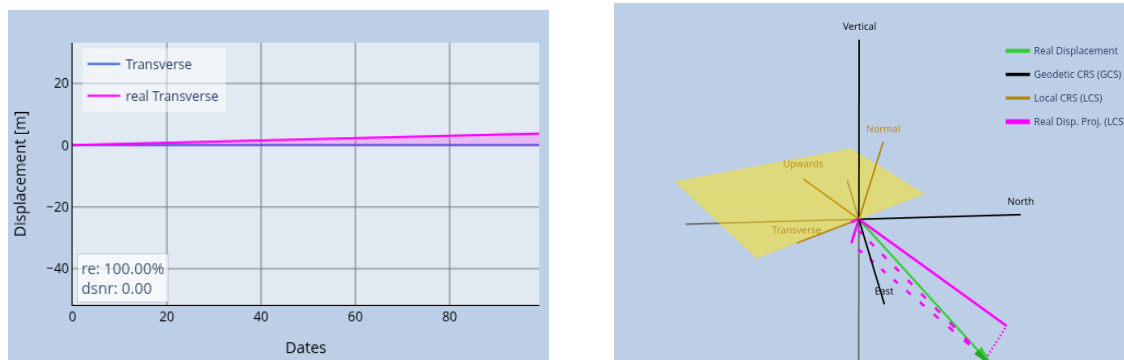


Figure 9. Displacement decomposition for the misaligned slope coordinate system case. **(top-left)** True synthetic displacement projections in the East, North, and Vertical directions. **(top-right)** LOS direction with added noise. **(middle-left)** Upwards, **(middle-right)** Normal, and **(bottom-left)** Transverse components of the decomposed displacement. The horizontal axis labeled “Dates” denotes synthetic acquisition epoch. The numerical solution is shown in blue, and the deviations from the true noise-free solution—if no displacement were neglected and there were no noise—are shown in pink. **(bottom-right)** 3D plot of the true synthetic displacement direction and the coordinate system.

3.4. Null Line Alignment

Finally, we consider the case in which the constrained axis is aligned with the null direction. In this configuration, displacement along the constrained axis is unobservable by the two LOS geometries and therefore does not bias the two estimated components in the orthogonal plane [3].

We consider a true synthetic displacement direction with an aspect of 60° and an elevation of -20° , and set the coordinate system to $\alpha = 179^\circ$ and $\beta = 8^\circ$. For these values of α and β , the Upwards direction points approximately along the null direction. Therefore, the third Equation (8) imposes zero Upwards displacement, i.e., $D = 0$ and $(a', b', c') = (1, 0, 0)$.

As shown in Figure 10, the solution is not affected by the neglected component, being the RE of the Transverse and Normal directions close to 0. They are not exactly 0, because the Upwards direction is not exactly aligned with the null direction, due to the discrete values that can be set in the dashboard for α and β .

The DSNR values for the Transverse and Normal components are 52.09 and 90.42, respectively, indicating that both estimated components are strongly signal dominated under the prescribed synthetic noise model.

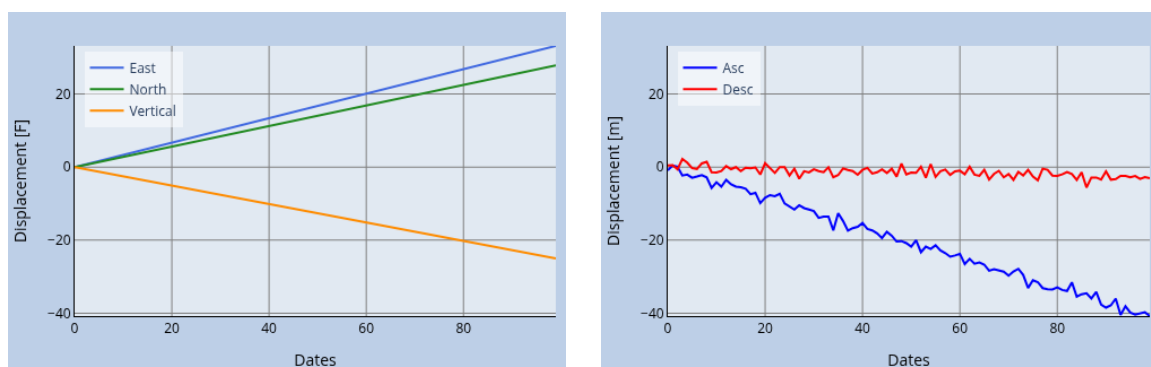


Figure 10. Cont.

