

## Article

# Multipath Mitigation in Single-Frequency Multi-GNSS Tightly Combined Positioning via a Modified Multipath Hemispherical Map Method

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**Abstract:** Multipath is a source of error that limits the Global Navigation Satellite System (GNSS) positioning precision in short baselines. The tightly combined model between systems increases the number of observations and enhances the strength of the mathematical model owing to the continuous improvement in GNSS. Multipath mitigation of the multi-GNSS tightly combined model can improve the positioning precision in complex environments. Interoperability of the multipath hemispherical map (MHM) models of different systems can enhance the performance of the MHM model due to the small multipath differences in single overlapping frequencies. The adoption of advanced sidereal filtering (ASF) to model the multipath for each satellite brings computational challenges owing to the characteristics of the multi-constellation heterogeneity of different systems; the balance efficiency and precision become the key issues affecting the performance of the MHM model owing to the sparse characteristics of the satellite distribution. Therefore, we propose a modified MHM method to mitigate the multipath for single-frequency multi-GNSS tightly combined positioning. The method divides the hemispherical map into  $36 \times 9$  grids at  $10^\circ \times 10^\circ$  resolution and then searches with the elevation angle and azimuth angle as independent variables to obtain the multipath value of the nearest point. We used the k-d tree to improve the search efficiency without affecting precision. Experiments show that the proposed method improves the mean precision over ASF by 10.20%, 10.77%, and 9.29% for GPS, BDS, and Galileo satellite single-difference residuals, respectively. The precision improvements of the modified MHM in the E, N, and U directions were 32.82%, 40.65%, and 31.97%, respectively. The modified MHM exhibits greater performance and behaves more consistently.

**Keywords:** multi-GNSS; tightly combined model; multipath mitigation; MHM; ASF



**Citation:** Tao, Y.; Liu, C.; Tong, R.; Zhao, X.; Feng, Y.; Wang, J. Multipath Mitigation in Single-Frequency Multi-GNSS Tightly Combined Positioning via a Modified Multipath Hemispherical Map Method. *Remote Sens.* **2024**, *16*, 4679. <https://doi.org/10.3390/rs16244679>

Academic Editor: Amir Allahviridi-Zadeh

Received: 14 October 2024

Revised: 8 December 2024

Accepted: 13 December 2024

Published: 15 December 2024



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## 1. Introduction

The Global Navigation Satellite System (GNSS) double-difference technique can quickly provide precise position information and achieve mm or even higher precision on a global level after the carrier phase ambiguities are fixed [1,2]. Therefore, the double-difference technique is widely applied in fields such as deformation monitoring [3–6]. Most of the errors can be mitigated in a short baseline using the double-difference technique, including the satellite clock error, receiver clock error, tropospheric delay, and ionospheric delay [7–9]. However, multipaths that depend on the station observation environment

cannot be eliminated using existing methods, including differential techniques and parameterization [10]. Therefore, several researchers have proposed site selection, and hardware- and software-based methods for mitigating multipaths.

Firstly, site selection reduces the impact of multipath effects, mainly by choosing open environments, but it cannot avoid the limitations of objective conditions [11]. Secondly, hardware-based multipath suppression techniques are mainly performed by improving the “narrow correlator” [12] of the receiver or using a multipath suppression antenna [13]; however, these methods cannot eliminate multipaths and the instruments are costly [14]. Finally, for software-based methods that exploit the spatio-temporal repeatability of multipath effects in static environments, sidereal filtering (SF) and a multipath hemispherical map (MHM) can be implemented to generate multipath mitigation models for real-time GNSS data processing [15].

Genrich and Bock (2006) proposed SF to mitigate a multipath based on the constant environment of the station and the repeatability of GPS satellites [13]. The wavelet decomposition technique was applied to extract a carrier-phase multipath from GPS double-difference observation residuals. Advanced sidereal filtering (ASF) uses each satellite repeat period instead of the mean repeat period to effectively mitigate multipaths [16]. The orbital repeat period shift time method was used to calculate the repeat period of satellites, which has the advantages of simple calculation and better orbital real-time performance [17]. Zhong et al. (2010) and Ye et al. (2015) used the “zero mean” assumption and the ASF method to effectively mitigate a multipath [9,18]. However, calculating the repeat period for each satellite undoubtedly increases the complexity of multipath mitigation in the case of large differences in repeat periods. In addition, the operational period of each satellite changes slowly over time, reducing the real-time effect of mitigating multipaths [19].

Therefore, lookup table methods, including MPS (multipath stacking) and MHM, effectively mitigate the multipath by calculating the mean of the multipath in a higher-resolution discrete grid based on the fact that the multipath depends only on the orbital positions in the sky when the environment of the main multipath is kept constant [15,20]. However, an excessively high resolution causes insufficient multipath mitigation in part of the grid, and an excessively low resolution causes a high-frequency multipath in the grid to be difficult to mitigate because of the sparse distribution characteristics of satellite orbits. The multipath hemispherical model based on trend surface analysis (T-MHM) fits the multipath in the grid through a linear regression model, which was effectively mitigated for both high-frequency and low-frequency multipaths [21,22], but there was a pathological problem with the regression equations solving in some of the grids [11]. Tian et al. (2023) used the least squares method to mitigate the multipath and improve the positioning precision by utilizing the spatial autocorrelation characteristic of the multipath [4]. The resolution of the grid limits the mitigation effect of the multipath with different frequencies using MHM and its extensions [23], or it is difficult to fit a complex nonlinear multipath model using a certain established function model; the multipath also consumes a large amount of computer memory during the modeling process, which leads to insufficient processing efficiency. Ren et al. (2023) proposed an MHM model with geographic cut-off elevation constraints that mask unsatisfactory signals while reducing low-frequency multipaths, which helps GNSS deformation monitoring in complex environments [24]. Pan et al. (2024) used multipath modeling as a regression task to fit multipaths in the spatial domain using a machine learning approach, demonstrating that machine learning-based multipath mitigation methods are effective and easy to use [25].

With the rapid development of multi-frequency multi-constellation GNSS, more than 130 GNSS satellites are available. The latest generation of GNSS satellites has three or more frequencies. It is expected that receivers developed by manufacturers will primarily support two signal frequencies: 1575.42 MHz and 1176.45 MHz since the major systems have overlapping frequency signals (except GLONASS), increasing the compatibility of GNSS positioning. The overlapping frequencies between BDS-3/GPS/Galileo increase the probability of a tight combination of the three systems [26]. The tightly combined model

adds additional observations and improves the strength of the model by utilizing a single-reference satellite to process observations from all the system satellites involved [27,28]. In addition, the increasing number and frequency of satellites not only increases the number of GNSS observations but also reduces the effect of the multipath on positioning precision [14]. Therefore, in dense vegetation and tall building environments where observation satellites are obscured, the application of the tightly combined model [29] and multipath mitigation have important significance. Geng et al. (2024) established an interoperable MHM mitigating multipath using the overlap frequency signals of GPS, Galileo, and BDS-3, showing that the interoperable GPS/Galileo/BDS-3 MHM is able to improve the spatial resolution, modeling efficiency, and correction performance [30]. The carrier phase residuals based on GPS, BDS-3, and Galileo overlapping frequency show a strong correlation at the same spatial position. Lu et al. (2023) proposed that the multipath joint modeling and correction method can improve the positioning performance more than the independent modeling and correction [31].

Taking into account the above multiple factors, we extracted the multipath from the single-frequency multi-system tightly combined double-difference observation equations and established an MHM model with the station as the center and elevation and azimuth as the axes. We then divided the hemispherical map into  $36 \times 9$  grids at  $10^\circ \times 10^\circ$  resolution. The elevation and azimuth angles of the satellites were input into the model and the K-nearest neighbor (KNN) algorithm was utilized to search for the nearest points and obtain the values of the corresponding multipaths. Previous KNN search algorithms have low search efficiency, and in this study, we used the k-d tree to improve search efficiency. The k-d tree was proposed by Jon Louis Bentley in 1975 [32]. It is a tree data structure (binary tree) used in multidimensional space to solve the problem of data organization and search in high-dimensional space, which can significantly speed up the search operations in multidimensional space, especially when the data volume is large, such as nearest neighbor search, range search, and point queries.

The rest of this paper is organized as follows. Section 2 describes the principles of the tightly combined model and the orbital repeat period of multi-GNSS satellites. The proposed method for multipath mitigation is described in Section 3. Section 4 verifies the performance of the proposed method through experiments. Finally, Section 5 summarizes the study.

