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Doppler Shift Estimation Method for Frequency Diverse Array Radar Based on Graph Signal Processing

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Abstract: In this paper, a novel Doppler shift estimation method for frequency diverse array (FDA) radar based on graph signal processing (GSP) theory is proposed and investigated. First, a well-designed graph signal model for a monostatic linear FDA is formulated. Subsequently, spectral decomposition is conducted on the constructed signal model utilizing graph Fourier transform (GFT) techniques, enabling the extraction of the target's Doppler shift parameter through spectral peak search. A comprehensive series of simulation experiments demonstrates that the proposed method can achieve the accurate estimation of target parameters even under low signal-to-noise ratio (SNR) conditions. Furthermore, the proposed method exhibits superior performance compared to the MUSIC algorithm, offering enhanced resolution and estimation accuracy. Additionally, the method is highly amenable to parallel processing, significantly reducing the computational burden associated with traditional procedures.

Keywords: graph signal processing; frequency diverse array; Doppler shift estimation; graph Fourier transform



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1. Introduction

Frequency diverse array (FDA) radar has the advantages of automatic beam scanning, high spectrum utilization and strong anti-multipath interference capability [1–3], which is the focus of radar detection, target imaging and parameter estimation research [4–6]. FDA can significantly improve the spatial resolution and parameter estimation performance of radar systems by introducing small frequency offsets between different antenna elements, and, at the same time, have the characteristics of a strong anti-jamming ability and adaptability to complex environments [7]. This technique has important application potential in the fields of communication, navigation, and imaging [8–10]. However, the high technical complexity and signal processing requirements of FDA radars pose new challenges for theoretical research and engineering implementation [11].

In recent years, FDA radar has been a research hotspot in modern radar systems, especially in range–angle parameter estimation, where it has made significant research progress [12–14]. FDA radar achieves flexible beam regulation and efficient parameter estimation by precisely controlling the transmit frequency components [15]. However, the existing studies focus more on the estimation of range and angle parameters, and there is a relative lack of research on velocity parameters. In terms of velocity estimation, FDA radars

still face many challenges, such as the Doppler shift estimation accuracy in low signal-to-noise ratio (SNR) environments and the target velocity resolution in complex backgrounds. As an important characterization of target motion information, the accurate estimation of Doppler shift is crucial for the acquisition of target velocity information, the analysis of motion state, and the optimization of comprehensive system performance. When FDA radar is applied to moving target detection, due to the motion of the target, the signal received by the radar will generate a Doppler shift, which will have an important impact on the subsequent measurement of the target distance and angle. Therefore, Doppler shift estimation is particularly critical in target detection [16,17]. The Doppler shift is directly related to the velocity of the target, and, by accurately estimating the Doppler shift, the velocity information of the target can be obtained to achieve target tracking and localization. Meanwhile, it also facilitates the subsequent target range and angle measurements, and by compensating for the Doppler shift, the range and azimuth of the target can be estimated more accurately. Therefore, estimating the Doppler shift is crucial for the further development of FDA radar technology.

Existing Doppler shift estimation methods mainly include fast Fourier transform (FFT)-based spectral estimation methods, autoregressive model-based parameter estimation methods, and high-resolution methods such as MUSIC (MUltiple SIgnal Classification) and ESPRIT (Estimation of Signal Parameters via Rotational Invariance Technique) [18]. Although these approaches achieve some degree of enhancement in Doppler shift estimation precision, their effectiveness substantially diminishes under challenging low SNR conditions (such as the SNR falls below -10 dB). In addition, these methods usually require high computational complexity, which makes it difficult to meet the demand of real-time processing. In practical applications, a Doppler shift estimation method with high resolution in complex environments is urgently needed in order to balance high accuracy and efficiency.

In this paper, a Doppler shift estimation method based on graph signal processing (GSP) is proposed on the basis of FDA radar. The method sparsely represents the FDA Doppler shift vector by constructing the graph adjacency matrix, spectrally decomposes the signal using the graph Fourier transform (GFT), establishes the spectral peak search function, and finally achieves the Doppler shift estimation. Compared with the traditional method, this method has the following advantages: (1) by exploiting the sparsity and local smoothness of graph signals, the anti-noise performance of the algorithm can be effectively improved; (2) it improves the discriminative ability in multi-target cases, and it can effectively differentiate the Doppler shift components of the targets that are closer to each other; and (3) the algorithm has lower complexity, and it is suitable for real-time processing scenarios.

Our main contributions are as follows:

- (1) We propose a novel graph model for FDA radar signals based on GSP theory and present a corresponding algorithm to address the Doppler shift estimation problem in the slow-time domain.
- (2) Based on GSP theory and our previous research, we establish a corresponding graph signal model to represent the signal correlation across the array. By exploiting the sparsity and intrinsic correlation of the graph model, the anti-noise performance of the algorithm can be effectively improved. Comparative analysis reveals that our approach outperforms the conventional MUSIC algorithm, exhibiting superior Doppler resolution and enhanced estimation accuracy.
- (3) The proposed method in this paper is designed to be deployable through parallel processing, where an increase in the number of parallel threads leads to a more

significant improvement in computational efficiency. This feature makes it particularly advantageous for handling large-scale data operations.

2. Signal Model

2.1. The Transmission Beam Characteristics of the FDA

Consider the monostatic linear frequency offset FDA radar shown in Figure 1, which consists of M transmitter and receiver co-located antennas that are horizontally aligned in a straight line with uniform spacing d to form a one-dimensional uniform linear array. Compared with the conventional phased array radar, the significant characteristic of the FDA radar is the existence of a small frequency offset in the carrier frequency of the adjacent array elements, which forms a unique frequency modulation characteristic during transmission.

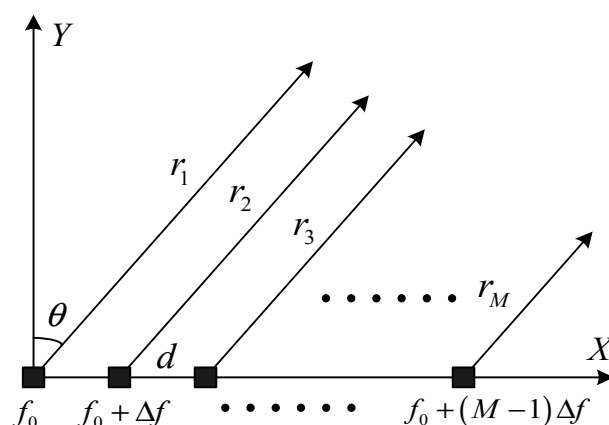


Figure 1. Schematic diagram of a monostatic linear frequency offset FDA radar.