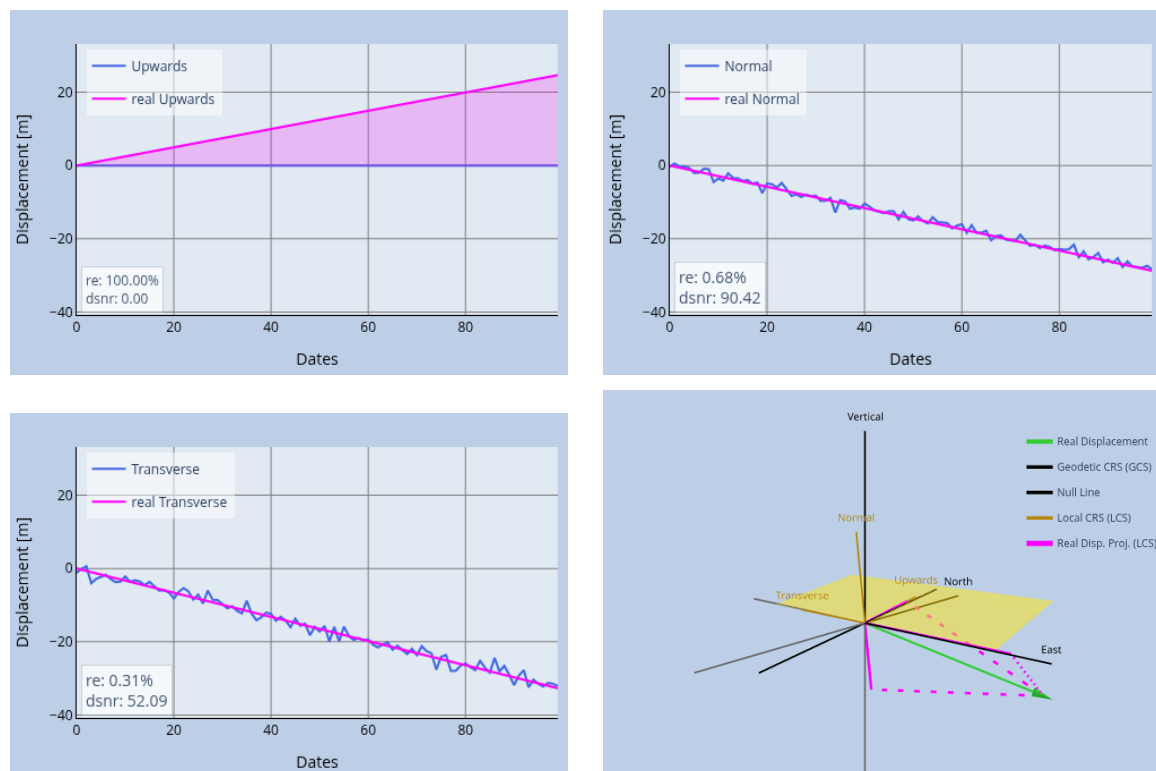


Figure 10. Displacement decomposition for the null line aligned coordinate system case. **(top-left)** True synthetic displacement projections in the East, North, and Vertical directions. **(top-right)** LOS direction with added noise. **(middle-left)** Upwards, **(middle-right)** Normal, and **(bottom-left)** Transverse components of the decomposed displacement. The horizontal axis labeled “Dates” denotes synthetic acquisition epoch. The numerical solution is shown in blue, and the deviations from the true noise-free solution—if no displacement were neglected and there were no noise—are shown in pink. **(bottom-right)** 3D plot of the true synthetic displacement direction and the coordinate system.

A null-aligned coordinate system is geometrically optimal for isolating the unobservable component when no physically meaningful deformation plane is available. However, it is generally less practical for interpretation because the resulting Upwards, Transverse, and Normal axes are not directly associated with conventional geographic or geomorphological directions. The broadly used ENV coordinate system, which is often close to the null-aligned frame for conventional near-polar SAR acquisitions, therefore remains a practical and interpretable approximation.