## 2. Principle

### 2.1. GPS/Galileo/BDS Tightly Combined Model and Multipath Extraction

The selection method of the reference satellite is used as the basic condition to distinguish between loosely and tightly combined models in the process of establishing multi-GNSS double-difference observation equations. Selecting reference satellites separately in each system is a loosely combined model, and selecting a common reference satellite in all systems is a tightly combined model.

The difference between the reference satellites and the other satellites in the system eliminates most of the systematic errors, but each double difference in the system consumes the observations of the three reference satellites in the GPS/Galileo/BDS-3 loosely combined model.

Early experiments on GPS/Galileo have demonstrated that there is differential inter-system bias (DISB) when different types of receivers observe L1/E2 and L5/E5a signals at both ends of the baseline [28], and the existence of DISB is mainly caused by hardware delays associated with different receivers and satellite signals of different systems. In this regard, the DISB must be estimated and corrected for GPS/Galileo/BDS tightly combined observations using different receiver types. The GPS/Galileo/BDS tightly combined double-difference model that considers DISB can be obtained by choosing a GPS satellite as a reference satellite between the GPS/Galileo/BDS systems. Table 1 lists the GPS/Galileo/BDS system signal frequencies and wavelengths.

**Table 1.** Frequency and wavelength of GPS/Galileo/BDS signals.

| GNSS Systems      | Frequency Band | Frequency (MHz) | Wavelength (cm) |
|-------------------|----------------|-----------------|-----------------|
| GPS/Galileo/BDS-3 | L1/E1/B1C      | 1575.42         | 19.03           |
| GPS               | L2             | 1227.60         | 24.42           |
| GPS/Galileo/BDS-3 | L5/E5a/B2a     | 1176.45         | 25.48           |
| Galileo/BDS-3     | E5b/B2b        | 1207.14         | 24.83           |

When the receiver type is different, the DISB of the GPS/Galileo/BDS-3 tightly combined mode is more stable and can be corrected in advance using estimation. When using overlapping frequency data from GPS/Galileo/BDS-3 of the same type of receiver for the tightly combined mode, the DISB is close to 0 [26]. Therefore, the GPS/Galileo/BDS-3 tightly combined double-difference model without DISB can be obtained by selecting a GPS satellite as the common reference satellite in GPS/Galileo/BDS-3:

$$\begin{cases} P_{kl,i}^{1_G^{sG}} = \rho_{kl}^{1_G^{sG}} + e_{kl,i}^{1_G^{sG}} \\ \phi_{kl,i}^{1_G^{sG}} = \rho_{kl}^{1_G^{sG}} + \lambda_i N_{kl,i}^{1_G^{sG}} + \varepsilon_{kl,i}^{1_G^{sG}} \\ P_{kl,i}^{1_G^{qE}} = \rho_{kl}^{1_G^{qE}} + e_{kl,i}^{1_G^{qE}} \\ \phi_{kl,i}^{1_G^{qE}} = \rho_{kl}^{1_G^{qE}} + \lambda_i N_{kl,i}^{1,qE} + \varepsilon_{kl,i}^{1_G^{qE}} \\ P_{kl,i}^{1_G^{bC}} = \rho_{kl}^{1_G^{bC}} + e_{kl,i}^{1_G^{bC}} \\ \phi_{kl,i}^{1_G^{bC}} = \rho_{kl}^{1_G^{bC}} + \lambda_i N_{kl,i}^{1,bC} + \varepsilon_{kl,i}^{1_G^{bC}} \end{cases} \quad (1)$$

where  $\phi$  is the carrier phase observation,  $P$  is the pseudo-range observation,  $\rho$  is the geometric distance between the satellite and GNSS receiver,  $\lambda$  is the frequency wavelength,  $N$  is ambiguity,  $e$  and  $\varepsilon$  are the observation noise of the pseudo-range and carrier phase,  $1_G$  indicates that one GPS satellite is used as a common reference satellite,  $s_G$ ,  $q_E$ , and  $b_C$  are GPS, Galileo, and BDS non-reference satellites. The GNSS receivers  $k$  and  $l$  receive the pseudo-range and carrier phase observations of the  $s$  satellites of the GPS,  $q$  satellites of the Galileo, and  $b$  satellites of the BDS, at the overlap frequency  $i$  ( $i = 1, 2, \dots, f$ ).

The double-difference technique eliminates most of the systematic errors, including orbital error, receiver clock error, and satellite clock error, and the tropospheric and ionospheric delays are ignored in short baseline positioning. Therefore, the carrier phase double-difference residuals are mainly multipath and noise, and the double-difference residuals for each satellite need to be extracted from the observation equations to mitigate the multipath. The double-difference equation is linearized to obtain the following equation:

$$V = Ax + By - L \quad (2)$$

where  $x$  is the unknown baseline vector;  $y$  is the integer ambiguity;  $A$  and  $B$  are the design matrices of the unknown baseline vector and integer ambiguity;  $L$  is the carrier phase constant term;  $V$  is the double-difference residuals.

## 2.2. Orbital Repeat Period of Multi-GNSS Satellites

The motion of the satellite relative to the station is repeatability; therefore, the repeatability of the multipath effect depends on the orbital motion of the satellite. The repeat period of the multipath is essentially the same as that of the satellite relative to the station. Therefore, SF and its extension methods rely on the time delay of the satellite.

We analyze the orbital repeat period of multi-GNSS by calculating the amount of time delay for different satellite orbital repeats from the broadcast ephemeris in 2022. The angular velocity  $n$  of the satellite can be obtained at each ephemeris broadcasting moment based on the orbital radius and angular velocity correction provided in the ephemeris:

$$n = \sqrt{\mu}/a^{\frac{3}{2}} + n_c \quad (3)$$



where  $a$  is the orbital half-length axis of the different satellites,  $\mu$  is the gravitational constant of Earth for the different satellites, and  $n_c$  is the mean angular velocity correction. The orbital repeat period of different system satellites was calculated as following:

$$t = R \times T_e - O \times 2\pi \cdot n \quad (4)$$

where  $T_e$  is the rotational period of the Earth, which is calculated to be 86,164.0996 s through the mean solar time,  $R$  and  $O$  are integers prime to each other,  $R$  is the rotation cycle of the Earth, and  $O$  is the number of orbits accomplished in one orbital repeat period of the satellite.  $R$  and  $O$  for different constellations of satellites are listed in Table 2.

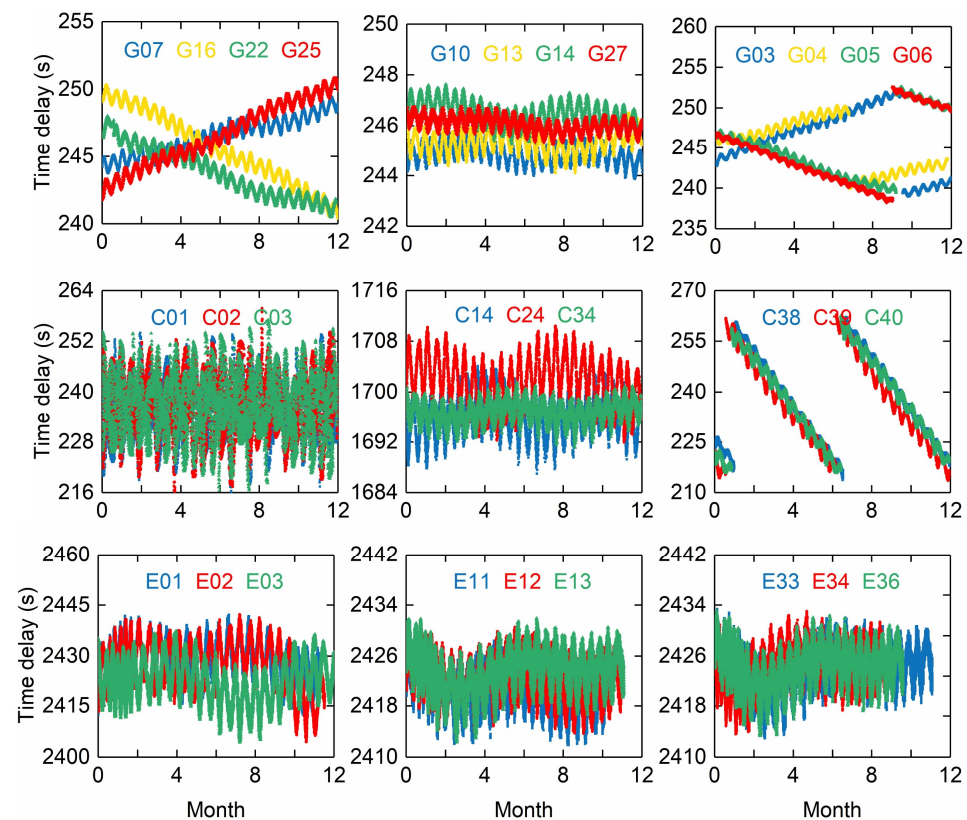
**Table 2.** Values of  $R$  and  $O$  for different constellations of satellites.  $R$  means the rotation cycle of the Earth;  $O$  means the number of orbits accomplished in one orbital repeat period of the satellite.

| Satellites   | $R$ (Cycle) | $O$ (Cycle) |
|--------------|-------------|-------------|
| GPS          | 1           | 2           |
| BDS GEO/IGSO | 1           | 1           |
| BDS MEO      | 7           | 13          |
| Galileo      | 10          | 7           |

In a stable environment, the multipath is only related to the geometric relationship between the receiver, the reflection source, and the satellite position. Therefore, it is necessary to consider the satellite's operating period and calculate the satellite's sub-satellite point. Due to the heterogeneous characteristics of multi-GNSS constellations, the operating period of each type of satellite is considered separately. Therefore, it is necessary to calculate the time delay of the repeat period of each satellite's sub-satellite point. The satellite's time delay is calculated based on the orbital parameters given by the broadcast ephemeris, and the single-difference residuals are aligned according to the time delay of each satellite to implement the ASF method to reduce the subsequent multipath impact.