In order to describe more specifically, taking the first transmit antenna as the reference array element, the signal form of the m -th transmitting array element can be expressed as follows:

$$s_m(t) = e^{j2\pi f_m t}, m = 1, 2, \dots, M, t \in [0, T_p] \quad (1)$$

where $f_m = f_0 + (m - 1)\Delta f$ is the carrier frequency of the m -th transmitting array element, f_0 is the reference carrier frequency, Δf is the frequency offset far less than the radar operating carrier frequency, and T_p is the pulse duration.

The electromagnetic waves emitted by each of the FDA elements are coherently superimposed at a point in space to form the main lobe of the beam, while they are decohered in other regions to form the side lobe. Assuming that the target satisfies the far-field condition, the beam radiated by each antenna in the array can be viewed as a set of parallel waves, and the angles are all θ . Let the first array element be the reference array element, and the distance between it and the target point is r ; then, the distance between the m -th array element and the target is

$$r_m = r - (m - 1)d \sin \theta \quad (2)$$

Then, the phase of the signal transmitted by the m -th array at the target can be expressed as

$$\psi_m = 2\pi f_m \left(t - \frac{r_m}{c} \right) = 2\pi (f_0 + (m - 1)\Delta f) \left(t - \frac{r - (m - 1)d \sin \theta}{c} \right) \quad (3)$$

Then, the phase difference between the signal when the m -th array reaches the target and the signal when the reference array reaches the target is

$$\begin{aligned}\Delta\psi &= \psi_m - \psi_1 = 2\pi f_m \left(t - \frac{r_m}{c}\right) - 2\pi f_1 \left(t - \frac{r_1}{c}\right) \\ &= 2\pi \left(\frac{(m-1)f_0 d \sin\theta}{c} + (m-1)\Delta f t - (m-1)\frac{\Delta f r}{c} + (m-1)^2 \frac{\Delta f d \sin\theta}{c} \right)\end{aligned}\quad (4)$$

The first term in Equation (4) is the same as the conventional phased array phase difference and is related to the target angle. The last three terms are new terms due to the introduction of the frequency increment Δf . The quadratic term $(m-1)^2 \Delta f d \sin\theta / c$ can be neglected due to $M\Delta f \ll f_0$ and the far-field assumption $Md \sin\theta \ll r$. Equation (4) can be simplified as

$$\Delta\psi = 2\pi(m-1) \left(\frac{f_0 d \sin\theta}{c} + \Delta f t - \frac{\Delta f r}{c} \right) \quad (5)$$

The electromagnetic field emitted by the array antenna is the vector sum of the electromagnetic field emitted by each array element of the array, and it is known that the neighboring array elements are proportional to each other by the signal expression of each array element shown in Equation (5). The beam direction diagram of the FDA can be obtained according to the proportional summation formula. Assuming that each array element of the FDA has an omnidirectional direction map with a weighting factor of 1. Then, the signal at the target point is a superposition of the signals of each array element, and the phase of each array element is in a simplified form by neglecting the quadratic term:

$$x(t) = e^{j2\pi f_0 \left(t - \frac{r}{c}\right)} \sum_{m=1}^M e^{j2\pi(m-1) \left(\frac{f_0 d \sin\theta}{c} + \Delta f t - \frac{\Delta f r}{c} \right)} \quad (6)$$

Equation (6) can be simplified to

$$x(t) = e^{j2\pi f_0 \left(t - \frac{r}{c}\right)} e^{j\pi(M-1) \left(\frac{f_0 d \sin\theta}{c} + \Delta f t - \frac{\Delta f r}{c} \right)} \frac{\sin \left[M\pi \left(\frac{f_0 d \sin\theta}{c} + \Delta f t - \frac{\Delta f r}{c} \right) \right]}{\sin \left[\pi \left(\frac{f_0 d \sin\theta}{c} + \Delta f t - \frac{\Delta f r}{c} \right) \right]} \quad (7)$$

By normalizing and modulating Equation (7), the amplitude expression of the transmitted beam pattern of the FDA is obtained:

$$P_{FDA}(t, r, \theta) = \left| \frac{\sin \left[M\pi \left(\frac{f_0 d \sin\theta}{c} + \Delta f t - \frac{\Delta f r}{c} \right) \right]}{\sin \left[\pi \left(\frac{f_0 d \sin\theta}{c} + \Delta f t - \frac{\Delta f r}{c} \right) \right]} \right| \quad (8)$$

where N , f_0 , d , and Δf are all system parameters, and the transmitting beam pattern of the FDA is determined by time t , distance r , and angle θ . This enables the FDA to estimate the distance and angle of the target and to scan the space automatically. The comparison between the FDA transmitting beam pattern ($t = 0$) and the phased array transmitting beam pattern is shown in Figure 2, in which the normalized amplitude is calculated by utilizing the $20\log_{10}$ conversion. Figure 2a,b are the beam pattern under polar coordinates, and Figure 2c,d are the beam pattern under rectangular coordinates. For the phased array ($\Delta f = 0$), there is no such small frequency offset between the arrays, so the beam direction pattern of the traditional phased array is only related to the angle. The array beam pattern has range-angle dependence and periodicity after introducing frequency offset between antennas.

