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A Novel Joint Denoising Strategy for Coherent Doppler Wind Lidar Signals

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Abstract: Coherent Doppler Wind Lidar (CDWL) is an effective tool for measuring the atmospheric wind field. However, CDWL is affected by various noises, which can reduce the usable value of the received echo signal. This paper proposes a novel joint denoising algorithm based on SVD-ICEEMDAN-SCC-MF to remove noises in CDWL detection. The SVD-ICEEMDAN-SCC-MF consists of singular value decomposition (SVD), improved complete ensemble empirical mode decomposition with adaptive noise (ICEEMDAN), Spearman correlation coefficient (SCC), and median filtering (MF). Specifically, the SVD first separates the signal from the noise by retaining the main feature (large singular value) and removing the remained components (small singular value) to achieve the initial signal reconstruction. Then, ICEEMDAN is used for decomposition to distinguish the intrinsic mode function (IMF) of the signal and the noise. The SCC of the retained components is calculated to determine the correlation of the reconstructed signal. Furthermore, low correlation components of the reconstructed signal are denoised again by median filtering (MF). Finally, the complete denoised signal is obtained by combining the components after MF and the high correlation components in the previous stage. The validity of the SVD-ICEEMDAN-SCC-MF is verified in simulated and real data, and the denoising effect is significantly better than other algorithms. In simulation cases, the SNR_{out} of the proposed method is improved by 20.5117 dB at most, from -5 dB to 15.5117 dB, and the $RMSE$ is only 0.5174. After denoising the power spectrum of the real CDWL signal, the detection range is extended from 3 km to more than 3.6 km.

Keywords: coherent Doppler wind lidar signal denoising; SVD; ICEEMDAN; Spearman correlation coefficient; median filtering



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1. Introduction

The Coherent Doppler Wind Lidar (CDWL) is an important wind field detection device that obtains atmospheric wind field information based on the Doppler frequency shift effect, with a rich history of usage. In the 1980s, the Kane research group at Stanford University developed a Coherent Doppler Wind Lidar with a wavelength of 1.06 μm , achieving wind field detection at 600 m and cloud detection at 2.7 km [1]. In 2002, LMCT released the WindTracer, a commercial Coherent Doppler Wind Lidar system based on

a 2.0 μm wavelength. NASA used the commercial WindTracer system to detect wind shear, clear-air turbulence, and other phenomena, and modeled aircraft wake vortices at airports in 2009 [2]. In 2017, Wang et al. from the University of Science and Technology of China (USTC) successfully developed the world's first Coherent Doppler Wind Lidar capable of simultaneously observing the atmospheric depolarization ratio and atmospheric wind field [3]. CDWL benefits from its high measurement accuracy, wide detection range, and strong anti-interference ability. It is widely used in environmental pollution monitoring [4–6], aerospace safety [7–9], meteorological research [10–15], and wind energy optimization [16,17]. However, in the actual CDWL detection, the intensity of the lidar echo signal decays proportionally to the square of the detection distance, and the instrument, thermal noise of the processing circuit, and background light interference interfere with the echo signal of CDWL, resulting in a significant reduction in detection accuracy [18]. Therefore, it is important to find a denoising algorithm that can improve the echo signal accuracy, both for wind speed inversion and expanding the detection range of CDWL.

Noise in CDWL signals is typically non-linear and non-stationary due to its continuous amplitude and random phase behavior in the time domain [19]. In early denoising studies, Fourier transform (FT) was used to reduce the noise of the lidar echo signal due to its good frequency domain analysis capability [20–22]. FT cannot provide time–frequency localized information, which easily results in signal distortion [23,24]. To employ time–frequency features effectively, Fang et al. [25] use the wavelet thresholding method to improve the denoising of LIDAR signals, which mainly deals with the details and global features of the signal flexibly through multi-resolution analysis. Then, the wavelet method has rapidly expanded in the field of denoising [26,27], but its denoising performance is still affected by the selection of wavelet bases and decomposition layers. Huang et al. [28] propose empirical mode decomposition (EMD), which removes noise by decomposing the signal into intrinsic modal functions for comprehensive analysis without prior parameter selection [29,30]. However, EMD suffers from modal aliasing and endpoint effects.

In recent years, Torres et al. [31] proposed the complete ensemble empirical mode decomposition with adaptive noise (CEEMDAN), which refines some of the limitations of EMD [32,33], which effectively suppresses the phenomena of mode aliasing and endpoint effect by adapting the noise model and realizing the complete decomposition. CEEMDAN still has residual noise and pseudo-modality in denoising [34,35]. Subsequently, variational modal decomposition (VMD) effectively avoids the problem of modal aliasing [36,37]; algorithms such as VMD-WOA achieve denoising by simply removing the high-frequency components containing noise. It filters out some useful information at the same time, resulting in signal distortion [38,39]. To summarize, the existing denoising methods are still unable to effectively decompose the signal and cannot make full use of the modal components, resulting in the signal being prone to distortion, and the accurate inversion of the atmospheric wind field cannot be realized.

In order to solve the above signal decomposition and the problem of not being able to utilize each modal component fully, this paper proposes a novel joint denoising algorithm based on SVD-ICEEMDAN-SCC-MF. The algorithm consists of singular value decomposition (SVD), improved complete ensemble empirical mode decomposition with adaptive noise (ICEEMDAN), the Spearman correlation coefficient (SCC), and median filtering (MF). The noisy signal is processed using SVD to achieve initial denoising, and the reconstructed signal is used as a reference. The signal is decomposed using ICEEMDAN, and the selected components are used to calculate the SCC. MF is performed on the components with low correlation with the reconstructed signal, which achieves the purpose of multiple denoising but also maximizes the preservation of the signal's detail information and avoids signal distortion. The final denoised signal is reconstructed using median-filtered components and

unprocessed high-correlation components. The effectiveness of our method is validated through multiple experiments, and the inversion accuracy of the CDWL signal is improved by processing its power spectrum.

2. Principle of Coherent Doppler Wind Lidar

The schematic of the Coherent Doppler Wind Lidar system used in this study is shown in Figure 1. The CDWL system consists of a laser transmitter module and a laser receiver module. The transmitter and receiver modules of the telescope of the CDWL used in this study are integrated. In the laser emission module, the seed source generates line-polarized light with a center frequency of f_0 , a portion of which is output as a local oscillator signal, while a portion of it is divided as an emission signal. The transmit signal is modulated by an acousto-optic modulator (AOM), which produces a frequency shift of the f_{AOM} to obtain a signal with a frequency of $f_0 + f_{AOM}$, which is amplified in a fiber optic amplifier to obtain the output of the power-amplified laser pulse. After the laser is emitted into the atmosphere, due to the Doppler shift effect, the echo signal will produce a Doppler shift in the atmospheric wind field with a frequency of f_d , at which time the receiver module receives the echo signal with a frequency of $f_0 + f_{AOM} + f_d$. The echo signal is coherently mixed with the local oscillator signal and converted into an intermediate frequency electrical signal with frequency $f_{AOM} + f_d$ by a balanced detector, which is sampled by the acquisition card for the next analysis and processing [40,41].

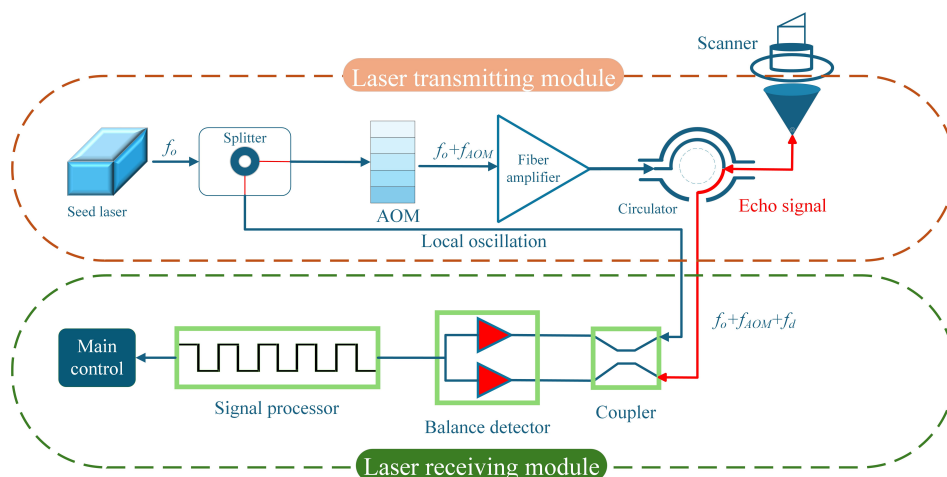


Figure 1. Schematic diagram of Coherent Doppler Wind Lidar.

In this study, we use a CDWL system manufactured by Hefei Technovo Lidar Hi-Tech Co. Ltd. in Hefei, China (model KC-WL-2006 Lidar). The CDWL system is pulse-triggered, with a laser wavelength of 1550 nm, a center frequency of 80 MHz, a sampling frequency of 1 GHz, a distance resolution of 60 m, an optimal detection distance of 3000 m, and an optimal wind speed measurement range of 62 m/s. The velocity azimuth display (VAD) method is used to retrieve the horizontal wind field in the CDWL scanning measurement mode. The main parameters of the Coherent Doppler Wind Lidar system are shown in Table 1.