Figure 1 shows the time delays of orbital repeat periods of the GPS, BDS, and Galileo satellites for 2022. GPS satellites G10, G13, G14, and G27 change within 4 s in a year. The remaining satellites show variations of less than 15 s in a year and no more than 2 s in a month. This indicates that GPS satellites have precise orbit control. The time delays of the geostationary orbit (GEO) satellites of the BDS were in the range of 215–260 s, with a maximum of 12 s of daily variation. The medium earth orbit (MEO) satellite has a time delay of approximately 1700 s and operates more stably. The time delay of the inclined geosynchronous orbit (IGSO) satellite was in the range of 210–260 s, with a variation of no more than 20 s a month. The Galileo satellite time delays are above 2400 s and do not change for more than 30 s in a year. The time delays exhibit significant variations with satellite operation time, and ASF modeling of the multipath requires the calculation of time delays for each satellite. In addition, there are significant orbital maneuvers for GPS satellites and IGSO satellites of BDS, with maneuver adjustment periods of approximately 12 and 6 months, respectively. The variable time delays and satellite maneuvers will limit the application of ASF in real time.

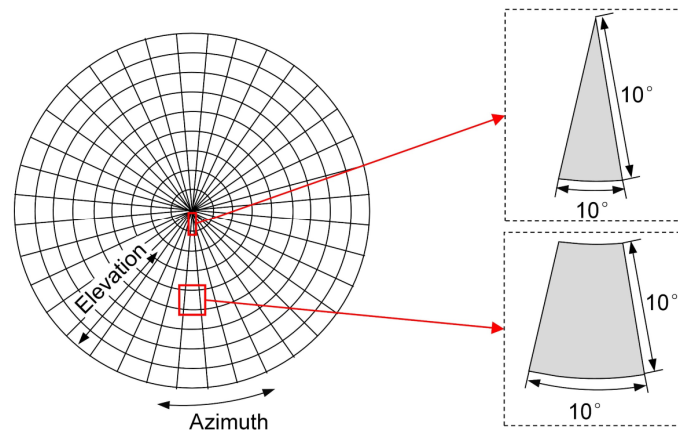
The time delays of most of the satellites in the figure exhibit short-period perturbations and long-period linear properties. The time delays of satellites of different systems vary considerably, and those of satellites of the same system are still not the same. Some satellites are affected by satellite maneuvers. It is difficult to achieve multi-GNSS multipaths that are mitigated using repeatability modeling methods in the coordinate domain. This limits the advantages of ASF for multipath mitigation.



**Figure 1.** Time delays of orbital repeat period of GPS, BDS, and Galileo satellites in 2022.

### 3. Proposed Method

The established modified MHM model is based on the mechanism of spatial repeatability of multipaths, while the temporal repeatability of multipaths is derived from spatial repeatability. The modified MHM model was established with the station as the center and elevation and azimuth angles as the coordinate axes. We divided the hemispherical map into  $3 \times 3$ ,  $6 \times 6$ , and  $60 \times 60$  grids according to  $20^\circ \times 20^\circ$ ,  $10^\circ \times 10^\circ$ , and  $1^\circ \times 1^\circ$  resolutions in the range of elevation angles of  $30^\circ$ – $90^\circ$  and azimuth angles of  $120^\circ$ – $180^\circ$ , respectively. After experimentation, we found that the precision difference between the three resolution grid models is very small (the improvements in the single-difference residual multipath mitigation of DOY92 corresponding to  $20^\circ \times 20^\circ$ ,  $10^\circ \times 10^\circ$ , and  $1^\circ \times 1^\circ$  are 36.67%, 36.68%, and 36.68%, respectively), but the times to search for all points for the  $20^\circ \times 20^\circ$ ,  $10^\circ \times 10^\circ$ , and  $1^\circ \times 1^\circ$  resolution grids are 3:45:23, 0:15:35, and 1:23:15, respectively. This indicates that the grid resolution has no effect on the precision of the multipath model, but affects the efficiency of the search, which is best using the  $10^\circ \times 10^\circ$  resolution. Therefore, we divided the hemispherical map into  $36 \times 9$  grids at a  $10^\circ \times 10^\circ$  resolution, as shown in Figure 2. Satellite single-difference residuals were loaded into this model based on their positions. In the subsequent multipath mitigation process, the elevation and azimuth angles of the satellite were input into the model to search for the nearest point. The searched multipath values were introduced into the observation equation for multipath mitigation, realizing the purpose of real-time multipath mitigation.



**Figure 2.** Hemispherical map with two typical grids.

Previously, the search method generally used the K-nearest neighbor (KNN) algorithm [33], which was implemented by traversing all points. Calculating the distance of query points from all points is very time consuming when the dataset is large. Therefore, to improve the efficiency of KNN, we can consider using special data structures to store data to reduce the number of distance calculations. In this study, a k-d tree was used to increase the KNN search efficiency. The basic principle of the KNN is as follows:

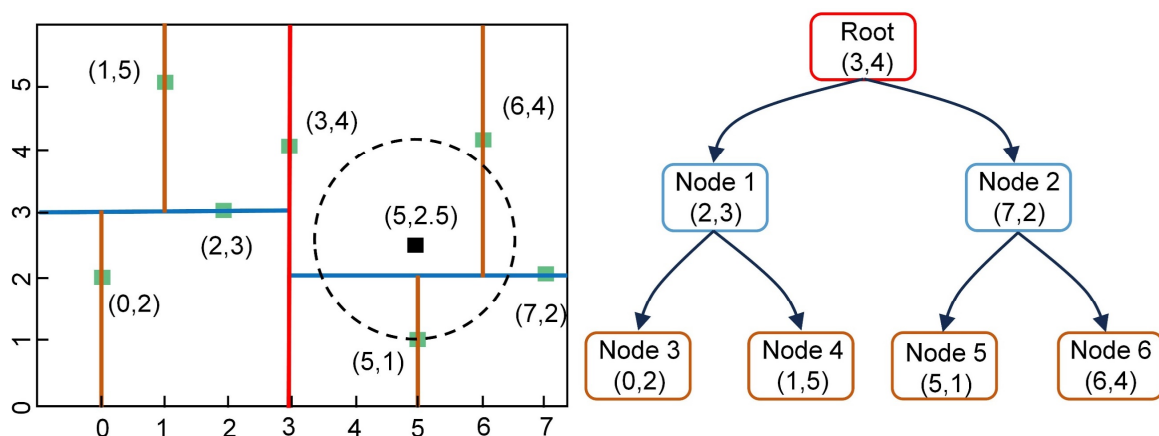
$$D = \{d_1, d_2, \dots, d_c\} \text{ KNN}_{k_1} \Rightarrow \{m_1, m_2, \dots, m_u, \dots, m_{k_1}\} \quad (5)$$

where  $D$  presents all  $c$  data points; the nearest  $k_1$  points of query point  $d$  in  $D$  are found by KNN search. The composition of  $\text{KNN}_{k_1}$  must satisfy the following conditions.