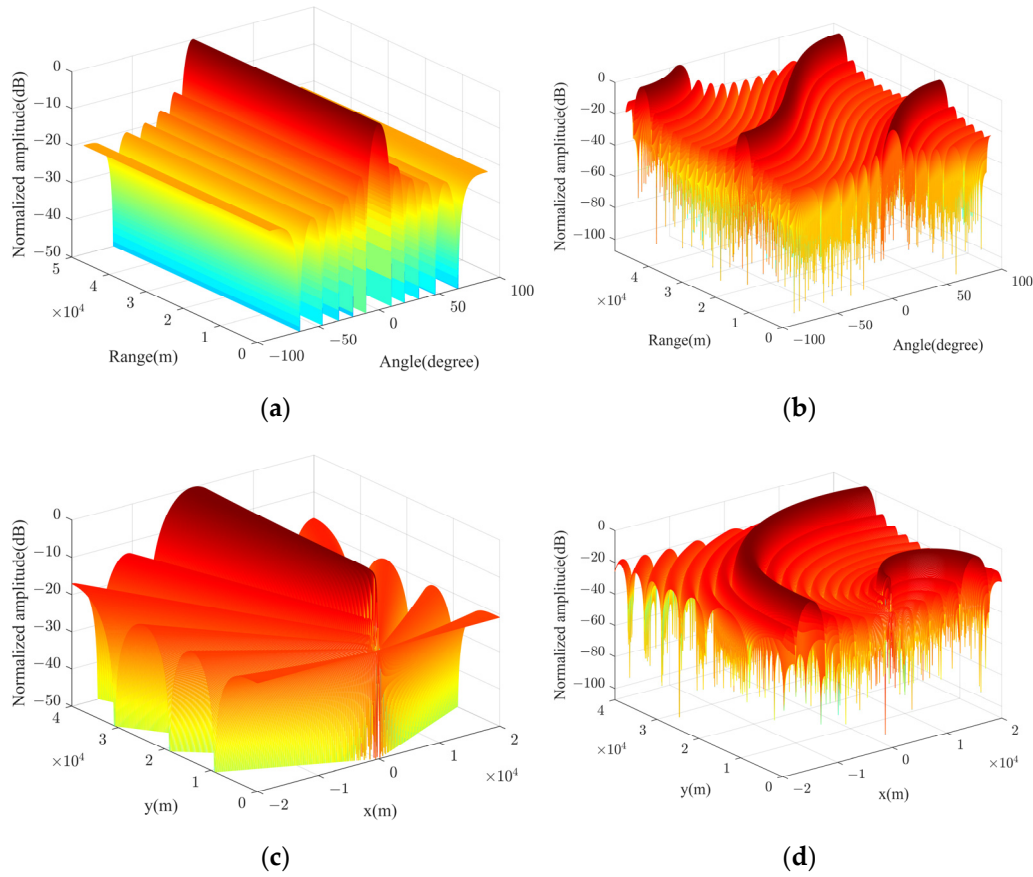


Figure 2. Transmit beam pattern: (a) phased array beam pattern (polar coordinates); (b) FDA beam pattern (polar coordinates); (c) phased array beam pattern (Cartesian coordinates); and (d) FDA beam pattern (Cartesian coordinates). The shade of color in the figure reflects the amplitude of the beam pattern. A pronounced gradation towards deeper red tones signifies an augmentation in intensity, whereas a gradational shift towards lighter blue tones denotes a reduction.

2.2. FDA Receiver Structure

The transmitted signal is reflected by the target, undergoes attenuation, phase change, and superposition with additive noise, resulting in an echo signal. For a far-field target with an azimuth of θ and an initial distance of r , the signal received on the m -th array element is

$$\begin{aligned} \tilde{y}_m(t) &= \zeta e^{j2\pi f_m \left\{ t - \frac{2[r - (m-1)d \sin \theta]}{c} \right\}} + n_m(t) \\ &\approx \zeta e^{j2\pi f_m t} e^{-j2\pi f_0 \frac{2r}{c}} \cdot e^{j2\pi [(m-1) \frac{2f_0 d \sin \theta}{c} - (m-1) \frac{2r \Delta f}{c}]} + n_m(t) \end{aligned} \quad (9)$$

where ζ represents the amplitude of the echo signal, and $n_m(t)$ is the noise component. It can be seen from Equation (9) that the echo signal received by each array element contains M frequencies, and $e^{j2\pi(m-1)\Delta f t}$ is related to m , resulting in the accumulated equivalent beam pattern changing with time. In order to eliminate the time dependence, the echo signal is processed by single frequency reception [19], as shown in Figure 3.

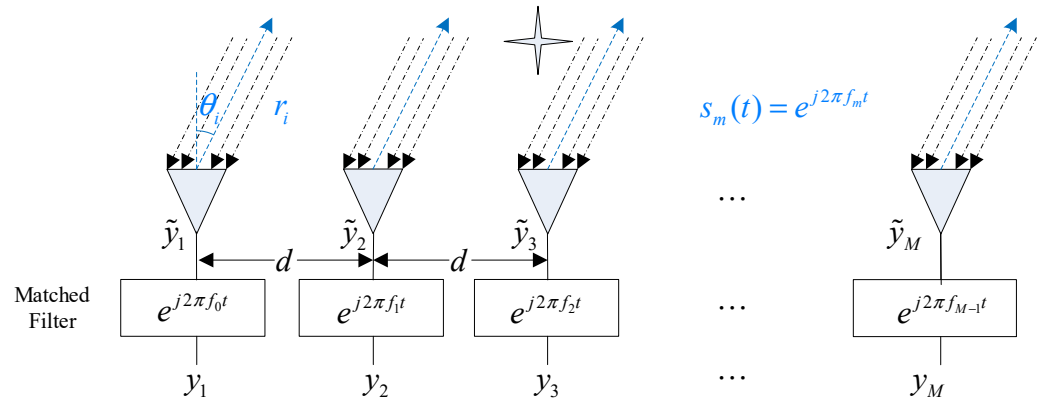


Figure 3. The diagram of signal processing procedures at the receiver end.

The application of matched filtering to echo signals can not only eliminate the time dependence but also separate signals of different frequencies, simplifying the processing of received signals. Therefore, the output signal finally obtained by the m -th array can be expressed as

$$\begin{aligned} y_m &= \zeta' e^{j2\pi[(m-1)\frac{2f_0 d \sin \theta}{c} - (m-1)\frac{2r\Delta f}{c}] + \omega_m} \\ &= \zeta' e^{j\frac{4\pi}{\lambda}(m-1)(f_0 d \sin \theta - \frac{r\Delta f}{f_0})} + \omega_m \\ &= \zeta' e^{j(m-1)\phi} + \omega_m \end{aligned} \quad (10)$$

where $\phi = \frac{4\pi}{\lambda}(f_0 d \sin \theta - \frac{r\Delta f}{f_0})$, $\zeta' = \zeta \exp(-j4\pi f_0 r/c)$ combines the round trip distance delay term from reference array to target, and ω_m is the noise component after matching filter. According to Equation (10), it is found that the range-angle information of the target is hidden in the data vector output by the receiving array, and there is a coupling relationship [20]. According to the phase difference between the elements of the above formula, the guiding vector of a uniform linear frequency array is defined as

$$\mathbf{a}(\theta, r) = [1, e^{j\phi}, \dots, e^{j(m-1)\phi}] \quad (11)$$

The signal model is extended to the Doppler domain, assuming that the target moves along the radar radial with velocity v , the FDA radar emits K pulses, and the pulse repetition period is T_{pr} ; then, the distance from the radar to the target when the k pulse is emitted is $r(t_k) = r - vt_k$. For a moving target, although the carrier frequency of each transmitting array element in the FDA is different, the echo Doppler frequency difference between array elements is very small due to the frequency offset $\Delta f \ll f_0$. When the frequency offset meets the condition of Equation (12), the Doppler expansion effect in the pulse can be ignored [21]:

$$\left| 2\pi \frac{2vf_M}{c} KT_{pr} - 2\pi \frac{2vf_0}{c} KT_{pr} \right| \leq \frac{\pi}{4} \quad (12)$$

where K represents the number of accumulated pulses, KT_{pr} is the coherent processing time, f_0 is the reference carrier frequency of FDA radar (the minimum carrier frequency in the transmitting array element), and f_M represents the carrier frequency of the M -th transmitting array element (the maximum carrier frequency in the transmitting array element). The physical meaning of this condition means that the maximum phase difference in the Doppler change due to the frequency increment Δf does not exceed $\pi/4$ rad. Therefore, the Doppler shift matrix can be approximated as $e^{j2\pi \frac{2vf_0}{c}(k-1)T_{pr}}$. Then, the moving target signal model of FDA radar is expressed as [22]

$$y_m(k) = \zeta e^{j(m-1)\psi_0} e^{j2\pi f_d(k-1)T_{pr}} + \omega_m \quad (13)$$

where $f_d = 2vf_0/c$ is the target Doppler frequency. In order to correctly solve the Doppler shift information of the target, the graph structure is constructed by using the moving target signal model of the FDA radar.

3. Doppler Shift Estimation Algorithm Based on Graph Signal Processing

3.1. Preliminaries

The graph is a representation that describes the relationship between data structures, generally consisting of a group of vertices and edges connecting vertices, usually represented by vertex set $\mathbf{V} = \{v_1, v_2, \dots, v_M\}$ and edge set $\xi = \{e_{ij}\}$, denoted as $G = (\mathbf{V}, \xi)$. The vertex of the graph represents the node in the actual network, and the edge represents the connection between the node i and the node j in the network. In order to better describe the correlation between nodes in the graph, the adjacency matrix $\mathbf{A}$ is introduced to replace the edge set. Each element a_{ij} in $\mathbf{A}$ represents the weight of the edge between vertices i and j . Then, the graph signal can be defined as a mapping between the vertex set $\mathbf{V}$ of the graph $G = (\mathbf{V}, \mathbf{A})$ in which it resides, and a set of complex signals $\mathbf{s} = [s_1, s_2, \dots, s_M]^T \in \mathbb{C}^M$ [23], denoted as $s : V \rightarrow \mathbb{C} | v_m \rightarrow s_m$.

Next, the definition of graph Fourier transform (GFT) is given. The traditional Fourier transform is performed by representing the signal as a linear combination of a set of orthogonal bases whose coefficients represent the properties of the signal in the frequency domain. Similarly, the GFT is based on this idea, but it uses the eigenvectors of the adjacency matrix of the graph as the orthogonal basis, thereby projecting the graph signal onto these eigenvectors and obtaining the representation of the graph signal in the graph spectral domain. Assuming that the adjacency matrix can be diagonalized, then the eigen decomposition of $\mathbf{A}$ can be expressed as

$$\mathbf{A} = \mathbf{T}\mathbf{\Lambda}\mathbf{T}^{-1} \quad (14)$$

where $\mathbf{T}$ is the eigenvector matrix of $\mathbf{A}$, each column vector of $\mathbf{T} = [\mathbf{t}_1, \mathbf{t}_2, \dots, \mathbf{t}_M]$ is the eigenvector of $\mathbf{A}$, and the diagonal matrix $\mathbf{\Lambda}$ contains the corresponding eigenvalues. Then, the graph Fourier transform of the graph signal $\mathbf{s}$ is expressed as

$$\tilde{\mathbf{s}} = \mathbf{F}\mathbf{s} = \mathbf{T}^{-1}\mathbf{s} \quad (15)$$

where $\mathbf{F} = \mathbf{T}^{-1}$ is the graph Fourier transform matrix, and $\tilde{\mathbf{s}} = [\tilde{s}_1, \tilde{s}_2, \dots, \tilde{s}_M]$ is the graph Fourier transform coefficient of the graph signal $\mathbf{s}$. Then, the graph signal $\mathbf{s}$ can be represented in a form similar to the classical signal processing Fourier transform

$$\mathbf{s} = \mathbf{F}^{-1}\tilde{\mathbf{s}} = \mathbf{T}\tilde{\mathbf{s}} = \tilde{s}_1\mathbf{t}_1 + \tilde{s}_2\mathbf{t}_2 + \dots + \tilde{s}_M\mathbf{t}_M \quad (16)$$

3.2. GFT Applied to Doppler Shift Estimation

The foremost task in this study is the construction of an appropriate graph model for the aforementioned FDA, which serves as a critical foundation for subsequent derivations. Based on the definitions given in Section 3.1 and taking into account the topological structure of the FDA, a cyclic directed graph, as illustrated in Figure 4, is initially constructed. Specifically, the graph signal is denoted as $\mathbf{y}_m = [y_m(1), y_m(2), \dots, y_m(K)]^T$, and the elements of this vector are determined by Equation (13), in which the vertex set represents the nodes of the array, and the edge set represents the signal correlation within the phase domain across the array. Herein, we construct a sparse adjacency matrix denoted as $\mathbf{A}$ instead of the edge set, since the constructed graph is directional. Comprehensively

considering the graph topology, degrees of freedom, and computational complexity, the adjacency matrix $\mathbf{A}$ is formulated as detailed in Equation (17).

$$\mathbf{A} = \frac{1}{2} \begin{bmatrix} 0 & a_{1,2} & 0 & \cdots & a_{1,K} \\ a_{2,1} & 0 & a_{2,3} & \ddots & 0 \\ 0 & a_{3,2} & \ddots & \ddots & \vdots \\ \vdots & \ddots & \ddots & 0 & a_{K-1,K} \\ a_{K,1} & 0 & \cdots & a_{K,K-1} & 0 \end{bmatrix} \quad (17)$$

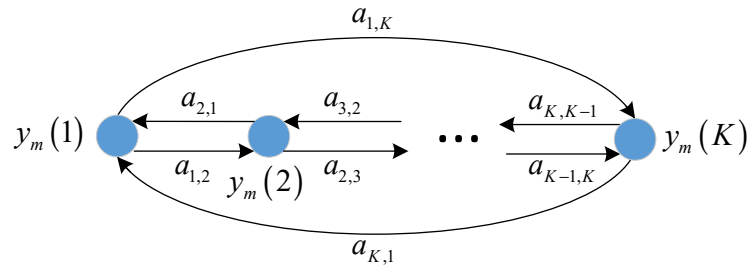


Figure 4. The constructed graph signal model and the relationship diagram of adjacency matrix weights.