4. Discussion

This study provides a consistent geometric formulation and an interactive numerical tool for examining the sensitivity of constrained InSAR displacement decomposition using ascending and descending observations. The problem is relevant because two LOS observations contain only two independent displacement combinations, whereas many geological and engineering applications require components expressed in physically meaningful global or local coordinate systems. Fuhrmann and Garthwaite [25] demonstrate that interpreting LOS displacement as purely vertical without accounting for horizontal motion may introduce substantial error. The present analysis addresses the related, but distinct, problem of estimating two displacement components from two LOS observations after prescribing a third component or deformation plane.

The present dashboard does not currently implement the projection of a single LOS observation onto an arbitrary prescribed displacement direction. Such a projection is straightforward when the displacement direction is independently known, but any un-

certainty in that direction is transferred directly into the estimated magnitude. Samiei-Esfahany et al. [26] analyzed the effect of horizontal deformation on InSAR-derived subsidence estimates and showed that neglecting horizontal motion may bias the inferred vertical displacement. Their study illustrates that even apparently simple LOS-to-Vertical conversion requires a physical assessment of the horizontal deformation field and the acquisition geometry.

This study focuses on the case in which two independent ascending and descending LOS observations are available. These observations allow two independent displacement components to be estimated within a prescribed plane, or a three-component vector to be obtained conditionally after introducing an additional constraint. The resulting estimate is therefore less strongly constrained than a single-geometry projection but does not constitute an unconstrained three-dimensional reconstruction. Brouwer and Hanssen [3,19] introduced a null-direction-aligned coordinate system in which the unobservable displacement component is isolated from the two observable components.

A widely used approximation is to impose zero North displacement and estimate the East and Vertical components from ascending and descending LOS observations. This choice is motivated by the weak sensitivity of conventional near-polar SAR geometries to North–South motion and by the proximity of the North axis to the null direction. If the constrained axis is exactly aligned with the null direction, the two components estimated in the orthogonal plane are not biased by an arbitrary displacement component along that null direction. The null component itself nevertheless remains unobservable and cannot be recovered from the two LOS measurements.

For conventional right-looking, near-polar SAR geometries, the zero-North assumption provides a practical and readily interpretable approximation because the North axis is often close to the null direction. This approximation has been used to separate regional Vertical and East–West deformation in the Beijing Plain [22] and the Po Plain [23]. Its validity is not guaranteed by the acquisition geometry alone: it must also be supported by the expected deformation mechanism or independent observations. When significant North–South motion is plausible, the associated bias should be quantified rather than implicitly neglected.

The examples in Section 3.1 illustrate two distinct effects of the zero-North assumption. When the true displacement is dominated by Vertical and East–West motion, the imposed constraint is approximately satisfied, and the deterministic bias remains small for the selected acquisition geometry. In contrast, when the true displacement contains a dominant North–South component, the constraint is strongly violated and systematic errors are transferred into the estimated Vertical and East components. This deterministic bias must be distinguished from the effect of random noise. As the true displacement direction approaches the null direction, its projection onto both LOS vectors decreases, and the same LOS noise level has a stronger influence on the decomposed time series. Although Section 2.2 provides the formal covariance-propagation expression, the numerical examples do not perform a full epoch-by-epoch observational uncertainty analysis; instead, the DSNR is calculated from the resulting time series to illustrate the effect of the prescribed synthetic noise.

The analytical formulation and dashboard allow users to evaluate two different effects. The noise-free constrained solution quantifies the deterministic bias caused by a mismatch between the assumed and actual displacement plane. The noisy synthetic realizations illustrate how a prescribed random LOS error is transferred into the estimated components. Accordingly, the numerical experiments characterize the propagation of the prescribed synthetic LOS noise rather than a complete observational uncertainty model. Together,

these outputs provide a practical means of evaluating the suitability of the zero-North assumption for a specified displacement direction and acquisition geometry.

The same framework can be applied to local coordinate systems associated with engineered structures. When a deformation plane is supported by structural geometry or geotechnical information, the displacement component perpendicular to that plane may be prescribed and the two in-plane components estimated from the LOS observations. This strategy has been applied to elongated infrastructures and geological structures, including railway embankments and salt-dome deformation [24,27]. The physical justification and uncertainty of the selected plane must nevertheless be documented because the decomposition is conditional on that choice.