$$\|d, d_m\| \leq \|d, d_j\| \quad (6)$$

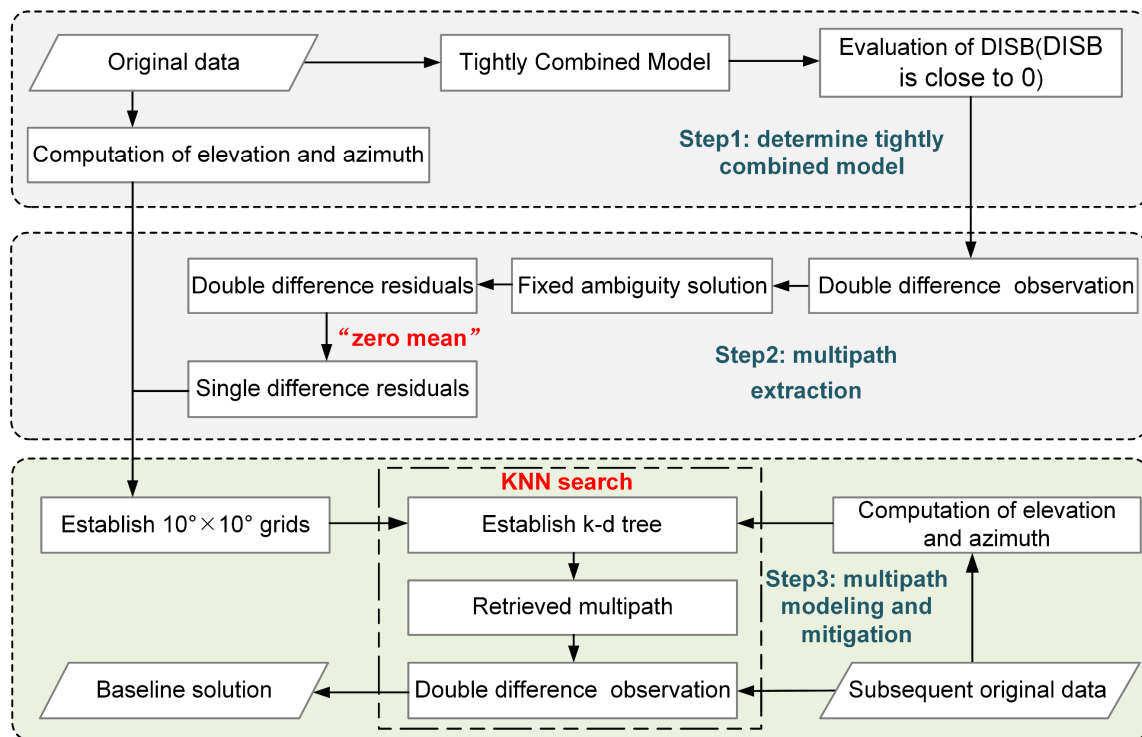
where  $\|d, d_m\|$  is the present distance between  $d$  and  $d_m$ ;  $j \in [1, c] - \text{KNN}_{k_1}$ .

The k-d tree iteratively splits the space with hyperplanes and constructs binary trees, allowing the KNN to search for the logarithmic time complexity. At each level of the binary tree, the data were divided into two groups based on the dimension with the highest variance. A k-d tree must be constructed before performing a KNN search, as shown in Figure 3. The first step is to construct the root node in a hyperrectangle that contains all data points in space; the next step is to select a node on the coordinate axes, create a hyperplane perpendicular to the coordinate axes through the node, and divide the hyperrectangle into two subregions; the third step is to repeat the second step in the case of at least one data point in the subregion; otherwise, stop the implementation.



**Figure 3.** Construction process of the k-d tree.

The process of multipath mitigation in multi-GNSS tightly combined positioning based on the modified MHM model is shown in Figure 4.



**Figure 4.** The processing flow of the modified MHM model.

Step 1: Determine the tightly combined model. After evaluating the size and stability of the DISB estimated from the GPS/BDS/Galileo tight combination, the DISB of the same type of receiver is close to zero, so we use the tightly combined multi-GNSS model.

Step 2: Multipath extraction. The double-difference observation equation is first composed, then the fixed ambiguity rate is enhanced by a priori baseline vectors to obtain the carrier phase double-difference residuals, and further the single-difference residuals are obtained by the “zero-mean” assumption.

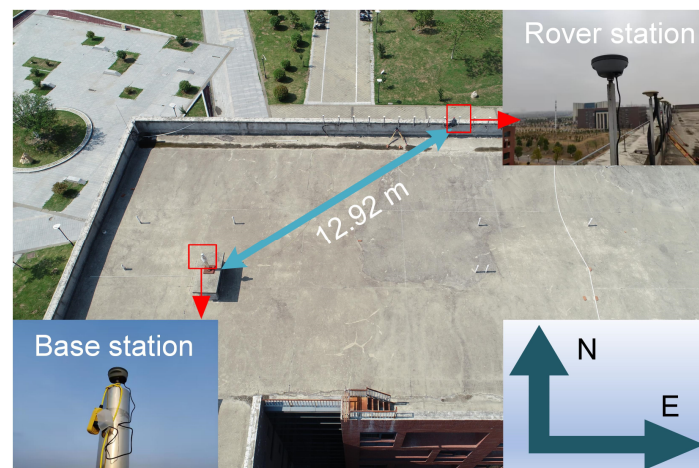
Step 3: Multipath modeling and mitigation. The above single-difference residuals are formed into a  $10^\circ \times 10^\circ$  grid according to the elevation and azimuth angles, and a k-d tree is built in each grid. The elevation and azimuth angles of the satellite on the subsequent original data were used as query points by using the KNN search in the k-d tree to obtain the retrieved multipath, which is brought into the double-difference observation equation for real-time multipath mitigation. Finally, baseline precision was evaluated.

## 4. Experiment Analysis

### 4.1. Data Collection

In order to verify the multipath mitigation performance of the modified MHM method for GPS/Galileo/BDS tightly combined positioning, the baseline length between the two stations was set to be 12.92 m during data collection, and both the base station and the rover station were erected on the roof of the School of Spatial Information and Geomatics Engineering, Anhui University of Science and Technology, China. The observation environment is illustrated in Figure 5. The multipath error mainly comes from the wall and the ground, the height of the base station antenna is about 2.3 m, and the height of the wall around the rover station is about 1.5 m. The north side of the rover station is at ground level, with the receiver at a height of about 17 m above ground level, and the south side is on the roof of the building, with a distance of about 13 m from the opposite wall. This multipath error

source can meet the requirements we deal with in the experiment. We use two receivers of the same type for orientation processing to eliminate the antenna phase-center variation, and the DISB between the systems can be negligible in the short baseline. Ionospheric and tropospheric errors can be eliminated using the double-difference observation equation, and ambiguity is fixed using the LAMBDA algorithm [34]. The receivers we use can receive the GPS triple-frequency, Galileo triple-frequency, and BDS five-frequency observations. Data were collected from 23 March to 16 April 2022 (DOY82-DOY106) for a total of 25 days, which covers the minimum time required for Galileo satellites with a repeat period of ten days, with a data sampling rate of 1 s, a satellite cutoff elevation angle of  $15^\circ$ , and a PDOP threshold set to 8.



**Figure 5.** Observation environment around the station.

#### 4.2. Single-Difference Residuals Analysis

Before conducting formal experiments, it is necessary to distinguish between factors due to poor satellite geometry and the multipath due to ground reflectors. Figure 6 shows the position dilution of precision (PDOP), horizontal dilution of precision (HDOP), vertical dilution of precision (VDOP), and the number of observable satellites for multi-GNSS. The number of satellites is maintained between 20 and 30 in the entire period, which is more observable than the single systems because of the tightly combined multi-GNSS single-frequency observation. The PDOP, VDOP, and HDOP describe the strength of the satellite geometric distribution on the positioning precision. To avoid poor satellite results in the spatial geometric distribution, an empirical value of eight was set as the PDOP threshold when the data were collected. As can be seen in Figure 6, both the PDOP and HDOP are below 1.5, while the VDOP is below 1, indicating that the positioning measurements are sufficiently accurate in both the horizontal and vertical directions. Therefore, it can be considered that the main factor affecting positioning precision is the multipath.

As the elevation angle of the satellite decreases, the GNSS signal is interfered with more ground reflectors during propagation; therefore, the multipath signal becomes more complex. It is necessary to evaluate multipaths from different elevation angles and satellite systems. The double-difference residuals are converted to single-difference residuals based on the “zero-mean” assumption, and the elevation angle corresponding to the single-difference residuals at each epoch is solved. The single-difference residuals are analyzed instead of the multipath because the single-difference residuals are mainly composed of the multipath and random noise, and the total sum of the random noise is 0. The elevation angles were grouped from low to high at  $5^\circ$  intervals, and the root mean square (RMS) of the single-difference residuals was calculated for each group. Figure 7 demonstrates the effect of single-difference residuals with increasing elevation angle. The RMS of the single-difference residuals of the three systems gradually decreased as the elevation angle increased, and eventually remained between 20 mm and 30 mm. Fluctuations were observed between  $45^\circ$



and  $55^\circ$  and between  $85^\circ$  and  $90^\circ$ . The differences in the single-difference residuals with the elevation angle among the three systems were small.