Table 1. Coherent Doppler Wind Lidar system parameters.

Transmitter		Transceiver		Data Acquisition	
Wavelength	1550 nm	Laser mode	Pulse	Sampling frequency	1 GHz
Pulse energy	145 μ J	Scan mode	Conical	Sampling points	400
Pulse repetition	10 KHz	Elevation angle	60°	Range resolution	60 m
Pulse width	400 ns	Step angle	90°	Gate number	128

3. SVD-ICEEMDAN-SCC-MF Methodology

The SVD-ICEEMDAN-SCC-MF algorithm consists of four parts: SVD, ICEEMDAN, the Spearman correlation coefficient (SCC), and median filtering (MF). They cooperate with each other to achieve multiple noise reduction and retain detailed information while reducing noise. Firstly, the SVD is used for preliminary noise reduction, and the signal after preliminary noise reduction provides a reference for the later calculation of SCC. Then, ICEEMDAN is used to decompose the reconstructed signal. The third step calculates the SCC of each decomposed component. The low correlation components are median filtered rather than directly discarded, which avoids signal distortion and loss of useful information and achieves the purpose of multiple denoising. Finally, the signal is reconstructed by the high-correlation components and the median-filtered components.

The SVD algorithm can effectively distinguish between the main components of the signal and the noise components through the singular values. The effective signal usually corresponds to a large singular value, while the noise is usually random and dispersed, and its energy is distributed throughout the signal, which corresponds to a smaller singular value. The use of the cumulative variance ratio allows for the automatic selection of the appropriate number of principal components, and by setting a reasonable threshold, effective denoising can be achieved. Typically, the threshold is set to 0.9. In cases of extreme noise, the threshold can be dynamically adjusted based on the denoising performance to achieve better results. This property of SVD makes it very robust to deal with random noise. ICEEMDAN achieves more stable decomposition and the better separation of different components in the signal by repeatedly adding white noise multiple times and averaging the results. Autonomous determination of the number of IMFs generated by the decomposition enables adequate decomposition of the signal. By introducing a step-by-step processing approach, each decomposition extracts one IMF from the signal, gradually reducing the complexity of the signal and more accurately extracting its true components. After preliminary denoising with SVD, part of the noise is removed, and the reconstructed signal after SVD denoising is used as a reference for calculating the SCC, ensuring the accuracy of SCC calculation. Additionally, the thorough and detailed decomposition of ICEEMDAN further enhances the reliability of the SCC calculation. For the low-correlation components obtained through the SCC calculation, further denoising operations are performed instead of discarding them. Therefore, even if signal components orthogonal to the reconstructed signal appear, applying median filtering to these components to remove potential residual noise is more beneficial for ensuring the accuracy of signal denoising. Because of these characteristics, the algorithm demonstrates excellent robustness and accuracy.

3.1. SVD

SVD is a matrix decomposition method that is important and widely used in matrix theory in the field of mathematics to simplify and remove redundancies from complex matrices for information processing [42], and it is an effective full-band denoising tool. The essence of SVD is to transform signals into matrices and decompose complex matrices based on the principle of the low-rank approximation of matrices [43]. The singular values obtained by SVD decomposition reflect the intrinsic properties of the signal, especially the importance of the signal, and through the selection and retention of appropriate singular values, SVD can achieve the purpose of signal denoising. SVD has good stability and is commonly used to eliminate random noise in non-linear and non-stationary signals, and has been widely used in the fields of signal noise reduction, information compression, and feature extraction [44].

Step 1. Construction of Hankel matrix

Let the signal Y containing noise be

$$Y = \{y(1), y(2), \dots, y(N)\}, \quad (1)$$

The Hankel matrix is chosen as the trajectory matrix for the SVD algorithm, and the Hankel matrix corresponding to the signal Y is

$$H = \begin{bmatrix} y(1) & y(2) & \cdots & y(n) \\ y(2) & y(3) & \cdots & y(n+1) \\ \vdots & \vdots & \ddots & \vdots \\ y(m) & y(m+1) & \cdots & y(N) \end{bmatrix}, \quad (2)$$

$N = m + n - 1, 1 < n < N, m \geq 2, n \geq 2$ in Equation (2). In order to achieve the adequate identification and separation of signal and noise and to improve the denoising effect of the Hankel matrix construction method, an appropriate matrix order should be chosen:

$$m = \begin{cases} (N+1)/2, & N \text{ is an odd number} \\ N/2, & N \text{ is an even number} \end{cases}, \quad (3)$$

According to Equation (3), deducing from equation $n = N + 1 - m$ yields n .

Step 2. Singular value decomposition

For an $m \times n$ matrix H , an SVD decomposition yields two normalized orthogonal matrices $U_{m \times m}$, $V_{n \times n}$ and diagonal matrix $S_{m \times n}$,

$$H = USV^T, \quad (4)$$

In the formula, U and V are the eigenvector matrices corresponding to singular values:

$$S = [\text{diag}(\sigma_1, \sigma_2, \dots, \sigma_r), \mathbf{0}] \in \mathbf{R}^{m \times n}, \quad (5)$$

S is the singular value matrix of the signal Y , where $\sigma_1 \geq \sigma_2 \geq \dots \geq \sigma_r$.

Step 3. Singular value selection

The singular values $\sigma_1, \sigma_2, \dots, \sigma_r$ contained in the matrix $S_{m \times n}$ are processed by the cumulative variance ratio as in Equation (6). A threshold value of 0.9 is set, and by traversing the calculation, the calculation stops when the cumulative variance ratio reaches the set threshold, at which point k is the number of singular values to be retained. After choosing the appropriate singular values, the processed matrix is obtained:

$$CVR = \frac{\sum_{i=1}^k \sigma_i^2}{\sum_{i=1}^r \sigma_i^2} \quad (6)$$

where k is the number of singular values currently selected.

Step 4. Signal Reconstruction

The reconstructed trajectory matrix after the denoising process is set as

$$H_d = \begin{bmatrix} y_d(1,1) & y_d(1,2) & \cdots & y_d(1,n) \\ y_d(2,1) & y_d(2,2) & \cdots & y_d(2,n) \\ \vdots & \vdots & \ddots & \vdots \\ y_d(m,1) & y_d(m,2) & \cdots & y_d(m,n) \end{bmatrix}, \quad (7)$$

The $y_d(m, n)$ in Equation (7) are the elements of the reconstructed trajectory matrix, the reconstructed matrix H_d is averaged diagonally, and the denoised signal Y_d can be obtained by converting the matrix back to a one-dimensional time series.

3.2. ICEEMDAN

ICEEMDAN is a modal decomposition algorithm further optimized on the basis of CEEMDAN, which eliminates the pseudo-modalities generated by CEEMDAN and solves some of its limitations. Unlike CEEMDAN, which directly adds Gaussian white noise during the decomposition process, ICEEMDAN incorporates white noise as part of the complete noise ensemble. By introducing white noise, it helps address the issues of decomposition instability and mode mixing. The modal components obtained by ICEEMDAN achieve a detailed decomposition of the signal, which can better deal with noise and non-linear features in the signal [45,46]. The flowchart of the ICEEMDAN is shown in Figure 2.

$M(\cdot)$ denotes the operator used to compute the local mean of the signal. $E_k(\cdot)$ denotes the operator of the k th IMF component obtained from the EMD decomposition, and its main function is to describe the EMD decomposition of the signal and extract the IMF. The coefficient β_k denotes the signal-to-noise ratio of the k th level.

Step 1. Add white noise to the original signal X to obtain

$$X^{(i)} = X + \beta_0 E_1(W^{(i)}), \quad (8)$$

$X^{(i)}$ denotes the noise-aided signal generated after the i th addition of white noise. $W^{(i)}$ is the realization of the i th Gaussian white noise with zero mean and unit variance.

Step 2. The empirical modal decomposition is used to compute the local mean of $X^{(i)}$, and the computed mean gives the first residual value

$$r_1 = \frac{1}{N} \sum_{i=1}^N M(X^{(i)}), \quad (9)$$

where N is the number of data points of signal X .