Subsequently, the next step is to solve for the non-zero elements in the adjacency matrix $\mathbf{A}$. Since the non-zero elements of each row in the adjacency matrix are functions of the Doppler shift parameter f_d , and incorporating the constraints that represent the correlation between adjacent nodes, it is straightforward to determine the adjacency weights in the i -th row of the adjacency matrix by applying the linear phase principle, which can be corroborated by analogous methodologies extensively discussed in references [24,25].

$$\left. \begin{aligned} a_{i,i+1} &= e^{-j2\pi f_d T_{pr}}, a_{i,K} = e^{-j2\pi f_d (K-1)T_{pr}}, & i &= 1 \\ a_{i,1} &= e^{j2\pi f_d (K-1)T_{pr}}, a_{i,i-1} = e^{j2\pi f_d (K-1)T_{pr}}, & i &= K \\ a_{i,i-1} &= e^{j2\pi f_d T_{pr}}, a_{i,i+1} = e^{-j2\pi f_d T_{pr}}, & i &= \text{else} \end{aligned} \right\} \quad (18)$$

Based on the above derivation, the FDA radar Doppler steering vector $\mathbf{y}_m$ is a special eigenvector $\mathbf{E}_u$ of the adjacency matrix $\mathbf{A}$, which corresponds to the unit eigenvalue, i.e., $\mathbf{A}\mathbf{y}_m = \mathbf{y}_m$. Similar to the covariance matrix, we extract the Doppler information of the target by using the GFT on $\mathbf{y}_m$.

Specifically, $\mathbf{A} = \mathbf{T}\mathbf{\Lambda}\mathbf{T}^{-1}$ is obtained by eigen decomposition of the graph adjacency matrix. When the search unit corresponds to the correct Doppler, the GFT coefficient $\tilde{\mathbf{Y}} = \mathbf{T}^{-1}\mathbf{Y}$ outputs a one-hot vector with only one element and zero other elements, whose only nonzero entries correspond to the eigenvector $\mathbf{E}_u$. The other eigenvectors are orthogonal to the target echo signal. When the search unit corresponds to the incorrect Doppler, $\mathbf{y}_m$ is not the eigenvector of $\mathbf{A}$, and the output does not have a one-hot structure [25]. Therefore, we construct the spectral peak search function of (19) to estimate the Doppler shift information of the target.

$$\mathbf{P}^{-1}(f_d) = \left\| \tilde{\mathbf{Y}}_- \right\|_2^2 = \sqrt{\sum_{i \neq I} |\mathbf{T}^{-1}\mathbf{Y}|^2} \quad (19)$$

where $\|\cdot\|_2$ denotes the 2-norm. $\tilde{\mathbf{Y}}_-$ means that the element with the largest absolute value of $\tilde{\mathbf{Y}}$ is discarded, and the purpose of searching for the peak is achieved by removing the I -th largest eigenvalue and taking the reciprocal. Finally, the spectral peak function $\mathbf{P}(f_d)$ is computed by traversing the search unit until the maximum response function is obtained;

that is, the value of the parameter f_d to be estimated is obtained. The proposed method is summarized in Algorithm 1.

Algorithm 1 Doppler shift estimation algorithm based on GSP

Input: FDA radar Doppler steering vector $\mathbf{y}_m$

Output: $f_d \leftarrow \arg\max \mathbf{P}(f_i)$

1: **initialize:** $M, f_0, T_p, Q, K, N_{\text{grid}}$

2: **for** $f[i] \leftarrow i = 1$ to N_{grid} **do**

3: Compute adjacency matrix $\mathbf{A}(f_i)$ using (18)

4: Computer eigenmatrix: $\mathbf{T}^{-1} \leftarrow \mathbf{A} = \mathbf{T} \mathbf{\Lambda} \mathbf{T}^{-1}$

5: Compute function: $\mathbf{P}(f_i) \leftarrow \frac{1}{\sqrt{\sum_{i \neq l} |\mathbf{T}^{-1} \mathbf{Y}|^2}}$

6: **end for**

7: **return** GSP spectrum $\mathbf{P}(f_i) \leftarrow i = 1$ to N_{grid}

As mentioned earlier, $\mathbf{y}_m$ is an eigenvector (with unit eigenvalues) of the adjacency matrix $\mathbf{A}$. Therefore, the spectral decomposition $\mathbf{A} = \mathbf{T} \mathbf{\Lambda} \mathbf{T}^{-1}$ causes the graph Fourier transform coefficients $\tilde{\mathbf{Y}} = \mathbf{T}^{-1} \mathbf{Y}$ to be nonzero only at the position of the unit eigenvalue corresponding to the eigenvector $\mathbf{y}_m$. According to the method in [26], the concept of the normalized Doppler shift $f_{d,s} = (2v/c)f_0 T_{pr}$ is introduced in this paper. Assuming a moving target with a Doppler shift of 0.2, the effect can be observed in Figure 5a when the search unit corresponds to the correct Doppler shift, where the energy of $\tilde{\mathbf{Y}}$ (its norm) is concentrated on a GFT coefficient. On the contrary, for the incorrect Doppler shift, the effect can be observed in Figure 5b, where $\tilde{\mathbf{Y}}$ is energy distributed over multiple GFT coefficients. It is this result that motivates us to construct the spectral peak search function, and the final estimation result is shown in Figure 5c.

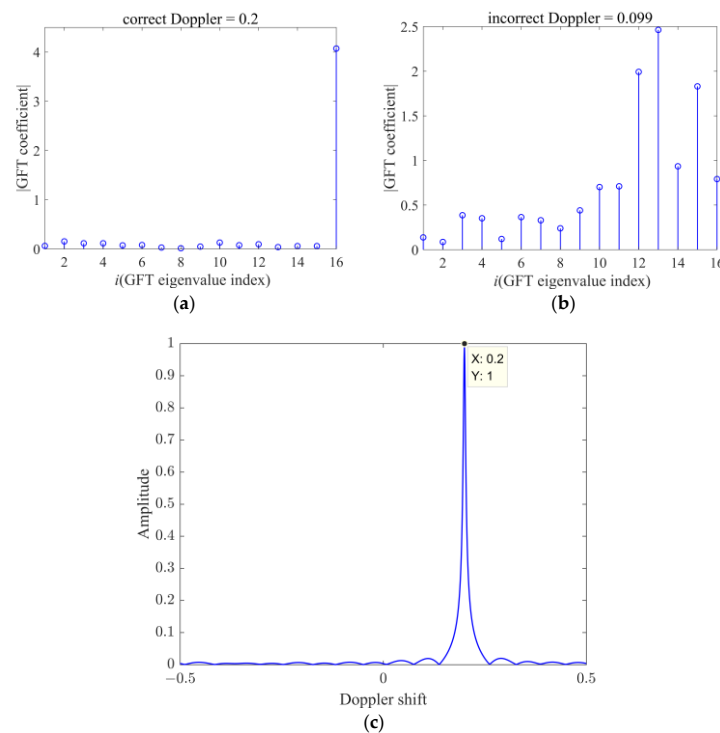


Figure 5. Graph Fourier transform energy distribution and spectral estimation results: (a) graph Fourier transform corresponds to the correct Doppler shift; (b) graph Fourier transform corresponds to the incorrect Doppler shift; and (c) spectral estimation results.

