remote sensing

MDPI

Correction

Correction: Prokop et al. Low Overlapping Point Cloud Registration Using Line Features Detection. *Remote Sens.* 2020, *12*, 61

Miloš Prokop †, Salman Ahmed Shaikh * and Kyoung-Sook Kim

National Institute of Advanced Industrial Science and Technology, Tokyo 135-0064, Japan; m.prokop@sms.ed.ac.uk (M.P.); ks.kim@aist.go.jp (K.-S.K.)

* Correspondence: shaikh.salman@aist.go.jp; Tel.: +81-80-3557-4442

† Current address: College of Science and Engineering, The University of Edinburgh, Edinburgh, UK.

Text Correction

There was an error in the original publication [1]. The text of the manuscript differed with the project code, available at: https://github.com/Milos9304/LowOverlapPCRegistration (accessed on 1 June 2025), in the metric function used to evaluate closeness of points on the parabolic cylinder. While the original manuscript suggested using the close-form integral of arc-length function, this choice in practical implementation leads to numerical instabilities, and an approximation of the integral, satisfying the properties of a metric function, was found to provide more reliable results.

A correction has been made to Section 3. Point Cloud Registration Using Line Features Detection, 3.5.1. Finding First Translational Parameter, 4th paragraph, by insertion:

"which has a closed-form expression (see Appendix A.1)

$$arclength\left(y^A, y^B\right) = \left[\frac{y}{2}\sqrt{1+4K^4y^2} + \frac{1}{4K^2}sinh^{-1}\left(2K^2y\right)\right]_{y=y^A}^{y=y^B} \tag{7}$$

However, due to numerical inefficiency and round-off errors related to calculation of the inverse hyperbolic sine, $\sinh^{-1}(\cdot)$, the use of Equation (7) in the actual implementation is discouraged and an approximation should be used in practice (see Appendix A.1) for details."

Another insertion has been performed in Appendix A. Parabolic Cylinder, 1st paragraph:

(i) We show how to find the closed-expression for arc length on parabolic surface, explain its practical numerical issues and propose a numerically stable approximation;

(ii) Another insertion has been performed below that, in the same paragraph:

"*Appendix A.1. The Arc Length on the Surface of Parabolic Cylinder and Its Approximation*
We begin with the arc length formula, initially introduced in Equation (6):

$$arclength(y^A, y^B) = \int_{y^A}^{y^B} \sqrt{1+\left(\frac{dz}{dy}\right)^2}\,dy$$

Given the parabolic cylinder $z = -K^2y^2 + H$ (Equation (3)) we compute the derivative $\frac{dz}{dy} = -2K^2y$ and thus $\left(\frac{dz}{dy}\right)^2 = 4K^4y^2$. Substituting into the arc length formula:

$$arclength\left(y^A, y^B\right) = \int_{y^A}^{y^B} \sqrt{1+4K^4y^2}\,dy$$

Received: 11 July 2025
Accepted: 21 July 2025
Published: 5 December 2025

Citation: Prokop, M.; Shaikh, S.A.; Kim, K.-S. Correction: Prokop et al. Low Overlapping Point Cloud Registration Using Line Features Detection. *Remote Sens.* 2020, *12*, 61. *Remote Sens.* **2025**, *17*, 3929. https://doi.org/10.3390/rs17243929

Copyright: © 2025 by the authors. Licensee MDPI, Basel, Switzerland. This article is an open access article distributed under the terms and conditions of the Creative Commons Attribution (CC BY) license (https://creativecommons.org/licenses/by/4.0/).

To evaluate this integral, we use the standard result ([30], p. 1035):

$$\int \sqrt{a^2 + x^2}\, dx = \frac{x}{2}\sqrt{a^2 + x^2} + \frac{a^2}{2} sinh^{-1}\left(\frac{x}{a}\right) + C$$

In our case, we substitute $a^2 = 1$, $x = 2K^2 y$, so we let:

$$u = 2K^2 y \quad \Rightarrow \quad dy = \frac{du}{2K^2}$$

Rewriting the integral:

$$\int \sqrt{1 + 4K^4 y^2}\, dy = \int \sqrt{1 + u^2} \frac{1}{2K^2}\, du = \frac{1}{2K^2} \int \sqrt{1 + u^2}\, du$$