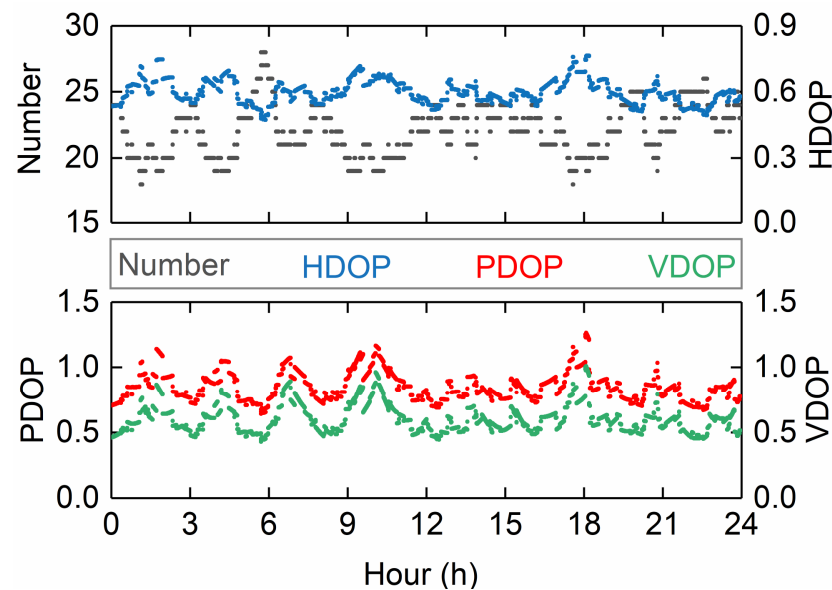


Figure 6. PDOP, HDOP, VDOP, and the number of observable satellites for multi-GNSS.

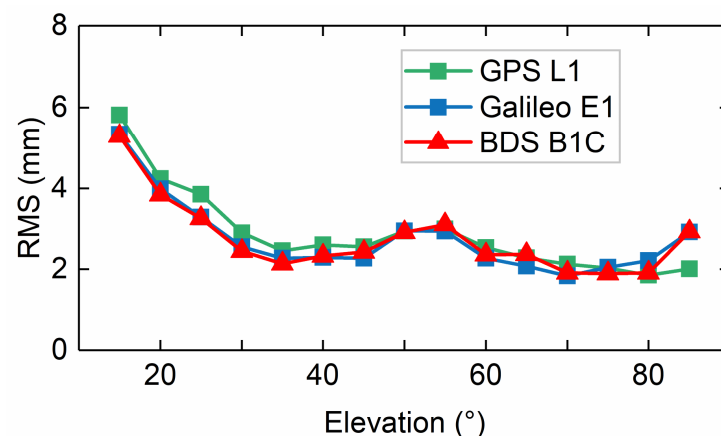
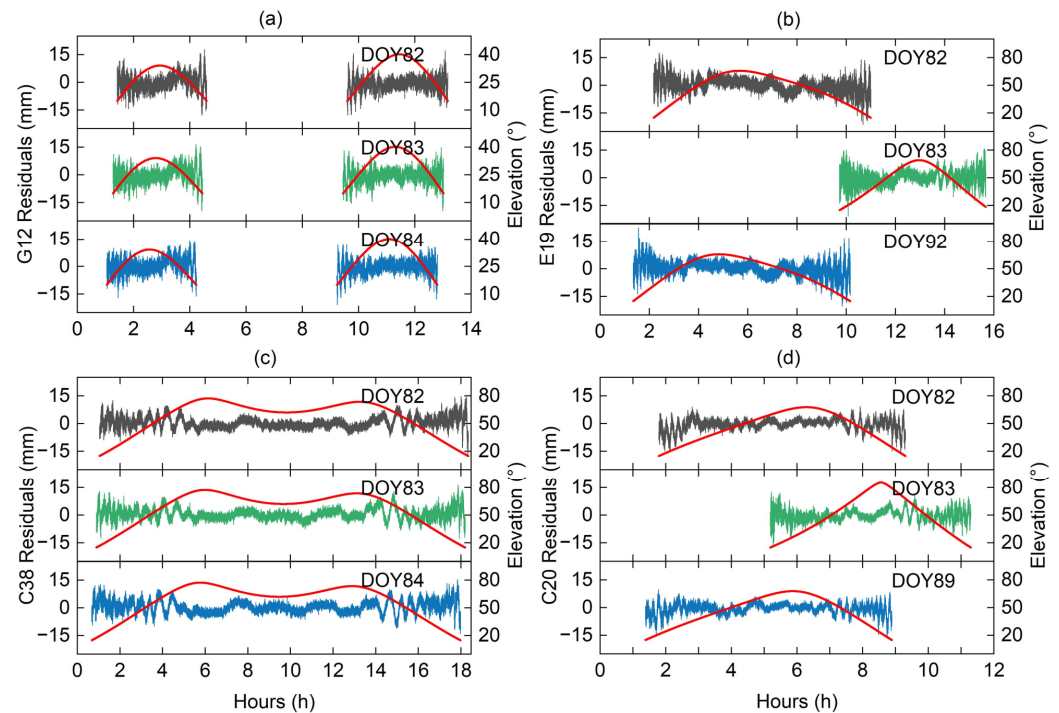


Figure 7. RMS of single-difference residuals for different elevation.

Figure 8a shows the single-difference residuals of the GPS satellite in DOY82–84, and it can be seen that the trend of single-difference residuals of this satellite for three consecutive days is highly correlated. The single-difference residuals of the previous day after removing this time delay are highly correlated with the current single-difference residuals because there is a time delay between the satellite single-difference residuals of the previous day and the current day. Time delays can be calculated using the broadcast ephemeris. Figure 8b shows the distribution of single-difference residuals of Galileo satellites in DOY82, DOY83, and DOY93, which have inconsistent visibility and lower correlation of their single-difference residuals in two adjacent days. Galileo satellites have a more significant correlation between DOY82 and DOY92, with a correlation of 0.6436, despite a time interval of 10 days, and a much longer time delay than GPS satellites. Figure 8c,d show single difference residual results for the MEO and IGSO satellites of the BDS, respectively. From the distribution of the single-difference residuals of DOY82, DOY83, and DOY89, it can be concluded that there is no obvious similarity between the single-difference residuals of DOY82 and DOY83, and there are differences in the visibility of the satellites. However, the single-difference residuals of DOY82 and DOY89 present a strong correlation, and it is verified that the orbital repeat period of the MEO satellites of

the BDS is 7 days. The fluctuations of the single-difference residuals of IGSO satellites in DOY82, DOY83, and DOY84 shows the strongest correlation in two consecutive days, and their time delays are also closer to those of GPS, which show repeatability within a sidereal day. However, the correlation of the single-difference residuals of DOY82 and DOY84 was lower than that of the two adjacent days.

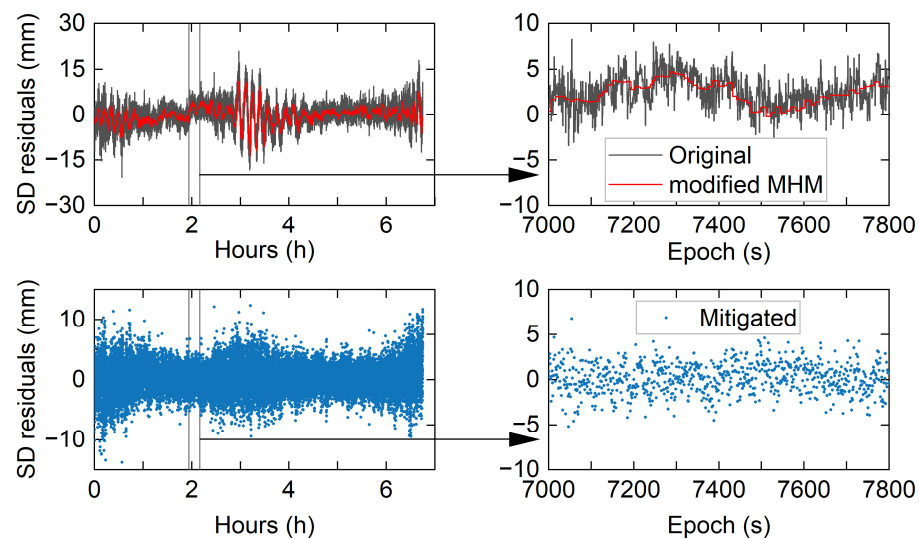


**Figure 8.** Single-difference residuals and elevation of GPS MEO G12 (a), Galileo MEO E19 (b), BDS-3 MEO C38 (c) and IGSO C20 (d).