3.3. Computational Complexity Analysis

Combined with the analysis of the above algorithms, it can be seen that the Doppler frequency shift estimation algorithm based on the graph signal processing proposed in this paper has many similarities with the classical super-resolution algorithm, Multiple Signal Classification (MUSIC) algorithm [27]. In order to better compare the computational efficiency of the two algorithms, this section gives the computational complexity analysis of the two types of algorithms, respectively. In the following, the GSP algorithm is analyzed and explained. Assuming that the algorithm traverses N_{grid} points, the computational complexity of the main steps is analyzed as follows:

1. $O(2KN_0)$ operations are needed to construct the joint adjacency matrix $\mathbf{A}(f_i) \in \mathbb{C}^{K \times K}$, where N_0 represents the amount of computation required to calculate the nonzero elements of the adjacency matrix;
2. The eigen decomposition of the adjacency matrix is calculated according to Equation (5), which requires $O(K^3)$ operations;
3. To calculate the graph Fourier transform of the echo signal according to Equation (6), $O(K^2)$ operations are required;
4. The spectral peak corresponding to the scanning point is calculated according to Equation (9), which requires $O(K)$ operations.
5. The computational complexity analysis of the MUSIC algorithm is as follows:
6. The computational complexity of the covariance matrix $\mathbf{R} \in \mathbb{C}^{K \times K}$ mainly depends on the operation of matrix multiplication, which requires $O(QK^2)$ operations;
7. The eigenvalue decomposition of $K \times K$ covariance matrix $\mathbf{R}$ requires $O(K^3)$ operations;
8. Calculate the spectral function, which involves matrix multiplication and matrix inversion. $O(K^2)$ operations are required for each scanned grid point, so the total complexity is $O(N_{\text{grid}}K^2)$.

Therefore, the total computational complexity is about $O(N_{\text{grid}}(2KN_0 + K^3))$, where the computational complexity mainly depends on the adjacency matrix construction and its eigen decomposition calculation. In contrast, the computational complexity of the MUSIC algorithm is about $O(QK^2 + K^3 + N_{\text{grid}}K^2)$, where Q represents the number of snapshots. The computational complexity of the MUSIC algorithm mainly depends on the covariance matrix, eigen decomposition and spectral peak search calculation. According to the above analysis, the computational complexity of the proposed algorithm is similar to that of the classical two-dimensional MUSIC algorithm, which belongs to the same order of magnitude.

4. Experimental Results and Analysis

In this chapter, we conduct a series of numerical simulations to verify the performance difference in our proposed graph signal processing algorithm compared to the traditional MUSIC algorithm. Simulation parameter settings are shown in Table 1:

Table 1. Simulation parameter table.

Parameter	Value
f_0	1 GHz
Δf	30 kHz
M	6
Number of snapshots	128
Number of pulses	10
T_{pr}	0.1 ms

4.1. Effectiveness Analysis of Algorithms

In order to verify the effectiveness of the algorithm, it is assumed that there exists a moving target in the far field, which is in a complex Gaussian white noise background and has a Doppler shift of $f_{d,s} = 0.2$. Three different scenarios are analyzed in this section. In the first scenario, SNR = 0 dB, when the signal and the noise have equal power and the background interference is high, and it is used to verify the performance of the algorithm under low SNR conditions. In the second scenario, SNR = 5 dB, when the signal is dominant and can highlight the target signal characteristics more clearly, and it is used to analyze the performance of the algorithm under medium SNR conditions. The last scenario adds an interfering signal, which is characterized by a fixed frequency with a certain amplitude, and it is used to examine the robustness of the algorithm in complex scenarios where strong interference exists. The results are shown in Figure 6.

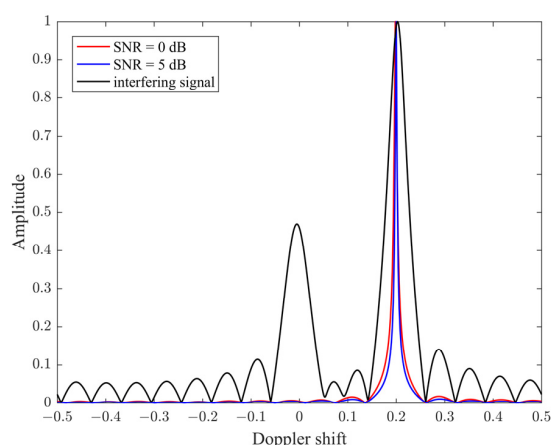


Figure 6. Algorithm performance in different SNR and interference scenarios.

From the results in Figure 6, the proposed algorithm exhibits strong target detection capability and anti-interference performance in all three scenarios. Under the condition of a low SNR, the algorithm is still able to effectively detect the Doppler shift in the target signal despite the strong noise, with high detection accuracy, showing good stability. As the SNR increases, the characteristics of the target signal become more obvious, and the algorithm shows good discrimination ability in this scenario and is able to accurately extract the Doppler shift in the target. In the environment of strong interference signals, the detection accuracy decreases, and another peak appears in the result to indicate the second source (interference signal), but the algorithm is still able to effectively separate the target signal from the interference signal, which verifies its robustness and applicability in complex backgrounds, and provides reliable support for target detection in real scenarios.