Using the standard integral result (Equation (7)):

$$\frac{1}{2K^2}\left(\frac{u}{2}\sqrt{1 + u^2} + \frac{1}{2} sinh^{-1}(u)\right) = \frac{1}{4K^2} u\sqrt{1 + u^2} + \frac{1}{4K^2} sinh^{-1}(u)$$

Substitute $u = 2K^2 y$ back in and we get the final, close formula form:

$$arclength\left(y^A, y^B\right) = \left[\frac{y}{2}\sqrt{1 + 4K^4 y^2} + \frac{1}{4K^2} sinh^{-1}\left(2K^2 y\right)\right]_{y=y^A}^{y=y^B}$$

Despite the availability of the exact analytical arc length expression (Equation (7)), in our implementation we do not employ it due to its numerical and computational drawbacks. Specifically, evaluating the inverse hyperbolic sine function, $sinh^{-1}(\cdot)$, at high precision introduces instability for small $\left|y^A - y^B\right|$, where the function becomes highly sensitive to floating-point errors. This was empirically observed in our experiments, where results for closely spaced line intersections showed significant fluctuations and degraded reliability in fitness scoring. Instead, our code, that was used to run experiments in this paper, used a modified metric to evaluate "closeness" of the points y^A, y^B, defined by the integral

$$\int_{y^A}^{y^B} y\sqrt{1 + 4K^4 y^2}\, dy \tag{A2}$$

whose closed-form expression avoids inverse trigonometric or logarithmic components and is computationally simpler and more numerically stable. While this function no longer corresponds to the true arc length (Equation (A3)), it retains monotonicity and smoothness, and its weighting naturally penalizes larger distances. This is desirable in our scoring context, as it discourages the matching of distant or unrelated features while preserving correct ordering locally. Moreover, in the limit as $\left|y^A - y^B\right| \to 0$, the expression becomes asymptotically equivalent to the true arc length, ensuring that local comparisons remain consistent with the underlying geometry. Indeed, the close formula of Equation (A2) can be evaluated the following way: Given the indefinite integral of Equation (A2)

$$\int y\sqrt{1 + 4K^4 y^2}\, dy$$

we substitute

$$u = 1 + 4K^4 y^2, \text{ so that } du = 8K^4 y\, dy$$

Solving for dy, we get

$$y\, dy = \frac{1}{y8K^4} du$$

Thus, the integral becomes

$$\int y\sqrt{1+4K^4y^2}\,dy = \int \sqrt{u}\frac{1}{8K^4}\,du + C = \frac{1}{8K^4}\int u^{1/2}\,du + C$$

Integrating:

$$\frac{1}{8K^4}\frac{u^{3/2}}{3/2} = \frac{1}{12K^4}u^{3/2}$$

Substituting back $u = 1 + 4K^4y^2$, we get the closed-form expression:

$$\int_{y^A}^{y^B} y\sqrt{1+4K^4y^2}\,dy = \frac{\left(1+4K^4y_B^2\right)^{3/2} - \left(1+4K^4y_A^2\right)^{3/2}}{12K^4}$$

which practically serves viable, and numerically more stable, alternative to the parabolic *arclength* function."

References

In the original publication, ref. 30 was not cited.

30. Bronshtein, I.N.; Semendyayev, K.A.; Musiol, G.; Mühlig, H. *Handbook of Mathematics*; Springer: Berlin/Heidelberg, Germany, 2007; ISBN 978-3-540-72122-2. Available online: https://books.google.com.co/books?id=gCgOoMpluh8C (accessed on 1 June 2025).

The authors state that the scientific conclusions are unaffected. This correction was approved by the Academic Editor. The original publication has also been updated.

Reference

1. Prokop, M.; Shaikh, S.A.; Kim, K.-S. Low Overlapping Point Cloud Registration Using Line Features Detection. *Remote Sens.* **2020**, *12*, 61. [CrossRef]

Disclaimer/Publisher's Note: The statements, opinions and data contained in all publications are solely those of the individual author(s) and contributor(s) and not of MDPI and/or the editor(s). MDPI and/or the editor(s) disclaim responsibility for any injury to people or property resulting from any ideas, methods, instructions or products referred to in the content.