Step 3. Subtracting the residuals of the first stage from the original signal yields the first component of the signal ($k = 1$), denoted IMF_1

$$IMF_1 = X - r_1, \quad (10)$$

Step 4. When $k = 2$, compute the local mean of r_1 by EMD plus Gaussian white noise and compute the second residual

$$r_2 = \frac{1}{N} \sum_{i=1}^N M(r_1 + \beta_1 E_2(W^{(i)})), \quad (11)$$

The second mode can be derived from the difference between r_1 and r_2 , i.e.,

$$IMF_2 = r_1 - r_2, \quad (12)$$

Step 5. When $k = 3, 4, \dots, K$, calculate the k th residual and k th mode, respectively

$$r_k = \frac{1}{N} \sum_{i=1}^N M(r_{k-1} + \beta_{k-1} E_k(W^{(i)})), \quad (13)$$

$$IMF_k = r_{k-1} - r_k, \quad (14)$$

Step 6. Repeat the computation until all the IMFs are obtained, and finally, the original signal is decomposed into K IMFs and a r_n , and X can be denoted as

$$X = \sum_{k=1}^K IMF_k + r_n, \quad (15)$$

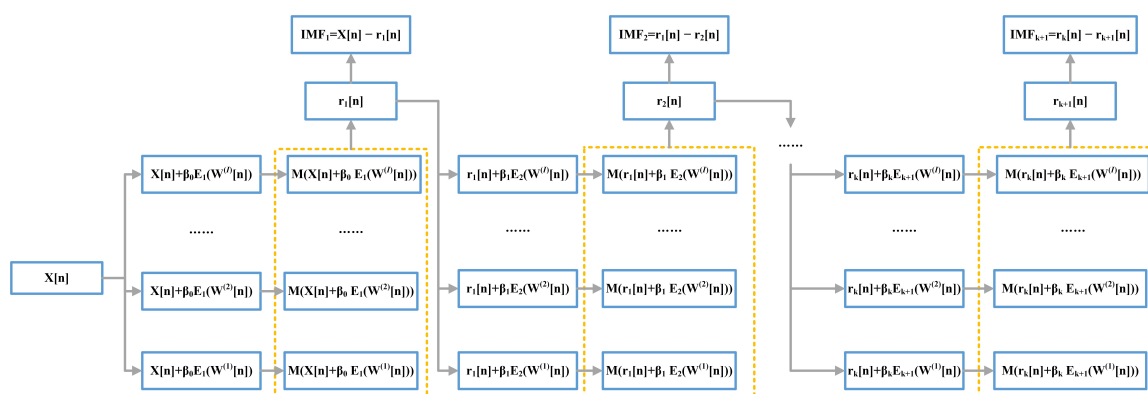


Figure 2. Flowchart of ICEEMDAN algorithm.

3.3. Spearman Correlation Coefficient

The Spearman correlation is a non-parametric measure of rank correlation that measures the trend correlation between variables. It performs significantly better than Pearson's correlation in measuring the correlation of variables in a non-linear process [47,48].

The Spearman correlation coefficient (SCC) method is a rank-based method that lets X and Y be independent and identically distributed datasets with N data each, and X_i and Y_i represent the i th value in X and Y , respectively, with $i = 1, 2, \dots, N$. SCC firstly arranges the data in the two datasets in ascending or descending order to obtain the ranked set x, y of the two sets of data, where x_i and y_i represent the ordering of X_i in X and Y_i in Y , respectively. The difference in the corresponding elements in the set x, y is made to obtain the ranking difference set d ,

$$d_i = x_i - y_i, \quad (16)$$

The Spearman correlation coefficient r_s is calculated as follows:

$$r_s = \frac{\sum_{i=1}^N (x_i - \bar{x})(y_i - \bar{y})}{\sqrt{\sum_{i=1}^N (x_i - \bar{x})^2 \sum_{i=1}^N (y_i - \bar{y})^2}}, \quad (17)$$

From Equation (17), in reality, the connection between the variables has little effect, and the following is obtained:

$$r_s = 1 - \frac{6 \sum_{i=1}^N d_i^2}{N(N^2 - 1)}, \quad (18)$$

The value of r_s ranges from $[-1, 1]$, and the larger its absolute value, the stronger the correlation; see Table 2.

Table 2. Trends in correlation coefficients.

r_s	Trends
$0 < r_s \leq 1$	positive correlation
0	irrelevant
$-1 \leq r_s < 0$	negative correlation

3.4. Median Filtering

Median filtering (MF) is a non-linear processing method with good edge preservation characteristics and the ability to suppress impulse noise, which is a typical representative of non-linear filtering methods [49].

The basic principle of median filtering is to replace the value of any point in a sequence of numbers with the median of the values of the points in a domain of that point. Assuming a set of data $x_1, x_2, \dots, x_n$ with the n numbers being in order of numerical size to obtain $x_1 \geq x_2 \geq \dots \geq x_n$, then the median of this sequence is defined as follows:

$$y = \text{med}(x_1, x_2, \dots, x_n) = \begin{cases} x_{\frac{n+1}{2}}, & n \text{ is an odd number} \\ \frac{1}{2}(x_{\frac{n}{2}} + x_{\frac{n}{2}+1}), & n \text{ is an even number} \end{cases} \quad (19)$$

MF can effectively remove impulse noise and parasitic oscillations, which can be easily adaptive and can further improve the filtering effect.

3.5. SVD-ICEEMDAN-SCC-MF Algorithm

The joint denoising algorithm SVD-ICEEMDAN-SCC-MF takes advantage of the excellent characteristics of the above methods in signal denoising, especially in dealing with non-linear and non-stationary signals, and achieves multiple denoising of noise-containing signals through the mutual cooperation of each denoising algorithm. The flowchart of the SVD-ICEEMDAN-SCC-MF algorithm is shown in Figure 3.

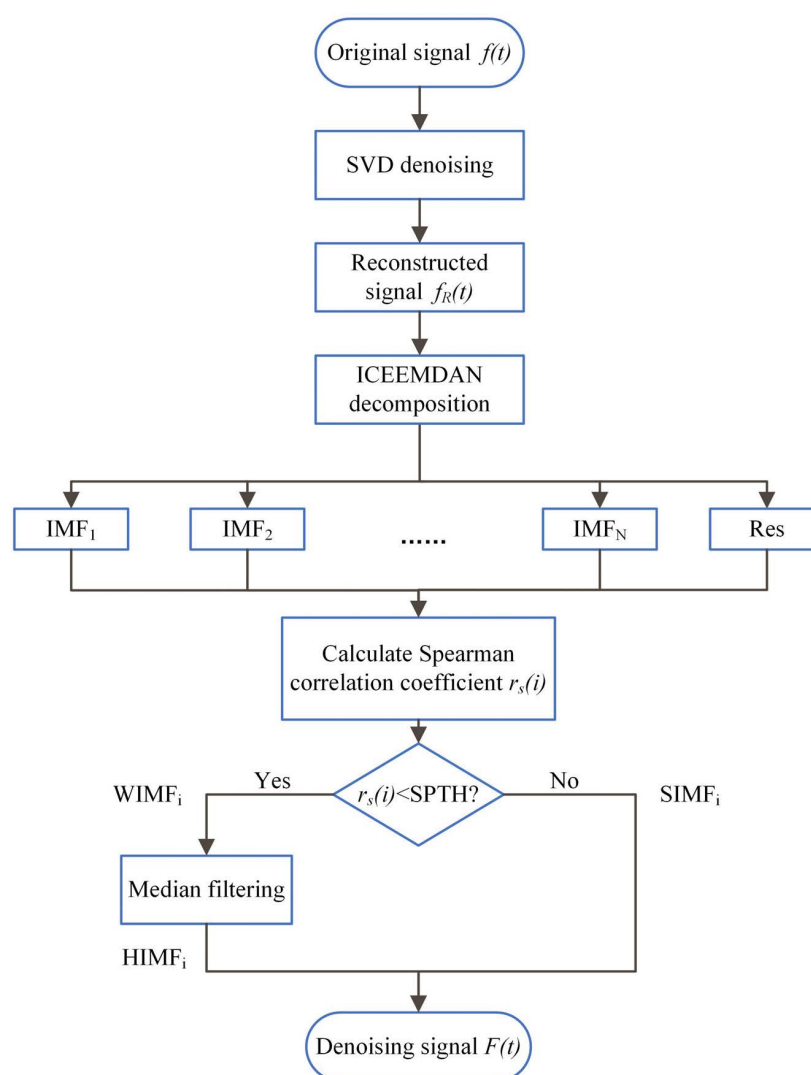


Figure 3. Flowchart of SVD-ICEEMDAN-SCC-MF denoising algorithm.

According to the above principle, the specific steps of the SVD-ICEEMDAN-SCC-MF algorithm are as follows:

Step 1. Input the original signal $f(t)$.

Step 2. The original signal $f(t)$ is preliminarily denoised according to Equations (1)–(7) to realize the preliminary processing of random noise and other interferences, and then the signal is reconstructed to obtain the reconstructed signal $f_R(t)$.

Step 3. Perform ICEEMDAN decomposition of the reconstructed signal $f_R(t)$ according to Equations (8)–(15) to obtain a number of eigenmode functions $IMF_i (i = 1, 2, \dots, N)$ and a residual Res .

Step 4. Calculate the Spearman correlation coefficient $r_s(i)$ for each component according to Equation (18).

Step 5. Determine whether the absolute value of the correlation coefficient $r_s(i)$ is less than the threshold SPTH.

- If yes, the component is weakly correlated with the reconstructed signal $f_R(t)$, then the component is ready for median filtering, which is noted as $WIMF_i$.
- If no, then the component is strongly correlated with the reconstructed signal $f_R(t)$, then the component is not processed, and the component is recorded as $SIMF_i$.

Step 6. The IMF component $WIMF_i$ with a weak correlation is processed by median filtering and denoised again to obtain $HIMF_i$.

Step 7. Reconstruct the signal from the median filtered IMF component $HIMF_i$ and the correlated IMF component $SIMF_i$.

Step 8. Obtain the denoised signal $F(t)$.