4.2. Performance of Parameter Estimation Under Different SNR Conditions

Next, the root mean square error (RMSE) of Doppler estimation is compared between the MUSIC algorithm and the proposed method under different SNRs. The SNR is $-20 \sim 20$ dB, and the RMSE is expressed as follows:

$$RMSE_{f_d} = \sqrt{\frac{1}{I} \sum_{i=1}^I (\hat{x}_i - x_i)^2} \quad (20)$$

where I is the number of Monte Carlo experiments at each SNR, where the value is 500. $\hat{x}_i$ is the estimated value of the i -th experiment; x_i is the true value of the i -th experiment. At the same time, to better evaluate the estimation performance of the algorithm, we introduce the Cramer–Rao Lower Bound (CRLB) as a comparison standard [13].

As shown in Figure 7, at the low SNR, the estimation accuracy of the proposed algorithm in the Doppler dimension is higher than that of the MUSIC algorithm. The main reason is that the estimation of the covariance matrix of the MUSIC algorithm is more affected by noise at a low SNR, which leads to the inaccurate estimation of the signal subspace [20]. In the GSP algorithm, the noise mainly affects the results of the GFT but does not affect the construction of the adjacency matrix. The number of nodes in the graph adjacency matrix is mainly determined by the number of pulses K , and the more accumulated pulses, the better the estimation effect.

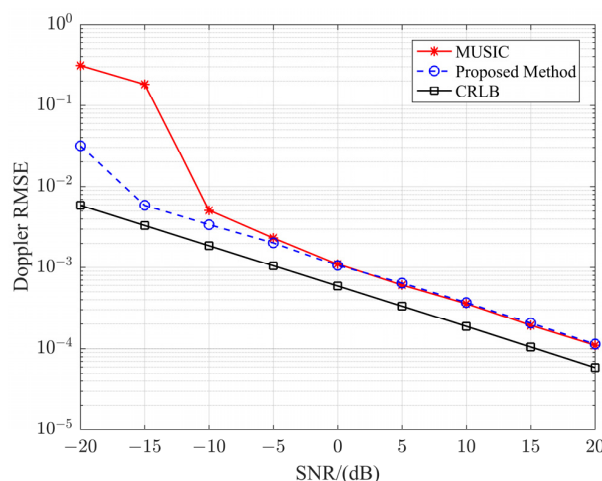


Figure 7. Doppler RMSE versus SNR.

4.3. Performance of Parameter Estimation Under Different Number of Snapshots and Pulses

In the following, the performance is analyzed for different numbers of snapshots Q and number of pulses K , where $\text{SNR} = 0$. It can be seen from Figure 8 that the more sampling points there are, the more information will be received, and the better the Doppler estimation effect will be. However, with the infinite increase in sampling points, the RMSE curve will gradually become flat. In Figure 9, the greater number of accumulated pulses that can be obtained, the higher the estimation accuracy of the algorithm. In addition, compared with increasing the number of snapshots, increasing the number of pulses is more obvious to improve the estimation accuracy. In general, the estimation accuracy of the proposed algorithm in the Doppler dimension is higher than that of the MUSIC algorithm.

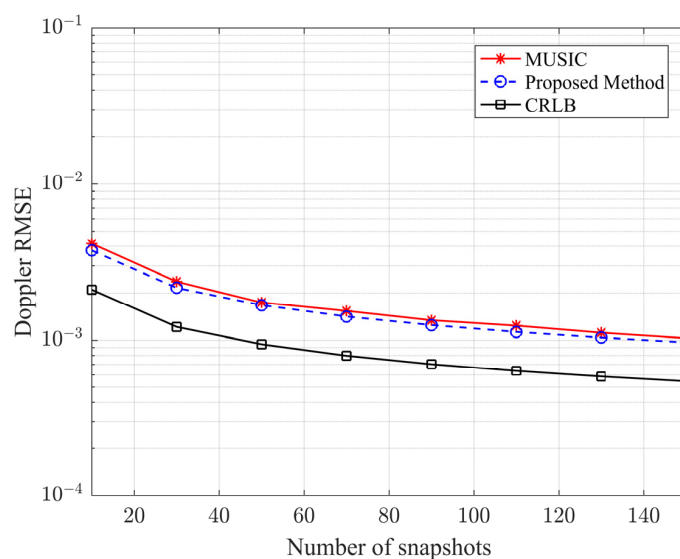


Figure 8. Doppler RMSE versus number of snapshots.

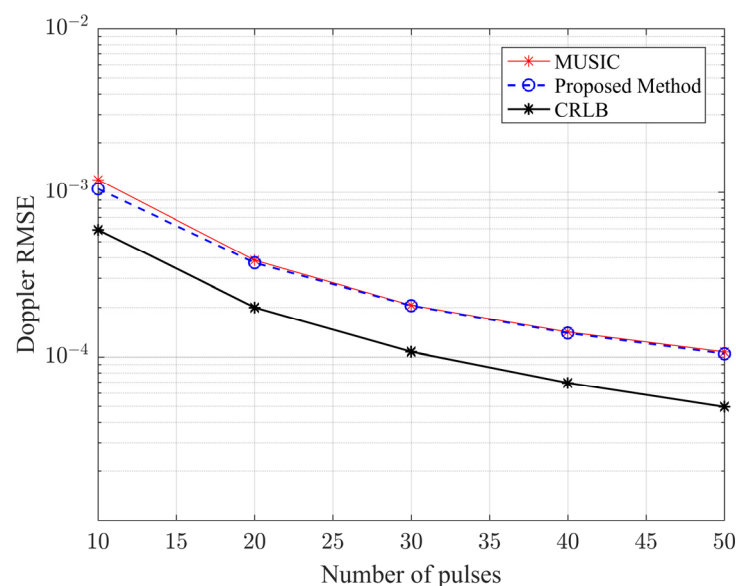


Figure 9. Doppler RMSE versus number of pulses.

4.4. Analysis of Algorithm Resolution

To further analyze the parameter estimation accuracy, this experiment compares the performance of the proposed algorithm and the MUSIC algorithm on the resolution to show the resolution ability of different methods, where $K = 128$, $\text{SNR} = 0$, and the target Doppler shifts are $f_{d,s1} = -0.1$ and $f_{d,s2} = 0.2$. From Figure 10, the proposed method has sharper peaks in the Doppler dimension than the super-resolution MUSIC algorithm, which reflects its super-resolution performance. The enhancement of the proposed method can be interpreted as follows: under low signal-to-noise ratio (SNR) conditions, the MUSIC algorithm is highly susceptible to noise interference due to its heavy reliance on the accuracy of covariance matrix estimation. In contrast, the proposed method transforms the echo signal into a well-designed graph-based formulation, which allows the main components of the signal to appear as relatively concentrated spectral peaks in the graph domain, while noise components are distributed across a wider spectral range. By leveraging the power of this transform domain method, the proposed approach can effectively identify the true frequency of the signal, thereby suppressing noise interference.