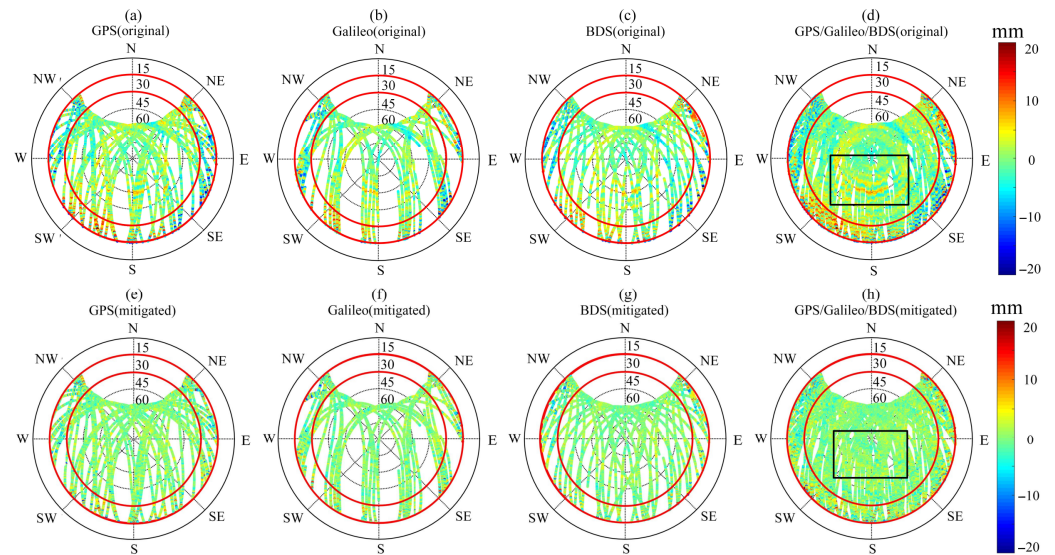
#### 4.3. Multipath Mitigation by Modified MHM

To analyze the multipath mitigation performance of the modified MHM method in multi-GNSS single frequency combined positioning, we first mitigated the multipath on the single-difference residuals, and second compared the differences between single and multiple systems in the coordinate series. Figure 9 shows the original single-difference residuals for the G12 satellite and the multipath obtained by the modified MHM and the residuals mitigated by the modified MHM. As shown in the figure and local figure, the single residuals obtained by the modified MHM were consistent with the trend of the original residuals. The noise at both ends of the mitigated residuals is relatively large, which is consistent with the previous analysis. In the low elevation angle region, the signal-to-noise ratio of the satellite is small, and the noise is larger.

Figure 10 shows the hemispherical maps of the original single difference residual and that mitigated using the modified MHM method. GPS, Galileo, and BDS satellites have severe multipath effects at elevation angles of 15–30°, and the single difference residual varies significantly with the elevation angle within the black rectangle in Figure 10d. After modified MHM mitigation of the multipath, as shown in Figure 10h, the color changes from red-blue to yellow-green in the region of 15–30° elevation angle, and the multipath was mitigated in the region above 30°.



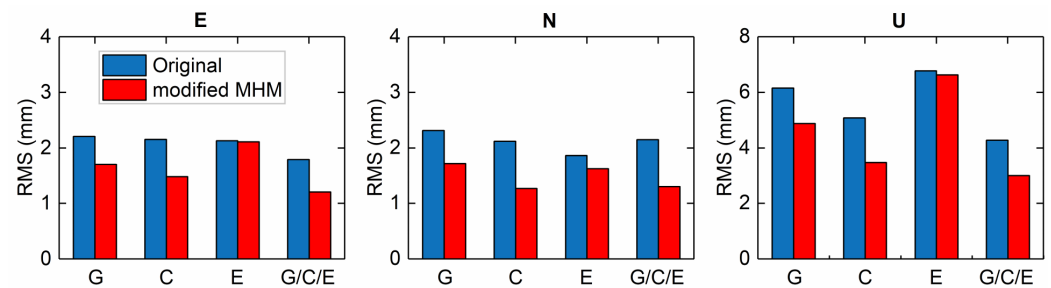
**Figure 9.** Original single-difference residuals for the G12 satellite and the multipath obtained by modified MHM and the residuals mitigated by modified MHM.



**Figure 10.** Hemispherical maps of the original single difference residual (a–d) and mitigated by using the modified MHM method (e–h) for GPS, Galileo, BDS, and GPS/Galileo/BDS combined observations, respectively. (The black rectangles in (d,h) are used to mark the areas in the sky map where multipath errors and improvements are evident).

We processed the GPS, BDS, Galileo, and GPS/BDS/Galileo combined observations to distinguish between single- and multiple-system multipath mitigation performances. Figure 11 shows the RMS of the original residuals and the residuals mitigated by the modified MHM of DOY92 in the E/N/U directions. G, C, E, and G/C/E represent GPS, BDS, Galileo, and GPS/BDS/Galileo, respectively. In the E direction, the RMS of the original residuals of GPS/BDS/Galileo is significantly lower than that of a single system, which is 1.79 mm; after multipath mitigation, the RMS of GPS/BDS/Galileo decreases by 0.41 mm, 0.36 mm, and 0.34 mm, respectively, compared to GPS, BDS, and Galileo. In the N direction, the RMS of the original residuals of Galileo is lower than that of the single systems and multiple systems, but the RMS of the residuals after multipath mitigation of BDS is the lowest. In the U direction, the original RMS of GPS/BDS/Galileo is 1.88 mm, 0.81 mm, and 2.50 mm lower than that of GPS, BDS, and Galileo, respectively; after mitigating the multipath, the GPS/BDS/Galileo is reduced to 3.00 mm, significantly lower than the

other results. We mitigated the multipath of seven consecutive days using the modified MHM method and obtained the following results. Table 3 displays the mean precision improvements of GPS, BDS, Galileo, and GPS/BDS/Galileo. We found that the precision improvements of GPS/BDS/Galileo are generally higher than those of the single systems. However, it should be noted that in the U direction, the RMS of BDS is higher than that of GPS/BDS/Galileo, before and after multipath mitigation. This unexpected result explains that the increase in the number of satellites does not necessarily lead to an improvement in positioning precision, but because of the increase in the number of satellites, the impact of the multipath effect will also increase.



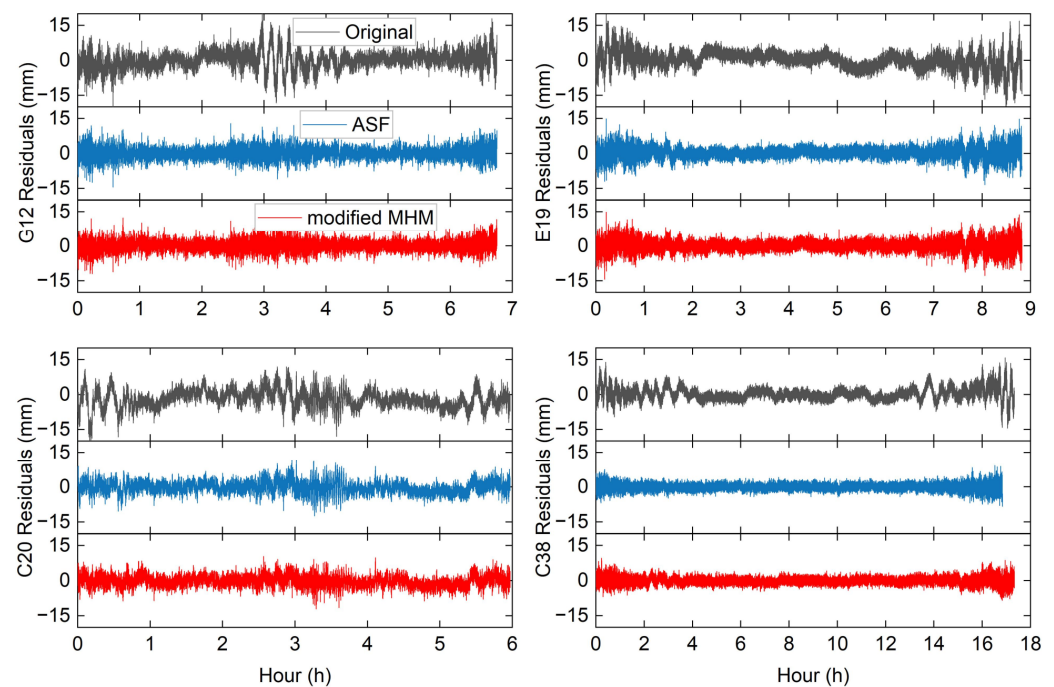
**Figure 11.** RMS of the original residuals and residuals mitigated by the modified MHM of DOY92 in the E/N/U directions.