4. Results

4.1. Evaluation Indicators

In order to quantitatively compare the denoising performance of different algorithms, the output signal-to-noise ratio (SNR_{out}) and root-mean-square error ($RMSE$) are used as performance evaluation metrics. SNR_{out} reflects the evaluation of the useful information content in the denoised signal, the larger the value of SNR_{out} , the more useful the information content in the denoised signal. Similarly, the smaller the $RMSE$, the higher the similarity between the two signals and the better the denoising effect of the signal [50]:

$$SNR_{out} = 10 \log \left(\frac{\sum_{n=1}^N x^2(n)}{\sum_{n=1}^N [x'(n) - x(n)]^2} \right) \quad (20)$$

$$RMSE = \sqrt{\frac{1}{N} \sum_{n=1}^N [x'(n) - x(n)]^2} \quad (21)$$

where $x(n)$ is the original signal, $x'(n)$ is the denoised signal, and N is the signal length.

4.2. Analogue Signal Denoising Experiment

In order to verify the effectiveness of the SVD-ICEEMDAN-SCC-MF algorithm, denoising simulations are carried out using three typical signals, namely, Bumps, Blocks, and Heavy sine. Because Gaussian white noise is widely present in real-world systems, it can be used to evaluate algorithm performance with clear theoretical significance, help to establish standardized testing methods, facilitate fair comparison between different algorithms, and, as it covers all frequencies and is difficult to remove, test the generalization ability of algorithms. Therefore, Gaussian white noise is added to the original signal, as shown in Figure 4. In the figure, the blue line represents the original signal without noise, while the red line represents the noisy signal after adding noise, where $SNR_{in} = 5$ dB.

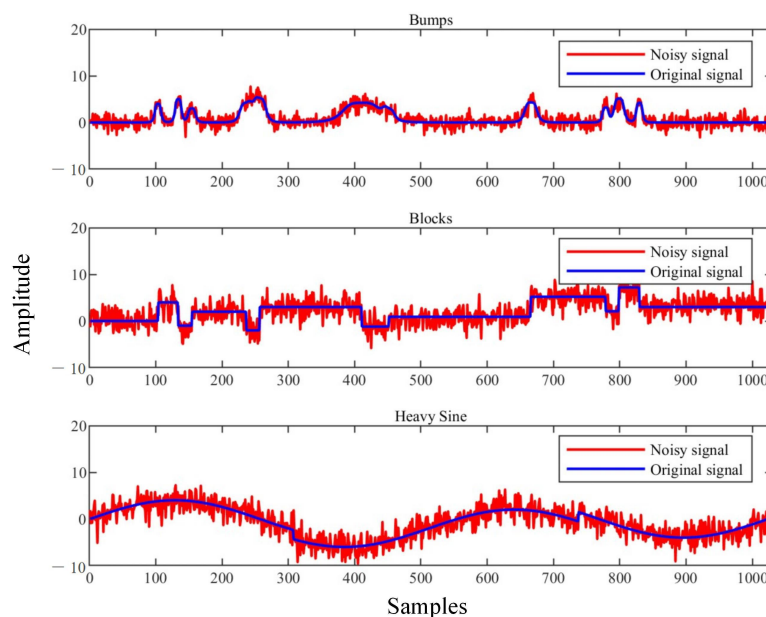


Figure 4. Three typical signals.

Here, the performance of the SVD-ICEEMDAN-SCC-MF is compared with the CEEMDAN-PE-WT [51], NGO-VMD [52], EMD-WT [53], and WT(db4) [54] algorithms using the Bumps signal as an example. The denoising performance of the above denoising method is tested at $SNR_{in} = 5$ dB, and the results are shown in Figure 5.

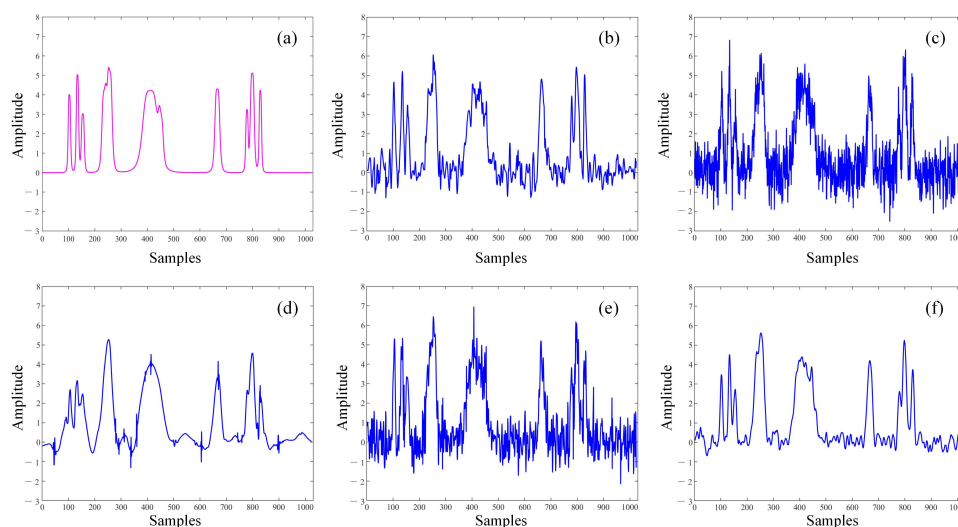


Figure 5. Comparison of denoising results of five denoising methods. (a) Original Bumps signal (without adding Gaussian white noise), (b) CEEMDAN-PE-WT, (c) NGO-VMD, (d) EMD-WT, (e) WT, and (f) SVD-ICEEMDAN-SCC-MF.

Figure 5 shows that the denoising performance of the five denoising methods varies greatly, and our SVD-ICEEMDAN-SCC-MF works best. After the denoising process, NGO-VMD exhibits the most noise in its results, and it is difficult to recognize the contour of the original signal from the denoising results; at the same time, its SNR_{out} is the lowest at 7.0298 dB. The WT is slightly better than the NGO-VMD, but again, the noise is still noticeable, and it is difficult to distinguish the original signal, with an SNR_{out} of 7.3145 dB.

Compared with the above two denoising methods, EMD-WT has greatly improved the denoising effect, the contour of the original signal is basically recognizable, and the SNR_{out} is simultaneously improved to 11.6061 dB, but the denoised signal is seriously distorted, and a lot of detailed information is lost. Compared with it, CEEMDAN-PE-WT retains part of the original signal information better, and its SNR_{out} is 12.0807 dB, but the denoised signal still contains a lot of “burrs”. The SVD-ICEEMDAN-SCC-MF algorithm proposed in this paper shows the best denoising effect. While the signal is denoised, the details are well preserved, and the original signal is basically restored. The SNR_{out} is as high as 15.5011 dB, and the $RMSE$ is 0.3021.

The SNR_{out} and $RMSE$ of CEEMDAN-PE-WT, NGO-VMD, EMD-WT, WT, and SVD-ICEEMDAN-SCC-MF to the Bumps signal after SNR_{in} with -5 , 0 , 5 , 10 , and 15 dB denoising are shown in Figure 6. As shown in the figure, the SVD-ICEEMDAN-SCC-MF algorithm proposed in this paper exhibits the best performance when processing Bumps signals. The SNR_{out} fold line is always above the other four folds, and the $RMSE$ fold line is always below the other four folds. As the SNR_{in} increases, the SNR_{out} and $RMSE$ perform even better. Even when the SNR_{in} is very poor at -5 dB, the SNR_{out} improves to 7.8962 dB, an improvement of nearly 13 dB, much higher than the other four algorithms, and while having the best SNR_{out} , it also has the lowest $RMSE$ of the five at 0.7250. The same optimal performance is observed in the other cases with different SNR_{in} , in particular, the lower the SNR_{in} , the larger the difference between the other four-fold lines roughly and the SVD-ICEEMDAN-SCC-MF fold line, indicating the better performance of the SVD-ICEEMDAN-SCC-MF algorithm. A slight exception is EMD-WT, which gradually increases the gap between its SNR_{out} fold line and the SVD-ICEEMDAN-SCC-MF fold line with increasing SNR_{in} but is always below SVD-ICEEMDAN-SCC-MF.

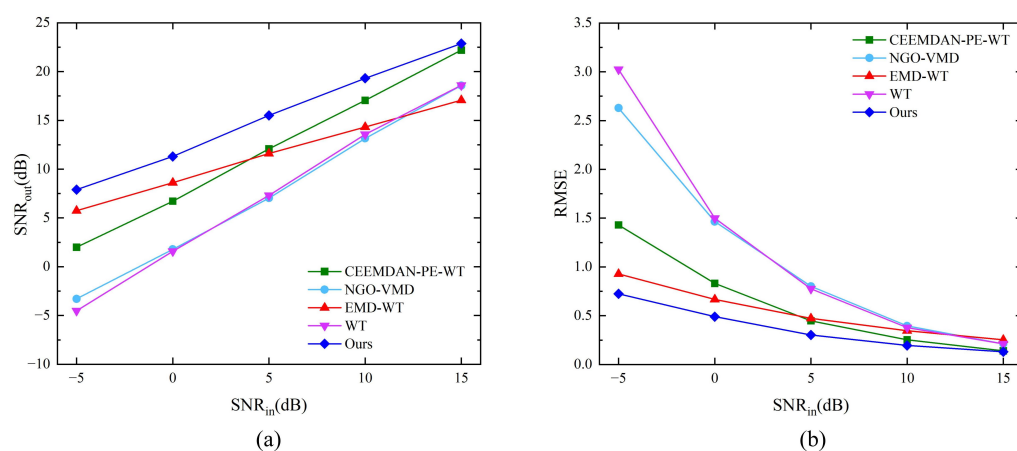


Figure 6. (a) SNR_{out} of Bumps signal after denoising at different SNR_{in} ; (b) $RMSE$ of Bumps signal after denoising for different SNR_{in} .