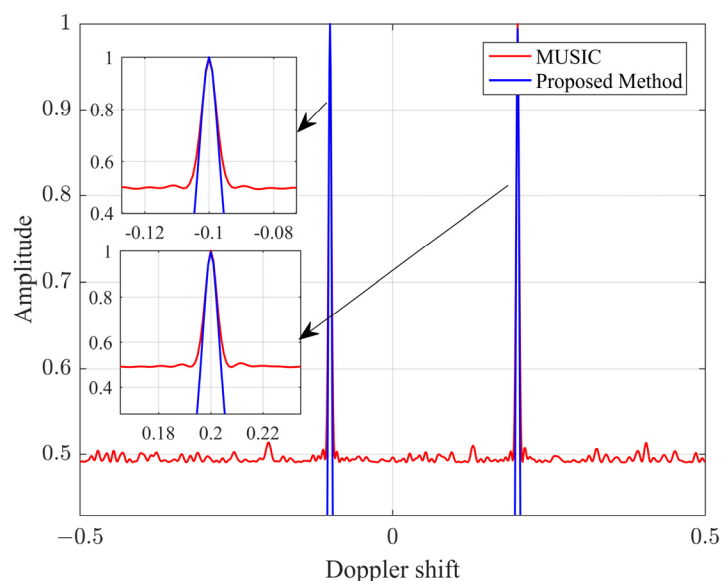


Figure 10. Comparison of Doppler shift resolution between MUSIC and the proposed method.

4.5. Algorithm Time Complexity Analysis

Finally, the computational time of the algorithm is given, and it can be seen from Section 3.3 that the complexity of the GSP algorithm increases with the increase in the search grid, which is mainly due to the need to recalculate the adjacency matrices and perform the feature decomposition in processing each search unit, which leads to a large number of core algorithmic processes within the search domain, thus bringing about a huge computational load in the serial computation. However, these large-scale small matrices are well suited for parallel accelerated processing, and the computational parameters between each search domain grid are not related to each other, which can significantly reduce the overall computational complexity, enhance the computational speed of the algorithm, and effectively shorten the computation time by massively parallelizing the solving operation. Moreover, since the eigen decomposition of the adjacency matrix A is deterministic for each search grid when the graph model is established, it is feasible to precompute the eigenvalues of matrix A to enhance algorithmic efficiency. In addition, by optimizing the storage and operation form of the adjacency matrix, such as constructing it into an efficient sparse matrix representation, the memory occupation can be further reduced, and the computational efficiency can be improved. The specific calculation time of the algorithm is shown in Table 2.

Table 2. Comparison of operation time of MUSIC, GSP, and parallel GSP algorithms (unit: second).

Algorithm	N_{grid}	5001	10,001	20,001	50,001
MUSIC		0.1692	0.3374	0.6723	1.6614
GSP		9.0967	18.3223	36.6538	90.5574
Parallel GSP		1.5374	1.7955	2.5643	3.9337

Among them, the Doppler shift search range is $-0.5 \sim 0.5$, the size of the search unit is changed by setting different search steps, and the Parallel GSP algorithm adopts 16-core CPUs for parallel computation. From Table 2, the MUSIC algorithm has the highest computational efficiency, the serial GSP algorithm has the lowest computational efficiency, the GSP algorithm after parallel optimization processing significantly improves the computational efficiency, and the more the number of parallel threads, the more obvious the effect of the enhancement, which is very conducive to the large-scale data computation.

5. Discussion

In this paper, we propose a novel graph model for FDA radar signals based on GSP theory and present a corresponding algorithm to address the Doppler shift estimation problem in the slow-time domain. The main innovations of this work include establishing a new graph-based signal model to address the Doppler shift estimation problem, reframing the Doppler frequency estimation challenge as the identification of spectral peaks within a graph-based framework, and achieving better anti-noise performance compared to the conventional algorithms. Additionally, the proposed method in this paper is designed to be deployable through parallel processing, thereby substantially enhancing computational efficiency through the utilization of parallel computing techniques.

Comparative analysis and simulation results reveals that the proposed approach outperforms the conventional MUSIC algorithm, exhibiting superior Doppler resolution and enhanced estimation accuracy. The enhancement in performance is attributable to the following factors: (1) by exploiting the sparsity and local smoothness of graph signals, the anti-noise performance of the algorithm can be effectively improved; (2) by leveraging the power of the graph signal processing, the proposed approach can effectively identify

the true frequency of the signal utilizing the established graph-based framework, thereby improving the discriminative ability in multi-target cases. However, it can be seen from Section 3.3 that the complexity of the GSP algorithm increases with the increase in the search grid, which is mainly due to the need to recalculate the adjacency matrices and perform the feature decomposition in processing each search unit. To enhance the efficiency of the algorithm, an effective approach is to conduct preprocessing procedures initially, such as precomputing the eigenvalues of the adjacency matrix or exploiting the matrix sparsity, this presents a promising area for our further research.

Furthermore, while this paper proposes a novel graph-based algorithm for Doppler shift estimation in FDA radar, the validation of the algorithm in this study is solely based on theoretical analysis and simulation experiments, rather than on measured data from real-world application scenarios. Additionally, when applying the proposed algorithm to practical scenarios, it is necessary to account for the impacts of non-ideal factors such as unstructured array configurations and multipath interference. These unresolved issues represent important directions for our future research.

6. Conclusions

In this study, we propose a novel GSP-based Doppler shift estimation algorithm to address the problem of estimating the parameters in FDA radar. The proposed method demonstrates superior performance over traditional methods, as evidenced by comprehensive theoretical analysis and experimental validation. A graph signal model to represent the Doppler shift vector is established, and a well-designed spectral searching approach is proposed, enabling the accurate extraction of the target's Doppler shift parameter. Simulation results indicate that the proposed method effectively solves the estimation performance degradation compared to the MUSIC algorithm, particularly under a low SNR and limited snapshot conditions. Future research directions may focus on the integration of deep learning techniques with GSP-based methodologies to enhance the robustness, as well as investigating its applicability in more complex scenarios.

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Abbreviations

The following abbreviations are used in this manuscript:

FDA	Frequency diverse array
GSP	Graph signal processing
GFT	Graph Fourier transform
SNR	Signal-to-Noise Ratio
FFT	Fast Fourier transform
MUSIC	MUltiple SIgnal Classification
ESPRIT	Estimation of Signal Parameters via Rotational Invariance Technique
CRLB	Cramer–Rao Lower Bound
RMSE	the Root Mean Square Error

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