The bias caused by an incorrect local deformation plane is controlled by two factors: the magnitude of the true displacement component that violates the imposed constraint and the orientation of the constrained axis relative to the null direction. A small angular misalignment may therefore have either a negligible or a substantial effect depending on the displacement magnitude and the geometric amplification factor. Brouwer and Hanssen [19] also show that random-noise sensitivity depends on the orientation of the true displacement relative to the null direction. The signal-to-noise definition used in [19] is therefore distinct from the time-series DSNR adopted in this study.

In this study, we define a slope-oriented local coordinate system in which displacement is expressed along the uphill direction of maximum slope (Upwards), the horizontal contour direction (Transverse), and the direction normal to the slope surface (Normal). This representation is physically meaningful for landslides, engineered slopes, embankments, and other processes governed by local topography. A downslope movement is represented by a negative Upwards component. Related InSAR-based monitoring approaches have been used to interpret slope instability and infrastructure deformation [24,28].

As with other local coordinate system strategies, the robustness of this slope-aligned decomposition depends on how well the defined axes match the actual displacement vector. Even slight misalignments between the assumed slope-normal coordinate system and the true movement direction can lead to error redistribution across components and affect interpretability (see Section 3.3.2).

The reliability of slope-oriented decomposition depends on the accuracy with which the local aspect and slope angle represent the actual deformation plane. Errors in the digital elevation model, spatial-scale mismatch, local surface irregularity, or a displacement mechanism that does not follow the topographic gradient may all produce coordinate-system misalignment. The dashboard allows these effects to be explored by varying α , β , and the true displacement direction. This is particularly important for unstable slopes, open-pit mines, tailings dams, and engineered embankments.

This study has three main limitations. First, the numerical examples are based on synthetic data and are designed to isolate geometric effects; they are therefore not intended to account for observational error sources such as atmospheric artifacts, decorrelation, phase-unwrapping errors, reference-frame inconsistencies, or temporal-interpolation errors. Second, the examples presented in the manuscript use one representative ascending–descending acquisition geometry. Different SAR geometries lead to different numerical values of the null direction, transfer factors, and noise amplification, although the analytical relationship between the constrained axis and the null direction remains unchanged. The dashboard allows users to repeat the same sensitivity analysis for different user-defined heading and incidence configurations, coordinate-system orientations, displacement directions, and imposed constraints. Third, the dashboard is intended as a sensitivity-analysis tool and not as a substitute for application-specific validation with independent observa-

tions. Future observational applications should therefore combine the proposed sensitivity analysis with independent validation data whenever available.

5. Conclusions

The choice of coordinate system controls both the interpretability and the sensitivity of constrained InSAR displacement decomposition. When the constrained axis is aligned with the null direction, an arbitrary displacement component along that axis does not bias the two estimated components in the orthogonal plane. The null-direction component itself remains unobservable and cannot be recovered from the two LOS observations. The resulting vector must therefore be interpreted as the observable projection of the true displacement rather than as a complete three-dimensional reconstruction.

The ENV system provides a practical and readily interpretable approximation for conventional near-polar SAR geometries when North–South motion is expected to be small. Because North is often close to the null direction, imposing zero North displacement may limit the transfer of the unobservable component into the East and Vertical estimates. This approximation is nevertheless process and geometry dependent and should not be treated as universally valid.

As the true displacement direction approaches the null direction, the observable LOS signal decreases and random measurement errors are amplified relative to the signal. The reliability of the estimated components therefore decreases unless complementary observations or stronger physical information are available. This noise-related loss of precision must be distinguished from deterministic bias caused by an incorrect constraint.

The constrained decomposition is unbiased with respect to the imposed plane only when the true displacement satisfies the constraint, that is, when its component along the constrained axis equals the prescribed value. If the actual displacement contains a nonzero component that has been incorrectly fixed to zero, the estimation error lies along the null direction and is redistributed among the reported components according to the observation geometry.

Consequently, the relative errors in the noise-free decomposed solution are controlled by the unmodeled displacement component introduced by the imposed constraint, by the orientation of the constrained axis relative to the null direction, and by the magnitude of the true component used to normalize the error. The influence of random LOS noise is assessed separately through the DSNR and, in formal applications, through covariance propagation.

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Data Availability Statement: No observational InSAR datasets were used in this study. The synthetic input data, numerical outputs, analysis scripts, source code, Dockerfile, and tagged software release used to reproduce the manuscript figures are available in the GitHub repository [21]. The interactive decomposition dashboard is available at <https://decomposition.detektia.com> (accessed on 1 June 2026).