Table 3 lists the SNR_{out} and $RMSE$ of these five algorithms when processing Blocks and Heavy sine signals with different SNR_{in} . The data before the parentheses in the table are SNR_{out} in dB, and the data inside the parentheses are $RMSE$. It can also be concluded that the SVD-ICEEMDAN-SCC-MF has the highest SNR_{out} and the lowest $RMSE$ at different SNR_{in} . The lower the SNR_{in} , the more obvious the advantage is, especially when dealing with Heavy sine signals, where the SNR_{out} is improved by 20.5117 dB at $SNR_{in} = -5$ dB, and at $SNR_{in} = 15$ dB, the SNR_{out} is as high as 26.6361 dB. Through the analysis of Figures 5 and 6 and Table 3, it can be concluded that the SVD-ICEEMDAN-SCC-MF algorithm proposed in this paper achieves excellent denoising performance for different types of signals and signals with different input signal-to-noise ratios. In various scenarios,

its SNR_{out} and $RMSE$ are both optimal. The SVD-ICEEMDAN-SCC-MF algorithm not only effectively removes noise but also preserves the detailed information of the signal, avoiding signal distortion. In contrast, the denoising performance of NGO-VMD and WT is relatively poor, failing to effectively eliminate noise. EMD-WT exhibits significant distortion, with the loss of detailed information in the signal. CEEMDAN-PE-WT still contains some noise that cannot be effectively removed.

Table 3. Comparison of denoising performance of five denoising methods with different SNR_{in} .

Signal	SNR_{in}	CEEMDAN-PE-WT	NGO-VMD	EMD-WT	WT	SVD-ICEEMDAN-SCC-MF (Ours)
Blocks	−5	1.2838 (2.5620)	−3.4867 (4.4372)	6.7147 (1.3710)	−4.6555 (5.0763)	9.2925 (1.0189)
	0	6.1068 (1.4701)	3.9012 (1.8954)	10.3173 (0.9055)	0.9083 (2.6752)	12.7184 (0.6868)
	5	10.5029 (0.8864)	6.6384 (1.3831)	10.7627 (0.8603)	6.4205 (1.4182)	14.1945 (0.5795)
	10	15.8144 (0.4809)	12.0596 (0.7409)	14.6569 (0.5494)	12.3039 (0.7204)	16.6830 (0.4351)
	15	19.3774 (0.3191)	16.3799 (0.4506)	17.2319 (0.4085)	17.0252 (0.4183)	19.8713 (0.3014)
Heavy sine	−5	1.6675 (2.5468)	−3.5589 (4.6485)	8.1924 (1.2016)	−4.6197 (5.2524)	15.5117 (0.5174)
	0	6.3061 (1.4930)	1.8292 (2.4998)	12.8485 (0.7030)	0.4070 (2.9446)	16.5291 (0.4602)
	5	11.4302 (0.8277)	6.9385 (1.3882)	16.6291 (0.4549)	6.1674 (1.5171)	21.8651 (0.2490)
	10	17.2320 (0.4244)	11.6688 (0.8052)	22.3915 (0.2343)	12.7069 (0.7145)	23.0056 (0.2183)
	15	22.4048 (0.2340)	16.9834 (0.4367)	23.9941 (0.1948)	18.3387 (0.3736)	26.6361 (0.1437)

While comparing SNR_{out} and $RMSE$, the computation time is compared to reflect the cost of computation, and the results are shown in Figure 7. The time of NGO-VMD in processing Bumps, Blocks, and Heavy sine signals is 29.171 s, 25.322 s, and 34.227 s, respectively, which is too large compared to the other four algorithms. In the process of processing these three typical signals, WT is always the shortest time and NGO-VMD is the longest time, and it is nearly dozens of times that of other algorithms. The computation time of the SVD-ICEEMDAN-SCC-MF algorithm is shorter than those of CEEMDAN-PE-WT and NGO-VMD, and the difference is very small compared to those of EMD-WT and WT in the processing of the Blocks and the Heavy sine signals. Considering the SNR_{out} , $RMSE$, and computation time, the SVD-ICEEMDAN-SCC-MF algorithm proposed in this paper has optimal performance.

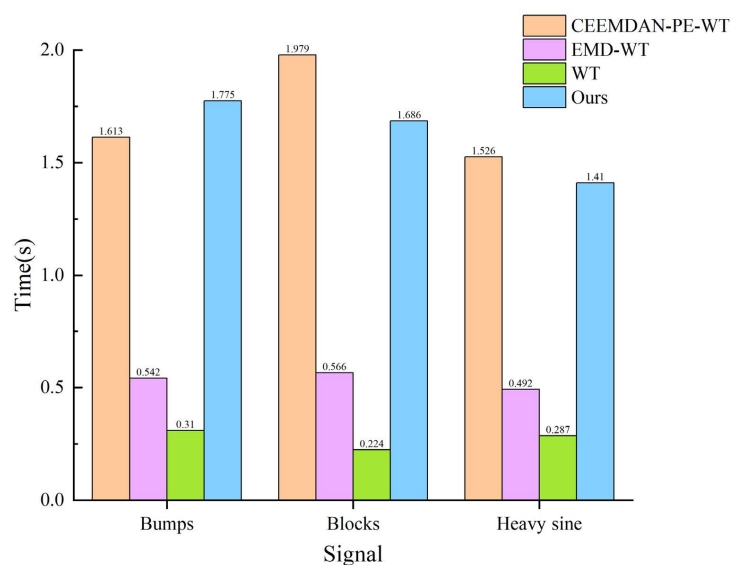


Figure 7. Time is taken by five different algorithms to process different signals (in seconds).

4.3. CDWL Simulation Signal Denoising Experiment

The CDWL echo signal simulation model is used to simulate the ideal CDWL echo signal. According to the echo signal Equation (22), the emitted laser is configured with a wavelength of 2 μm , a local oscillator power of 1 mW, an emitted laser pulse width of 400 ns, and an acousto-optic frequency shift of 55 MHz to simulate the CDWL echo signal. The echo signal's power spectrum will be supplemented with Gaussian white noise with an SNR_{in} of -1 dB. Using the above five denoising methods, the synthesized signal is denoised, and the results are compared as shown in Figure 8:

$$S(t) = 2(P_{LO})^{1/2} \exp(i2\pi v_{IF}t) \sum_m \left[P_T \left(t - \frac{2R_m}{c} \right) \right]^{1/2} \times \exp \left[i\varphi \left(t - \frac{2R_m}{c} \right) \right] \tilde{K}_m \exp(-i2kv_m t) \quad (22)$$

In the formula, P_{LO} is the local oscillator optical power, P_T is the emitted laser pulse, v_{IF} is the acousto-optic frequency shift, R_m is the distance between the center of each layer and the receiving plane, k is the wave number, v_m is the radial wind speed, and $\tilde{K}_m$ is a circular random variable consisting of atmospheric parameters.

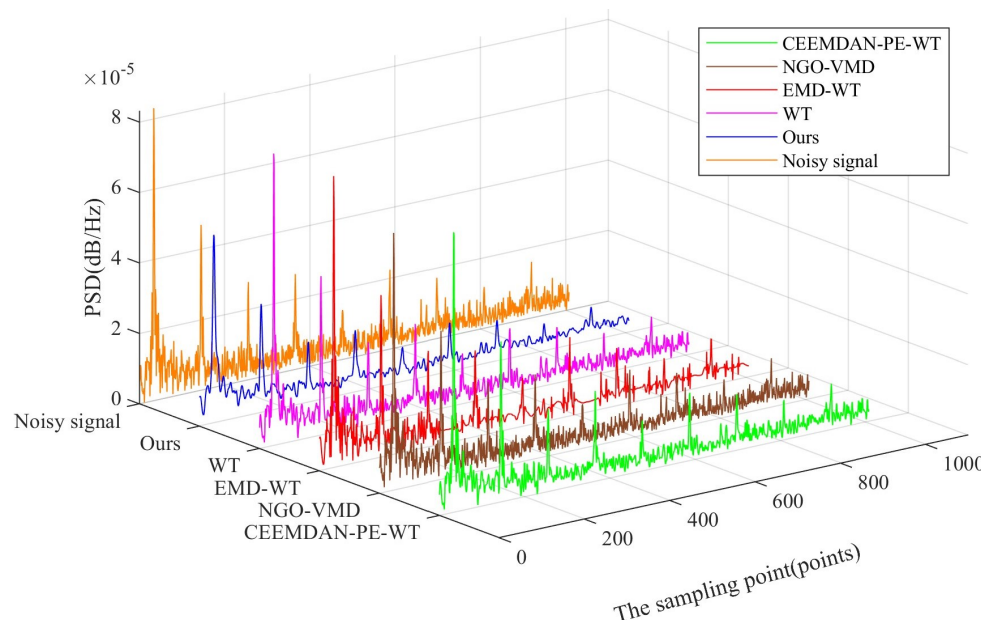


Figure 8. The denoising comparison of the power spectrum of the echo signal simulation model using five denoising methods.