**Table 3.** Mean precision improvement of GPS, BDS, Galileo, and GPS/BDS/Galileo.

| Direction | Mean Precision Improvement (%) |       |         |                 |
|-----------|--------------------------------|-------|---------|-----------------|
|           | GPS                            | BDS   | Galileo | GPS/BDS/Galileo |
| E         | 18.41                          | 25.56 | 5.60    | 29.19           |
| N         | 18.22                          | 29.76 | 12.80   | 31.42           |
| U         | 17.61                          | 29.20 | 7.39    | 24.73           |

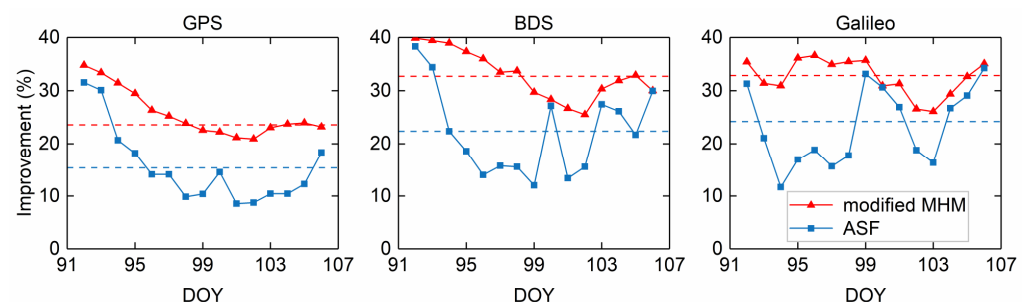
#### 4.4. Comparison with Other Methods in the Observation Domain and the Coordinate Domain

To compare the multipath mitigation performance of the ASF and modified MHM methods for multi-GNSS, experiments were conducted separately. The results of four representative satellites before and after multipath mitigation with single-difference residuals were selected from the three systems. Figure 12 shows the original single-difference residuals of G12, E19, C20, and C38 and the residuals mitigated by the ASF and modified MHM methods. The original single-difference residuals mainly included multipath and random noise. After multipath mitigation of the ASF and modified MHM methods, the residuals only contain random noise and the residual multipath, and the mitigation effect can be intuitively seen according to the fluctuation of the residual series. It can be intuitively found that the amplitude changes of the G12, C38, and E19 residuals are relatively small, indicating that the multipath has been eliminated compared with the original residuals and after multipath mitigation by ASF and modified MHM, respectively. After the modified MHM mitigates the multipath, the RMS values of G12, C20, and E19 are 2.29 mm, 1.38 mm, and 2.28 mm, respectively. After ASF mitigates the multipath, the RMS values of G12, C20, and E19 are 2.37 mm, 1.32 mm, and 2.27 mm, respectively. We found that the difference between the two methods was not significant. As for the residual of C38 after multipath mitigation, the RMS of the modified MHM is 0.27 mm lower than that of the ASF, and the performance of the modified MHM is slightly better than that of the ASF. It is worth mentioning that the MEO repeat period of BDS causes a significant “end effect” after ASF processes multipath, while the MHM approach is not affected.



**Figure 12.** Original single-difference residuals of G12, E19, C20, and C38 and the residuals mitigated by ASF and modified MHM methods.

We calculated the precision improvement of all satellite single-difference residuals after multipath mitigation by ASF and modified MHM for 25 consecutive days to further verify the difference between the two methods. Figure 13 shows the precision improvement of the GPS, BDS, and Galileo single-difference residuals. The red and blue dashed lines represent the mean improvements in ASF and modified MHM, respectively. As can be seen from the figure, in terms of GPS single-difference residuals, although the precision improvement of both methods is declining, the precision improvement of the modified MHM is significantly greater than that of the ASF intuitively, and the mean precision improvement of the modified MHM is 10.17% higher than that of the ASF. For BDS single-difference residuals, the precision improvement of the modified MHM is greater than that of the ASF within 15 days, and the fluctuation of the ASF is significantly larger, with mean precision improvements of 32.93% and 22.16% for ASF and modified MHM, respectively. For Galileo, the fluctuation trends of the modified MHM and ASF were similar, but in DOY93–98, DOY102, and DOY103, the mean precision improvement of ASF was 15.12% lower than that of modified MHM. The mean precision improvement of the modified MHM over ASF was 10.20%, 10.77%, and 9.29% for the single-difference residuals for the GPS, BDS, and Galileo satellites, respectively.



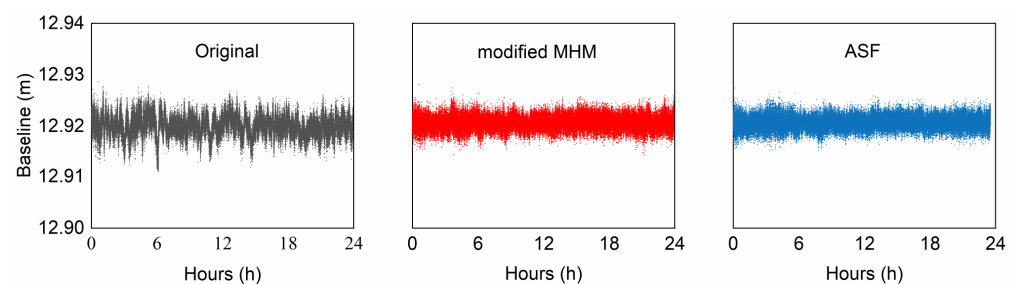
**Figure 13.** Precision improvement of GPS, BDS, and Galileo single-difference residuals, respectively.

The performance of ASF can be influenced by two main factors: (1) the precision of multipath extraction from carrier phase residuals; (2) the precision of each satellite's repeat

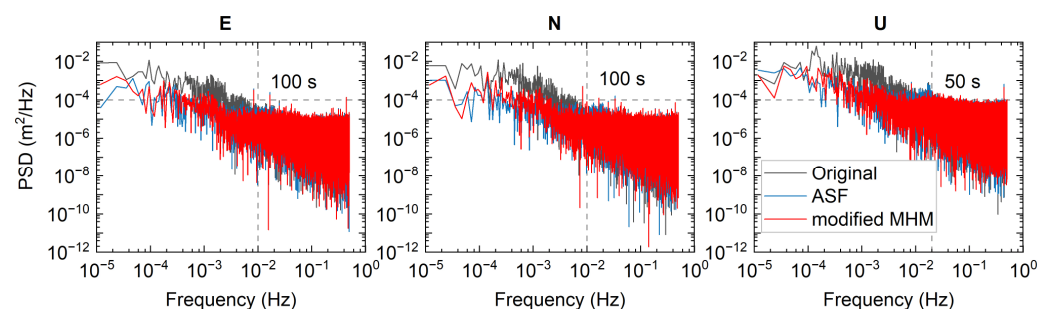


period. Additionally, multipath effects are most similar in adjacent repeat periods and tend to diverge over time. Although the repeat period of GPS satellites is relatively more stable compared to BDS-3 and Galileo (Figure 1), the multipath differences continue to increase, resulting in a gradual decline in ASF performance. In contrast, the repeat periods of BDS and Galileo are not fully consistent with GPS and exhibit greater variability. This inconsistency leads to abrupt changes in ASF performance on DOY 100 and DOY 99. In summary, statistical analysis of the single-difference residuals before and after multipath mitigation shows that the MHM has greater performance and behaves more consistently.

The purpose of the multipath mitigation of double-difference observations is to improve the positioning precision, and the mitigated double-difference observations are used to fix the ambiguity and further estimate the baseline coordinates. The baseline coordinates of the station were determined based on the single epoch model, and the precise reference coordinates of the station were averaged over multiple consecutive days without mitigation for the multipath model. Uncorrected double-difference observations were also used to generate baseline coordinates for comparison purposes. Figure 14 shows the DOY92 original baseline series and the series after multipath mitigation using the ASF and modified MHM methods. The distribution of the baseline series can be seen in Figure 15. After mitigating the multipath, the series obeys the normal distribution, indicating that the multipath has been eliminated. The E, N, and U components are differenced with accurate reference station coordinates and then are fast Fourier transformed to obtain the power spectral density (PSD) to further compare the differences between the two methods in the frequency domain.



**Figure 14.** Original baseline series and the series after multipath mitigation by using ASF and modified MHM methods in DOY92.