Conflicts of Interest: Authors Álvaro Hernández-Cabezudo, Alfredo Fernández-Landa, Jaime Sánchez-Fernández, Vrinda Krishnakumar and Candela Sancho were employed by the company De-tektia Earth Surface Monitoring S.L. The remaining authors declare that the research was conducted in the absence of any commercial or financial relationships that could be construed as a potential conflict of interest.

Abbreviations

The following abbreviations are used in this manuscript:

InSAR	Interferometric Synthetic Aperture Radar
LOS	Line-of-Sight
ENV	East, North, Vertical
UTN	Upwards, Transverse, Normal
RE	Relative Error
DSNR	Displacement Signal-to-Noise Ratio

Appendix A. Decomposition in Particular Coordinate Systems

Two independent LOS observations determine only two independent displacement combinations. To estimate two physically interpretable components, we prescribe the displacement along a third direction. In the examples below, the constrained component is set to zero, which is equivalent to assuming that the true displacement lies within the selected deformation plane. The resulting solution is conditional on this assumption, and any nonzero true component along the constrained axis produces an estimation error in the null direction.

Appendix A.1. Decomposition in the East-Vertical Plane

Under the assumption that the deformation takes place in the East-Vertical plane, we have to impose that there is no deformation in the North direction by setting $D = 0$ and $(a, b, c) = (0, 1, 0)$ in Equation (3). Therefore, the system of Equation (4) is reduced to:

$$\begin{aligned} d_{LOS}^A &= -\sin \theta^A \cos \psi^A \cdot d_{East} + \cos \theta^A \cdot d_{Vert} , \\ d_{LOS}^D &= -\sin \theta^D \cos \psi^D \cdot d_{East} + \cos \theta^D \cdot d_{Vert} . \end{aligned} \quad (A1)$$

Solving for d_{East} and d_{Vert} we get:

$$\begin{aligned} d_{East} &= \frac{\cos \theta^D d_{LOS}^A - \cos \theta^A d_{LOS}^D}{\cos \theta^A \cos \psi^D \sin \theta^D - \cos \theta^D \cos \psi^A \sin \theta^A} , \\ d_{Vert} &= \frac{\sin \theta^D \cos \psi^D d_{LOS}^A - \sin \theta^A \cos \psi^A d_{LOS}^D}{\cos \theta^A \cos \psi^D \sin \theta^D - \cos \theta^D \cos \psi^A \sin \theta^A} . \end{aligned} \quad (A2)$$

Appendix A.2. Decomposition in a General Vertical-Planimetric Plane

In this case, we aim to decompose the displacements into vertical and planimetric components. We assume knowledge of the planimetric displacement direction and set the angle α of the coordinate system accordingly, see Figure 2. By setting $\beta = 90^\circ$, we orient the coordinate system such that the Upwards direction aligns with the Vertical and the Normal direction aligns with the planimetric direction. Therefore, the $\mathbf{R}^{-1}$ matrix, see (6), is given by:

$$\mathbf{R}^{-1} = \begin{pmatrix} 0 & 0 & 1 \\ \cos \alpha & -\sin \alpha & 0 \\ \sin \alpha & \cos \alpha & 0 \end{pmatrix} . \quad (A3)$$

Since the Transverse direction is orthogonal to the plane of deformation, we set the displacement in this direction to zero by setting $D = 0$ and $(a', b', c') = (0, 1, 0)$. Applying this to the system of Equation (9), we obtain:

$$d_{LOS} = u'_1 d_{Up} + u'_3 d_{Normal} .$$

Using (1), (8) and (A3), we have that:

$$\begin{aligned} d_{LOS}^A &= \cos \theta^A d_{Up} + \sin \theta^A W^A d_{Normal} , \\ d_{LOS}^D &= \cos \theta^D d_{Up} + \sin \theta^D W^D d_{Normal} , \end{aligned} \quad (A4)$$

where:

$$W^X = \cos \alpha \sin \psi^X - \sin \alpha \cos \psi^X .$$

Solving for d_{Up} and d_{Normal} gives:

$$\begin{aligned} d_{Up} &= \frac{\sin \theta^D W^D d_{LOS}^A - \sin \theta^A W^A d_{LOS}^D}{\cos \theta^A \sin \theta^D W^D - \cos \theta^D \sin \theta^A W^A} , \\ d_{Normal} &= \frac{\cos \theta^A d_{LOS}^D - \cos \theta^D d_{LOS}^A}{\cos \theta^A \sin \theta^D W^D - \cos \theta^D \sin \theta^A W^A} . \end{aligned} \quad (A5)$$

When $\alpha = 90^\circ$ (A5) is reduced to (A2), with the Normal and Upwards components corresponding to the East and Vertical components, respectively, see Figure 2.