As shown in Figure 8, the SVD-ICEEMDAN-SCC-MF algorithm shows the best denoising performance by comparing with the power spectrum of the synthesized signal 'Noisy signal'. After processing, the nine main peaks of the signal are clearly visible, and the power spectrum of the signal is smoother. There is basically no noise around the peaks that can cause interference, and the main peaks are obvious, which is conducive to the subsequent data processing. EMD-WT and CEEMDAN-PE-WT also have better results, but the processed signal still has much noise, which interferes with the identification of the main peaks. In particular, EMD-WT still has a high number of indistinguishable noise "spikes" between the 660th and 1000th data points, and the main peaks are difficult to recognize. The WT and NGO-VMD denoising effects generally still retain more noise. Therefore, it can be concluded that the SVD-ICEEMDAN-SCC-MF algorithm proposed in this paper has advantages in denoising the simulation model of CDWL echo signals.

4.4. Real CDWL Signal Denoising Experiment

In order to verify the effectiveness of the SVD-ICEEMDAN-SCC-MF algorithm in denoising, a field observation experiment was conducted on 11 April 2024 using Coherent Doppler Wind Lidar at Shandong Normal University, Changqing District, Jinan City, Shandong Province, China (latitude 36.5° N, longitude 116.8° E). The weather on the day of the test was clear, and the ambient air AQI was 76, which is good air quality. The specific parameters of the CDWL are shown in Section 2. The signal is sampled by the acquisition card, and then the signal is divided into several consecutive distance gates in chronological order. The data processing module processes the signal to estimate the power spectrum, extracts the Doppler shift, and calculates the atmospheric wind field information from the processed Doppler shift data. The physical diagram of the CDWL is shown in Figure 9.

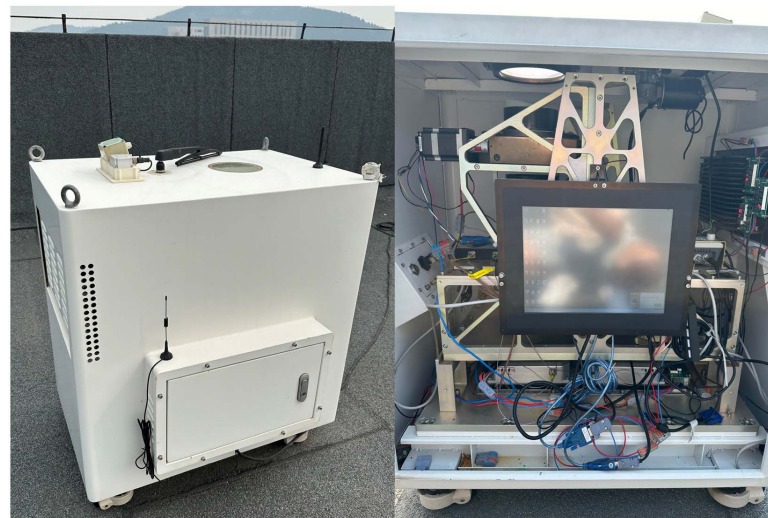


Figure 9. Coherent Doppler Wind Lidar instrument diagrams.

CDWL inverts the wind speed by analyzing the Doppler shift of the echo signal, but the quality of the echo signal is affected by various interferences. As the distance gradually increases, the signal quality gradually deteriorates, and the noise in the power spectrum at the farther distance gates drowns out the main peak of the signal, which leads to a serious impact on the accuracy of the subsequent wind speed calculations. According to the Spearman rank correlation coefficient criterion and experimental tests, the threshold $SPTH = 0.1$ is selected to meet the very weak correlation defined by the correlation criterion. The MF window length of 5 is selected through several experimental tests. Taking the 0° echo signal as an example, under the optimal parameters, the signal's power spectrum is denoised using the SVD-ICEEMDAN-SCC-MF algorithm. The decomposition results of ICEEMDAN are shown in Figure 10, where 10 IMF components and one residual are generated through the ICEEMDAN decomposition. Calculating the Spearman correlation coefficient for each of these components yields absolute value results ranging from 0.0171 to 0.7186. The absolute values of the correlation coefficients of the 1st, 2nd, 3rd, 5th, and 10th IMF components are 0.0171, 0.0309, 0.0788, 0.0513, and 0.0768, respectively, which are less than the thresholds, and thus these components are processed with median filtering.

To verify the effectiveness of denoising, the denoising results of the true CDWL echo signal's power spectrum at four angles (0° , 90° , 180° , and 270°) are compared after applying the SVD-ICEEMDAN-SCC-MF algorithm. The denoising results for the 1st to 60th distance gates, which are within a distance of 3.6 km, are shown in Figure 11. In order to show the denoising results clearly, part of the data of each orientation signal is analyzed separately. The denoising results of the 51st–60th distance gates at distances of 3 km to 3.6 km are shown in

Figure 12. The data of points 6165–6264 totaling 100 points in the denoising result of the 56th distance gate with a distance of about 3.36 km are shown in Figure 13.

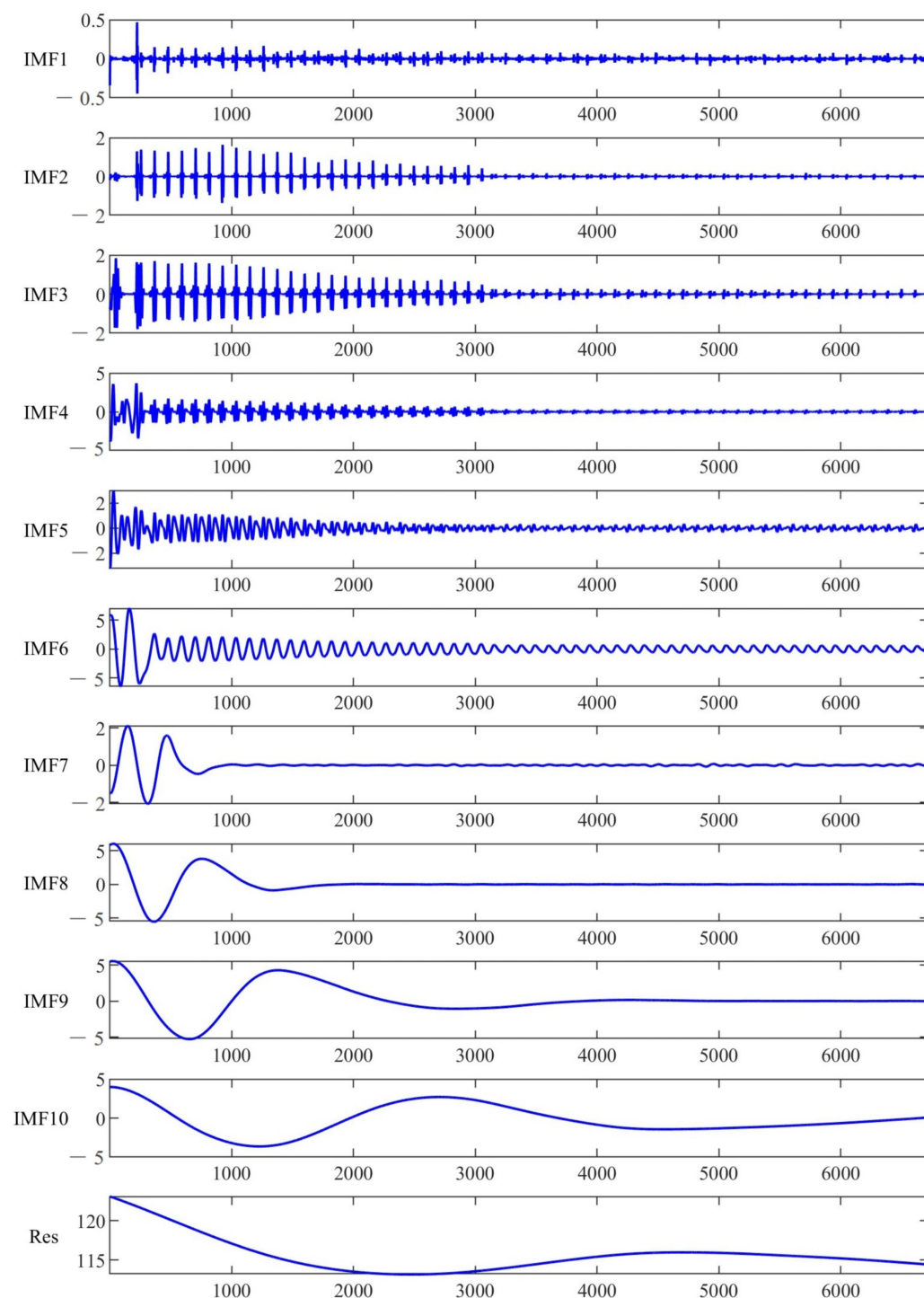


Figure 10. The ICEEMDAN decomposition results of the 0° signal power spectrum.