**Figure 15.** PSD of the original series and the series mitigated by both methods.

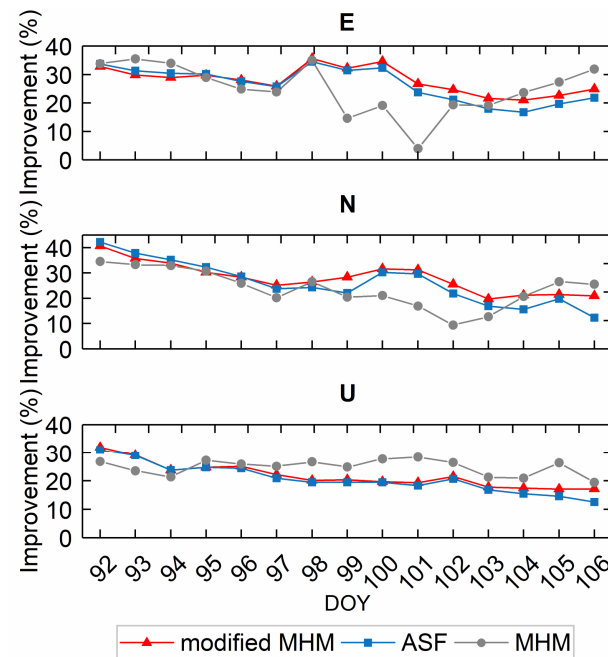
Figure 15 illustrates the PSD of the original series and the series mitigated by both methods, and both methods can eliminate most of the middle- to high-frequency multipath. In the frequency below 0.01 Hz, the PSD of the two are more similar on E/N directions, but in the range of 0.01–0.02 Hz, the PSD of the modified MHM is lower than that of the ASF in the U direction. Compared with the ASF, the modified MHM has a positive effect on mitigating the middle-to-high-frequency multipath in the period of 50–100 s.

In order to compare and analyze the differences between our method and the MHM in terms of accuracy and computational efficiency, we added experiments on the MHM.

The multipath depends on the position of the satellite and not on the specific satellite, based on the spatio-temporal correlation of the multipath. The MHM method for multipath mitigation is as follows. The single-difference residuals of GPS, Galileo, and BDS for a total of 10 days are denoised to obtain multipaths, and the multipaths are placed in a grid. The grid is defined by elevation and azimuth angle in a carrier coordinate system [15]. The grids were divided according to certain elevation and azimuth angle intervals. The MHM model is established by taking the mean value of the single-difference residuals in the grid as the multipath of the grid. In order to achieve more accurate multipath error reduction, considering the large amount of data in the multi-GNSS environment, we adopted an MHM grid resolution of  $0.1^\circ \times 0.1^\circ$ . Such a high resolution helps to improve the refinement of the model, thereby more effectively capturing and reducing the multipath effect. In the subsequent multipath mitigation process, the grid corresponding to the elevation and azimuth angles of the satellite is found in the MHM model, and the multipath of the grid is introduced into the observation equation for multipath mitigation. Comparing the computational efficiency of MHM and the modified MHM, we found that, excluding the time for reading and saving data, the time for establishing the MHM model was 6:30:05 in the range of elevation angles of  $30^\circ$ – $90^\circ$  and azimuth angles of  $120^\circ$ – $180^\circ$ . The grid search times of  $20^\circ \times 20^\circ$ ,  $10^\circ \times 10^\circ$ , and  $1^\circ \times 1^\circ$  were 3:45:23, 0:15:35, and 1:23:15, respectively. Therefore, the k-d number enhanced the search method with a  $10^\circ \times 10^\circ$  grid resolution is the most efficient.

The coordinate precision improvement statistics results after multipath mitigation based on the three models are in Figure 16. The coordinate components for 15 consecutive days were statistically analyzed to further distinguish the differences between the ASF and modified MHM methods for multipath mitigation in continuous data. Figure 16 shows the precision improvement of the original components and components after multipath mitigation by both methods from DOY92–DOY106. In the E direction, the precision improvements of the ASF and modified MHM are more similar in DOY92–97, but after DOY92, the precision improvements of the modified MHM are greater than those of the ASF. In the N direction, after DOY95, the precision improvement of the modified MHM was higher than that of the ASF. In the U direction, after DOY96, the precision improvements of the modified MHM are greater than those of the ASF. The mean precision improvement of the modified MHM is 27.94%, 28.06%, and 21.81% in the E, N, and U directions, respectively. The mean precision improvements of the modified MHM over the ASF were 1.40%, 2.00%, and 1.11% in the E, N, and U directions, respectively. The precision improvement of MHM is more variable in the E and N directions, and the precision improvement in the E direction of DOY99–101 and the N direction of DOY100–102 is significantly lower than that of the ASF and the improved MHM. In the U direction, the whole precision performance of MHM is better. The mean precision improvement of MHM in the E, N, and U directions are 24.97%, 23.85%, and 24.768%. The experimental results show that the mean improvement of the modified MHM is 2.97% and 4.21% higher than those of the MHM. However, in the U direction, the average improvement of the MHM is improved by 2.95% higher than that of the modified MHM. Nevertheless, from the overall performance point of view, the modified MHM still performs better than the MHM in multipath mitigation.

SF mainly relies on the satellite repeat period to mitigate the multipath. ASF, as an extension of the SF method, calculates the repeat period of each satellite separately to enhance the multipath mitigation performance. For multi-GNSS multipath mitigation, the calculation of the repeat period of different systems becomes cumbersome. At the same time, the calculation method of the repeat period is inevitably affected by satellite maneuvers. The satellite repeat period changes slowly over time. All of the above reasons will cause the performance of ASF in correcting the multipath to continue to decrease. The modified MHM is not affected by the satellite repeat period, so the modified MHM method performs better than the ASF method in mitigating the multipath.



**Figure 16.** Precision improvement of original components and the components after multipath mitigation by three methods from DOY92-DOY106.

## 5. Conclusions

We proposed a modified MHM method to mitigate the multipath for single-frequency multi-GNSS tightly combined positioning. In the case of single overlapping frequency, the difference of the multipath between different systems is small; the performance of the multipath model can be improved by interoperation between multi-GNSS systems. Due to the sparsity of satellite orbits and the slight orbit offset, the traditional MHM mitigating multipaths may suffer from omissions. To avoid the shortcomings of multipath mitigation, we use KNN to search for multipaths. We adopt the multi-dimensional data structure of k-d tree to speed up the KNN search operation in the space, and also build a grid with  $10^\circ \times 10^\circ$  resolution to improve the search efficiency while ensuring the precision. The experimental results are as follows:

We used a modified MHM for multipath mitigation of the GPS, BDS, Galileo, and GPS/BDS/Galileo combinations. It can be observed that the precision improvement of the GPS/BDS/Galileo combination was higher than that of a single system. However, the RMS of BDS was higher than that of GPS/BDS/Galileo in the U direction. This indicates that the increased satellite signals may also lead to an increased multipath effect, which in turn decreases the positioning precision.

We mitigated the multipath of the GPS/BDS/Galileo combination using ASF and modified MHM and compared it with the uncorrected values. The mean precision of the modified MHM over the ASF was improved by 10.20%, 10.77%, and 9.29% for the GPS, BDS, and Galileo satellites' single-difference residuals, respectively. The mean precision of the MHM in the E, N, and U directions increased by 1.40%, 2.00%, and 1.11%, respectively, compared with the ASF. The MHM outperforms the ASF for multipath mitigation in the case of the GPS\Galileo\BDS combination. The conclusion is that the modified MHM has better performance and more consistent behavior, and can improve positioning precision in quasi-static environments.

**Author Contributions:** Methodology, Y.T., C.L. and R.T.; software, Y.T. and C.L.; validation, Y.T., R.T. and J.W.; formal analysis, C.L., R.T., X.Z. and Y.F.; data curation, Y.T., R.T., X.Z. and Y.F.; writing—original draft preparation, Y.T.; writing—review and editing, C.L., X.Z. and J.W. All authors have read and agreed to the published version of the manuscript.

**Funding:** This research was funded by the Key Project of Natural Science Research in Universities of Anhui Province (grant numbers 2022AH050849, KJ2020A0312, and KJ2021A0443), the Youth Project of Anhui Natural Science (grant number 2008085QD179), the Natural Science Foundation of Beijing (grant number 8222011), and the Science and Technology Research Project of Colleges and Universities in Hebei Province (grant number ZD2021023).

**Data Availability Statement:** Upon reasonable request, the data that support the findings of this study are available from the corresponding author.

**Conflicts of Interest:** The authors declare no conflicts of interest.

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