Appendix A.3. Decomposition in the General UTN Coordinate System

In a general case, α and β can take any value between 0 and 360 degrees and 0 and 90 degrees, respectively, see Figure 2. In this case, the Upwards and Normal directions describe a plane in which the real displacement is expected to occur and the Transverse direction is orthogonal to it, always parallel to the horizontal plane.

From (9) we have that:

$$\begin{aligned} d_{LOS}^A &= u'_1 d_{Up} + u'_2 d_{Transverse} + u'_3 d_{Normal} , \\ d_{LOS}^D &= u'_1 d_{Up} + u'_2 d_{Transverse} + u'_3 d_{Normal} , \\ D &= a' d_{Upwards} + b' d_{Transverse} + c' d_{Normal} . \end{aligned}$$

where the components of the unit vectors are given by (A7). Fixing the displacement in the Transverse direction to zero by setting $D = 0$ and $(a', b', c') = (0, 1, 0)$, we get:

$$\begin{aligned} d_{Up} &= \frac{u'_3 d_{LOS}^A - u'_3 d_{LOS}^D}{u'_1 u'_3 - u'_3 u'_1} , \\ d_{Normal} &= \frac{u'_1 d_{LOS}^A - u'_1 d_{LOS}^D}{u'_3 u'_1 - u'_1 u'_3} . \end{aligned} \quad (A6)$$

Appendix B. Null Direction

The null direction represents the direction along which ground motion cannot be detected by the available satellite observations [3]. It is defined as the intersection of the perpendicular planes to the LOS vectors in the two acquisition geometries: ascending and descending, Figure A1. Any displacement occurring purely along this null direction produces zero observable change in both LOS measurements, creating an inherent blind spot in InSAR-based deformation monitoring.

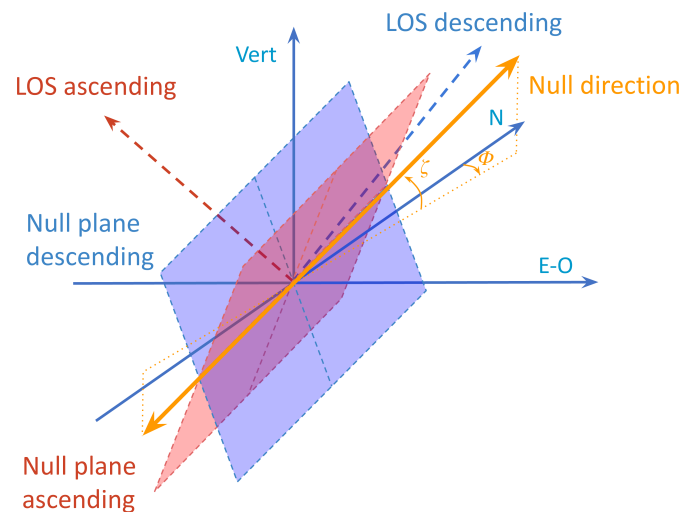


Figure A1. Null direction, formed by the intersection of the planes perpendicular to the LOS vectors in the two acquisition geometries: ascending and descending. ϕ and ζ are the azimuth and elevation angles of the null direction defined in the ENV coordinate system, respectively.

Given our LOS unit vectors for ascending and descending geometries described in the ENV coordinate system (1), we have that their perpendicular direction is given by:

$$\hat{\mathbf{N}} = (N_1, N_2, N_3) .$$

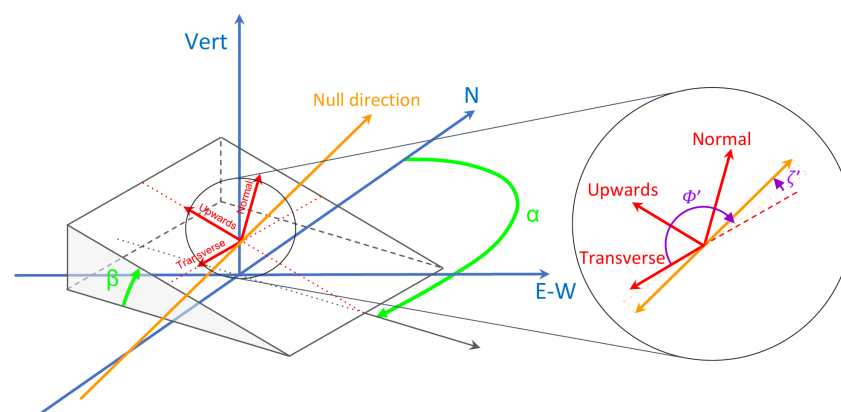


Figure A2. Null direction described by the azimuth and elevation angles ϕ' and ζ' with respect to the UTN coordinate system. The azimuth is measured from the Transverse direction towards the Upwards direction in the plane defined by them. The elevation is measured from that plane in the Normal direction. The UTN coordinate system itself is defined by the aspect and slope angles α and β with respect to the ENV coordinate system.

As a general case, we can express the null direction in the UTN coordinate system. Given our LOS unit vectors for ascending and descending geometries described in the UTN coordinate system, we have that the null direction is given by:

$$\hat{\mathbf{N}}' = (N'_1, N'_2, N'_3) = \hat{\mathbf{u}}_{LOS}^A \times \hat{\mathbf{u}}_{LOS}^D = \begin{pmatrix} u_2^A u_3^D - u_3^A u_2^D \\ u_3^A u_1^D - u_1^A u_3^D \\ u_1^A u_2^D - u_2^A u_1^D \end{pmatrix}, \quad (\text{A7})$$

where the components of the unit vectors are given by (8) with $\mathbf{R}$ being the rotation matrix from the ENV coordinate system to the UTN coordinate system defined with the azimuth and elevation angles α and β (6).

Given the definitions of both ϕ' and ζ' in the general UTN coordinate system, we have that:

$$\tan \phi' = \frac{N'_1}{N'_2},$$

$$\tan \zeta' = \frac{N'_3}{\sqrt{N'^2_1 + N'^2_2}}. \quad (\text{A8})$$

Therefore,

$$\frac{\tan \zeta'}{\cos \phi'} = \frac{N'_3}{N'_2}. \quad (\text{A9})$$

Figure A3 shows the values of the weights (A8) and (A9) as a function of the azimuth and elevation angles α and β of the UTN coordinate system.

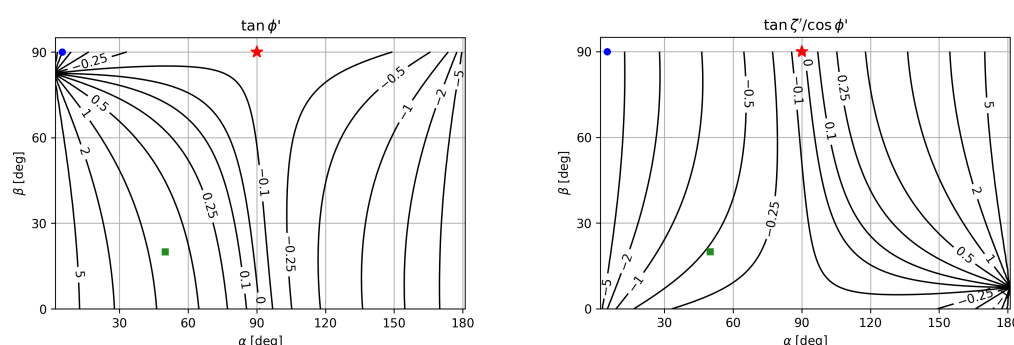


Figure A3. $\tan \phi'$ and $\tan \zeta' / \cos \phi'$ as a function of α and β . The red star represents $\alpha = 90^\circ$ and $\beta = 90^\circ$ with $\tan \phi' \simeq -0.13$ and $\tan \zeta' / \cos \phi' \simeq -0.02$, the blue circle $\alpha = 5^\circ$ and $\beta = 90^\circ$ with $\tan \phi' \simeq -2.02$ and $\tan \zeta' / \cos \phi' \simeq -15.21$, and the green square $\alpha = 50^\circ$ and $\beta = 20^\circ$ with $\tan \phi' \simeq 0.76$ and $\tan \zeta' / \cos \phi' \simeq -0.47$.

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