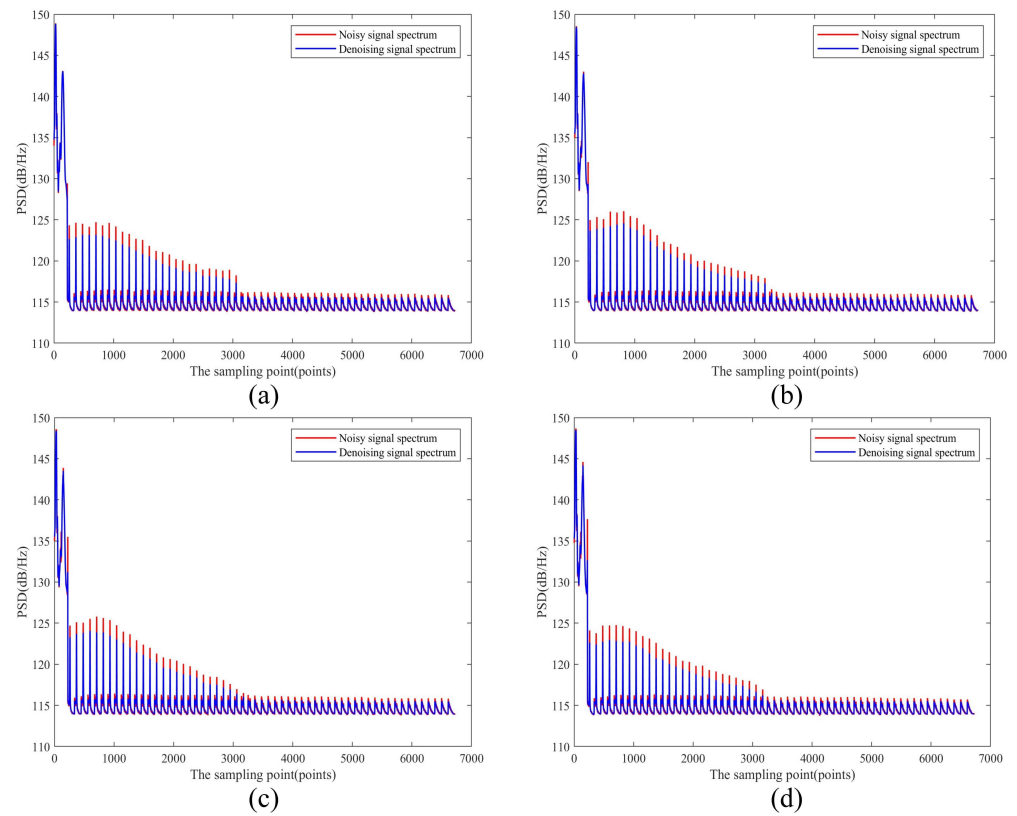


Figure 11. Denoising results for the power spectrum of the 1st–60th distance gate signals. (a) 0° echo signal, (b) 90° echo signal, (c) 180° echo signal, (d) 270° echo signal.

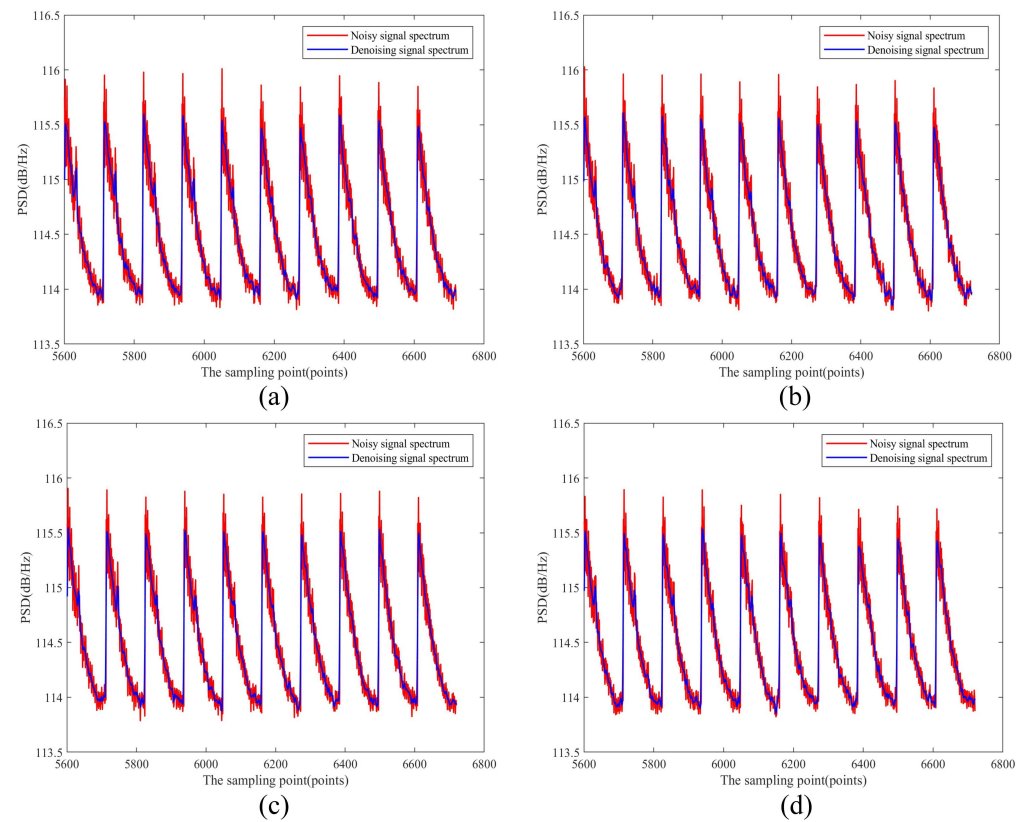


Figure 12. The 51st–60th distance gate signal power spectrum denoising results. (a) 0° echo signal, (b) 90° echo signal, (c) 180° echo signal, (d) 270° echo signal.

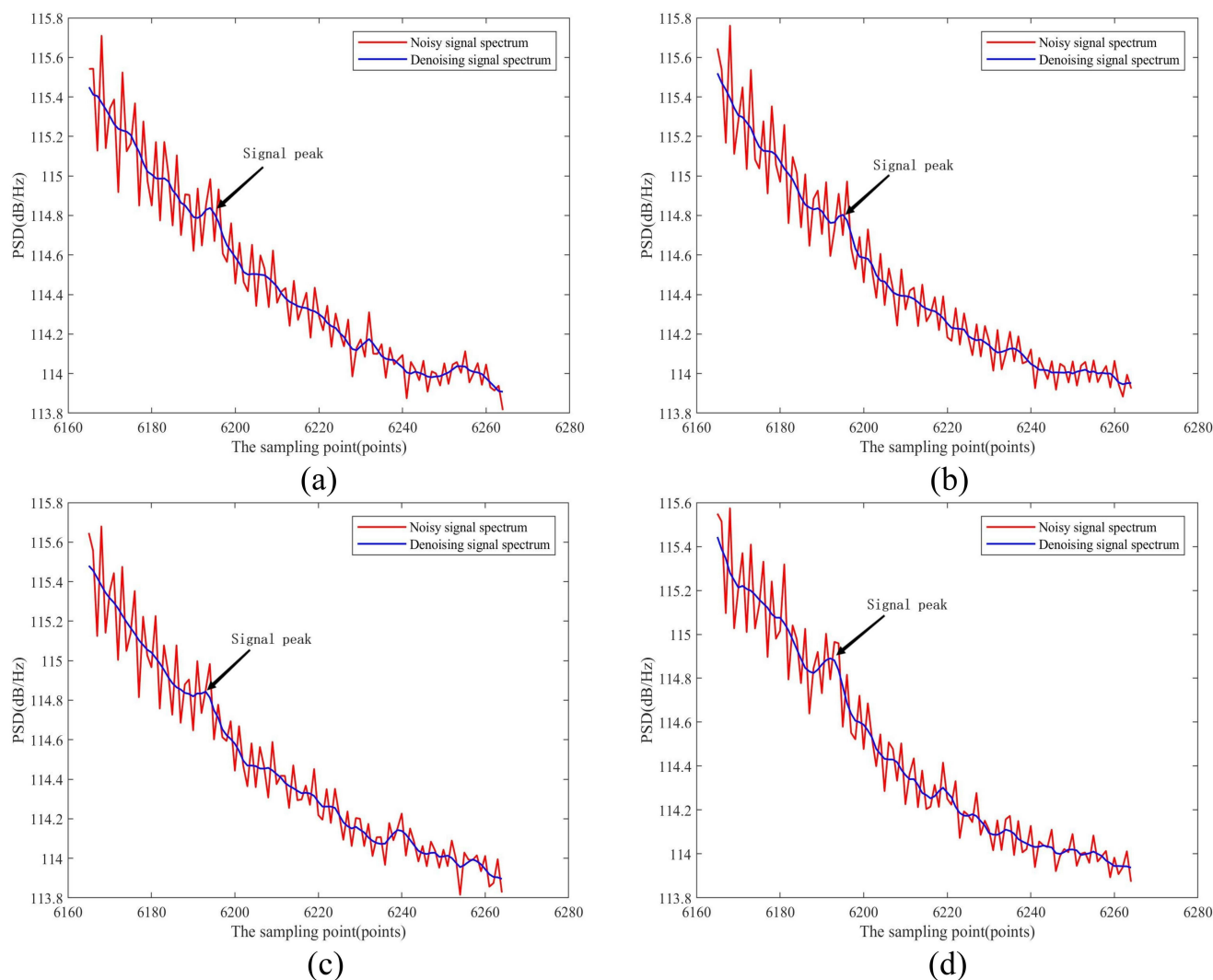


Figure 13. Denoising results of data at points 6165–6264 in the 56th distance gate. (a) 0° echo signal, (b) 90° echo signal, (c) 180° echo signal, and (d) 270° echo signal.

At a distance of about 2 km, the signal peaks in the power spectrum are disturbed by gradually increasing noise, and the noise begins to be noticeable. The signal peaks are gradually swamped with increasing detection distance and atmospheric turbulence. According to Figure 11, the signal power spectrum shows some denoising effect in all four directions, and the power spectra of the 51st–60th distance gates are extracted as in Figure 12, where the noise is well removed, and the signal's power spectrum shows an excellent denoising effect in terms of smoothness and completeness. In the closer signal, the signal peaks are clearer, and in the farther signal, the noise is effectively removed, and the signal is processed to obtain the signal peaks at the far distance. Take the data of the 56th distance gate as an example and select points 6165–6264 for a total of 100 points of data to zoom in and analyze, as shown in Figure 13. In the noisy signal, the signal peak is completely overwhelmed by the noise, and it is impossible to invert the wind speed accurately, while after the denoising process of the SVD-ICEEMDAN-SCC-MF algorithm, the denoising effect is obvious, and the signal peak is well restored. The CDWL transmitter elevation angle is 60°, the time to turn 90° is about 6 s, and the measurement points in the four directions are separated by 2–3 km in horizontal distance. Therefore, there will be a slight difference in wind speed magnitude, and the peak positions obtained may be slightly different but basically the same. By identifying the horizontal level of the

signal peak roots with less noise contamination at close range, the positions of other signal peaks can be determined. The peak locations of the four directions are (6194,114.838), (6195,114.804), (6193,114.843), and (6192,114.891). The corresponding inverse radial wind speed magnitudes are 0.6355 m/s, 0.124 m/s, 1.395 m/s, and 2.1545 m/s, respectively.

Through the comparison in Section 4.2, we can observe that the SNR_{out} of the EMD-WT algorithm is consistently lower than that of the SVD-ICEEMDAN-SCC-MF proposed in this paper, while its $RMSE$ is consistently higher than that of the SVD-ICEEMDAN-SCC-MF. However, under low SNR_{in} conditions below 5 dB, EMD-WT outperforms CEEMDAN-PE-WT, NGO-VMD, and WT. And the computation time of EMD-WT is always smaller than that of SVD-ICEEMDAN-SCC-MF. So EMD-WT is used for real CDWL denoising in order to compare the performance of EMD-WT and SVD-ICEEMDAN-SCC-MF in real CDWL signal denoising. In order to show the denoising results more clearly, the 51st–55th distance gates will be extracted from the denoised experimental data for comparison, and the result graph is shown in Figure 14. As shown in Figure 14, there are serious distortions in the denoising results of EMD-WT. The signal's power spectrum is over-smoothed, causing the peaks of the signal to disappear. Furthermore, significant residual noise remains poorly suppressed. After processing with the SVD-ICEEMDAN-SCC-MF algorithm, the signal peaks are well preserved, and the noise is effectively removed.

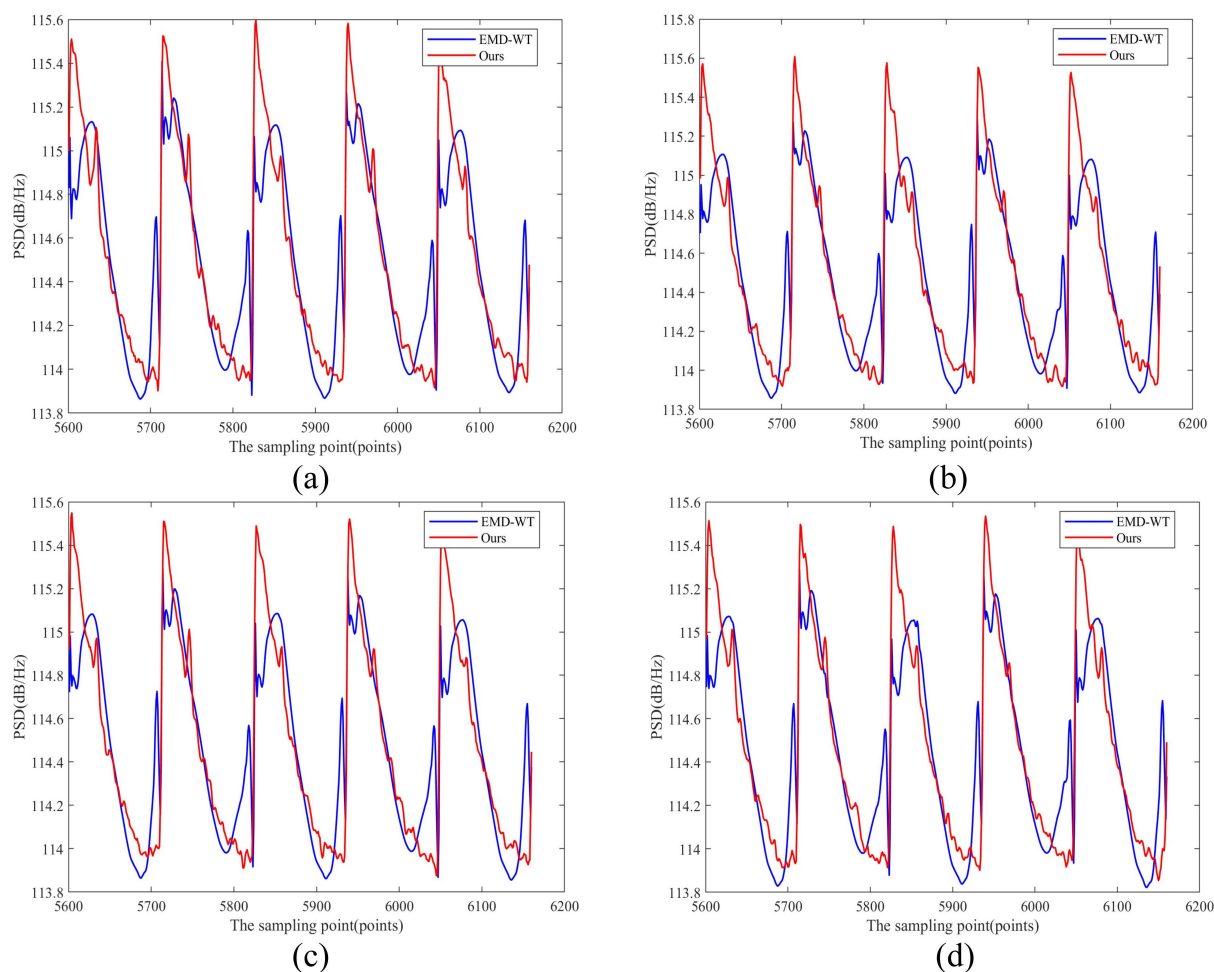


Figure 14. Denoising results of SVD-ICEEMDAN-SCC-MF and EMD-WT on the power spectrum of the real CDWL signal (51st–55th distance gates). (a) 0° echo signal, (b) 90° echo signal, (c) 180° echo signal, (d) 270° echo signal.

The performance of the denoising results in the four directions and the comparison results of denoising real CDWL signals by SVD-ICEEMDAN-SCC-MF and EMD-WT mutually verify the effectiveness as well as the robustness of the denoising of the SVD-ICEEMDAN-SCC-MF algorithm. After SVD-ICEEMDAN-SCC-MF denoising, the effective detection range of CDWL is increased from 3 km to more than 3.6 km.

5. Conclusions

This paper proposes a new joint CDWL signal denoising algorithm called SVD-ICEEMDAN-SCC-MF. The method first uses SVD for preliminary denoising of the input signal, then applies ICEEMDAN decomposition to the reconstructed signal, and calculates the Spearman correlation coefficients for each of the components generated by the decomposition. The components smaller than the threshold are processed by median filtering to achieve the purpose of multiple denoising, and finally, the signal is reconstructed using the median-filtered components and the unprocessed components with high correlation. The practicality and accuracy of the proposed method are demonstrated by denoising experiments on several analog and real signals under different conditions. The denoising performance of SVD-ICEEMDAN-SCC-MF is compared with several other algorithms by denoising three typical signals, Bumps, Blocks, and Heavy sine, and SVD-ICEEMDAN-SCC-MF outperforms the other algorithms by both the SNR_{out} and $RMSE$. Especially for the Heavy sine signal with $SNR_{in} = -5$ dB, the processed $SNR_{out} = 15.5117$ dB shows an improvement of 20.5117 dB, and the $RMSE$ is only 0.5174. In the comparative experiments of denoising the simulation model of the CDWL echo signal, the denoising effect is much better than other algorithms, which accurately shows the nine main signal peaks. Finally, the real CDWL signal is processed using SVD-ICEEMDAN-SCC-MF, which makes the signal peaks of the distance gates at close distances clearer and restores the signal peaks of the distance gates at long distances that are buried by noise. By comparing the denoising results in four directions, the effectiveness of the denoising algorithm is demonstrated, and the effective detection range of CDWL is improved from 3 km to more than 3.6 km.

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